DERIVATIVE-*DEPENDENT* METRIC TRANSFORMATIONS AND SCALAR-TENSOR THEORIES

2nd mini-workshop on gravity and cosmology

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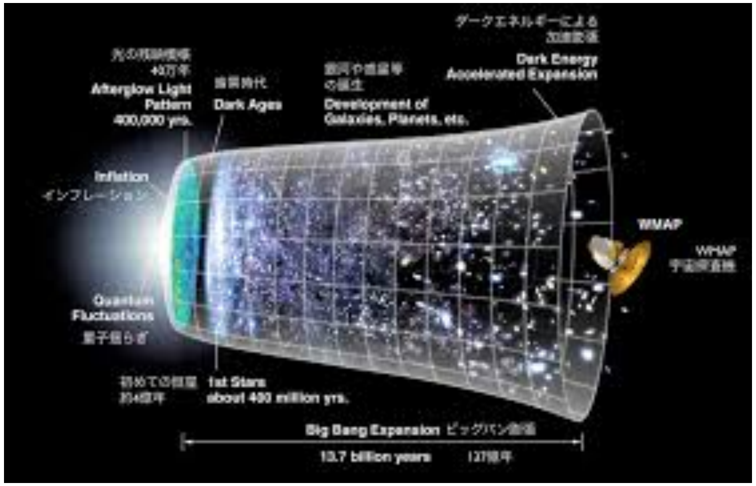
Outline

1. Introduction and motivation
2. Derivative-dependent transformation: Simple example
3. Meaning of the transformation: General analysis
4. Summary

What is the universe accelerated by?

- Cosmological constant
- Graviton mass
- Matter condensation
- Unknown scalar field(s)
- …



Horndeski theory

Horndeski '74, Nicolis et al & Deffayet et al '09, Kobayashi et al '11

- Most-general single scalar field and gravity theory, which field equations contain derivatives only up to the 2^{nd} order

$$S = \sum_{i=2}^{5} \int d^4x \sqrt{-g} \mathcal{L}_i$$

$$\mathcal{L}_2 = K(\phi, X)$$

$$\mathcal{L}_3 = -G_3(\phi, X) \Box \phi$$

$$\mathcal{L}_4 = G_4(\phi, X) R + G_{4X} \left[(\Box \phi)^2 - (\nabla_\mu \nabla_\nu \phi)^2 \right],$$

$$\mathcal{L}_5 = G_5(\phi, X) G_{\mu\nu} \nabla^\mu \nabla^\nu \phi - \frac{G_{5X}}{6} \left[(\Box \phi)^3 - 3 (\Box \phi) (\nabla_\mu \nabla_\nu \phi)^2 + 2 (\nabla_\mu \nabla_\nu \phi)^3 \right]$$

$$X = -g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

Horndeski theory Horndeski '74, Nicolis et al & Deffayet et al '09, Kobayashi et al '11

- Most-general single scalar field and gravity theory, which field equations contain derivatives only up to the 2nd order

$$\frac{\delta S}{\delta g_{\mu\nu}} = 0, \ \frac{\delta S}{\delta \phi} = 0 \quad \Longrightarrow \quad f(\ddot{g}_{\mu\nu}, \dot{g}_{\mu\nu}, g_{\mu\nu}, \ddot{\phi}, \dot{\phi}, \phi) = 0$$

GLPV theory

Gleyzes et al. (2014)

- Single scalar field and gravity theory beyond Horndeski, and it has the 3rd order field equations.
- In a particular gauge, unitary gauge, the 3rd order EoMs reduce to the 2nd order ones.

$$S = \sum_{i=2}^{5} \int d^4x \sqrt{-g} \mathcal{L}_i$$

$$\mathcal{L}_2 = K(\phi, X)$$

$$\mathcal{L}_3 = -G_3(\phi, X) \Box \phi$$

$$\mathcal{L}_4 = G_4(\phi, X) R + G_{4X} \left[(\Box\phi)^2 - (\nabla_\mu \nabla_\nu \phi)^2 \right] + F_4(\phi, X) \epsilon^{\mu\nu\rho}{}_{\sigma} \epsilon^{\mu'\nu'\rho'\sigma} \phi_\mu \phi_{\mu'} \phi_{\nu\nu'} \phi_{\rho\rho'}$$

$$\mathcal{L}_5 = G_5(\phi, X) G_{\mu\nu} \nabla^\mu \nabla^\nu \phi - \frac{G_{5X}}{6} \left[(\Box\phi)^3 - 3 (\Box\phi)(\nabla_\mu \nabla_\nu \phi)^2 + 2 (\nabla_\mu \nabla_\nu \phi)^3 \right]$$
$$+ F_5(\phi, X) \epsilon^{\mu\nu\rho\sigma} \epsilon^{\mu'\nu'\rho'\sigma'} \phi_\mu \phi_{\mu'} \phi_{\nu\nu'} \phi_{\rho\rho'} \phi_{\sigma\sigma'}$$

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- In a particular gauge, unitary gauge, the 3rd order EoMs reduce to the 2nd order ones.

$$\frac{\delta S}{\delta g_{\mu\nu}} = 0, \ \frac{\delta S}{\delta \phi} = 0 \quad \longrightarrow \quad f\left(\frac{d^3 g_{\mu\nu}}{dt^3}, \cdots, \frac{d^3\phi}{dt^3}, \cdots\right) = 0$$

In unitary gauge: Φ = t,

$$\frac{\delta S_{u.g}}{\delta h_{ij}} = 0 \quad \longrightarrow \quad f(\ddot{h}_{ij}, \dot{h}_{ij}, h_{ij}, t, N, N^i) = 0$$

Connection of Horndeski to GLPV

- Derivative-dependent *disformal* transformation

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + \Gamma(\phi, X)\partial_\mu\phi\partial_\nu\phi$$

$$\text{Horndeski} \subseteq \text{GLPV}$$

Transformed gravity

Zumalacarregui et al '14

- Derivative–dependent *conformal* transformation

$$\bar{g}_{\mu\nu} = \Omega^2(X,\phi) g_{\mu\nu}$$

Bekenstein '92,
Zumalacarregui et al '14

$$\mathcal{L}_E = \frac{\sqrt{-\bar{g}}}{16\pi G}\bar{R}[\bar{g}] + \sqrt{-g}(\mathcal{L}_\phi(g_{\mu\nu},\phi) + \mathcal{L}_m(g_{\mu\nu},\phi))$$

$$\Downarrow$$

$$\mathcal{L}_C = \frac{\sqrt{-g}}{16\pi G}(\Omega^2 R + \underline{6\Omega_{,\alpha}\Omega^{,\alpha}}) + \sqrt{-g}(\mathcal{L}_\phi + \mathcal{L}_m)$$

Beyond Horndeski/GLPV term appears!

New scalar-tensor theory?

Wellness of the theories

- Ostrogradski's theorem

Ostrogradski's theorem Ostrogradski (1850), Woodard '07

- "If the higher order time derivative Lagrangian is *non-degenerate*, there is at least one linear instability in the Hamiltonian of this system"

Non-degenerate = over 4th order EoMs

- example

$$L = \frac{1}{2}R^2 + S(R - \ddot{Q}) \rightarrow \frac{d^4Q}{dt^4} = 0$$

$$H_{phys} = \boxed{P_Q(P_S - P_R)} + S^2 + SR + \frac{1}{2}R^2$$

$$P_R \approx 0, \;\; R + S \approx 0$$

An extra d.o.f appears!

# of degree of freedom = (6-2)/2 = 2 = 1 **+ 1**

Horndeski/GLPV theory

- Horndeski : up to the 2^{nd} order EoMs
- GLPV : up to the 3^{rd} order EoMs

$$\frac{\delta S}{\delta g_{\mu\nu}} = 0, \ \frac{\delta S}{\delta \phi} = 0 \ \Longrightarrow \ f(\ddot{g}_{\mu\nu}, \dot{g}_{\mu\nu}, g_{\mu\nu}, \ddot{\phi}, \dot{\phi}, \phi) = 0$$

$$f(\frac{d^3 g_{\mu\nu}}{dt^3}, \cdots, \frac{d^3\phi}{dt^3}, \cdots) = 0$$

Degenerate theories

Horndeski/GLPV theory doesn't suffer from the *Ostrogradski's theorem*

Transformed theory

- 4^{th} order term by beyond Horndeski

$$\Omega^2 G_{\mu\nu} + 2\Omega(g_{\mu\nu}\Box\Omega - \Omega_{;\mu\nu}) + (6\Box\Omega - \Omega R)\Omega_{,X}\phi_{,\mu}\phi_{,\nu}$$
$$- g_{\mu\nu}\Omega_{,\alpha}\Omega^{,\alpha} + 4\Omega_{,\mu}\Omega_{,\nu} = 8\pi G(T^{\phi}_{\mu\nu} + T^{m}_{\mu\nu}),$$

$$\nabla_\mu(\Omega_{,X}\phi^{,\mu}(\Omega R - 6\Box\Omega)) + \Omega_{,\phi}(\Omega R - 6\Box\Omega) + \frac{1}{2}\frac{\delta\mathcal{L}_\phi}{\delta\phi} = 0,$$

Substitute it back into EoMs

- Trace of 1^{st} EoM

$$(6\Box\Omega - \Omega R)(\Omega - 2\Omega_{,X}X) = 8\pi GT$$

Transformed theory

- $$\Omega^2 G_{\mu\nu} + 2\Omega(g_{\mu\nu}\Box\Omega - \Omega_{;\mu\nu}) + T_{\mathrm{K}}\phi_{,\mu}\phi_{,\nu} - g_{\mu\nu}\Omega_{,\alpha}\Omega^{,\alpha} + 4\Omega_{,\mu}\Omega_{,\nu} = 8\pi G T^{\mathrm{tot}}_{\mu\nu},$$

$$\nabla_\mu(\phi^{,\mu} T_{\mathrm{K}}) + \frac{\Omega_{,\phi}}{\Omega_{,X}} T_{\mathrm{K}} - \frac{1}{2}\frac{\delta\mathcal{L}_\phi}{\delta\phi} = 0. \qquad 0,$$

Reduced to the 2nd order EoMs

Transformed theory

$$\mathcal{L}_C = \frac{\sqrt{-g}}{16\pi G}(\Omega^2 R + 6\Omega_{,\alpha}\Omega^{,\alpha}) + \sqrt{-g}(\mathcal{L}_\phi + \mathcal{L}_m)$$

Fact	Expectation
EoMs are the 4th order, which means the theory is non-degenerate	Linear instability of the Hamiltonian Extra degrees of freedom
EoMs can be reduced to the 2^{nd} order by the trace of field equation	Counter-example of the Ostrogradski's theorem and stable New scalar-tensor theory??

2. Derivative-dependent transformation

- Conformal + Disformal transformation Bekenstein '92

$$\boxed{g_{\mu\nu} \to \tilde{g}_{\mu\nu} = \mathcal{A}(\phi, X) g_{\mu\nu} + \mathcal{B}(\phi, X) \partial_\mu \phi \partial_\nu \phi}$$

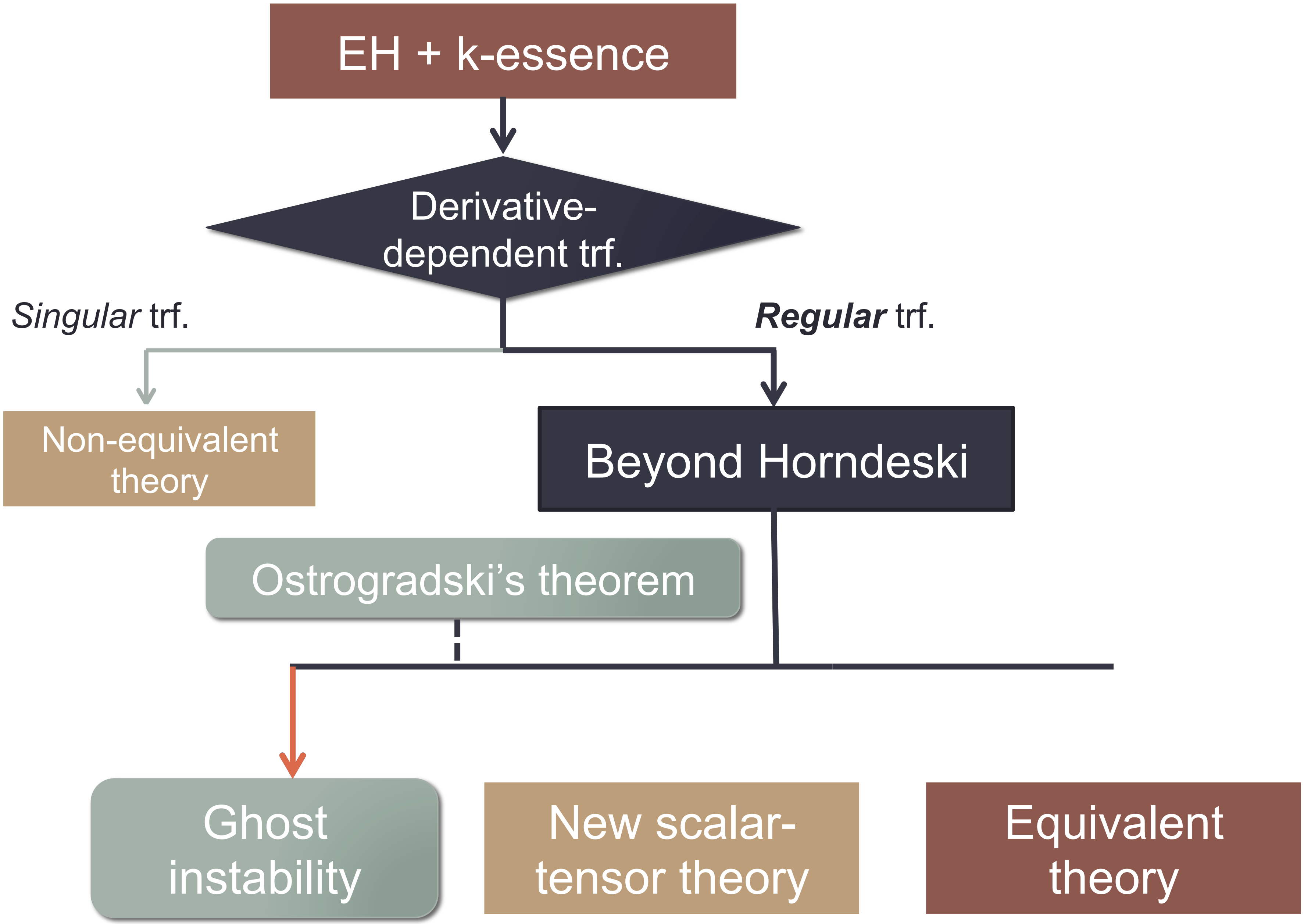
$$\det g \neq 0 \quad : \qquad \mathcal{A}(\mathcal{A} - \mathcal{B}X) \neq 0$$

$$\det \frac{\partial \tilde{g}_{\mu\nu}}{\partial g_{\alpha\beta}} \neq 0 \quad : \quad \mathcal{A}(\mathcal{A} - \mathcal{A}_X X + \mathcal{B}_X X^2) \neq 0$$

- Transforming E-H + K-essence in tilde system

$$\tilde{I}_{\text{total}} = \tilde{I}_{\text{EH}} + \int d^{d+1}x \sqrt{-\tilde{g}}\, \tilde{P}(\phi, \tilde{X})$$

$$\tilde{I}_{total}[\tilde{g}_{\mu\nu}, \phi] = I_{total}[g_{\mu\nu}, \phi]$$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- Transformed action in general gauge

$$\tilde{I}_{\rm total} = \tilde{I}_{\rm EH} + \int d^{d+1}x\sqrt{-\tilde{g}}\,\tilde{P}(\phi,\tilde{X})$$

$$\Downarrow$$

$$\tilde{I}_{total}[\tilde{g}_{\mu\nu},\phi] = I_{total}[g_{\mu\nu},\phi]$$

$$\begin{aligned}
\tilde{I}_{\rm EH} = \frac{M_p^2}{2}\int d^{d+1}x\sqrt{-g}\,\mathcal{A}^{(d-1)/2}\left(1-\frac{\mathcal{B}}{\mathcal{A}}X\right)^{1/2} \\
\times\Bigg\{R+\frac{\mathcal{B}}{\mathcal{A}-\mathcal{B}X}\left[\nabla_\mu\nabla_\nu\phi\,\nabla^\mu\nabla^\nu\phi-(\nabla^2\phi)^2\right]+\frac{d(d-1)}{4}\left[\nabla_\mu\ln\mathcal{A}\,\nabla^\mu\ln\mathcal{A}-\frac{\mathcal{B}}{\mathcal{A}-\mathcal{B}X}(\nabla^\mu\phi\,\nabla_\mu\ln\mathcal{A})^2\right] \\
-\frac{d-1}{2}\frac{\mathcal{B}}{\mathcal{A}-\mathcal{B}X}\nabla_\mu\ln\mathcal{A}\left[X\nabla^\mu\ln\left(\frac{\mathcal{B}}{\mathcal{A}}\right)+\nabla^\mu\phi\,\nabla^\nu\phi\,\nabla_\nu\ln\left(\frac{\mathcal{B}}{\mathcal{A}}\right)\right] \\
-\frac{\mathcal{B}}{\mathcal{A}-\mathcal{B}X}\left(\frac{1}{2}\nabla^\mu X+\nabla^\mu\phi\,\nabla^2\phi\right)\left[(d-1)\,\nabla_\mu\ln\mathcal{A}+\nabla_\mu\ln\left(\frac{\mathcal{B}}{\mathcal{A}}\right)\right]\Bigg\}, \qquad (11)
\end{aligned}$$

$\boxed{\text{Many beyond Horndeski terms}}$

Hamiltonian analysis in Unitary gauge

- Unitary gauge

$$\phi = t$$

- ADM variables

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -N^2dt^2 + \gamma_{ij}\left(dx^i + N^i dt\right)\left(dx^j + N^j dt\right)$$

- Transformation

$$\tilde{N}^2 = \mathcal{A}N^2 - \mathcal{B}, \quad \tilde{N}^i = N^i, \quad \tilde{\gamma}_{ij} = \mathcal{A}\gamma_{ij}$$

$$\mathcal{A} = \mathcal{A}(t, N),\ \mathcal{B} = \mathcal{B}(t, N)$$

- Action in unitary gauge

$$\tilde{I}^{\text{unitary}}_{\text{total}} = \tilde{I}^{\text{unitary}}_{\text{EH}} + \int dt d^d x\, N\sqrt{\gamma}\, A_2(t,N)$$

$$\tilde{I}^{\text{unitary}}_{\text{EH}} = \int dt\, d^d x N\sqrt{\gamma}\left[A_4(t,N)\left(\underline{K^2 - K^i{}_j K^j{}_i + (d-1)KL + \frac{d(d-1)}{4}L^2}\right) - U(t,N,\gamma)\right]$$

$$K_{ij} \equiv \frac{1}{2N}\left(\dot{\gamma}_{ij} - D_i N_j - D_j N_i\right)\ , \quad K \equiv \gamma^{ij} K_{ij}$$

$$\boxed{L \equiv \frac{\mathcal{A}_N}{\mathcal{A}}\left(\frac{\dot{N}}{N} - \frac{N^i}{N} D_i N\right) + \frac{\mathcal{A}_t}{\mathcal{A}N}}\ ,$$

$$U(t,N,\gamma) \equiv -B_4(t,N)\left[R^{(d)} - (d-1)D^2 \ln\mathcal{A} - \frac{(d-1)(d-2)}{4} D_i \ln\mathcal{A}\, D^i \ln\mathcal{A}\right]$$

$$A_4(t,N) \equiv -\frac{M_p^2}{2}\frac{N\mathcal{A}^{d/2}}{\sqrt{\mathcal{A}N^2 - \mathcal{B}}}\ ,$$

$$B_4(t,N) \equiv \frac{M_p^2}{2N}\mathcal{A}^{(d-2)/2}\sqrt{\mathcal{A}N^2 - \mathcal{B}}\ ,$$

The Hamiltonian & The Constraints

- Hamiltonian

$$\pi_\Phi = \frac{\delta I_{total}}{\delta \dot{\Phi}}$$

$$H = \int d^d x \left(\pi^{ij} \dot{\gamma}_{ij} + \pi_N \dot{N} - \mathcal{L} \right) = \int d^d x \left(\mathcal{H}_\perp + N^i \mathcal{H}_i^N \right)$$

$$\mathcal{H}_\perp = -N\sqrt{\gamma} \left[\frac{1}{A_4} \left(\frac{\pi^{ij}\pi_{ij}}{\gamma} - \frac{1}{d-1} \frac{\pi^2}{\gamma} \right) + \frac{\mathcal{A}_t}{N\mathcal{A}} \frac{\pi}{\sqrt{\gamma}} + A_2 - U(t, N, \gamma) \right]$$

$$\mathcal{H}_i^N = \mathcal{H}_i + \pi_N D_i N \ ,$$

$$\mathcal{H}_i \equiv -2\sqrt{\gamma} D_j \left(\frac{\pi^j{}_i}{\sqrt{\gamma}} \right) \ ,$$

- Constraints

$$\pi_i \approx 0\ , \quad \pi_N - \frac{\mathcal{A}_N}{\mathcal{A}}\pi \equiv \tilde{\pi}_N \approx 0$$

$$\dot{\pi}_i(x) \approx \{\pi_i(x), H'\}_P = -\mathcal{H}_i^N(x) \approx 0$$

$$\dot{\tilde{\pi}}_N(x) \approx \frac{\partial}{\partial t}\tilde{\pi}_N(x) + \{\tilde{\pi}_N(x), H'\}_P \equiv \mathcal{C} \approx 0$$

$$\mathcal{C} = \sqrt{\gamma}D_i\left(N^i\frac{\tilde{\pi}_N}{\sqrt{\gamma}}\right) + \frac{1}{\sqrt{\gamma}}\left[\left(\frac{N}{A_4}\right)_N + \frac{d\mathcal{A}_N N}{2\mathcal{A}A_4}\right]\left(\pi_{ij}\pi^{ij} - \frac{1}{d-1}\pi^2\right) + \mathcal{C}_U[t, N, \gamma, A_{2N}, B_{4N}]$$

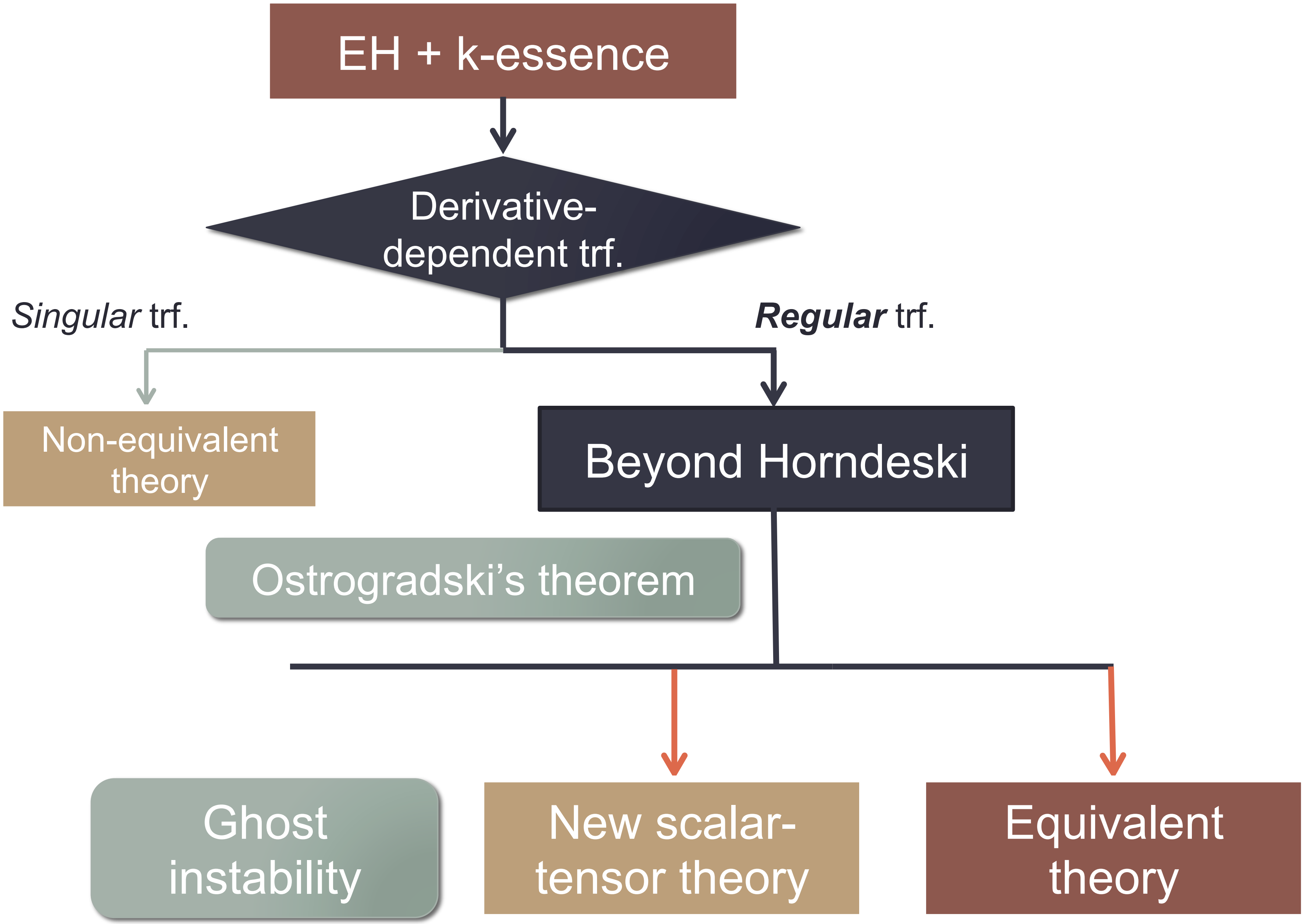
$$\mathcal{C}_U = \left(\frac{\delta}{\delta N(x)} - \frac{\mathcal{A}_N}{\mathcal{A}}\gamma_{ij}\frac{\delta}{\delta\gamma_{ij}(x)}\right)\int d^d y N\sqrt{\gamma}\,(A_2 - U(t, N, \gamma))\ .$$

$$\text{1st class}: \mathcal{H}_i^N,\ \pi_i \qquad \text{2nd class}: \tilde{\pi}_N,\ \mathcal{C}$$

- # of degrees of freedom

No extra d.o.f. appears

$$\# = \frac{1}{2}(20 - 6\times 2 - 2) = 3 = 2 + 1$$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- The Hamiltonian analysis tells that the transformed theory has no extra degrees of freedom, and would not suffer from the linear instability.

- To investigate the transformed theory is a new scalar-tensor theory or an equivalent theory, we may consider the meaning of the derivative−dependent transformation.

- We start from a very general metric + scalar theory including our simple example, and consider the meaning of the transformation.

3. Meaning of the transformation: General analysis

- Consider a scalar-tensor theory which contains up to m-th order g's derivatives and up to n-th order Φ's derivatives:

$$
\begin{aligned}
I' = \int d^{d+1}x \, \Big[& L(g_{\mu\nu}, \mathcal{R}_{\alpha\beta\gamma\delta}, \mathcal{R}_{\mu\alpha\beta\gamma\delta}, \cdots, \mathcal{R}_{\mu_1\cdots\mu_m\alpha\beta\gamma\delta}, \phi, \phi_\mu, \cdots, \phi_{\mu_1\cdots\mu_n}) + \Lambda^{\alpha\beta\gamma\delta}(\mathcal{R}_{\alpha\beta\gamma\delta} - R_{\alpha\beta\gamma\delta}) \\
& + \Lambda^{\mu\alpha\beta\gamma\delta}(\mathcal{R}_{\mu\alpha\beta\gamma\delta} - \nabla_\mu \mathcal{R}_{\alpha\beta\gamma\delta}) + \cdots + \Lambda^{\mu_1\cdots\mu_m\alpha\beta\gamma\delta}(\mathcal{R}_{\mu_1\cdots\mu_m\alpha\beta\gamma\delta} - \nabla_{(\mu_m}\mathcal{R}_{\mu_1\cdots\mu_{m-1})\alpha\beta\gamma\delta}) \\
& + \lambda^\mu(\phi_\mu - \nabla_\mu\phi) + \cdots + \lambda^{\mu_1\cdots\mu_n}(\phi_{\mu_1\cdots\mu_n} - \nabla_{(\mu_m}\phi_{\mu_1\cdots\mu_{m-1})}) \Big] ,
\end{aligned}
$$

- The action can be cast into the form

$$
I = \int d^{d+1}x \left[\frac{1}{2} \mathcal{K}_{AB} \dot{\Phi}^A \dot{\Phi}^B + M_A \dot{\Phi}^A - V \right]
$$

$$\cup$$

$$
\tilde{I}^{\text{unitary}}_{\text{EH}} = \int dt \, d^d x N \sqrt{\gamma} \left[A_4(t, N) \left(K^2 - K^i{}_j K^j{}_i + (d-1) K L + \frac{d(d-1)}{4} L^2 \right) - U(t, N, \gamma) \right]
$$

- General transformation including any order time derivatives would be also cast into the derivative-*independent* form if it is regular:

$$\tilde{\Phi}^A = F^A(\Phi, t), \quad (A = 1, 2, \cdots, \mathcal{N})$$

$$\det F^A_B \neq 0,\ \infty \qquad F^A_B = \frac{\partial F^A}{\partial \Phi^B}$$

How to reduce trf. derivative-independent

- two particle system

$$L = \frac{1}{2}\dot{Q_1}^2 + \frac{1}{2}\dot{Q_2}^2$$

$$q_1 = Q_1 + \epsilon\dot{Q_2}^2, \qquad q_2 = Q_2 \qquad\qquad Q_1 = q_1 - \epsilon\dot{q_2}^2, \qquad Q_2 = q_2$$

$$L = \frac{1}{2}\dot{Q_1}^2 + \frac{1}{2}R^2 + S(R - \dot{Q_2})$$

$$q_1 = Q_1 + \epsilon R^2, \qquad q_2 = Q_2, \qquad r = R, \qquad s = S.$$

- General transformation including any order time derivatives would be also cast into the derivative-*independent* form if it is regular:

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$$\det F^A_B \neq 0, \ \infty \qquad F^A_B = \frac{\partial F^A}{\partial \Phi^B}$$

$$\tilde{N}^2 = \mathcal{A} N^2 - \mathcal{B}, \quad \tilde{N}^i = N^i, \quad \tilde{\gamma}_{ij} = \mathcal{A} \gamma_{ij}$$

This transformation is a point transformation included in canonical transformation as long as F^A is regular. So, the physics in the two different frame should be the same.

- We can easily find the generator by comparing the Hamiltonians in the different frames

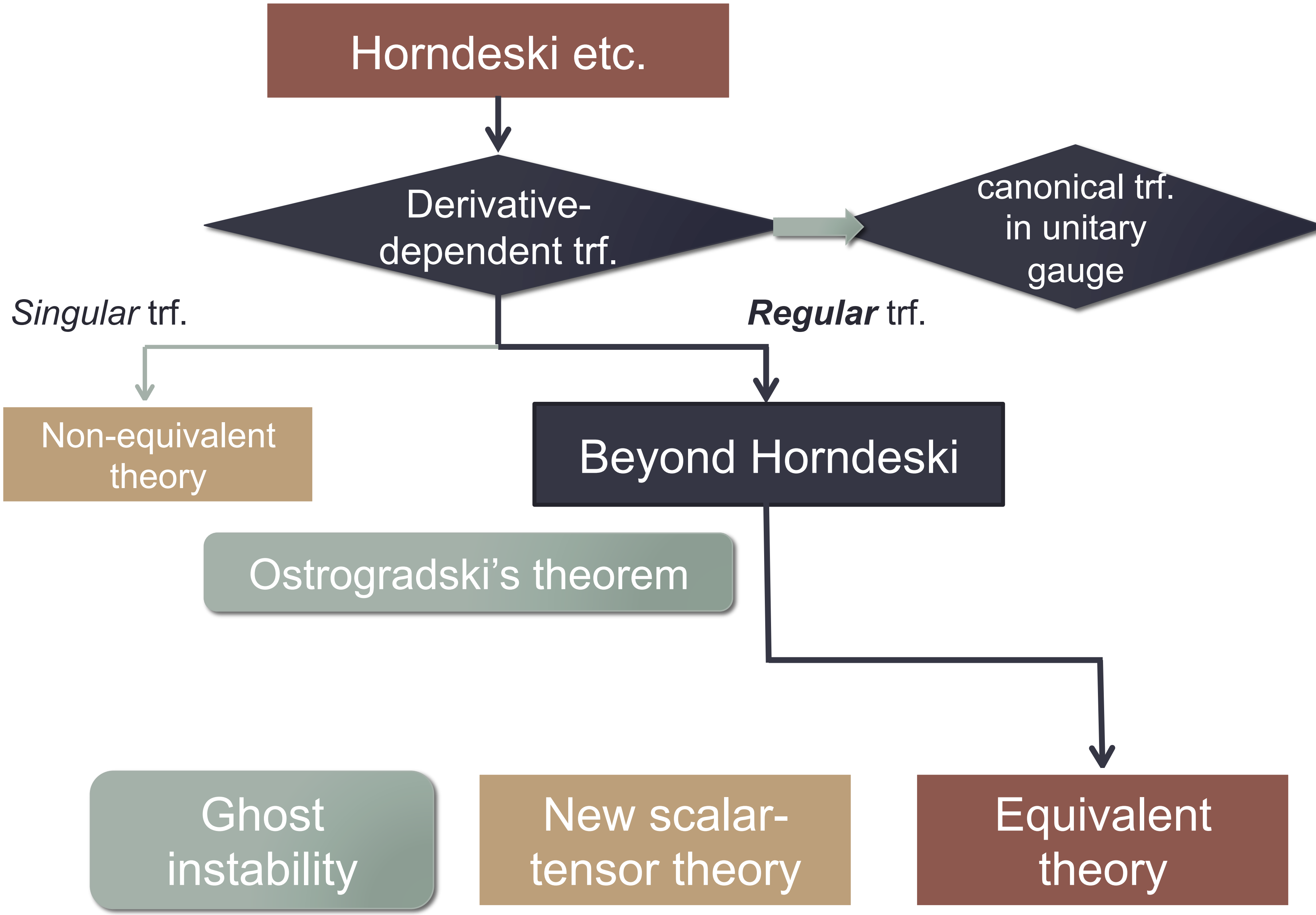
$$\tilde{H} = H + \frac{\partial \mathcal{G}}{\partial t} \qquad \mathcal{G}[\Pi, \tilde{\Phi}; t] = -\int d^d x \, \Pi_A \, G^A(\tilde{\Phi}, t),$$

$$\Phi^A = G^A(\tilde{\Phi}, t), \quad (A = 1, 2, \cdots, \mathcal{N})$$

$$\{\Phi^A(\vec{x}), \Phi^B(\vec{y})\}_P = 0, \quad \{\Phi^A(\vec{x}), \Pi_B(\vec{y})\}_P = \delta^A_B \delta^3(\vec{x} - \vec{y}), \quad \{\Pi_A(\vec{x}), \Pi_B(\vec{y})\}_P = 0.$$

$$\left\{\tilde{\Phi}^A(\vec{x}), \tilde{\Phi}^B(\vec{y})\right\}_P = 0, \quad \left\{\tilde{\Phi}^A(\vec{x}), \tilde{\Pi}_B(\vec{y})\right\}_P = \delta^A_B \delta^3(\vec{x} - \vec{y}), \quad \left\{\tilde{\Pi}_A(\vec{x}), \tilde{\Pi}_B(\vec{y})\right\}_P = 0,$$

We would never find new scalar-tensor theories as long as we consider the regular transformation, although the transformed theory may have quite non-trivial beyond Horndeski terms.



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4. Summary

- Derivative-dependent transformation, though it can create beyond Horndeski term, does not make any new scalar-tensor theory from known theories as long as the transformation is invertible and regular.
$$\mathcal{A}(\mathcal{A}-\mathcal{B}X)(\mathcal{A}-\mathcal{A}_X X+\mathcal{B}_X X^2)\neq 0$$
- In unitary gauge, regular derivative-dependent trf. reduces to point trf. included in the canonical trf.
- The result looks very non-trivial and may be possibly misleading in general gauge, but quite natural in unitary gauge.
- *Singular* transformation can create some new scalar-tensor theories (ex. mimetic DM Chamseddine et al'13)
$$\tilde{g}_{\mu\nu}=Xg_{\mu\nu}$$