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Causality
Hyperbolicity
& Shock formation
in Lovelock Theories

Causality, Hyperbolicity & Shock formation in Lovelock Theories

- Lovelock Theories
  = GR + (higher-curvature corrections)

  - EoM up to 2nd derivatives → Avoids ghost instability
  - From string theory?

- GR: Gravity propagate at c
- Lovelock: Faster/slower propagation than c

→ { Causality in Lovelock theories?
    Does EoM remain hyperbolic?

Causality, Hyperbolicity & Shock formation in Lovelock Theories

- Causality in Lovelock theories?
  - Can we define causality in this theory?
  - Can graviton escape from black hole interior?

- Does EoM remain hyperbolic?
  - Hyperbolic EoM = Wave equation
  - Determined by principal part of EoM
    - GR: Guaranteed to be hyperbolic
    - Lovelock: ?

- Shock formation due to variable sound speed?

Contents

1. Introduction
   – Lovelock theories
   – Characteristics
2. Questions
   – Can graviton escape from black hole interior?
   – Propagation on *plane wave solutions*
   – Propagation *around black holes*
   – Shock formation?
3. Summary

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1. Introduction
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Introduction: Lovelock theories

- Lovelock theories in d dimensions ($p \leq (d-1)/2$)

$$\mathcal{L} = R - \sum_p 2k_p \delta^{c_1 \ldots c_{2p}}_{d_1 \ldots d_{2p}} R_{c_1 c_2}{}^{d_1 d_2} \ldots R_{c_{2p-1} c_{2p}}{}^{d_{2p-1} d_{2p}}$$

$$= R - 8k_2 \left(R^2 - 4R_{ab}R^{ab} + R_{abcd}R^{abcd} \right) + \cdots$$

$$\left(\delta^{c_1 \ldots c_n}_{d_1 \ldots d_n} \equiv n! \delta^{c_1}_{[d_1} \ldots \delta^{c_n}_{d_n]} \right)$$

- EoM = Einstein eq. + correction

$$E^a{}_b \equiv G^a{}_b + B^a{}_b = 0$$

where

$$B^a{}_b = \sum_{p \geq 2} k_p \delta^{a c_1 \ldots c_{2p}}_{b d_1 \ldots d_{2p}} R_{c_1 c_2}{}^{d_1 d_2} \ldots R_{c_{2p-1} c_{2p}}{}^{d_{2p-1} d_{2p}}$$

Introduction: Characteristics

- A signal propagates on ***characteristic surface***

EoM of ψ : $0 = \nabla^2 \psi = g^{tt} \partial_t^2 \psi + \cdots$

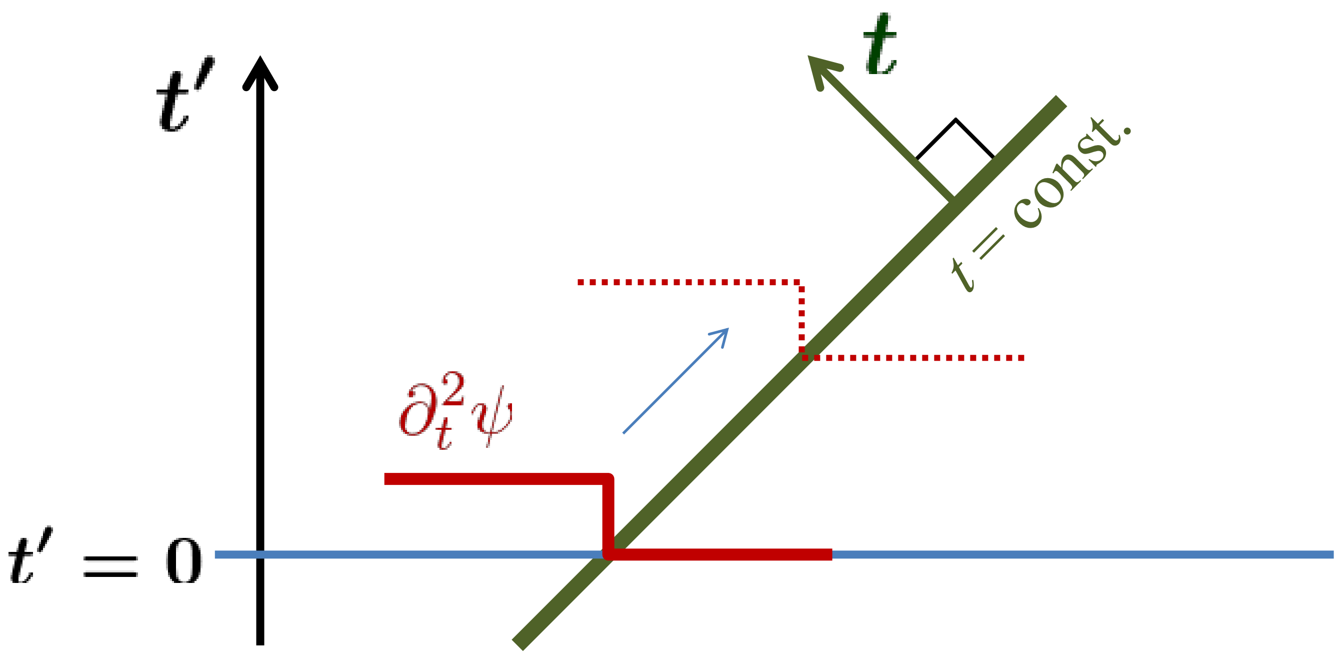
- $g^{tt} \neq 0$: $\partial_t^2 \psi$ uniquely determined
  → usual time evolution
- $g^{tt} = 0$: $\partial_t^2 \psi$ non-unique
  → $t =$ const. surface is *characteristic*

Introduction: Characteristics

- $g^{tt} = 0$: $\partial_t^2 \psi$ non-unique

  → $t =$ const. surface is *characteristic*

✓ *Characteristic surface* is a possible wave front



Introduction: Characteristics

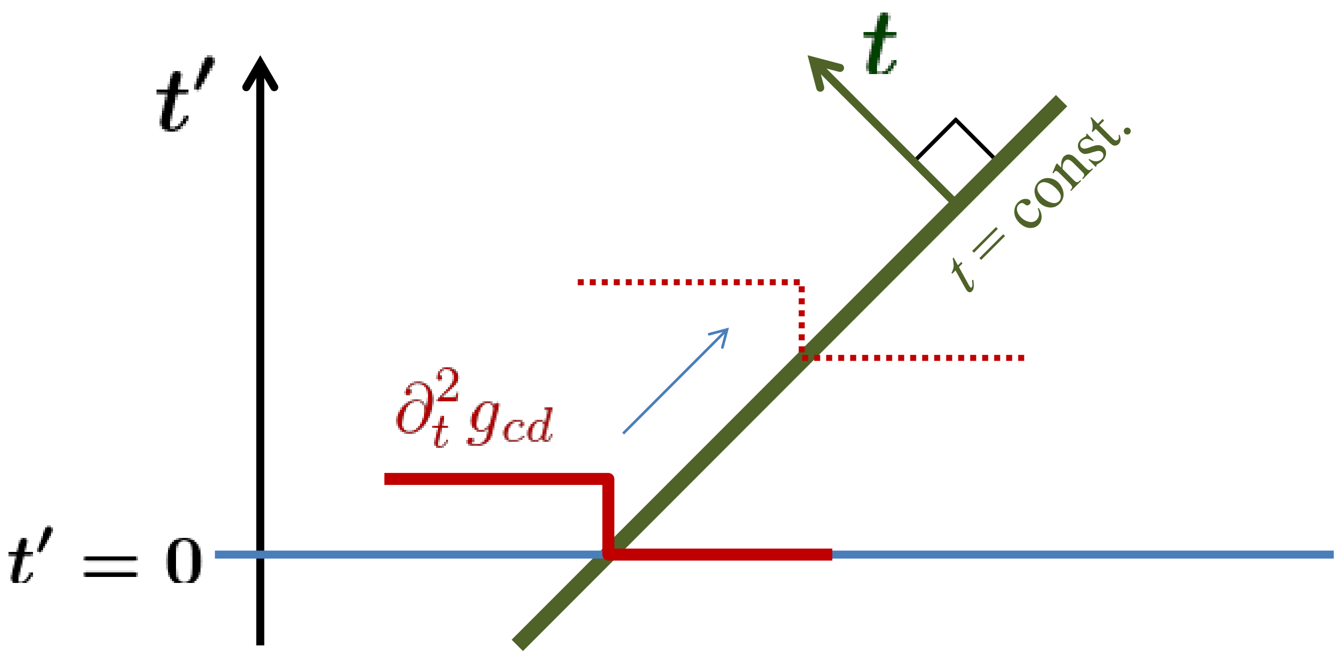
- Characteristics in Lovelock theories

[Aragone '87]
[Choquet-Bruhat'88]

$$0 = G_{ab} + B_{ab} = P_{ab}{}^{cd}\,\partial_t{}^2 g_{cd} + \cdots$$

$$P^{abcd} \sim g^{tt} g^{c(a} g^{b)d} + \mathcal{R}^{abcd}$$

Characteristic $\Leftrightarrow$ $\det P = 0$



Questions

1. Can graviton escape from black hole interior?

   ↑ No: Killing horizon is characteristic surface

2. Propagation on *plane wave solutions*

3. Propagation *around black holes*

- Does it obey causality?
- Is hyperbolicity maintained?

2. Propagation on *plane wave solutions*

More generally, we consider

Ricci-flat type N spacetimes

as backgrounds.

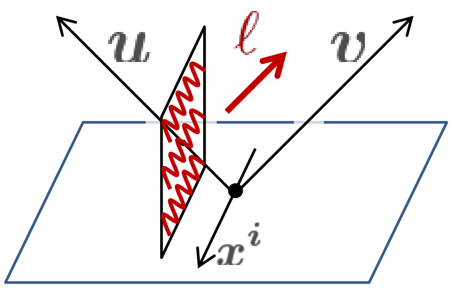
Only null(ℓ)-angular components of Riemann tensor are nonzero:

$$R_{\ell i \ell j} \equiv \Omega_{ij}$$

✓ Example: Plane wave solution [Boulware-Deser '85]

$$ds^2 = a_{ij} x^i x^j du^2 + 2 du dv + \delta_{ij} dx^i dx^j$$

$$\Rightarrow \; R_{\ell i \ell j} \propto a_{ij}$$



2. Propagation on *Ricci-flat type N spacetimes*

Proposition:

Characteristic surfaces are null w.r.t. "effective metrics":

$$G_I^{ab} = g^{ab} + \omega_I \ell^a \ell^b$$

$$\left(I = 1, \ldots, \frac{1}{2}d(d-3) \quad \omega_I = \omega_I(\Omega_{ij}) \right)$$

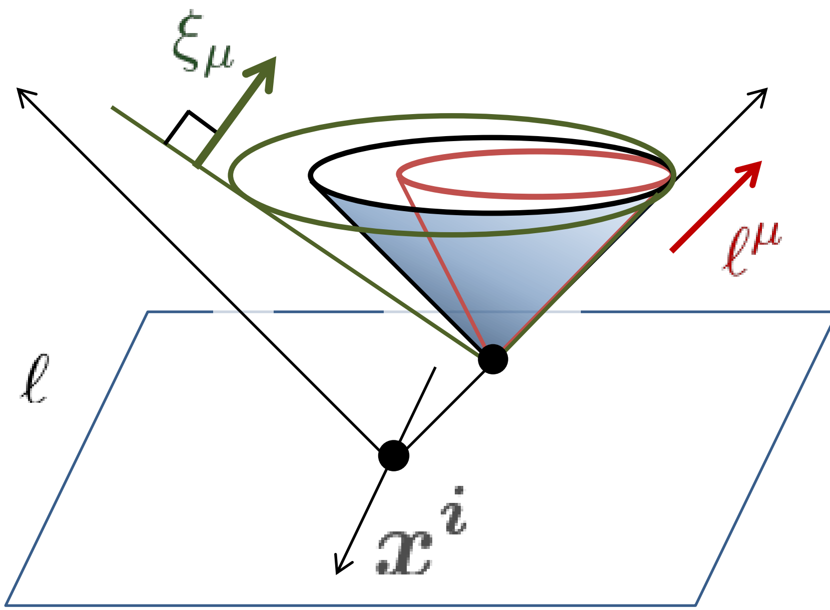
✓ $\det P = \prod_I G_I^{ab} \xi_a \xi_b = 0$

✓ ℓ : null w.r.t. G_I

$\Rightarrow$ Characteristic cones tangent to ℓ

✓ Nested characteristic cones

✓ Causality w.r.t. the largest cone



3. Propagation *around black holes*

- Static, maximally symmetric black holes

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Sigma^2$$

- Proposition:
  Characteristic surfaces are null w.r.t. "effective metrics":

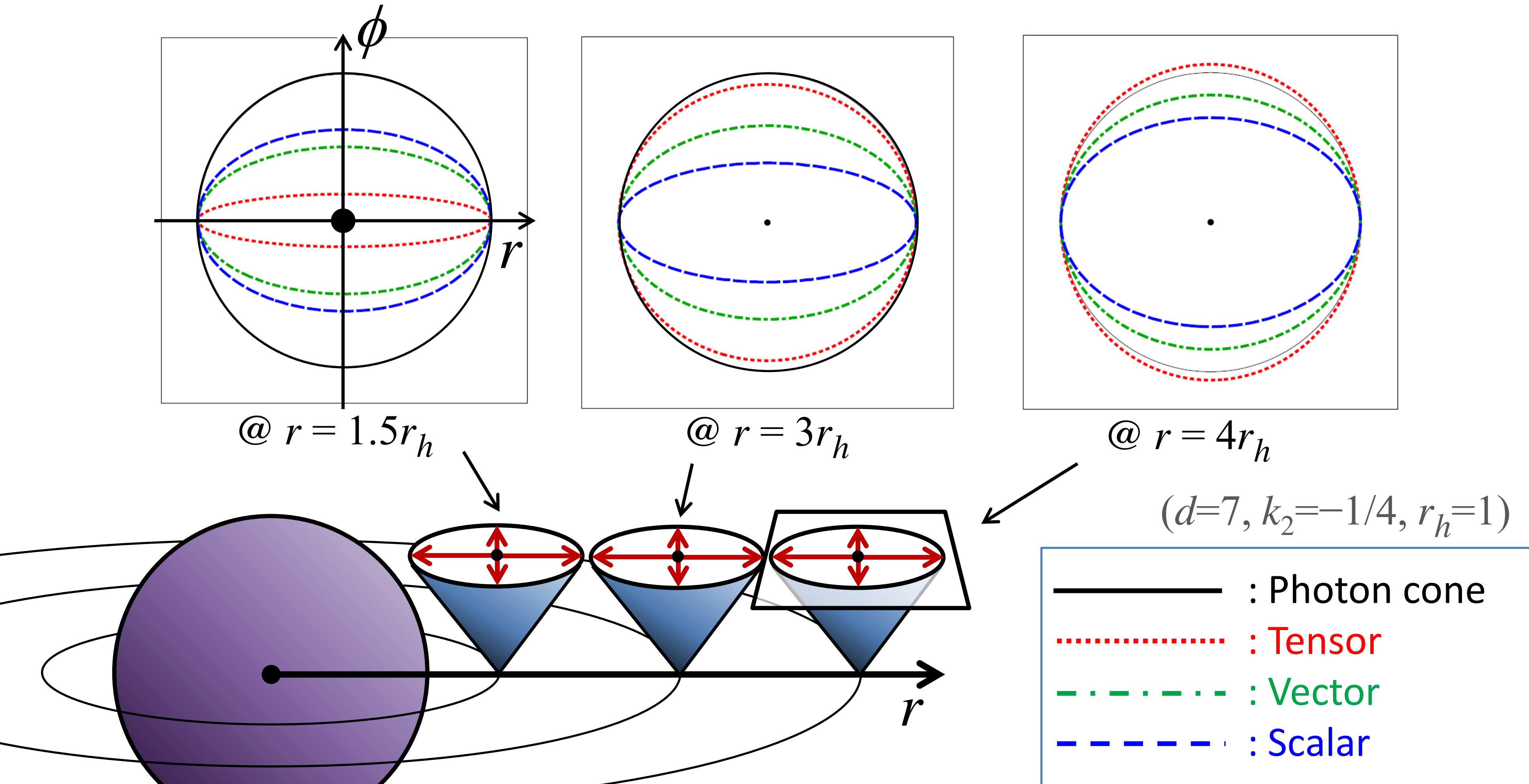
$$G^{A}_{\mu\nu}dx^\mu dx^\nu = -f(r)dt^2 + f(r)^{-1}dr^2 + \frac{r^2}{c_A(r)}d\Sigma^2$$

A : Tensor, Vector, Scalar modes

$c_A(r)$: (Propagation speed)^2 in Σ directions

3. Propagation *around black holes*

$$G^{A}_{\mu\nu}dx^{\mu}dx^{\nu} = -f(r)dt^2 + f(r)^{-1}dr^2 + \frac{r^2}{c_A(r)}d\Sigma^2$$



- Small BH $\Rightarrow c_A < 0$ near horizon

  $\Rightarrow$ Violation of hyperbolicity

$$\left(-\frac{\partial^2}{\partial t^2}+\frac{\partial^2}{\partial r_*^2}+\frac{f(r)c_A(r)}{r^2}\frac{\partial^2}{\partial\Sigma^2}\right)\Psi \equiv f(r)G_A^{\mu\nu}\partial_\mu\partial_\nu\Psi$$

$$\left(\frac{\partial^2}{\partial\Sigma^2}\simeq -l^2\right)$$

- Initial value problem is not well-posed

  ✓ $\omega^2 = -\alpha^2 l^2 \Rightarrow$ growing mode $\propto \exp(\alpha l t)$

  ✓ Perturb initial data with this mode as

$$\delta g_{\mu\nu}(t,r,x) \sim e^{-\sqrt{l}} e^{\alpha l t} \Rightarrow \begin{cases} \bullet\, t=0: \ \delta g, \partial^n\delta g = 0 \\ \bullet\, t>0: \ \delta g \to \infty \end{cases}$$

with $l \to \infty$

$\therefore$ Solution is not continuous w.r.t. initial data

No solution exists except for special initial conditions

Answers

1. Can graviton escape from black hole interior?

   ↑ No: Killing horizon is characteristic surface

2. Propagation on *plane wave solutions*

   ✓ Characteristics = Null w.r.t. effective metrics

   ✓ Causality w.r.t. the largest cone

3. Propagation *around black holes*

   ✓ Characteristics = Null w.r.t. effective metrics

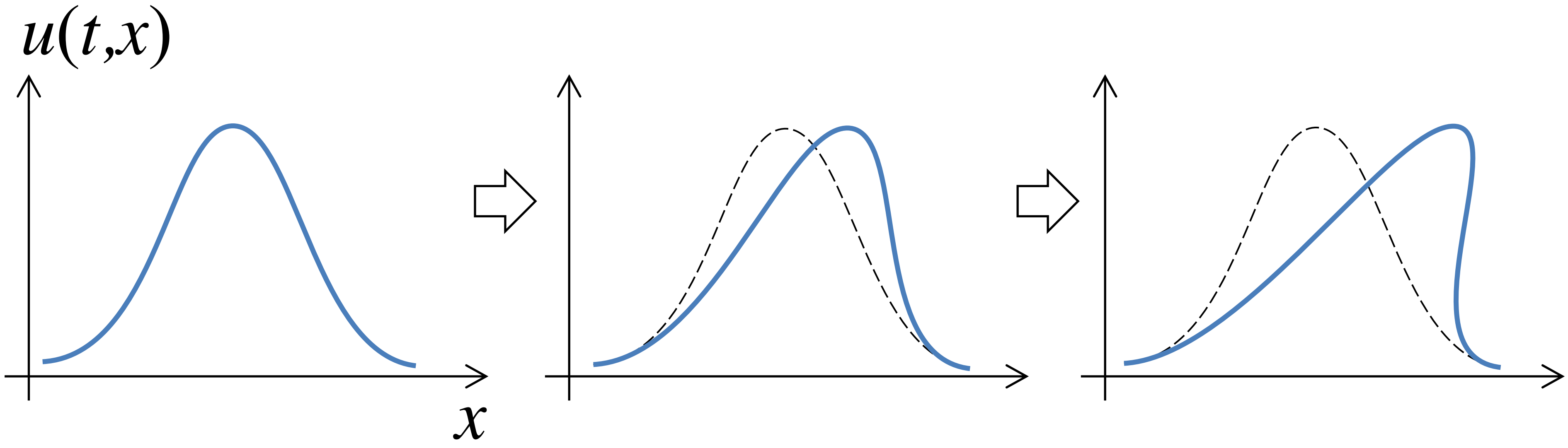
   ✓ Hyperbolicity violation near small BH horizons

Contents

Shock formation

- Sound speed $\neq$ const.
- Waveform distortion → Shock formation?

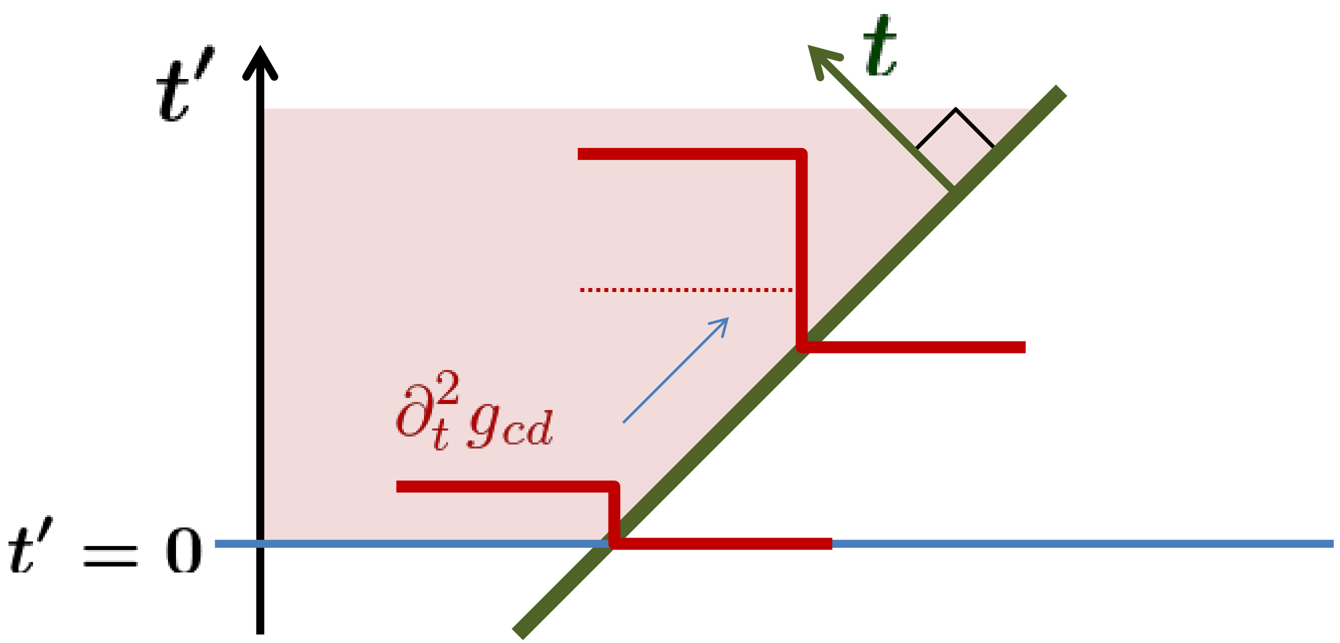
ex.) Burgers' equation $\partial_t u + u\,\partial_x u = 0$



Shock formation

- Propagation of discontinuity in $\partial^2 g_{ab}$
  - GR: Amplitude obeys linear eq.
  - Lovelock: Amplitude obeys nonlinear eq.

→ Amplitude blow up?



- EoM in Lovelock

$$E_{ab} \equiv G_{ab} + B_{ab} = 0$$

- Take discontinuous part

(discontinuous part)

$$[\,E_{ab}\,] = P_{ab}{}^{cd}\,[\partial_t^2 g_{cd}] = 0$$

$$\Rightarrow\ [\partial_t^2 g_{cd}] = \Pi(x^i)\, r_{cd} \qquad\qquad (\,P \cdot r = 0\,)$$

- Transport equation of amplitude $\Pi(x^i)$

$$[\,\partial_t E_{ab}\,] = 0 \quad \Rightarrow \quad \dot{\Pi} + M\,\Pi + N\,\Pi^2 = 0$$

$$N = 4\sum_{p\geq 2} p(p-1)k_p\, \delta^{0ikmpr_5\ldots p_{2p}}_{1jlnqs_5\ldots s_{2p}}\, \Gamma^0_{ij'} g^{jj'}\, r^l_k r^n_m r^q_p\, R^{s_5 s_6}_{r_5 r_6} \ldots R^{s_{2p-1}s_{2p}}_{r_{2p-1}r_{2p}}$$

$$\dot{\Pi} + M\,\Pi + N\,\Pi^2 = 0$$

$$\left(\Phi(s) = \int_0^s M(s')ds' \right)$$

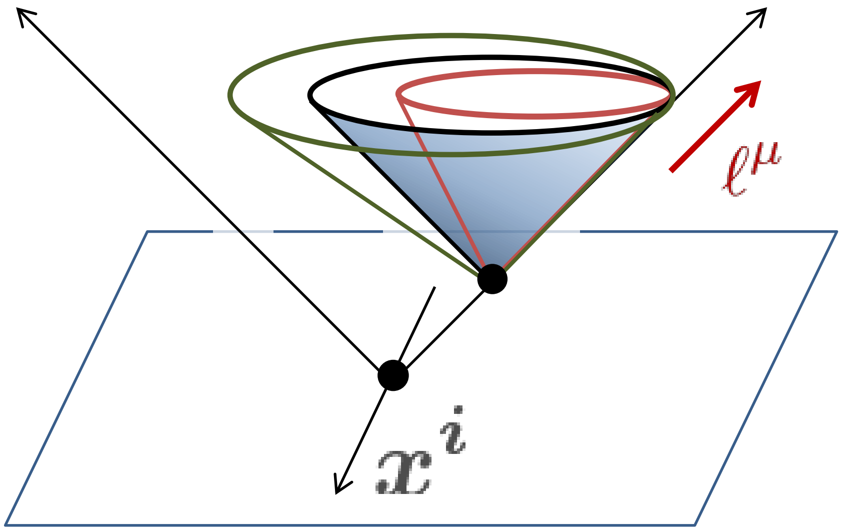
$$\Rightarrow \qquad \Pi(s) = \frac{\Pi(0)\, e^{-\Phi(s)}}{1 + \Pi(0) \int_0^s N(s') e^{-\Phi(s')} ds'}$$

- GR: $N = 0$ $\Rightarrow$ $\Pi(s)$ finite unless $e^{-\Phi}$ diverges
- Lovelock: $N \neq 0$ $\Rightarrow$ $\Pi(s)$ diverges at finite s for large $|\Pi(0)|$
  Shock formation

Shock formation

- Ricci-flat type N spacetimes
  - Along ℓ^μ : $N = 0$ → No shock
  - Along other directions: $N \neq 0$ → Shock formation

$$\Pi(u) \sim \frac{1}{u - u_0}$$



Shock formation

✓ Does a shock formation imply pathology?

- Violation of weak cosmic censorship?

  Shock = Naked singularity

- Stability of Minkowski in Lovelock theories?

  Flat background → $\boldsymbol{N = 0}$, stable?

- Shock = Weak solution in Lovelock theory

  Shock formation from smooth initial data?

Summary

- **Characteristics in Lovelock theories**
  - Can graviton escape from black hole interior? ← No
  - Propagation on plane wave & BH backgrounds
    - ✓ Characteristics = Null w.r.t. effective metrics
    - ✓ Causality w.r.t. the largest cone
    - ✓ Hyperbolicity violation near small BH horizons
- **Shock formation in Lovelock theories**
    - ✓ ∃ nonlinear term, shock forms for large initial data

?: Does ~~hyperbolicity~~ occur in time evolution?

?: Shock formation & evolution from smooth data?

?: Stability of Minkowski?

?: Shock formation in scalar-tensor theories?