Geometrical destabilization of heavy scalar fields during inflation

Sébastien Renaux-Petel

CNRS - IAP

With Krzysztof Turzynski (Warsaw)

Appeared today on the arXiv:1510.01281

Workshop, IAP, 06.09.2015
Outline

I. General mechanism:

‘geometrical destabilization’
of heavy fields

II. Minimal realization and observational consequences
Inflation in high-energy physics

Simplest hope of model-builders:

• one of the field is light and yields inflation.

• the other fields are extremely massive and decouple completely from the low energy effective field theory.

Extra field $\chi$ stabilized by a large mass

$$m_\chi \gg H$$

Inflationary direction $\phi$
Inflation in high-energy physics

Simplest hope of model-builders:

Even if displaced from its minimum, the heavy field rapidly rolls back towards its stabilizing value and does not affect inflation.

Extra field $\chi$ stabilized by a large mass $m_\chi \gg H$.
Mechanism of destabilization of heavy scalar fields:

the heavy field climbs up its potential, completely changing the inflationary picture.

Here:

Inflationary direction $\phi$

would-be stabilized field
Linear cosmological perturbation theory

\[ S = \int d^4x \sqrt{-g} \left( -\frac{1}{2} G_{IJ}(\phi^K) \nabla_\mu \phi^I \nabla^\mu \phi^J - V(\phi^I) \right) \]

Flat gauge: \[ \phi^I = \bar{\phi}^I(t) + Q^I(t, x) \]

\[ \mathcal{D}_t \mathcal{D}_t Q^I + 3H \mathcal{D}_t Q^I + \frac{k^2}{a^2} Q^I + M_{IJ} Q^J = 0 \]

\[ \mathcal{D}_t A^I = \dot{A}^I + \Gamma_{JK}^I \phi^J A^K \]

Mass matrix: Sasaki, Stewart, 95, Langlois, RP, 08

\[ M_{IJ} = V;_IJ - \mathcal{R}_{IKL}^J \phi^K \phi^L - \frac{1}{a^3} \mathcal{D}_t \left[ \frac{a^3}{H} \phi_I \phi_J \right] \]

Riemann curvature tensor of the field space metric
**Adiabatic/entropic decomposition**

Gordon et al, 00, Nibbelink and van Tent, 01

Projection along and perpendicular to the velocity direction:

\[
e^I_\sigma = \frac{\dot{\phi}^I}{\sqrt{G_{IJ} \dot{\phi}^I \dot{\phi}^J}} \equiv \frac{\dot{\phi}^I}{\dot{\sigma}}
\]

**Curvature** perturbation:

\[
\zeta = \frac{1}{\sqrt{2\epsilon}} Q_\sigma \quad \epsilon \equiv -\frac{\dot{H}}{H^2}
\]

**Entropic/isocurvature** perturbation:

\[
S = \frac{1}{\sqrt{2\epsilon}} Q_s
\]

(N=2 for simplicity)
Super-Hubble evolution

\[ \frac{\dot{\zeta}}{H} = 2\eta_\perp S \quad \text{with} \quad \eta_\perp \equiv -\frac{V,_{s}}{H\dot{\sigma}} \]

In general:

super-Hubble evolution of the curvature perturbation
Super-Hubble evolution

\[ \frac{\dot{\zeta}}{H} = 2\eta_\perp S \quad \text{with} \quad \eta_\perp \equiv -\frac{V_s}{H\dot{\sigma}} \]

- \( \eta_\perp \) reduces to \( \dot{\theta}/H \) in canonical 2-field inflation (trivial field space metric)
\[
\frac{\dot{\zeta}}{H} = 2\eta_\perp S
\]

with

\[
\eta_\perp \equiv -\frac{V',s}{H\dot{\sigma}}
\]

- \(\eta_\perp\) reduces to \(\dot{\vartheta}/H\) in canonical 2-field inflation (trivial field space metric)

- **Non-zero when the trajectory ‘bends’, i.e. deviates from a geodesic**

- **Dimensionless measure** of the adiabatic/entropic coupling
Super-Hubble evolution

\[ \frac{\dot{\zeta}}{H} = 2\eta_\perp S \quad \text{with} \quad \eta_\perp \equiv -\frac{V,s}{H\dot{\sigma}} \]

\[ \ddot{Q}_s + 3H\dot{Q}_s + m_{s(\text{eff})}^2 Q_s = 0 \]

\( S = \frac{1}{\sqrt{2\epsilon} Q_s} \)

effective entropic mass squared:

\[ \frac{m_{s(\text{eff})}^2}{H^2} \equiv \frac{V_{;ss}}{H^2} + 3\eta_\perp^2 + \epsilon R_{\text{field space}} M_{\text{Pl}}^2 \]

naive Hessian contribution

bending contribution

‘geometrical’ contribution

RP and Turzynski
1405.6195 (JCAP)
When the geometrical contribution is negative and large enough, it can render the entropic fluctuation tachyonic, even with a large ‘bare mass’, with potentially dramatic observational consequences.
Geometrical destabilization

\[ \frac{m_{s(\text{eff})}^2}{H^2} \equiv \frac{V_{;ss}}{H^2} + 3\eta_\perp + \epsilon R^{\text{field space}}M_{\text{Pl}}^2 \]

• Necessary condition for this ‘geometrical destabilization’: \( R^{\text{field space}} < 0 \)

Relevant when \( \epsilon \) and/or \( R^{\text{field space}}M_{\text{Pl}}^2 \) is/are large enough:

• At the end of inflation: \( \epsilon \rightarrow 1 \) always!

• Violation of slow-roll during inflation (features)
Geometrical destabilization

Relevant when $\epsilon$ and/or $R_{\text{field space}}^2 M_{Pl}^2$ is/are large enough:

$$R_{\text{field space}}^2 M_{Pl}^2 \sim (M_{Pl}/M)^2$$

where $M$ can be identified in simple setups with the scale of new physics beyond $H$

Quite legitimate to have:

$$M = \mathcal{O}(10^{-2}, 10^{-3}) M_{Pl}$$

(string scale, KK scale, GUT scale...)

Even for $\frac{V;ss}{H^2} \sim 100$

the effective mass becomes tachyonic when:

$$\epsilon \rightarrow \epsilon_c = 10^{-4} \quad \text{or} \quad 10^{-2}$$
After the critical point

- Similar to hybrid inflation (but different kinetic origin and kinetic effects).

- Theoretical uncertainties and model-dependence (beyond linear perturbation theory, stochastic inflation, production of primordial black holes, tachyonic preheating, inflating topological defects ...):
  - Inflation can end abruptly without impact on large scale fluctuations
  - Or a second phase of inflation
  - Or ...
Most conservative approach

- Inflation ends abruptly without impact on large scale fluctuations

- But here, this does have important consequences:
  - Inflation ends when $\epsilon \rightarrow \epsilon_c$
  - This can be several tens of e-folds (or more) before the standard end of inflation
  - The window on the potential probed by cosmological scales is changed, and so are the predictions.
Visualization (exaggerated numbers)

Standard end of inflation

Window on the potential probed by observable modes

$\epsilon \sim 1$

$V(\phi)$

$\Lambda^4$

$0.2$ $0.4$ $0.6$ $0.8$ $1.0$
Visualization (exaggerated numbers)

$V(\phi)$

$\Lambda^4$

$0.2$ $0.4$ $0.6$ $0.8$ $1.0$

New end of inflation

$\epsilon_c \ll 1$

New observational window

$\epsilon \sim 1$

Window on the potential probed by observable modes

Standard end of inflation

$\frac{\phi}{M_p}$
**Vizualization (exagerated numbers)**

New end of inflation

\( \epsilon_c \ll 1 \)

New observational window

Quite dramatic impact on observables!

Standard end of inflation
I. General mechanism:

‘geometrical destabilization’ of heavy fields

II. Minimal realization and observational consequences
**Minimal realization**

$$\mathcal{L} = -\frac{1}{2} (\partial \phi)^2 \left( 1 + 2 \frac{\chi^2}{M^2} \right) - V(\phi) - \frac{1}{2} (\partial \chi)^2 - \frac{1}{2} m_h^2 \chi^2$$

- Slow-roll model of inflation, with inflaton $\phi$
- Heavy field $\chi$ with $m_h^2 \gg H^2$
- Simple dimension 6 operator suppressed by a mass scale of new physics $M \gg H$
Minimal realization

\[ \mathcal{L} = -\frac{1}{2} (\partial \phi)^2 \left( 1 + 2 \frac{\chi^2}{M^2} \right) - V(\phi) - \frac{1}{2} (\partial \chi)^2 - \frac{1}{2} m_h^2 \chi^2 \]

- Simple dimension 6 operator suppressed by a mass scale of new physics \( M \gg H \)

- Generally expected from the effective theory point of view.

- Does correspond to lots of models in the literature, in which it is usually said: «chi is stabilized by a large mass» so let us put \( \chi = 0 \) (consistently with the equations of motion)
Minimal realization

\[ \mathcal{L} = -\frac{1}{2} (\partial \phi)^2 \left( 1 + \frac{2}{M^2} \frac{\chi^2}{M^2} \right) - V(\phi) - \frac{1}{2} (\partial \chi)^2 - \frac{1}{2} m_h^2 \chi^2 \]

- Apparently benign high-energy correction (small correction to the kinetic term) but ...

\[ R_{\text{field space}} \simeq -\frac{4}{M^2} \quad \text{for} \quad \chi \ll M \]

\[ \frac{m_{s(\text{eff})}^2}{H^2} = \frac{m_h^2}{H^2} - 4 \epsilon(t) \left( \frac{M_{\text{Pl}}}{M} \right)^2 \quad \text{along} \quad \chi = 0 \]

- The inflationary trajectory becomes unstable after \( \epsilon \rightarrow \epsilon_c \)
Observational predictions

• We developed a method to study the tachyonic growth in the linear regime (solving directly for the power spectrum)

  2 conservative approaches to address the subsequent theoretical uncertainty:

• Inflation ends abruptly without impact on large scale fluctuations

• At the critical point, we shift $\chi$ to the typical value $H_c/(2\pi)$

  and we follow the evolution of the coupled two-field system: 2nd phase of inflation with interesting properties!
Observational predictions

Prototypical example: Starobinsky potential

\[ V(\phi)/\Lambda^4 = \left(1 - e^{-\sqrt{2/3} \phi/M_{\text{Pl}}} \right)^2 \]

And large ‘stabilizing mass’:

\[ m_h^2 = 100 H_{\text{inf}}^2 \]
Perspectives and generalizations

- Study of concrete models in the literature (alpha-attractors, others)

- Similar discussion in N-field models, with (N-1) threats of tachyonic instabilities, and the Ricci scalar replaced by relevant projections of the Riemann tensor

- Even more dramatic impact on models with masses of order the Hubble parameter (typical in susy)

- Features in the potential can trigger the instability

- Links with constraints on primordial non-Gaussianities
Conclusion

• Very general mechanism: geometrical destabilization of heavy scalar fields

• One more line to the check-list of model-builders of inflation in realistic contexts: stabilization by a large mass need not be sufficient!

• Similar to the eta problem: higher-order operators suppressed by a large energy scale, even of Planck value, can substantially modify the inflationary dynamics, ruining the required flatness of the inflationary direction in the case of the eta problem, and the required large curvature of the orthogonal directions here.
Conclusion

• Modified observational predictions

• Modifies interpretations of cosmological constraints (on ns and r) in terms of fundamental physics

• New mechanism to end inflation

• Varied phenomenology

• Calls for new theoretical developments: kind of ‘kinetic hybrid inflation’
Self-advertisement

Primordial non-Gaussianities after Planck 2015: an introductory review

Invited review for the French Academy of Sciences

arXiv:1508.06740