Renormalization and Unitarity in Lifshitz scaling theory

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Unitarity and renormalizability

Conjecture by Llewellyn Smith (1974)

If no physical ghost propagation **predictability**

	Unitarity	= renormalizabiliy
Y-M theory	○	○
Weinberg-Salam model	○	○
Massive vector	×	×
4-Fermi	×	×

No counterexample!

Why Lifshitz scaling theory?

Check if this is purely quantum origin or not.

Less symmetric theory is better.
Otherwise symmetry may do something.

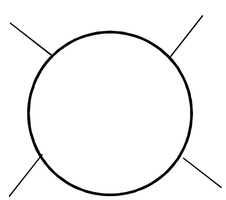
Why not break the Lorentz sym.?

Horava Lifshitz gravity

Check unitarity may be easier.

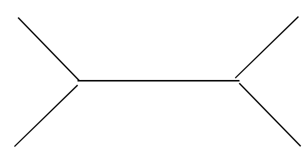
renormalization

$$\int d\omega d^d k \, V \frac{1}{\omega^2 - p^{2z}} V \ldots\ldots\ldots$$



unitarity

$$V \frac{1}{\omega^2 - p^{2z}} V$$



Contents

Renormalizability

The conventional Power Counting Renormalization (PCR) is not enough.
For renormalization (finite number of counter terms),
we need extended version of PCR.

Unitarity

Only with unitarity, optical theorem can be derived.
Optical theorem gives an inequality for scattering amplitude called “Unitary bound”.
Derive the condition where Unitary bound of tree diagram holds in UV limit.

Contents

Renormalizability

The conventional Power Counting Renormalization (PCR) is not enough.
For renormalization (finite number of counter terms),
we need extended version of PCR.

See the coincidence of these conditions

Unitarity

Only with unitarity, optical theorem can be derived.
Optical theorem gives an inequality for scattering amplitude called “Unitary bound”.
Derive the condition where Unitary bound of tree diagram holds in UV limit

Renormalizability

Conventional Power-counting
And
Extended Power-counting

Conventional PCR

Second order action

$$S_2 = \int dt d^d x \, \phi(-\partial_t^2 - (-\Delta)^z)\phi$$

$$[p] = 1, \quad [E] = z, \quad [\phi] = (d-z)/2$$

$$[dt] + d[dx] + 2[\partial_t] + 2[\phi] = 0$$

Interaction action

$$S_{int} = \lambda \int dt d^d x \, \partial_x^a \phi^b$$

$$[\lambda] = -[dt] - d[dx] - a[\partial_x] - b[\phi]$$

$$= z + d - a - b(d-z)/2 \qquad \geq 0$$

Conventional Power-Counting Renormalizaion (PCR) condition

Renormalization

Relevant terms in RG flow

Describe the IR behavior of theory.
Perhaps the conventional power-counting works

Finite number of counter terms

UV completion
Prediction
The conventional PCR is not enough

Unitarity in UV is expected to be related to the latter one!

Nonrenormalizable term with PCR

Example with d=3, z=5

$$S_2 = \int dt d^3x \, \phi(-\partial_t^2 - (-\Delta)^5)\phi \qquad [\phi] = -1$$

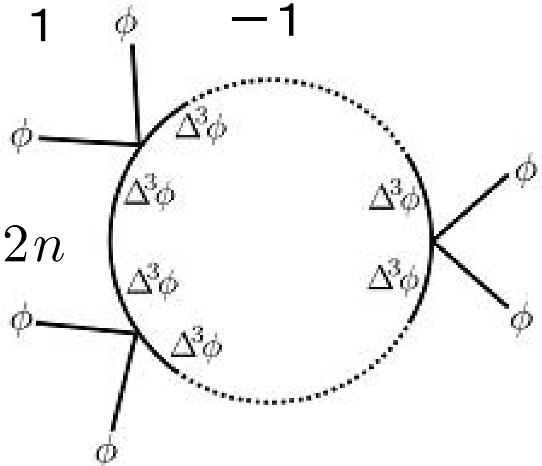
interaction term with [λ]=0 (satisfies the conventional PCR)

$$S_{int} = \lambda \int dt d^3x \, \phi^2(\Delta^3\phi)^2$$

$$[\lambda] = -\underbrace{[dt]}_{-5} - 3\underbrace{[dx]}_{-1} - 12\underbrace{[\partial_x]}_{1} - 4\underbrace{[\phi]}_{-1} = 0$$

1-loop 2n-point function

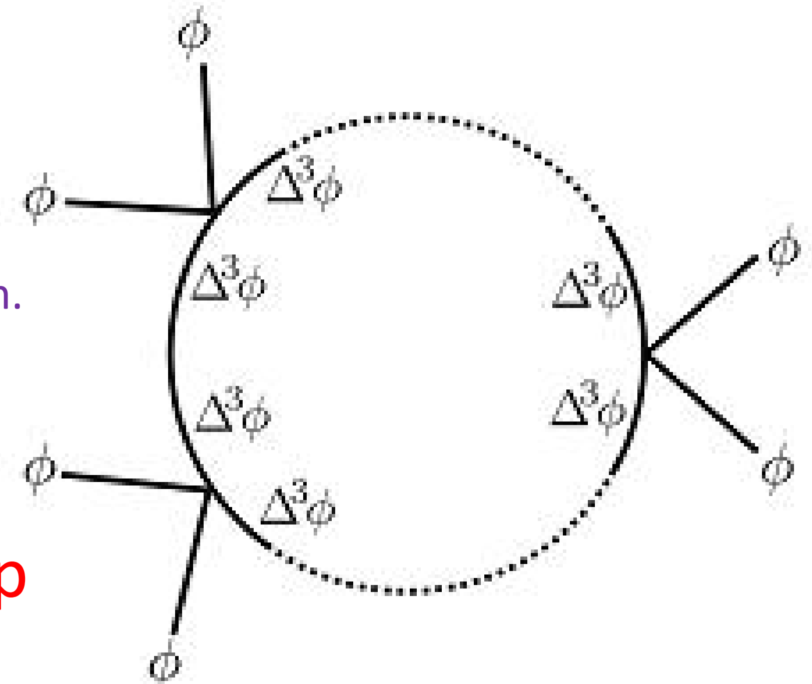
$$\int d\omega d^3k \left(\frac{1}{\omega^2 - p^{10}}\right)^n (p^{12})^n \sim \Lambda^{8+2n}$$



For any n, this diverges.

Infinite number of counter terms are required!

Extended PCR



We are looking the divergence of loop diagram.
Only internal lines are important.

$$S_{int} = \lambda \int dt d^3x \ \phi^2 \ \underline{(\Delta^3\phi)^2}$$

Related to loop

With d=3,z=5, if operator has dimension 8, it is marginal.

However, $(\Delta^3\phi)^2$,which is the part actually related to loop calculation,

is dimension 10.

Therefore, **not renormalizable**

Any portion must have less than 8 dimension.

↑

Inverse of $[dtd^dx]$

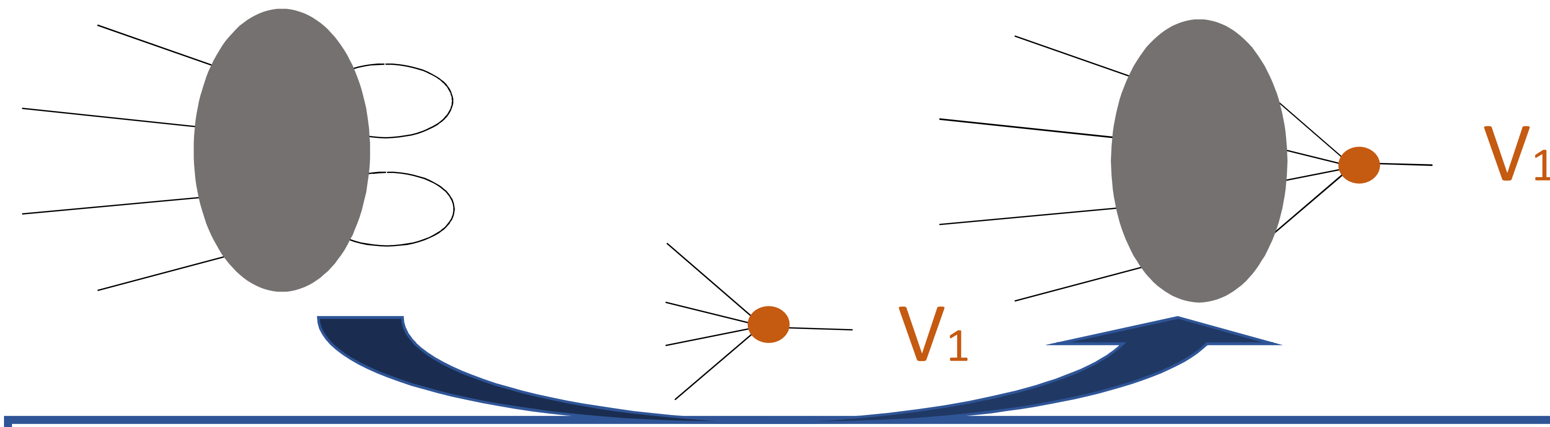
Extended PCR (marginal case)

Extended PCR

Any portion of interaction operator $< (\text{ or } \leq) \; d + z$

Which?

A diverging loop diagram



Add vertex V1. $S_{int} = \lambda \int dt d^3x (\partial_x^{a_1}\phi)\dots(\partial_x^{a_4}\phi)(\partial_x^{b}\phi)$

A portion of V1 operator is marginal.

Change the structure of external line. Need extra counter terms!

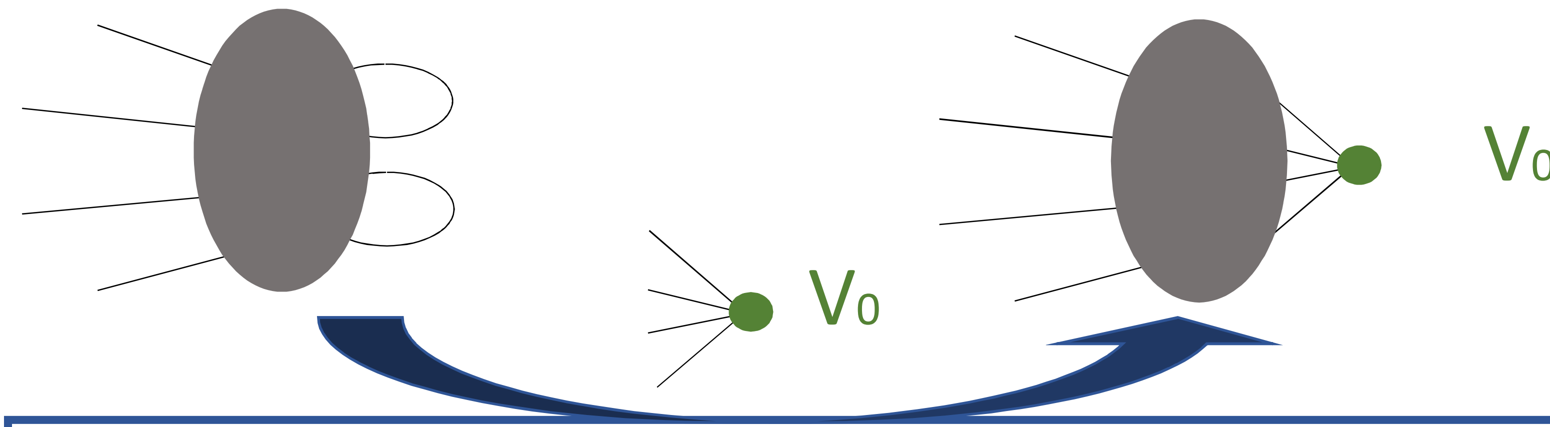
Extended PCR (marginal case)

Extended PCR

Whole of interaction operator $< (\text{ or } \leq) \; d + z$

Which?

A diverging loop diagram



Add quartic vertex V0. $S_{int} = \lambda \int dt d^3x (\partial_x^{a_1}\phi) \ldots (\partial_x^{a_4}\phi)$

Whole part of V0 operator is marginal.

The structure of external line is the same. Don't need extra counter terms!

Extended PCR (marginal case)

Extended PCR

Any portion of interaction operator $< (\text{ or } \leq) \; d + z$

Which?

For whole operator, $\leq$

For a portion of operator, $<$

whole

$$S_{int} = \lambda \int dt d^3x \, (\partial_x^a \phi) \ldots (\partial_x^b \phi) \ldots (\partial_x^c \phi)$$

A portion

Cubic interaction term

$$S_3 = \int dt d^d x \, (\partial_x^{a_1} \phi)(\partial_x^{a_2} \phi)(\partial_x^{a_3} \phi) \qquad a_1 \leq a_2 \leq a_3$$

$$a_1 + a_2 + a_3 \leq (5z - d)/2$$

$$a_2 + a_3 < 2z \;\Rightarrow\; a_2 + a_3 \leq 2z - 1$$

$a_1, \; a_2, \; a_3, \; z, \; d$ are integers

Extended PCR (marginal case)

Extended PCR

Any portion of interaction operator $< (\text{ or } \leq) \; d+z$

Which?

For whole operator, $\leq$

For a portion of operator, $<$

whole

$$S_{int} = \lambda \int dt d^3x \; (\partial_x^a \phi) \ldots (\partial_x^b \phi) \ldots (\partial_x^c \phi)$$

A portion

Quartic interaction term

$$S_4 = \int dt d^d x \, (\partial_x^{a_1} \phi)(\partial_x^{a_2} \phi)(\partial_x^{a_3} \phi)(\partial_x^{a_4} \phi) \qquad a_1 \leq a_2 \leq a_3 \leq a_4$$

$$a_1 + a_2 + a_3 + a_4 \leq 3z - d$$

$$a_2 + a_3 + a_4 < (5z - d)/2 \;\Rightarrow\; a_2 + a_3 + a_4 \leq (5z - d - 1)/2$$

$$a_3 + a_4 < 2z \;\Rightarrow\; a_3 + a_4 \leq 2z - 1$$

$a_1, \; a_2, \; a_3, \; a_4, \; z, \; d$ are integers

Unitarity

-Untarity bound-

Unitarity Bound (general discussion)

Unitarity

$$SS^\dagger = 1$$

$S = 1 + iT$

$$-i(T - T^\dagger) = TT^\dagger$$

Scattering amplitude $\mathcal{M}(i \to f)$

$$\langle f|T|i\rangle = \delta(E_i - E_f)\delta^d(\mathbf{p}_i - \mathbf{p}_f)\mathcal{M}(i \to f)$$

Orthonomal basis $|X\rangle$ $\sum_X |X\rangle\langle X| = 1$

$$-i[\mathcal{M}(i \to f) - \mathcal{M}(f \to i)^*]$$

$$= \sum_X \delta(E - E_X)\delta^d(\mathbf{p} - \mathbf{p}_X)\mathcal{M}(i \to X)\mathcal{M}(f \to X)^*$$

$i = f$

$$2\ \mathrm{Im}\ \mathcal{M}(i \to i) = \sum_X \delta(E - E_X)\delta^d(\mathbf{p} - \mathbf{p}_X)|\mathcal{M}(i \to X)|^2$$

Unitarity bound (perturbative)

Normalized n-particle state $|\mathbf{p}_1 ... \mathbf{p}_n\rangle$

$$\int \prod_{j=1}^{n} \frac{d^d p}{2E_{p_j}} |\mathbf{p}_1 ... \mathbf{p}_n\rangle\langle\mathbf{p}_1 ... \mathbf{p}_n| = 1$$

Normalized state with discrete parameter l

Orthonormal func. on constant E and P $h_l(\mathbf{p}_j)$

$$|E, \mathbf{P}, l\rangle = \int d\Pi_n h_l(\mathbf{p}_j) |\mathbf{p}_1 ... \mathbf{p}_n\rangle$$

$$d\Pi_n := \prod_{j=1}^{n} \frac{d^d p_j}{2E_j} \delta(E_1 + ... + E_n - E)\delta^d(\mathbf{p}_1 + ... + \mathbf{p}_n - \mathbf{P})$$

Scattering amplitude on constant E and P sub-space

$$\mathcal{M}(E, \mathbf{P}; l \to l')$$

$$\langle E, \mathbf{P}, l|T|E', \mathbf{P}', l'\rangle = \delta(E - E')\delta^d(\mathbf{P} - \mathbf{P}')\mathcal{M}(E, \mathbf{P}; l \to l')$$

Unitarity bound (l-basis)

$$\text{Im}\ \mathcal{M}(i \to i) = \sum_X \delta(E - E_X)\delta^d(\mathbf{p} - \mathbf{p}_X)|\mathcal{M}(i \to X)|^2$$

$$\langle E, \mathbf{P}, l|T|E', \mathbf{P}', l'\rangle = \delta(E - E')\delta^d(\mathbf{P} - \mathbf{P}')\mathcal{M}(E, \mathbf{P}; l \to l')$$

$$|\ \text{Im}\ \mathcal{M}(E, \mathbf{P}; l \to l)| = \sum_{l'} |\mathcal{M}(E, \mathbf{P}; l \to l')|^2$$

∥∧ ∨∥

$$|\mathcal{M}(E, \mathbf{P}; l \to l)| \qquad |\mathcal{M}(E, \mathbf{P}; l \to l')|^2$$

$l' = l$

$$|\mathcal{M}(E, \mathbf{P}; l \to l) \geq |\mathcal{M}(E, \mathbf{P}; l \to l)|^2$$

$$|\mathcal{M}(E, \mathbf{P}; l \to l)| \leq const.$$

Unitarity bound (l-basis)

$$\text{Im } \mathcal{M}(i \to i) = \textstyle\sum_X \delta(E - E_X)\delta^d(\mathbf{p} - \mathbf{p}_X)|\mathcal{M}(i \to X)|^2$$

$$\langle E, \mathbf{P}, l|T|E', \mathbf{P}', l'\rangle = \delta(E - E')\delta^d(\mathbf{P} - \mathbf{P}')\mathcal{M}(E, \mathbf{P}; l \to l')$$

$$|\ \text{Im } \mathcal{M}(E, \mathbf{P}; l \to l)| = \textstyle\sum_{l'} |\mathcal{M}(E, \mathbf{P}; l \to l')|^2$$

$\|\wedge$ $\vee\|$

$|\mathcal{M}(E, \mathbf{P}; l \to l)|$ $\qquad |\mathcal{M}(E, \mathbf{P}; l \to l')|^2$

$\|\wedge$

$const.$

$l' = l$

$$|\mathcal{M}(E, \mathbf{P}; l \to l) \geq |\mathcal{M}(E, \mathbf{P}; l \to l)|^2$$

$$|\mathcal{M}(E, \mathbf{P}; l \to l)| \leq const.$$

Unitarity bound (l-basis)

$$\mathrm{Im}\ \mathcal{M}(i\to i)=\sum_X \delta(E-E_X)\delta^d(\mathbf{p}-\mathbf{p}_X)|\mathcal{M}(i\to X)|^2$$

$$\langle E,\mathbf{P},l|T|E',\mathbf{P}',l'\rangle=\delta(E-E')\delta^d(\mathbf{P}-\mathbf{P}')\mathcal{M}(E,\mathbf{P};l\to l')$$

$$|\ \mathrm{Im}\ \mathcal{M}(E,\mathbf{P};l\to l)|=\sum_{l'}|\mathcal{M}(E,\mathbf{P};l\to l')|^2$$

$$\rotatebox{90}{$\geq$} \qquad\qquad\qquad \rotatebox{90}{$\leq$}$$

$$|\mathcal{M}(E,\mathbf{P};l\to l)| \qquad\qquad |\mathcal{M}(E,\mathbf{P};l\to l')|^2$$

$$\rotatebox{90}{$\geq$}$$

$$const.$$

$$|\mathcal{M}(E,\mathbf{P};l\to l')|\leq const.$$

Unitarity Bound

Two-particle Scattering

High energy limit

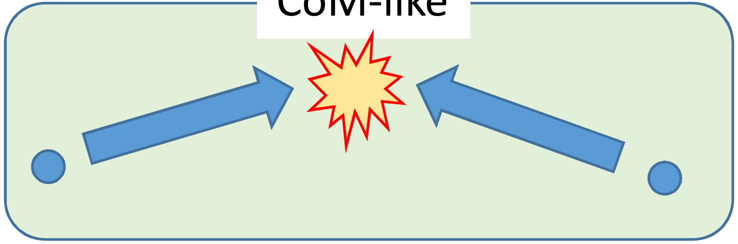
Relativistic theory

we can take CoM frame.
High energy limit => (E -> ∞,   P=0)

Lifshitz scaling theory

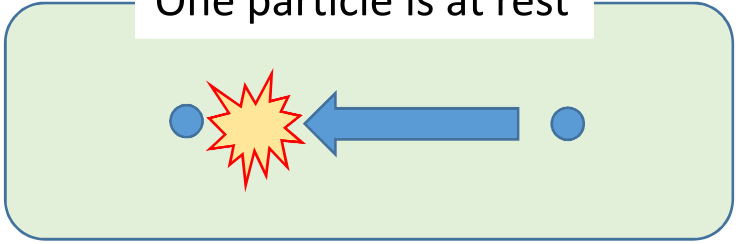
No Lorentz sym.
High energy limit with non-zero P (which can diverge) gives different constraint.

CoM-like



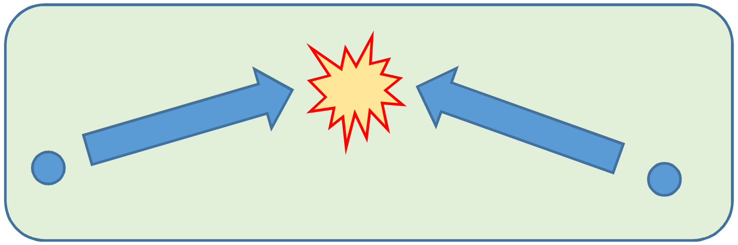
$|\alpha\rangle$

One particle is at rest



$|\beta\rangle$

CoM-like state $|\alpha\rangle$



Orthonormal func. on constant E and P $h_l(\mathbf{p}_j)$

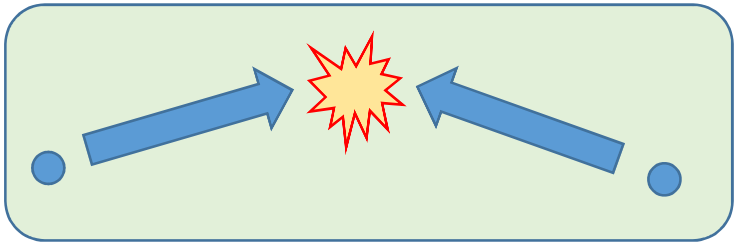
$$|\alpha\rangle = \int d\Pi \ \ h_\alpha(\mathbf{p}_j)|\mathbf{p_1}, \mathbf{p_2}\rangle$$

$$d\Pi := \frac{d^d p_1}{2E_1}\frac{d^d p_2}{2E_2}\delta(E_1 + E_2 - E)\delta^d(\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{P})$$

$$h_\alpha(p_j) = \frac{1}{\sqrt{N_\alpha(P)}} \times \begin{cases} 1 & (|\mathbf{p}_1| - |\mathbf{p}_2| \leq P/2) \\ 0 & (|\mathbf{p}_1| - |\mathbf{p}_2| > P/2) \end{cases}$$

$$N_\alpha = \int_{I_\alpha} \frac{d^d p_1}{2E_1}\frac{d^d p_2}{2E_2}\delta(E_1 + E_2 - E)\delta^d(\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{P})$$

CoM-like state $|\alpha\rangle$



Orthonormal func. on constant E and P $h_l(\mathbf{p}_j)$

$$|\alpha\rangle = \int d\Pi \;\; h_\alpha(\mathbf{p}_j)|\mathbf{p_1},\mathbf{p_2}\rangle$$

$$d\Pi := \frac{d^dp_1}{2E_1}\frac{d^dp_2}{2E_2}\delta(E_1+E_2-E)\delta^d(\mathbf{p}_1+\mathbf{p}_2-\mathbf{P})$$

$$h_\alpha(p_j) = \frac{1}{\sqrt{N_\alpha(P)}} \times \begin{cases} 1 & (|\mathbf{p}_1|-|\mathbf{p}_2| \le P/2) \\ 0 & (|\mathbf{p}_1|-|\mathbf{p}_2| > P/2) \end{cases}$$

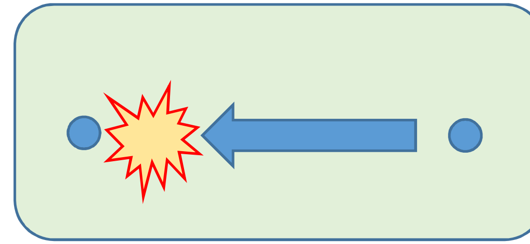
$$N_\alpha = \int_{I_\alpha} \frac{d^dp_1}{2E_1}\frac{d^dp_2}{2E_2}\delta(E_1+E_2-E)\delta^d(\mathbf{p}_1+\mathbf{p}_2-\mathbf{P})$$

P^d P^d P^{-z} P^{-d}

P^{-z} P^{-z}

$$\propto P^{d-3z}$$

One particle is at rest $|\beta\rangle$



$$h_\beta(p_j) = \frac{1}{\sqrt{N_\beta(P)}} \times \begin{cases} 1 & (|\mathbf{p}_1| \le \epsilon) \\ 0 & (|\mathbf{p}_1| > \epsilon) \end{cases}$$

$$p_1 = \mathcal{O}(1)$$
$$p_2 = \mathcal{O}(P)$$

P^0 P^d P^{-d}

$$N_\beta = \int_{I_\beta} \frac{d^d p_1}{2E_1} \frac{d^d p_2}{2E_2} \delta(E_1 + E_2 - E)\, \delta^d(\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{P})$$

P^{-z}

$$E_2 \simeq E \simeq P^z \;\Rightarrow\; E_1 + E_2 - E = \mathcal{O}(P^{z-1})$$

$$N_\beta \propto P^{-2z+1}$$

$$\mathcal{M}(\alpha \to \alpha)$$

Unitarity bound

$$const. \geq |\mathcal{M}(E, \mathbf{P}; \alpha \to \alpha)|$$

$$= |\int d\Pi(p) d\Pi(k) h_\alpha(p) h_\alpha(k) \mathcal{M}(\mathbf{p}_1, \mathbf{p}_2 \to \mathbf{k}_1, \mathbf{k}_2)|$$

$$\frac{1}{\sqrt{N_\alpha}}$$

$$d\Pi(p) := \frac{d^d p_1}{2E_1} \frac{d^d p_2}{2E_2} \delta(E_1 + E_2 - E) \delta^d(\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{P})$$

$$\sim N_\alpha \propto P^{d-3z}$$

$$\sim N_\alpha P^a \propto P^{d-3z+a}$$

Suppose $\mathcal{M}(\mathbf{p}_1, \mathbf{p}_2 \to \mathbf{k}_1, \mathbf{k}_2)| = P^a$

$$a \leq 3z - d$$

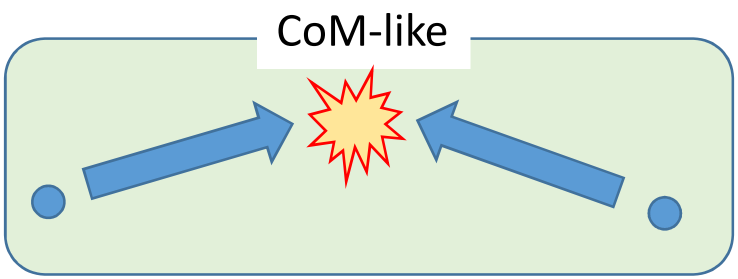
UB for scattering amplitude

$$\mathcal{M}(\mathbf{p}_1, \mathbf{p}_2 \to \mathbf{k}_1, \mathbf{k}_2)| = P^a$$

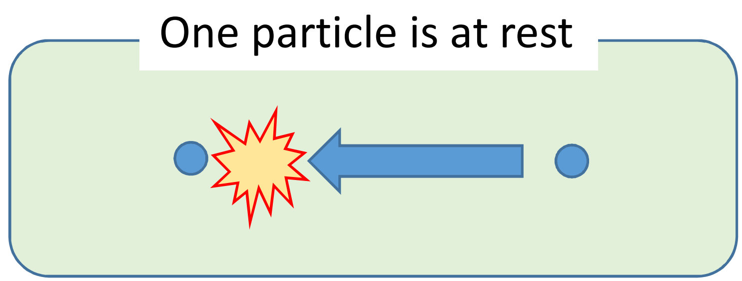
$$\mathcal{M}(\alpha \to \alpha) \qquad a \leq 3z - d$$

$$\mathcal{M}(\beta \to \beta) \qquad a \leq 2z - 1$$

$$\mathcal{M}(\alpha \to \beta) \qquad a \leq (5z - d - 1)/2$$



$$|\alpha\rangle$$



$$|\beta\rangle$$

Quartic interaction

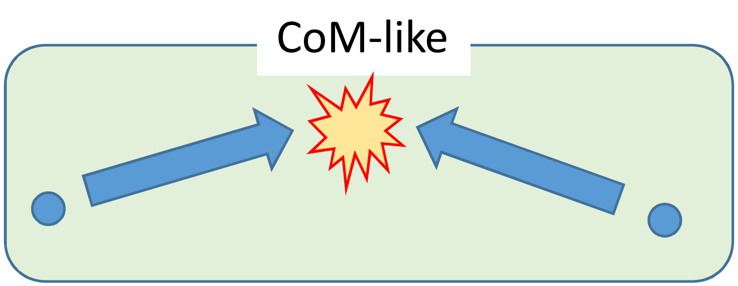
$$S_4 = \int dt d^d x \, (\partial_x^{a_1}\phi)(\partial_x^{a_2}\phi)(\partial_x^{a_3}\phi)(\partial_x^{a_4}\phi) \qquad a_1 \le a_2 \le a_3 \le a_4$$

$$\mathcal{M}(\mathbf{p}_1, \mathbf{p}_2 \to \mathbf{k}_1, \mathbf{k}_2)| = p_1^{a_1} p_2^{a_2} k_1^{a_3} k_4^{a_4} + [\text{ perm. }]$$

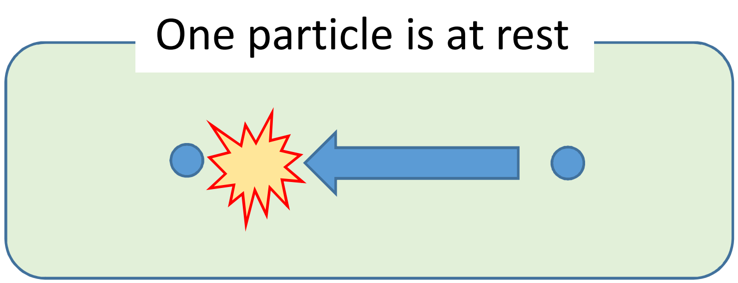
$$(\alpha \to \alpha) \to P^{a_1+a_2+a_3+a_4} \qquad a_1 + a_2 + a_3 + a_4 \le 3z - d$$

$$(\beta \to \beta) \to P^{a_3+a_4} \qquad a_3 + a_4 \le 2z - 1$$

$$(\alpha \to \beta) \to P^{a_2+a_3+a_4} \qquad a_2 + a_3 + a_4 \le (5z - d - 1)/2$$



$|\alpha\rangle$



$|\beta\rangle$

Extended PCR (marginal case)

Extended PCR

Any portion of interaction operator $< \; (\;\; \text{or} \;\; \leq) \; d+z$

Which?

For whole operator, $\leq$

For a portion of operator, $<$

whole

$$S_{int} = \lambda \int dt d^3x \; (\partial_x^a \phi) \ldots (\partial_x^b \phi) \ldots (\partial_x^c \phi)$$

A portion

Quartic interaction term

$$S_4 = \int dt d^d x \, (\partial_x^{a_1}\phi)(\partial_x^{a_2}\phi)(\partial_x^{a_3}\phi)(\partial_x^{a_4}\phi) \qquad a_1 \leq a_2 \leq a_3 \leq a_4$$

$$a_1 + a_2 + a_3 + a_4 \leq 3z - d$$

$$a_2 + a_3 + a_4 < (5z - d)/2 \;\Rightarrow\; a_2 + a_3 + a_4 \leq (5z - d - 1)/2$$

$$a_3 + a_4 < 2z \;\Rightarrow\; a_3 + a_4 \leq 2z - 1$$

$a_1, \, a_2, \, a_3, \, a_4, \, z, \, d$ are integers

Extended PCR (marginal case)

Extended PCR

Unitarity bound

$$(\alpha \to \alpha) \to P^{a_1+a_2+a_3+a_4} \qquad a_1 + a_2 + a_3 + a_4 \leq 3z - d$$

$$(\beta \to \beta) \to P^{a_3+a_4} \qquad a_3 + a_4 \leq 2z - 1$$

$$(\alpha \to \beta) \to P^{a_2+a_3+a_4} \qquad a_2 + a_3 + a_4 \leq (5z - d - 1)/2$$

$$a_1 + a_2 + a_3 + a_4 \leq 3z - d$$

$$a_2 + a_3 + a_4 < (5z - d)/2 \;\Rightarrow\; a_2 + a_3 + a_4 \leq (5z - d - 1)/2$$

$$a_3 + a_4 < 2z \;\Rightarrow\; a_3 + a_4 \leq 2z - 1$$

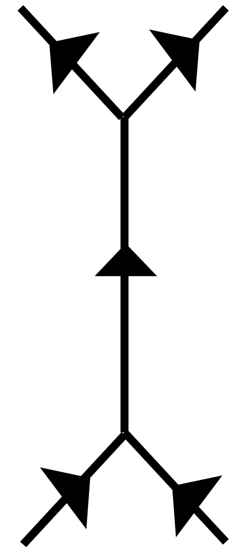
$a_1,\ a_2,\ a_3,\ a_4,\ z,\ d$ are integers

Cubic interaction

$$S_3 = \int dt d^d x \, (\partial_x^{a_1}\phi)(\partial_x^{a_2}\phi)(\partial_x^{a_3}\phi) \qquad a_1 \le a_2 \le a_3$$

$$\mathcal{M}(\mathbf{p_1}, \mathbf{p}_2 \to \mathbf{k}_1, \mathbf{k_2}) \propto \frac{V(\mathbf{p}_1, \mathbf{p}_2, \mathbf{P})V(\mathbf{k_1}, \mathbf{k_2}, \mathbf{P})}{E^2 - P^{2z}}$$

$$\sim (p_1^{a_1} p_2^{a_2} P^{a_3} k_1^{a_1} k_2^{a_2} P^{a_3} + \text{ perm. })/P^{2z}$$



S-channel

$$(\alpha \to \alpha) \to P^{2(a_1+a_2+a_3)-2z} \qquad a_1 + a_2 + a_3 \le (5z-d)/2$$

$$(\beta \to \beta) \to P^{2(a_2+a_3)-2z} \qquad a_2 + a_3 \le 2z - 1/2$$

Suppose $a_i, \ z$ are integers

$$a_2 + a_3 \le 2z - 1$$

$$(\alpha \to \beta) \to P^{a_1+2(a_2+a_3)-2z}$$

$$a_1 + 2a_2 + 2a_3 \le (9z - d - 1)/2$$

No additional condition

Extended PCR (marginal case)

Extended PCR

Any portion of interaction operator $< \; (\; \text{or} \; \leq) \; d + z$

Which?

For whole operator, $\leq$

For a portion of operator, $<$

$$S_{int} = \lambda \int dt d^3x \, \underbrace{(\partial_x^a \phi) \ldots \overbrace{(\partial_x^b \phi) \ldots (\partial_x^c \phi)}}$$

whole

A portion

Cubic interaction term

$$S_3 = \int dt d^d x \, (\partial_x^{a_1} \phi)(\partial_x^{a_2} \phi)(\partial_x^{a_3} \phi) \qquad a_1 \leq a_2 \leq a_3$$

$$a_1 + a_2 + a_3 \leq (5z - d)/2$$

$$a_2 + a_3 < 2z \;\Rightarrow\; a_2 + a_3 \leq 2z - 1$$

$a_1, \; a_2, \; a_3, \; z, \; d$ are integers

Extended PCR (marginal case)

Unitarity bound

$$(\alpha \to \alpha) \to P^{2(a_1+a_2+a_3)-2z}$$

$$a_1 + a_2 + a_3 \leq (5z - d)/2$$

$$(\beta \to \beta) \to P^{2(a_2+a_3)-2z}$$

$$a_2 + a_3 \leq 2z - 1/2$$

Suppose $a_i, \ z$ are integers

$$a_2 + a_3 \leq 2z - 1$$

$$(\alpha \to \beta) \to P^{a_1+2(a_2+a_3)-2z}$$

$$a_1 + 2a_2 + 2a_3 \leq (9z - d - 1)/2$$

No additional condition

$$a_1 + a_2 + a_3 \leq (5z - d)/2$$

$$a_2 + a_3 < 2z \Rightarrow a_2 + a_3 \leq 2z - 1$$

$a_1, \ a_2, \ a_3, \ z, \ d$ are integers

Summary

Renormalizability

The conventional Power Counting Renormalization (PCR) is not enough.
For renormalization (finite number of counter terms),
we need extended version of PCR.

See the coincidence of these conditions

Unitarity

Only with unitarity, optical theorem can be derived.
Optical theorem gives an inequality for scattering amplitude called “Unitary bound”.
Derive the condition where Unitary bound of tree diagram holds in UV limit