Hamiltonian Analysis of Nonprojectable Horava-Lifshitz gravity with $U(1)$ Symmetry

Yota Watanabe（Kavli IPMU, Tokyo）

Collaboration with

Shinji Mukohyama（YITP, K IPMU）

Ryo Namba（K IPMU）

Rio Saitou（ICRR, Tokyo）

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Horava Gravity(HG)

- Limit of description by GR+SM

  Dark Matter,   not UV complete

- Candidate for UV complete gravity : higher-∂ $\rightarrow$ propagator well converge

  $R^2_{\mu\nu\rho\sigma}$ $\rightarrow$ higher-∂_tbreak positivity of Hamiltonian

  $K^2_{ij} + R^z_{ijkl}$ $(\omega \sim k^z)$@UV $\rightarrow$ break Lorentz inv. @UV （HG）

- Power-counting renormalizability

  $\mathrm{z} \geq d$（spatial dimension）

  $[\mathrm{int.\,term}] \leq 2z$

  $x_i \rightarrow r x_i, t \rightarrow r^z t$: scaling dimention $[\partial_i] = 1, [\partial_t] = z$

“$N = N(t)$” $U(1)$HG

- HG：3 dof, scalar strong coupling@IR, cosmological pert. break down
- To elilminate scalar, impose new symmetry： Horava, Melby-Thompson(2010)

  lapse： $N = N(t) \to N(t) - A(t,x)/c^2$

  NLO of $1/c$ exp.（spatial scale<<time scale）

  t-diffeo： $\delta(ct) = cf(t) \to cf(t) - \frac{\alpha(t,x)}{cN}$

  $\to \delta_\alpha N_i = N\partial_i\alpha,\ \delta_\alpha A = N\partial_\perp\alpha,\ \delta_\alpha \nu = \alpha,\ \delta_\alpha\gamma_{ij} = 0,\ \delta_\alpha N = 0$：“$U(1)$”

  A ：transform as time component of 1-form, ν：auxiliary field
- “$N = N(t)$” $U(1)$HG: 2 dof Kluson(2011)
- $N = N(t)$ unnecessary with $a_i = \partial_i \ln N$ Blas,Pujolas,Sibiryakov(2010)

“$N = N(t,x)$” $U(1)$HG

$$S = \frac{M_{\rm Pl}^2}{2}\int dt d^d x \sqrt{\gamma} N\left[\widetilde{K}^{ij}\widetilde{K}_{ij} - \lambda\widetilde{K}^2 - V + \left(2\Omega - R + \eta_1 a_i a^i + \eta_2 D_i a^i\right)\sigma\right]$$

$\widetilde{K}_{ij}$: Extrinsic curvature with $U(1)$ inv. shift $\widetilde{N}^i = N^i - N D^i \nu$

$V = V\left(R_{ijkl}, D_i, a_i = \partial_i \ln N\right)$, upto ∂_i^{2z}

$\sigma = \frac{A}{N} - \partial_\perp \nu - \frac{1}{2} D^i \nu D_i \nu$: $U(1)$ inv. "gauge field" A $\omega^2 \propto k^6 (d=3)$@UV

- Dof of cosmo. linear pert. is 3 with $\eta_2 \neq 0$, $^\exists$strongly-coupled scalar
  Unnecessary to eliminate ∵HG connetcts GR@IR($\lambda \to 1$) by nonlinear

Wang, Wu(2011) Izumi, Mukohyama(2011) Gumrukcuoglu, Mukohyama, Wang(2012)

- Linear analysis: dof 2 w/ $\eta_2 = 0$ $\to$ 3@nonlinear？

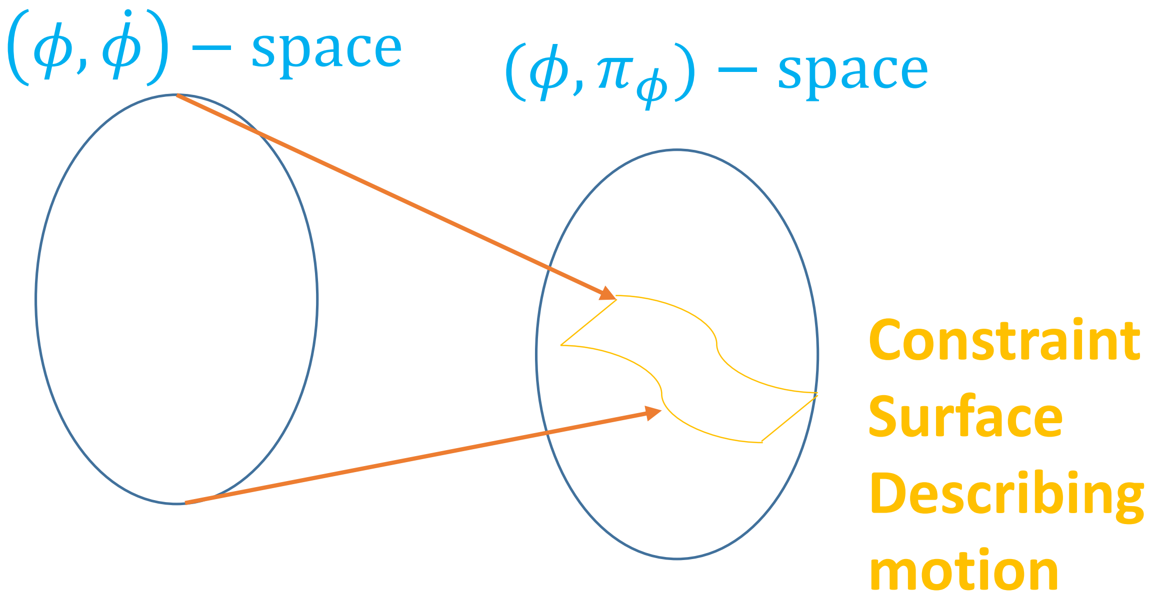
Dof-counting by Hamiltonian analysis

- Difficult to grasp constraints by EOM of unperturbed action

  → search constraints in Hamiltonian formalism（phase space）

- $\pi_i \equiv \frac{\delta S}{\delta \dot{N}^i} = 0 \rightarrow \dot{N}^i \neq \dot{N}^i(\pi_i)$
- $\pi_N \equiv \frac{\delta S}{\delta \dot{N}} = 0$
- $\pi_A \equiv \frac{\delta S}{\delta \dot{A}} = 0$
- $\pi_\nu \equiv \frac{\delta S}{\delta \dot{\nu}} = -J_A$

$$J_A \equiv \frac{M_{\rm Pl}^2}{2}\sqrt{\gamma}\left(2\Omega - R + \eta_1 a_i a^i + \eta_2 D_i a^i\right)$$



$$\phi_A \propto \frac{\lambda-1}{d\lambda-1} D^2\left(\frac{\pi}{\sqrt{\gamma}}N\right) + R^{ij}\mathcal{g}_{ijkl}\frac{\pi^{kl}}{\sqrt{\gamma}}N - D_i D_j\left(\frac{\pi^{ij}}{\sqrt{\gamma}}N\right) \quad \mathcal{g}_{ijkl} = \frac{1}{2}(\gamma_{ik}\gamma_{jl}+\gamma_{il}\gamma_{jk}) - \frac{\lambda}{d\lambda-1}\gamma_{ij}\gamma_{kl}$$

Secondary Constr.

$$\mathcal{H}_\perp = \frac{2}{\sqrt{\gamma}}\pi^{ij}\mathcal{g}_{ijkl}\pi^{kl} + \frac{\delta}{\delta N}\int\sqrt{\gamma}NV \qquad \mathcal{H}_i^N = -2\sqrt{\gamma}D_j\left(\frac{\pi_i^j}{\sqrt{\gamma}}\right) + \pi_N D_i N$$

- $\dot{C} \simeq \{C, H'\}_{\text{Poisson}}, \quad H' = H + \int (\text{Lagrange multiplier } \lambda^\alpha)(\text{constraint}_\alpha)$
- Time-consistency of constraints

  → New constr. *or* Known constr. *or* Eq. determining λ^α

  $$0 = \dot{\pi}_N \supset \left\{\pi_N, \int \lambda^1(\pi_\nu + J_A)\right\}_{\text{Poisson}} \propto \sqrt{\gamma}\left(\eta_2 D^2 + 2\eta_1 a^i D_i\right)\frac{\lambda^1}{N}$$

  $(\eta_1, \eta_2) \neq (0,0) \ \rightarrow \ J_A, \mathcal{H}_i^N$

  $(\eta_1, \eta_2) = (0,0) \ \rightarrow \ J_A, \mathcal{H}_i^N, \mathcal{H}_\perp \rightarrow \phi_A$

  Recall branches are $\eta_2 \neq (=) 0$ @ linear level
- First-class constr. not determine λ^α → gauge-fix. cond.

  How to show Poisson brackets of $V = V\left(R_{ijkl}, D_i, a_i = \partial_i \ln N\right) \not\approx 0$?

$\delta V/\delta g_{ij}$

- Variation of $F = F[g_{ij}, \pi^{ij}, s, V^i]$ under spacial diffeo. $x^i \to x^i + \xi^i(x)$

$$\delta F = \int \mathrm{d}^d x \left[\left(\frac{\delta F}{\delta g_{ij}}\right)_{\pi/\sqrt{g}} \delta g_{ij} + \frac{\delta F}{\delta(\pi^{ij}/\sqrt{g})}\delta\left(\frac{\pi^{ij}}{\sqrt{g}}\right) + \frac{\delta F}{\delta s}\delta s + \frac{\delta F}{\delta V^i}\delta V^i \right]$$

$$= \int \mathrm{d}^d x \xi^i \left\{ -2 g_{ik}\sqrt{g} D_j \left[\frac{1}{\sqrt{g}}\left(\frac{\delta F}{\delta g_{kj}}\right)_{\pi/\sqrt{g}}\right] + \frac{\delta F}{\delta(\pi^{jk}/\sqrt{g})} D_i \left(\frac{\pi^{jk}}{\sqrt{g}}\right) \right.$$

$$\left. + 2\sqrt{g} D_j \left(\frac{1}{\sqrt{g}}\frac{\delta F}{\delta(\pi^{il}/\sqrt{g})}\frac{\pi^{jl}}{\sqrt{g}}\right) + \frac{\delta F}{\delta s} D_i s + \frac{\delta F}{\delta V^j} D_i V^j + \sqrt{g} D_j \left(\frac{1}{\sqrt{g}}\frac{\delta F}{\delta V^i} V^j\right) \right\}$$

For $\delta F = 0$ with $\forall \xi^i$, we can convert $\delta F/\delta g_{ij}$ into other variations

$\delta V/\delta N$

$$\mathcal{g}_{ijkl}=\frac{1}{2}(\gamma_{ik}\gamma_{jl}+\gamma_{il}\gamma_{jk})-\frac{\lambda}{d\lambda-1}\gamma_{ij}\gamma_{kl}$$

- Without explicit calc., it is sufficient to show

$$\det\{\textstyle\int C_\alpha\varphi,\int C_\beta\chi\}\neq 0\,,\qquad \varphi,\chi:\text{scalar}$$

$$\because\ 0=\dot{C}_\alpha=\int\{C_\alpha,C_\beta\}\lambda^\beta+\cdots\qquad\rightarrow\qquad \lambda^\beta=\int\{C_\alpha,C_\beta\}^{-1}\cdots$$

- $\det\{\int C_\alpha\varphi,\int C_\beta\chi\}=\det\begin{pmatrix}0 & B\\ C & D\end{pmatrix}=(\det B)(\det B(\varphi\leftrightarrow\chi))$

$$\det\begin{bmatrix}\int d^dx\chi\frac{\delta}{\delta N}\int d^dy\varphi\frac{\delta}{\delta N}\int d^dz\sqrt{\gamma}NV & -\int d^dx\chi\left(\frac{\lambda-1}{d\lambda-1}\pi D^2+R^{ij}\mathcal{g}_{ijkl}\pi^{kl}-\pi^{ij}D_iD_j\right)\varphi\\ \int d^dx\chi\left(\frac{\lambda-1}{d\lambda-1}\pi D^2+R^{ij}\mathcal{g}_{ijkl}\pi^{kl}-\pi^{ij}D_iD_j\right)\varphi & \int d^dx\sqrt{\gamma}N[R^{ij}\chi+(\gamma^{ij}D^2-D^iD^j)\chi]\mathcal{g}_{ijkl}[R^{kl}\varphi+(\gamma^{kl}D^2-D^kD^l)\varphi]\end{bmatrix}$$

- $V\big(R_{ijkl},D_i,a_i=\partial_i\ln N\big)$: linear comb. of $a_ia^i, RD_ia^i, D^2a_iD^2a^i$
- π^{ij} terms remain nonzero $\rightarrow\det\not\approx 0$

Result

- dim. of phase space : $d^2 + 3d + 6$ $(\gamma_{ij}, \pi^{ij}, N^i, \pi_i, N, \pi_N, A, \pi_A, \nu, \pi_\nu)$

  1^{st} -class constr. : $2d + 2$ $(\mathcal{H}_i^N, \pi_i, \pi_A, \pi_\nu)$

  2^{nd}-class constr : $(\eta_1, \eta_2) \neq (0,0)$, 2 (π_N, J_A),

  $(\eta_1, \eta_2) = (0,0) + 2(\mathcal{H}_\perp, \phi_A)$
- dof $= [(\text{dim. of phase space}) - 2(1^{\mathrm{st}} - \text{class}) - (2^{\mathrm{nd}} - \text{class})]/2$

  $(\eta_1, \eta_2) \neq (0,0)$ dof$=(d^2 - d - 2)/2 + 1 \rightarrow$ $^{\exists}$scalar

  $(\eta_1, \eta_2) = (0,0)$ dof$=(d^2 - d - 2)/2$ transverse traceless tensor
- Linear analysis is insensitive to η_1 because $\eta_1 a_i a^i \sigma$ not affect quadratic action

Summary

- Scalar dof of “$N = N(t,x)$”$U(1)$HG is strong coupling

  $\rightarrow$ non-linear analysis is needed

- Performed Hamiltonian analysis to study constr. structure and dof

- Conditions for existence of scalar dof

	this analysis	linear analysis
scalar exist	$(\eta_1, \eta_2) \neq (0,0)$	$\eta_2 \neq 0$
scalar not exist	$(\eta_1, \eta_2) = (0,0)$	$\eta_2 = 0$

$$S \supset (\eta_1 a_i a^i + \eta_2 D_i a^i)\sigma$$