

Testing dark energy and modified gravity models with EFTCAMB: The Hořava gravity case

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ERC-NIRG project no.307934

Based on

NF, M. Raveri, D. Vernieri, B. Hu, A. Silvestri, (2015) arXiv:1508.01787,
B. Hu, M. Raveri, A. Silvestri, NF, PRD 91 (2015) 063524 [arXiv:1410.5807]
M. Raveri, B. Hu, NF, A. Silvestri, PRD 90 (2014) 043513 [arXiv:1405.1022]
B. Hu, M. Raveri, NF, A. Silvestri, PRD 89 (2014) 103530 [arXiv:1312.5742]

EFTCAMB webpage : <http://wwwhome.lorentz.leidenuniv.nl/hu/codes/>

“2nd mini-workshop on gravity and cosmology”
5-7 October 2015, IAP Paris



Outline

- *Motivation;*
 - *Dark Energy and Modify gravity: why and how*
- *Effective Field Theory for cosmic acceleration: an overview*
- *EFTCAMB/EFTCosmoMC;*
 - *Structure,*
 - *stability check.*
- *Testing gravity models:*
 - *an example: the Hořava gravity case.*

State of the art of Modern Cosmology

- Two Unknown components:
 - Dark Energy: SNIa, CMB, BAO, Galaxy Cluster counts (68.3%)
↓
fluid component, accounts for the recent accelerated expansion
 - Dark Matter: flat RCs, BBN, CMB, Lensing, LSS (26.8%)
↓
Only Gravitational interaction, Non-Baryonic, accounts for the missing matter
- Homogeneous & Isotropic Universe;
- Spatially Flat $\Omega_k \sim 0$;

↓
Dark Universe $\sim 95\%$

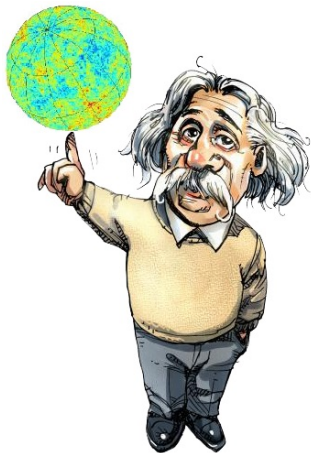
Best working model:

Λ CDM \rightarrow GR + FLRW + Λ +CDM

$\Lambda \rightarrow$ extra fluid : $w_\Lambda \equiv \frac{p_\Lambda}{\rho_\Lambda} = -1$

Why do we need to Modify Gravity?

- Inflation: Fine tuning problems in the early Universe;
 $\ddot{a} > 0 \rightarrow \phi$, scalar field.
- Late time accelerated expansion:
 - Dark Energy as an extra fluid:
 $w_{DE} \equiv p_{DE}/\rho_{DE} < -1/3$,
 - Geometrical modification of the gravitational sector.
- Quantum Gravity;
- Dark Matter issues: Observations vs N-body simulations;
 - Baryons in the N-body simulations,
 - CDM vs WDM,
 - MOND: Modification of the Newtonian dynamics at galactic scale.



Modifying General Relativity

How to modify GR:

- extra DoF(s): scalar, vector, tensor field(s);
- going beyond the 2nd order differential equations;
- diffeomorphism invariance breaking;
- higher than 4 dimensions;

Solar system constraints

- screening mechanisms
(Chameleon, Symmetron, k-mouflage, Vainshtein)

In the following we will focus on theories with

- an extra scalar and dynamical DoF;
- higher order field equations (in spatial derivatives);
- break diffeomorphism invariance;
- 4 dimensions.

Test gravity on cosmological scale

- Pletora of Dark Energy & Modified Gravity models
 - cosmological constant, quintessence, k-essence...
 - $f(R)$, Brans-Dicke, Galileon, GLPV, Hořava gravity....
- Model independent parametrizations to test gravity on cosmological scale, to name (among others) the most recent
 - Growth functions: μ and γ ,
[Silvestri *et al.* PRD 87, 104015 (2013)]
 - Parametrized Post Friedmann framework,
[Baker *et al.*, PRD 87, 024015 (2013)]
 - Effective Field Theory of Cosmic Acceleration,
[Gubitosi *et al.* JCAP 1302 (2013) 032
Bloomfield *et al.* JCAP 1308 (2013) 010]
 - Horndeski (and beyond) parametrization,
[Bellini & Sawicki, JCAP 1407 (2014) 050
Gleyzes *et al.* JCAP 1502 (2015) 018]

Effective Field Theory Action

- Operators are time-dependent spatial diffeomorphisms invariants;
- Unitary gauge: the extra scalar d.o.f. does not appear directly in the action, i.e. scalar field perturbations are vanishing;
- Jordan frame: directly related to observations;
- $S_m[\chi_i, g_{\mu\nu}]$: Validity of the Weak Equivalence Principle.

The action:

$$\mathcal{S}_{EFT} = \int d^4x \sqrt{-g} \left\{ \frac{m_0^2}{2} (1 + \Omega(t)) R + \Lambda(t) - c(t) \delta g^{00} + \frac{M_2^4(t)}{2} (\delta g^{00})^2 \right. \\ \left. - \frac{\bar{M}_1^3(t)}{2} \delta g^{00} \delta K - \frac{\bar{M}_3^2(t)}{2} \delta K^\mu{}_\nu \delta K^\nu{}_\mu + \frac{\hat{M}^2(t)}{2} \delta g^{00} \delta R^{(3)} \right. \\ \left. - \frac{\bar{M}_2^2(t)}{2} (\delta K)^2 + m_2^2(t) (g^{\mu\nu} + n^\mu n^\nu) \partial_\mu g^{00} \partial_\nu g^{00} + \dots \right\} + S_m[\chi_i, g_{\mu\nu}]$$

where e.g. $\delta A = A - A^{(0)}$

Stückelberg Field & the extra dynamical scalar DoF

Stückelberg technique: restoring the time diffeomorphism invariance by an infinitesimal time coordinate transformation

$$t \rightarrow t + \pi(x^\mu).$$

Making manifest the extra scalar DoF will modify all the EFT functions which are typically Taylor expanded in π according to

$$f(t) \rightarrow f(t + \pi(x^\mu)) = f(t) + \dot{f}(t)\pi + \frac{\ddot{f}(t)}{2}\pi^2 + \dots$$

Operators that are not fully diffeomorphism invariant transform according to the tensor transformation law, e.g.

$$\begin{aligned} g^{00} &\rightarrow \frac{\partial(t + \pi(x^\mu))}{\partial x^\mu} \frac{\partial(t + \pi(x^\mu))}{\partial x^\nu} g^{\mu\nu} \\ &= g^{00} - 2\dot{\pi} + 2\dot{\pi}\delta g^{00} - \dot{\pi}^2 - \frac{(\bar{\nabla}\pi)^2}{a^2} + \dots \end{aligned}$$

Action with the extra scalar DoF

The EFT action in conformal time with the π field manifest through the Stückelberg trick, up to second order operators, for $\{\Omega, \Lambda, c\}$, reads

$$\begin{aligned} S = \int d^4x \sqrt{-g} & \left\{ \frac{m_0^2}{2} [1 + \Omega(\tau + \pi)] R + \Lambda(\tau + \pi) \right. \\ & - c(\tau + \pi) a^2 \left[\delta g^{00} - 2 \frac{\dot{\pi}}{a^2} + 2\mathcal{H}\pi \left(\delta g^{00} - \frac{1}{a^2} - 2 \frac{\dot{\pi}}{a^2} \right) \right. \\ & \left. \left. + 2\dot{\pi} \delta g^{00} + 2g^{0i} \partial_i \pi - \frac{\dot{\pi}^2}{a^2} + g^{ij} \partial_i \pi \partial_j \pi - \left(2\mathcal{H}^2 + \dot{\mathcal{H}} \right) \frac{\pi^2}{a^2} + \dots \right] \right. \\ & \left. + \dots \right\} + S_m[g_{\mu\nu}], \end{aligned}$$

Deviations from GR are enclosed in the π -field equations:

$$A(\tau, k) \ddot{\pi} + B(\tau, k) \dot{\pi} + C(\tau) \pi + k^2 D(\tau, k) \pi + E(\tau, k) = 0$$

Advantages & Limitations

- Model independent framework to address the acceleration issue;
- Parametrization of DE/MG theories with a single extra scalar DoF ;
- The EFT functions are all **unknown** functions of time;
- Precise *mapping* between EFT functions and most of the single scalar field DE/MG models, e.g.

$$\int d^4x \frac{m_0^2}{2} (R + f(R)) \rightarrow \int d^4x \frac{m_0^2}{2} [(1 + f'_0) R + f_0 - R_0 f'_0]$$

then

$$\Omega(t) = f'_0, \quad \Lambda(t) = \frac{m_0^2}{2} f_0 - R_0 f'_0, \quad c(t) = 0$$

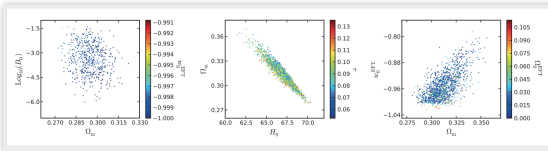
- Low energy description of cosmological phenomena;
- Only single scalar field \rightarrow No vector or tensor fields;
- Action does not describe higher-dimensional theories.

EFTCAMB website:

Webpage: <http://www.lorentz.leidenuniv.nl/hu/codes/>

Effective Field Theory with CAMB

By B. Hu, M. Raveri, N. Frusciante and A. Silvestri



EFTCAMB is a patch of the public Einstein-Boltzmann solver CAMB, which implements the Effective Field Theory approach to cosmic acceleration. The code can be used to investigate the effect of different EFT operators on linear perturbations as well as to study perturbations in any specific DE/MG model that can be cast into EFT framework. To interface EFTCAMB with cosmological data sets, we equipped it with a modified version of CosmoMC, namely EFTCosmoMC, creating a bridge between the EFT parametrization of the dynamics of perturbations and observations.

B. Hu, M. Raveri, N. Frusciante, A. Silvestri, PRD **89** (2014) 103530,
M. Raveri, B. Hu, N. Frusciante, A. Silvestri, PRD **90** (2014) 043513

EFTCAMB & EFTCosmoMC

- Patches of CAMB/CosmoMC;
- EFTCAMB evolves the full perturbative equations without relying on any quasi-static approximation;
- EFTCAMB evolves the tensor perturbative equations;
- EFTCAMB is compatible with massive neutrinos;
- Built-in models: designer- $f(R)$, minimally couple quintessence, (future) Hořava gravity;
- Built-in: several choices for the forms of the EFT functions;
- Built-in: several equation of state parametrizations, i.e. $w_{DE}(a)$;
- EFTCosmoMC: exploration of the parameter space performing comparison with several cosmological data sets: CMB data.

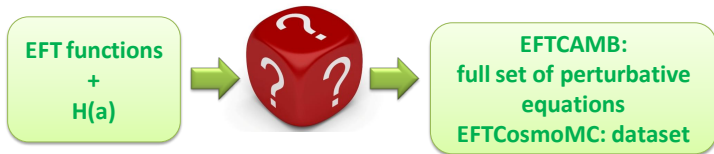
Stability check of perturbations

To ensure that the underlying theory of gravity is stable we place the following theoretical constraints:

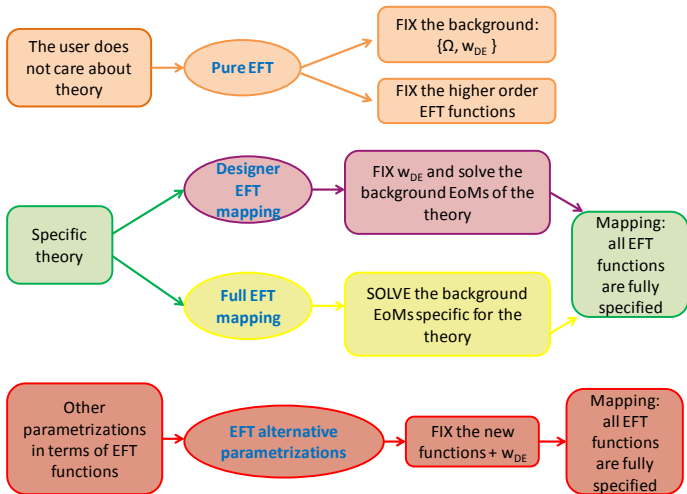
- $1 + \Omega > 0$: the effective Newtonian constant does not change sign;
- Effective scalar d.o.f. should not be a ghost;
- $c_s^2 \geq 0$: to prevent gradient instabilities (sub/super luminal propagation optional);
- $m_\pi^2 \geq 0$: to avoid tachyonic instabilities (optional);
- No tensor ghosts.

EFTCosmoMC: stability requirements become **viability priors**

How to fix the EFT functions?



EFTCAMB Structure



AN EXAMPLE: The Horava Gravity Case

[NF, M. Raveri, D. Vernieri, B. Hu, A. Silvestri, (2015) arXiv:1508.01787]

Why testing Lorentz Invariance?

- Quantum gravity description: one way is to assume LV;
[S. Liberati, Class. Quant. Grav. 30, 133001 (2013)]
- It is expected that LV theories will recover LI at long distance / low-energy scales;
- It is important to test LI at all scales, LV has been constrained:
 - within the Standard model of particle physics;
[V. A. Kostelecky and N. Russell, Rev. Mod. Phys. 83, 11 (2011)]
 - BBN constraints: $|G_{cosmo}/G_N - 1| < 0.38$ (99.7% C.L.)
[S. M. Carroll and E. A. Lim, PRD **70**, 123525 (2004)]
 - at post-newtonian level:
$$\alpha_1 < 3.0 \cdot 10^{-4}, \quad \alpha_2 < 7.0 \cdot 10^{-7} \quad (99.7\% \text{C.L.});$$

[C. M. Will, Living Rev. Rel. 17, 4 (2014)]
 - astrophysical scale (Binary pulsar system);
[K. Yagi et al., PRL., 112, 16, 161101 (2014)]
 - cosmological scale;
[B. Audren, et al., JCAP 1503 (2015) 016]

Hořava Gravity action

General Overview on Hořava gravity: Daniele & Thomas's Talks

We consider all terms contributing to linear cosmological perturbations

$$\begin{aligned} S_H = & \frac{1}{16\pi G_H} \int dx^4 \sqrt{-g} [K_{ij}K^{ij} - \lambda K^2 - 2\bar{\Lambda} + \xi \mathcal{R} + \eta a_i a^i \\ & + g_1 \mathcal{R}^2 + g_2 \mathcal{R}_{ij} \mathcal{R}^{ij} + g_3 \mathcal{R} \nabla_i a^i + g_4 a_i \nabla^2 a^i + g_5 \mathcal{R} \nabla^2 \mathcal{R} \\ & + g_6 \nabla_i \mathcal{R}_{jk} \nabla^i \mathcal{R}^{jk} + g_7 a_i \nabla^4 a^i + g_8 \nabla^2 \mathcal{R} \nabla_i a^i] + S_m[g_{\mu\nu}, \chi_i], \end{aligned}$$

where $a_i = \partial_i \ln N$.

- Power-counting renormalizability \rightarrow at least 6th-order spatial derivatives
- One extra scalar DoF, hidden inside the metric (unitary gauge).

[D. Blas et al., PRL 104 (2010) 181302]

We set constraints on the low-energy parameters.

Hořava Gravity mapping into EFT

We can map the low-energy action of Hořava gravity in the EFT language as follows:

$$(1 + \Omega) = \frac{2\xi}{(2\xi - \eta)},$$

$$c(\tau) = -\frac{m_0^2}{a^2(2\xi - \eta)}(1 + 2\xi - 3\lambda) \left(\dot{\mathcal{H}} - \mathcal{H}^2 \right),$$

$$\Lambda(\tau) = \frac{2m_0^2}{(2\xi - \eta)} \left[-\xi\bar{\Lambda} - (1 - 3\lambda + 2\xi) \left(\frac{\mathcal{H}^2}{2a^2} + \frac{\dot{\mathcal{H}}}{a^2} \right) \right],$$

$$\bar{M}_3^2 = -\frac{2m_0^2}{(2\xi - \eta)}(1 - \xi), \quad \bar{M}_2^2 = -2\frac{m_0^2}{(2\xi - \eta)}(\xi - \lambda),$$

$$m_2^2 = \frac{m_0^2\eta}{4(2\xi - \eta)}, \quad M_2^4(\tau) = \frac{m_0^2}{2a^2(2\xi - \eta)}(1 + 2\xi - 3\lambda) \left(\dot{\mathcal{H}} - \mathcal{H}^2 \right),$$

$$\bar{M}_1^3 = \hat{M}^2 = 0.$$

Note: We worked out the mapping also for the high-energy parameters.

Stability

- Theoretical conditions: no ghosts and gradient instabilities for scalar and tensor modes

$$0 < \eta < 2\xi, \quad \lambda > 1 \quad \text{or} \quad \lambda < \frac{1}{3},$$

- Observational constraints:
 - BBN $\rightarrow G_{\text{cosmo}} = \frac{(2\xi - \eta)}{3\lambda - 1} G_N$;
 - PPN parameters:

$$\alpha_1 = 4(2\xi - \eta - 2),$$
$$\alpha_2 = -\frac{(\eta - 2\xi + 2)(\eta(2\lambda - 1) + \lambda(3 - 4\xi) + 2\xi - 1)}{(\lambda - 1)(\eta - 2\xi)}.$$

Direct constraint on λ that reads:

$$\log_{10}(\lambda - 1) < -4.1 \text{ (99.7\%C.L.)}.$$

For the present analysis we consider two specific cases of Hořava gravity:

- H3, where we vary all three parameters $\{\lambda, \eta, \xi\}$;
- H2, where we set $\alpha_1 = \alpha_2 = 0$, this implies:

$$\eta = 2\xi - 2 \quad \rightarrow \quad \{\lambda, \eta\}.$$

The Background

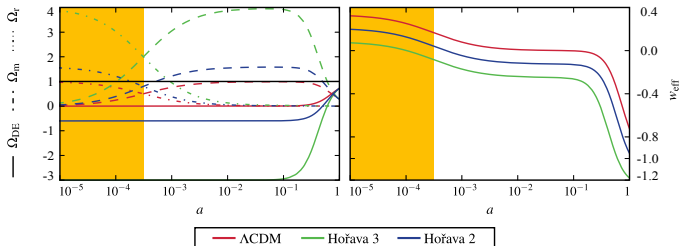
The background equation evolved by EFTCAMB:

$$\mathcal{H}^2 = \frac{(2\xi - \eta)}{3\lambda - 1} a^2 H_0^2 \left[\frac{\Omega_m^0}{a^3} + \frac{\Omega_r^0}{a^4} + \rho_\nu + \left(\Omega_{DE}^0 - 1 + \frac{3\lambda - 1}{2\xi - \eta} \right) \right]$$

and

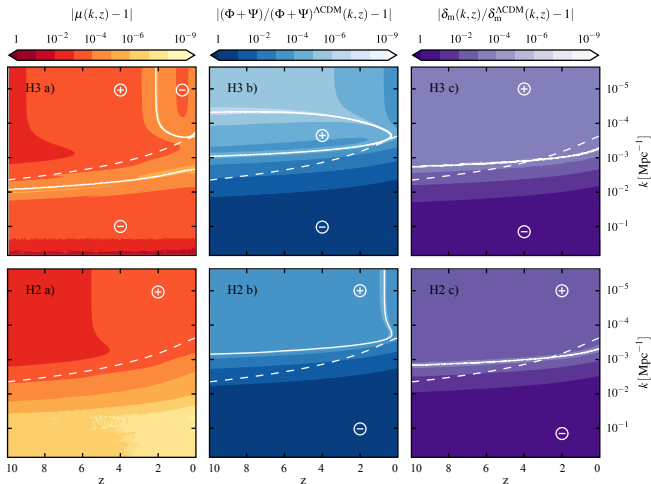
$$\Omega_i(a) = 8\pi G_N \frac{\rho_i}{3} \frac{a^2}{\mathcal{H}^2},$$

$$\Omega_{DE}(a) = \frac{2\xi}{2\xi - \eta} \frac{\bar{\Lambda}}{3} \frac{a^2}{\mathcal{H}^2} + 1 - \frac{3\lambda - 1}{2\xi - \eta}.$$

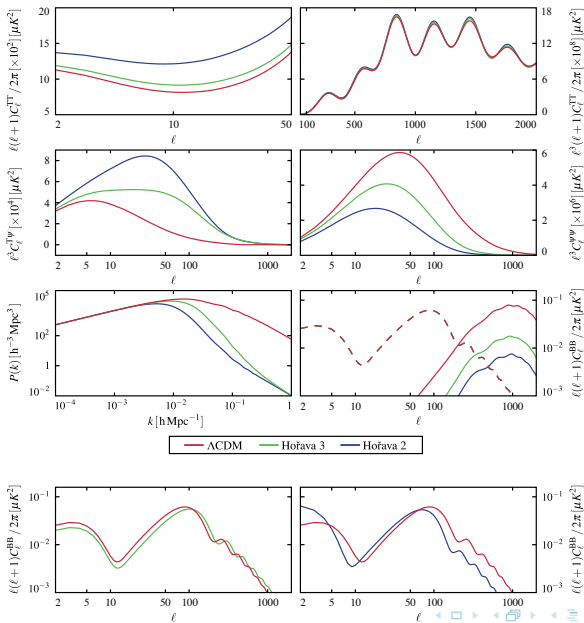


Gravitational potentials and matter distribution

- Φ, Ψ Newtonian gravitational potentials;
- $k^2\Psi \equiv -\mu(k, a)\frac{a^2}{2m_0^2}\rho_m\Delta_m$;
- $\Phi + \Psi \rightarrow$ lensing.



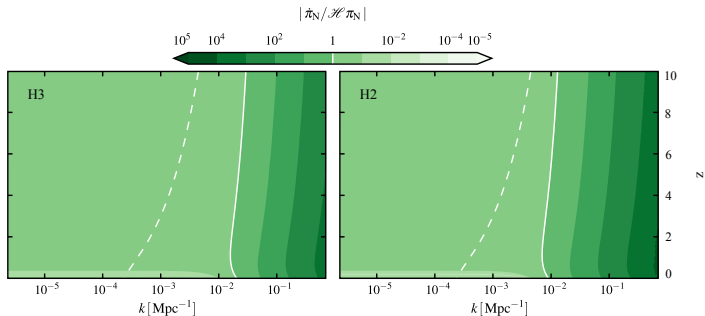
Cosmological observables



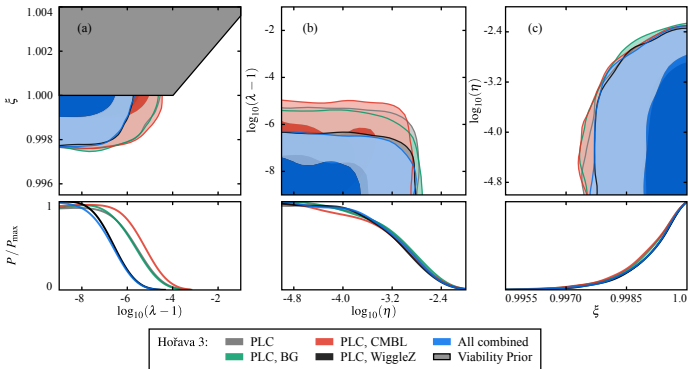
Quasi-staticity indicator

To quantify the deviations from QS for the dynamical scalar DoF, π , we define the quantity

$$\xi_N = \frac{\dot{\pi}_N}{\mathcal{H}\pi_N}$$



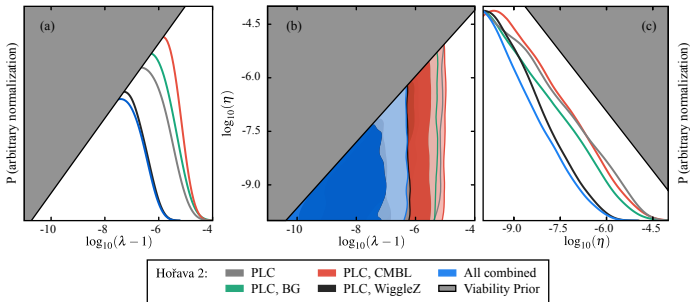
H3 Results



The bounds at 99.7% C.L from all data sets:

$$\begin{aligned} \xi - 1 &> -0.0038, & \log_{10}(\lambda - 1) &< -6.2, \\ \log_{10}(\eta) &< -2.4, & \Omega_{DE}^0 &= 0.69_{-0.02}^{+0.02}, \\ \alpha_1 &< 0.032, & \log_{10} \alpha_2 &< 10.8, \\ G_{\text{cosmo}}/G_N - 1 &< 0.0040. \end{aligned}$$

H2 Results



The bounds at 99.7% C.L from all data sets:

$$\log_{10}(\lambda - 1) < -5.9,$$

$$\log_{10}(\eta) < -6.1,$$

$$\Omega_{DE}^0 = 0.69_{-0.02}^{+0.02},$$

$$G_{\text{cosmo}}/G_N - 1 < 1.5 \times 10^{-6}.$$

Conclusion

- EFTCAMB/EFTCosmoMC:
 1. Evolves full dynamical perturbative scalar and tensor equations;
 2. It is compatible with massive neutrinos;
 3. Allows for model independent test of gravity at large scale;
 4. Allows to test specific DE/MG models (built-in: $f(R)$; minimally coupled quintessence; Hořava gravity);
 5. Built-in stability check.
- Hořava gravity analysis:
 1. We worked out a complete mapping of the theory;
 2. We get bounds on the parameters of the theory;
 3. We get improved upper bounds on G_{cosmo} ;
 4. We found that the effects of the modifications are quite dramatic;
 5. We found that the QS approximation is not safe to describe the evolution of perturbations.
- WHAT'S NEXT?
 1. Additional DE/MG models of Cosmological interest;
 2. Investigation of the validity of the QS approximation;
 3. **Release EFTCAMB 2015 SOON.**

