

Lorentz violation, causality, and black holes

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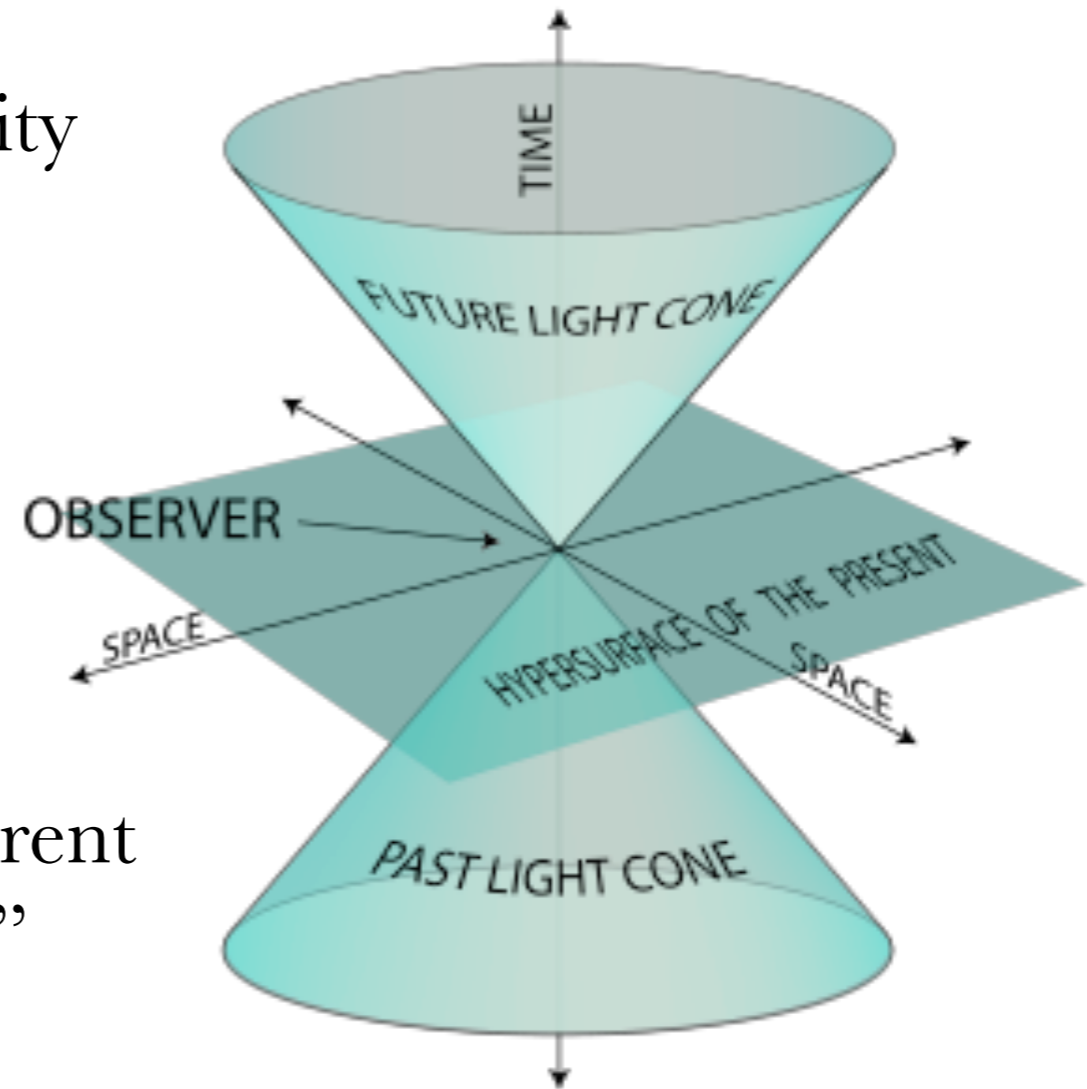
LV and causal structure

Causal structure in special relativity

- LV with linear dispersion relations

$$\omega \propto k$$

- Different modes have different speeds and different “light” cones
- But there are still “light” cones!





Einstein-aether theory

The action of the theory is

$$S_{\text{ae}} = \frac{1}{16\pi G_{\text{ae}}} \int d^4x \sqrt{-g} (-R - M^{\alpha\beta\mu\nu} \nabla_{\alpha} u_{\mu} \nabla_{\beta} u_{\nu})$$

where

$$M^{\alpha\beta\mu\nu} = c_1 g^{\alpha\beta} g^{\mu\nu} + c_2 g^{\alpha\mu} g^{\beta\nu} + c_3 g^{\alpha\nu} g^{\beta\mu} + c_4 u^{\alpha} u^{\beta} g_{\mu\nu}$$

and the aether is implicitly assumed to satisfy the constraint

$$u^{\mu} u_{\mu} = 1$$

- Most general theory with a unit timelike vector field which is second order in derivatives

T. Jacobson and D. Mattingly, Phys. Rev. D 64, 024028 (2001).



Einstein-aether theory

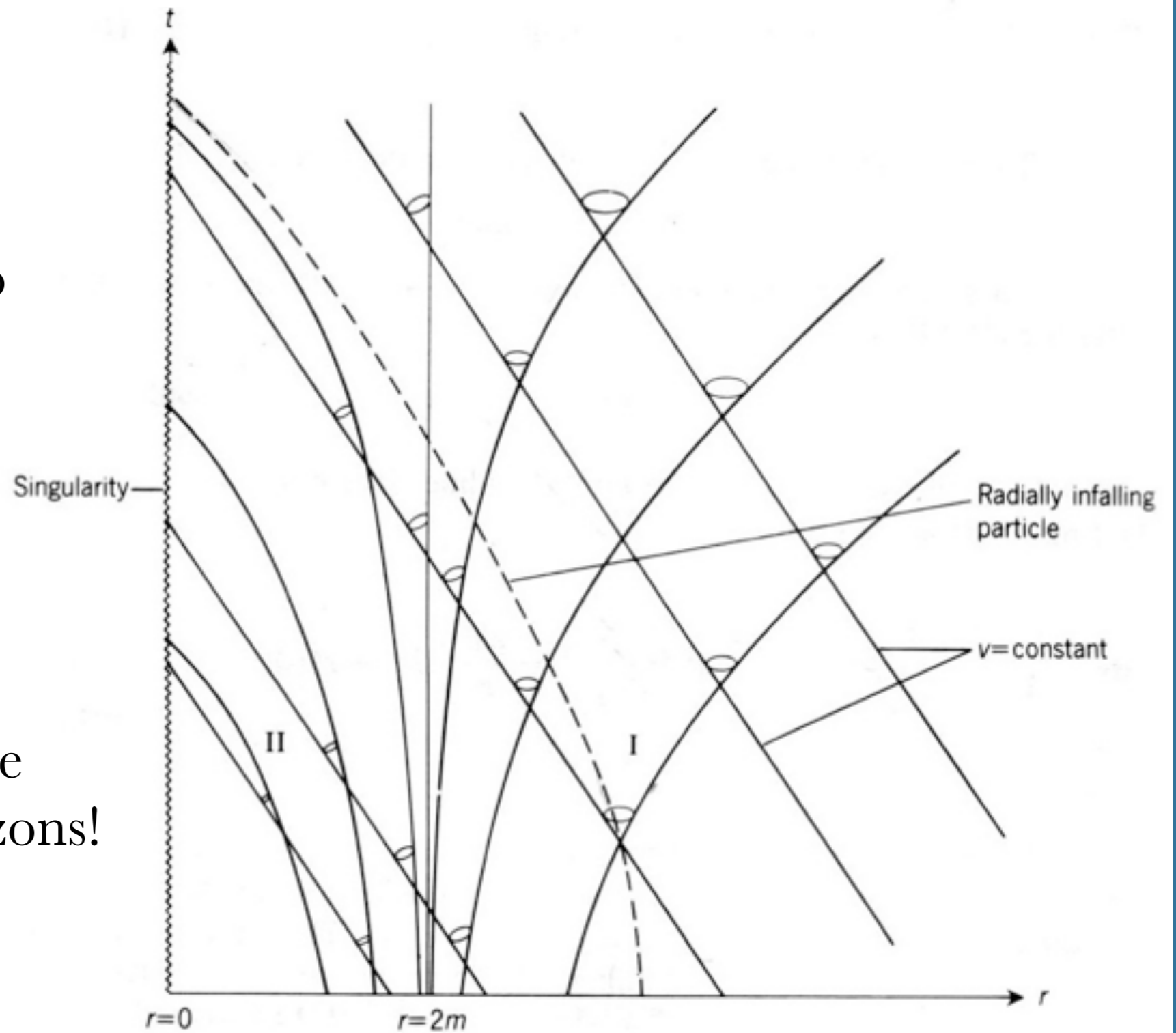
- Extensively tested and still viable
- It propagates a spin-2, a spin-1 and spin-0 mode.
- Linear dispersion relations.
- These modes travel at different speeds.
- We expect multiple horizons!



LV and black hole structure

What happens to black holes?

- They will have multiple horizons!





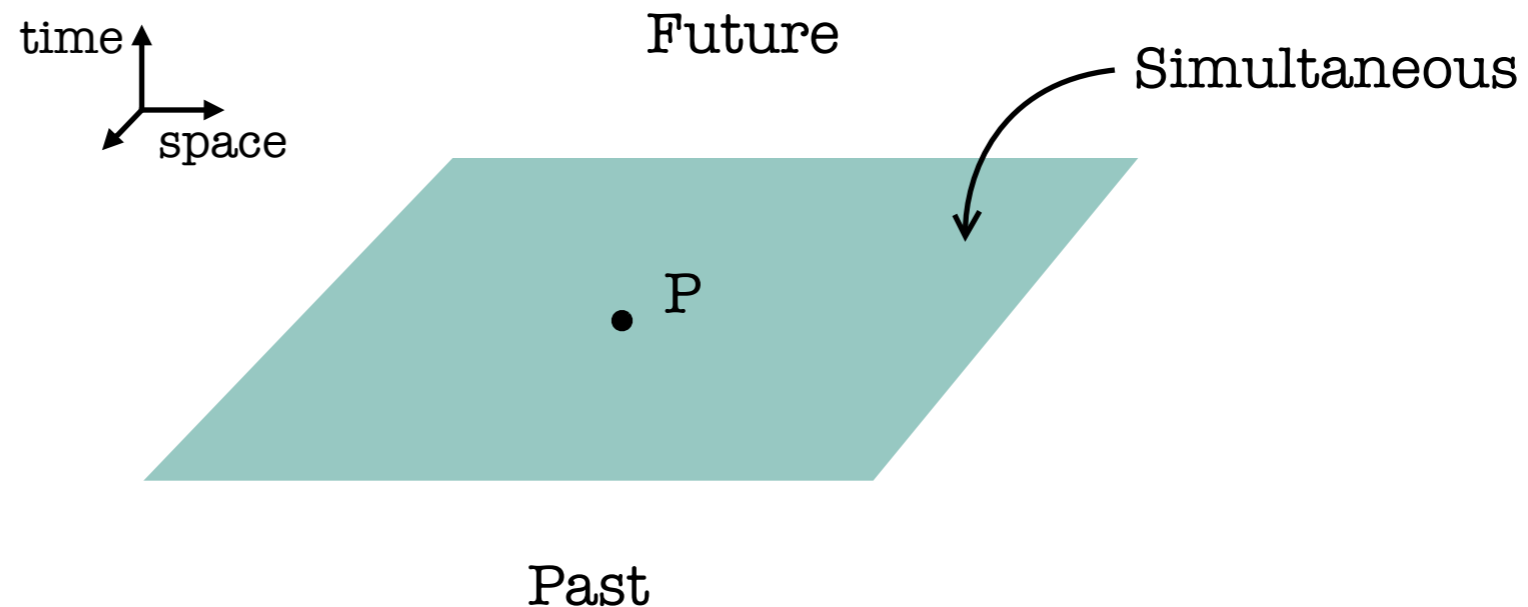
LV and black hole structure

- LV with non-linear dispersion relations

$$\omega^2 \propto k^2 + ak^4 + \dots$$

- No light cones!

Causal structure without relativity



No black holes at all??



Hypersurface orthogonality

Now assume
$$u_\alpha = \frac{\partial_\alpha T}{\sqrt{g^{\mu\nu} \partial_\mu T \partial_\nu T}}$$

and choose T as the time coordinate

$$u_\alpha = \delta_{\alpha T} (g^{TT})^{-1/2} = N \delta_{\alpha T}$$

Replacing in the action and defining one gets

$$S_\infty^{ho} = \frac{1}{16\pi G_H} \int dT d^3x N \sqrt{h} \left(K_{ij} K^{ij} - \lambda K^2 + \xi^{(3)} R + \eta a^i a_i \right)$$

with $a_i = \partial_i \ln N$ and the parameter correspondence

$$\frac{G_H}{G_\infty} = \xi = \frac{1}{1 - c_{13}} \quad \lambda = \frac{1 + c_2}{1 - c_{13}} \quad \eta = \frac{c_{14}}{1 - c_{13}}$$

T. Jacobson, Phys. Rev. D 81, 101502 (2010).



Horava-Lifshitz gravity

The action of the theory is

$$S_{HL} = \frac{1}{16\pi G_H} \int dT d^3x N \sqrt{h} \left(L_2 + \frac{1}{M_\star^2} L_4 + \frac{1}{M_\star^4} L_6 \right)$$

where

$$L_2 = K_{ij} K^{ij} - \lambda K^2 + \xi^{(3)} R + \eta a_i a^i$$

L_4 : contains all 4th order terms constructed with the induced metric h_{ij} and a_i

L_6 : contains all 6th order terms constructed in the same way

P. Hořava, Phys. Rev. D 79, 084008 (2009)

D. Blas, O. Pujolas and S. Sibiryakov, Phys. Rev. Lett. 104, 181302 (2010)



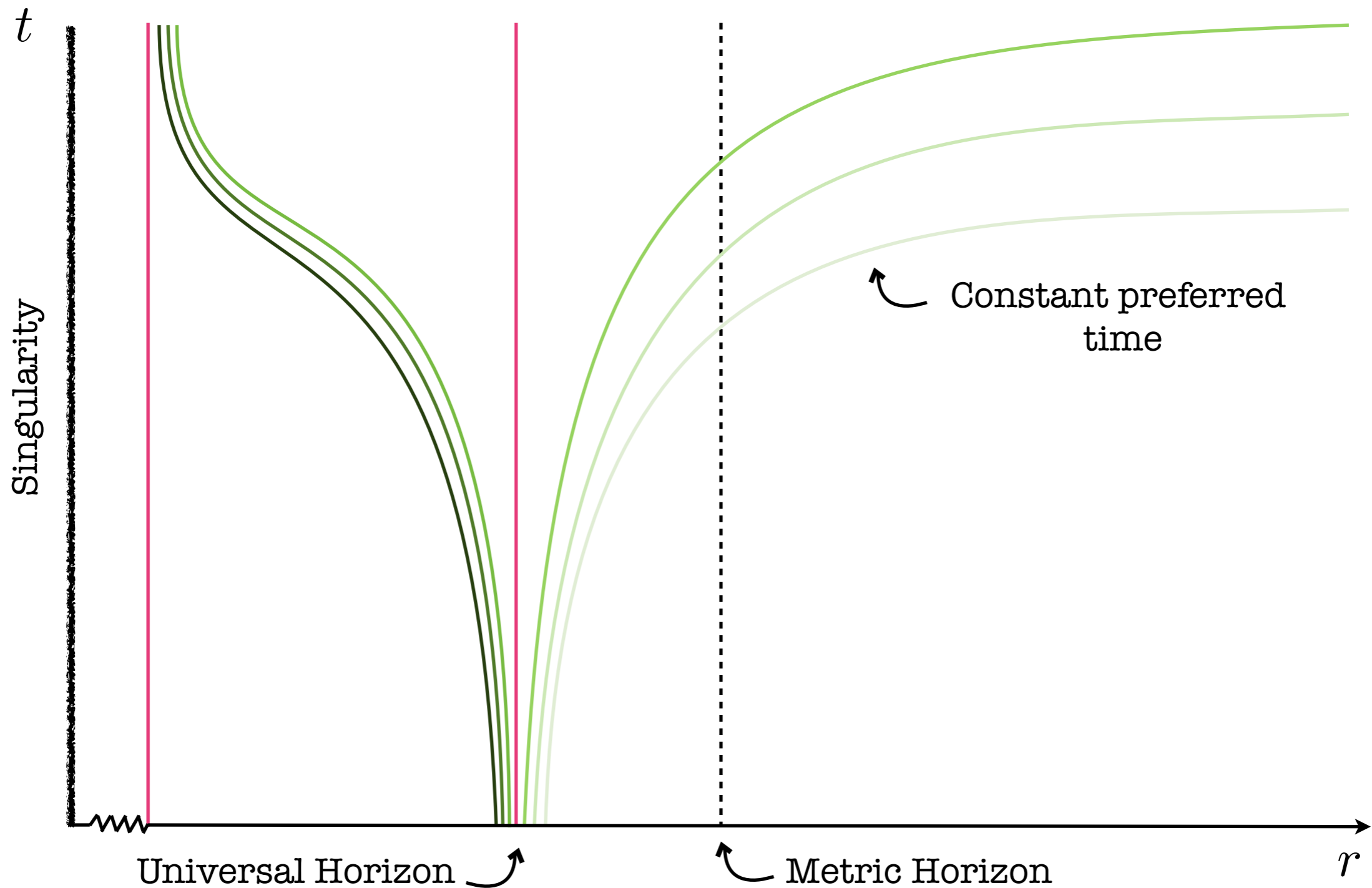
Horava-Lifshitz gravity

- Higher order terms contain higher order spatial derivatives: higher order dispersion relations!
- They modify the propagator and render the theory power-counting renormalizable
- All terms consistent with the symmetries will be generated by radiative corrections
- This version of the theory is viable so far
- “Low energy limit” is h.o. Einstein-aether theory!
- We expect no causal boundaries!



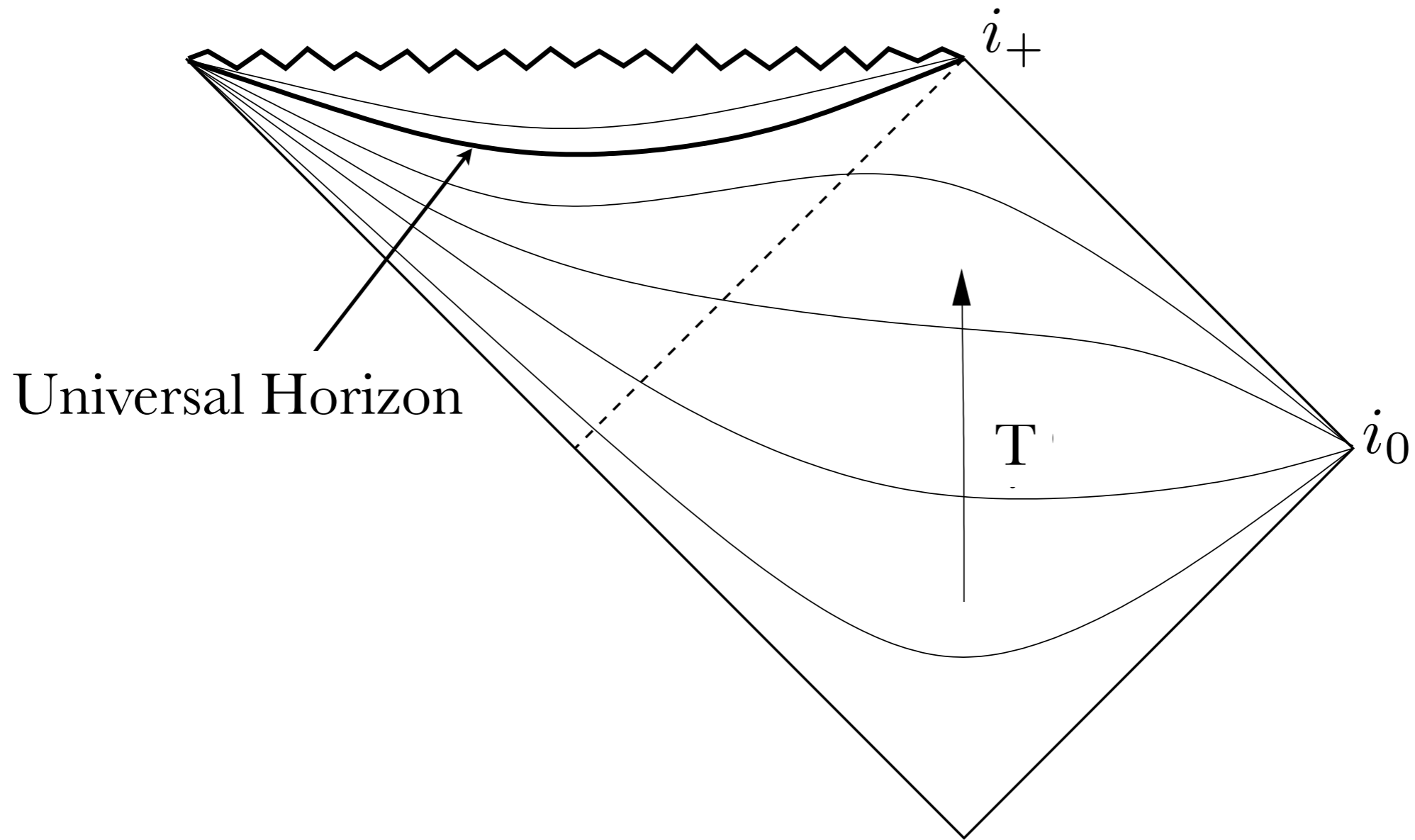
Spacetime diagram

E. Barausse, T. Jacobson and T.P.S., Phys. Rev. D 83, 124043 (2011)





Penrose diagram



Taken from D. Blas and S. Sibiryakov, Phys. Rev. D 84, 124043 (2011)



Rotating black holes

- Slowly rotating BHs in Einstein-aether theory do not have a preferred foliation.
- Slowly rotating BHs in Horava gravity have universal horizons.

T.P.S. and E. Barausse, Phys. Rev. Lett. 109, 181101 (2012)

T.P.S. and E. Barausse, Phys. Rev. D 87, 087504 (2013)

T.P.S. and E. Barausse, Class. Quant. Grav 30, 244010 (2013)

- 3d rotating black holes can have universal horizons even with flat asymptotics.
- Universal horizons can lie “outside” de Sitter horizons.

T.P.S., I. Vega and D. Vernieri, Phys. Rev. D 90, 044046 (2014)



What's next?

- A new “toolkit” is needed
- How do we define this horizon in full generality?
- Can we have a local definition when we have less symmetry?
- Is the universal horizon relevant to astrophysics?

M. Colombo, J. Bhattacharyya, and T.P.S., arXiv:1509.01558 [gr-qc]



Causally preferred foliation

Consider a manifold with a preferred foliation (\mathcal{M}, Σ, g)

Definition: Ordered foliation

- ◆ Every event in \mathcal{M} lies on a unique leaf
- ◆ Every pair of events has a unique causal relation

No preferred labeling implies invariance under $T \rightarrow \tilde{T}(T)$

Definition: Causal and acausal curves

Continuous, piecewise differentiable curve with tangent t^μ

causal, future directed if $u_\mu t^\mu > 0$

causal, past directed if $u_\mu t^\mu < 0$

acausal if $u_\mu t^\mu = 0$

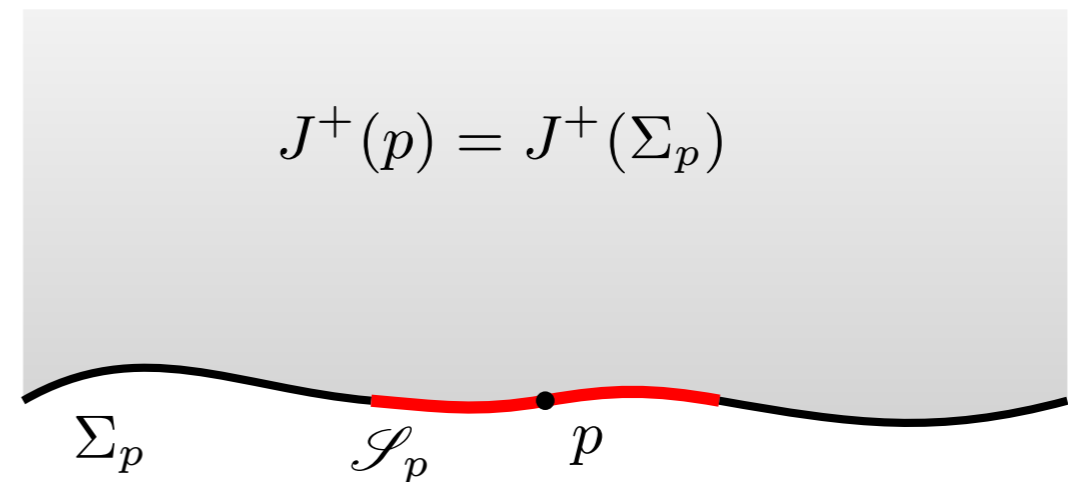
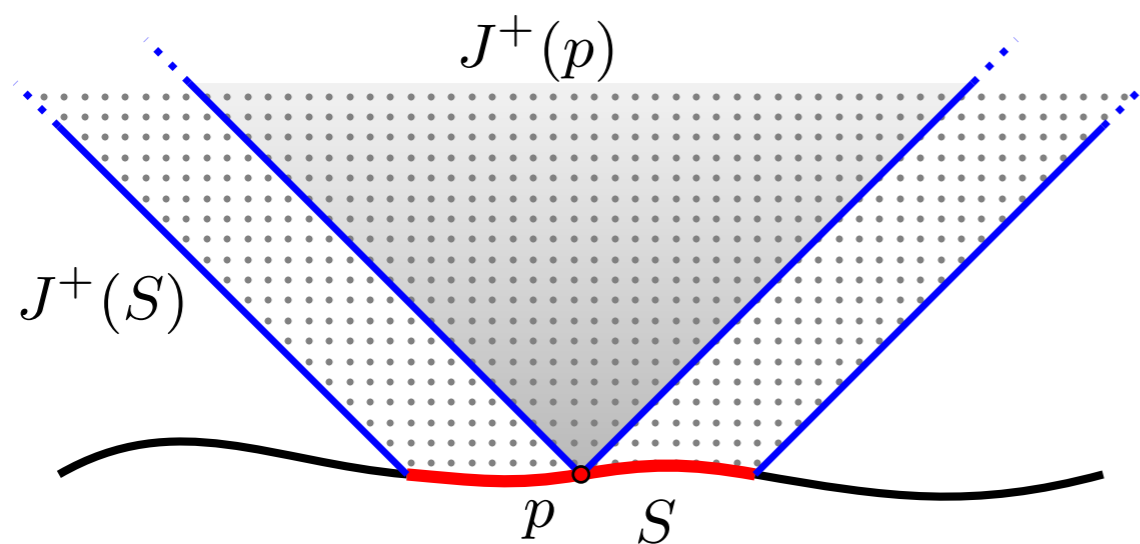


Future and Past

Definition: Future and Past

The future $J^+(p)$ of an event p is the set of all events that can be reached from p by a future directed causal curve.

Similarly for the Past.





Asymptotics

What's the analogue of asymptotic flatness?

projector:
$$p_{\mu\nu} = g_{\mu\nu} - u_\mu u_\nu$$

Conformally extended manifold: $\tilde{\mathcal{M}}$,
$$\tilde{p}_{\mu\nu} = \Omega^2 p_{\mu\nu}$$

point as spatial infinity of Σ_p : i_p
$$\tilde{\Sigma}_p = \Sigma_p \cup i_p$$

One also needs conditions for the foliation!

Key concept: ‘trivially foliated asymptotically flat end’

$\langle\langle \mathcal{M} \rangle\rangle$: open region in which every leaf has trivially foliated asymptotically flat end

$$\mathcal{I} = \bigcup_{p \in \langle\langle \mathcal{M} \rangle\rangle} i_p$$



Universal horizons

Future and past event horizons:

$$\mathcal{H}^+ \equiv \partial J^-(\mathcal{I}) \qquad \mathcal{H}^- \equiv \partial J^+(\mathcal{I})$$

It follows that

• $\langle\langle \mathcal{M} \rangle\rangle = J^-(\mathcal{I}) \cap J^+(\mathcal{I})$

• \mathcal{H}^\pm are leaves and boundaries of $\langle\langle \mathcal{M} \rangle\rangle$

Black hole: $\mathcal{B}(\mathcal{I}) \equiv \mathcal{M} \setminus J^-(\mathcal{I})$

White hole: $\mathcal{W}(\mathcal{I}) \equiv \mathcal{M} \setminus J^+(\mathcal{I})$

Key point: $\mathcal{H}^\pm(\mathcal{I}) \cap \mathcal{I} = \emptyset$



Local characterisation

Definition: Stationary spacetime (simplified)

There exist a killing vector that is causal in an open set \mathcal{X} that overlaps with $\langle\langle \mathcal{M} \rangle\rangle$ and ‘aligns’ with the aether asymptotically.

Theorem

$(u \cdot \chi) = 0$, $(a \cdot \chi) \neq 0$ form a set of necessary and sufficient conditions for a hypersurface to be a universal horizon

Key points:

- $\langle\langle \mathcal{M} \rangle\rangle = \mathcal{X}$
- χ is unique
- analogy with Killing horizon, $(a \cdot \chi) = \text{constant}$



Astrophysical relevance

Assume also axisymmetry

$$(u \cdot \varphi) = (a \cdot \varphi) = 0$$

Suppose there is a universal horizon

- Does there need to be a Killing horizon?
- Does it cloak the universal horizon?

$$V^\mu \equiv \chi^\mu + W \varphi^\mu \quad W \equiv -(\chi \cdot \varphi) / (\varphi \cdot \varphi) \quad (V \cdot \varphi) = 0$$

Iff $\chi_{[\mu} \varphi_{\nu} \nabla_{\kappa} \varphi_{\lambda]} = 0 \quad \varphi_{[\mu} \chi_{\nu} \nabla_{\kappa} \chi_{\lambda]} = 0$

then by Carter's rigidity theorem

$(V \cdot V) = 0$ is a null hypersurface where $W = \text{constant}$



Perspectives

- Black holes are of great interest in Lorentz-violating theories. New notion: “universal horizon”
- Non-trivial causal structure
- Is this horizon stable?
- Does it form from collapse?

D. Blas and S. Sibiryakov, Phys. Rev. D 84, 124043 (2011)

M. Saravani, N. Afshordi and R. B. Mann, Phys. Rev. D 89, 084029 (2014)