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Modified Gravity *inside* Astrophysical Bodies

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Outline

I. Introduction

II. Vainshtein screening mechanism,
and its failure *inside* a source

T. Kobayashi, Y. Watanabe and D. Yamauchi (2015)

III. Impact on the stellar structure

R. Saito, D. Yamauchi, S. Mizuno, J. Gleyzes and D. Langlois (2015)

Infrared modification of gravity

- The expansion of the universe is accelerated today,
  BUT our best theory (GR (EH) + SM) fails to explain it.

- Gravity should be a major player,
  BUT its nature is little known on cosmological scales;

  Gravity might not be described by GR.

Questions:

Is it possible **to modify GR on cosmological (IR) scales *without* spoiling its success in the solar-system observations?**

If possible**,**

is it possible **to test the modification of gravity?**

Fifth force – new long-range force

The first question is non-trivial.

IR modification also changes the gravitational force at shorter scales.

(Force sourced by energy-momentum tensor)

+ a light scalar dof universally coupled to matter
(scalar-tensor theories, f(R) gravity,…)

It also mediates **a new long-range force (fifth force)** at short scales.

+ a mass term for the graviton (spin 2 particle)
(massive gravity)

Massless multiplet 2 dof $\longrightarrow$ Massive multiplet 5 dof

(with Lorentz symmetry)

The helicity-0 mode does the same.

The fifth force violates **the equivalence principle**.

A scalar-type universal coupling (conformal coupling):

$$\mathcal{L} \supset -\frac{G_s}{2}(\partial\phi)^2 + \underline{\phi T}$$

Poisson equation for the scalar field

$$\nabla^2\phi = -4\pi G_s T \simeq 4\pi G_s \rho$$ **for a non-relativistic object**

$$\simeq 0$$ **for a relativistic object**

The light deflection observations ⟺ The earth experiments

Incompatible unless $G_s/G_N < \mathcal{O}(10^{-4})$

Is it impossible to get O(1) corrections on cosmological scales?

Screening mechanisms

Environmental effects can screen the fifth force.

$$\nabla^2\phi \quad + \cdots = 4\pi G_s \rho$$

Interaction terms are important!

Around a non-relativistic object,…

the scalar field can be **massive**.

➔ The fifth force becomes a short-range force.

Chameleon mechanism

the scalar field can be **“strong”** for derivative interactions.

(As gravity becomes “strong” inside a Schwarzschild radius.)

➔ A fifth force sourced by the object is suppressed.

Vainshtein mechanism

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Vainshtein mechanism

Vainshtein screening mechanism, and its failure *inside* a source

T. Kobayashi, Y. Watanabe and D. Yamauchi (2015)

Fifth force

No interactions

For a spherical non-relativistic object,

$$\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}(r^2\phi') = 4\pi G_s \rho$$

Enclosed mass

$$\Rightarrow \quad \phi' = \frac{G_s M(r)}{r^2} \quad ; \quad M(r) \equiv 4\pi \int r^2 \mathrm{d}r \rho$$

It mediates the fifth force with the inverse-square law.

$$\frac{\text{Fifth force}}{\text{Gravitational force}} = \frac{G_s}{G_N}$$

Large changes on cosmological scales ($G_s \sim G_N$) → Large modification of the local grav. law

Vainshtein screening mechanism

With derivative interactions

From shift symmetry,

$$\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}(r^2 J^r) = 4\pi G_s \rho$$

Suppose that a derivative coupling is added like

$$J^r = r\left[\frac{\phi'}{r} + \frac{1}{m^4}\left(\frac{\phi'}{r}\right)^3\right]$$

E.g., 4th Galileon term

$$\mathcal{L}_4 \sim \frac{G_s^{-1}}{m^4}(\nabla^2\phi)^2(\nabla\phi)^2$$

The inverse-square law is no longer valid for

$$\frac{\phi'}{r} \ll \frac{1}{m^4}\left(\frac{\phi'}{r}\right)^3 \Rightarrow r \ll r_V \equiv \left(\frac{G_s M}{m^2}\right)^{\frac{1}{3}}$$

Vainshtein radius

Vainshtein screening mechanism

$$J^r = r\left[\cancel{\frac{\phi'}{r}} + \frac{1}{m^4}\left(\frac{\phi'}{r}\right)^3\right] = \frac{G_s M(r)}{r^2}$$

$$\Rightarrow \quad \phi' = \frac{G_s M(r)}{r^2}\left(\frac{r}{r_V}\right)^2$$

The fifth force is suppressed inside the Vainshtein radius

$$\frac{\text{Fifth force}}{\text{Gravitational force}} = \frac{G_s}{G_N}\underline{\left(\frac{r}{r_V}\right)^2}$$

$G_s \sim G_N$ without changing the local gravitational law

Failure of screening inside a source

Another type of a universal coupling (**Disformal coupling**)

[M. Zumalacárregui & J. García-Bellido 2014]

$$\mathcal{L} \supset \Gamma(X)\partial_\mu\phi\partial_\nu\phi T^{\mu\nu}; \quad X \equiv -\frac{1}{2}(\partial\phi)^2$$

with a X-dependent coupling "constant" $\Gamma \quad (\sim m^{-2})$

- No symmetry to forbid it
- It appears in GLPV (beyond Horndeski) theories [Gleyzes+ 2014]

GLPV theories ≅ Horndeski theories + disformal coupling

Failure of screening inside a source

Importance of the X dependence:

$$\mathcal{L} \supset \Gamma(X)\partial_\mu\phi\partial_\nu\phi T^{\mu\nu}; \qquad X \equiv -\frac{1}{2}(\partial\phi)^2$$

$$= \Gamma(X)\dot{\phi}^2\rho$$ **for a non-relativistic object**

No coupling when ϕ does not depend on time,
but it depends on time when it explains the cosmic acceleration.

$$\phi(t,r) = \bar{\phi}(t) + \varphi(t,r) \simeq \bar{\phi}(t) + \varphi(r)$$

$$\Rightarrow \quad \Gamma(X)\dot{\phi}^2\rho \simeq \left[\underline{\Gamma_X(\bar{X})}\dot{\bar{\phi}}^2\right](\nabla\varphi)^2\rho$$

Failure of screening inside a source

$$\frac{\varphi'}{r} + \frac{1}{m^4}\left(\frac{\varphi'}{r}\right)^3 = \frac{G_s M(r)}{r^3} + \underbrace{(\Gamma_X \dot{\phi}^2)}_{\sim m^{-2}} \frac{M'(r)}{r^2}\left(\frac{\varphi'}{r}\right)$$

Failure of screening inside a source

In the "strong scalar force" regime, $\varphi'/r \gg m^2$

$$\cancel{\frac{\varphi'}{r}} + \frac{1}{m^4}\left(\frac{\varphi'}{r}\right)^3 = \cancel{\frac{G_s M(r)}{r^3}} + \underbrace{(\Gamma_X \overline{\dot{\phi}^2})}_{\sim m^{-2}} \frac{M'(r)}{r^2}\left(\frac{\varphi'}{r}\right)$$

$$\Rightarrow \quad (\varphi')^2 = m^4 (\Gamma_X \overline{\dot{\phi}^2}) G_s M'(r)$$

φ' ***is less suppressed inside a source,*** $M' \neq 0$

Failure of screening inside a source

Effective gravitational potential for matter

$$\tilde{\Phi} = \Phi - \Gamma \dot{\phi}^2$$

$$\tilde{\Phi}' \simeq \Phi' - \Gamma_X [(\varphi')^2]' \dot{\bar{\phi}}^2$$

$$= G_N \left(\frac{M}{r^2} - \underline{\epsilon M''} \right) \; ; \epsilon \equiv \frac{m^4 (\Gamma_X \dot{\bar{\phi}}^2)^2}{(G_N / G_s)}$$

Comparable to the GR term $(\epsilon \sim 1)$

Disformal coupling **= Nonlinear derivative** coupling to matter

The Vainshtein mechanism fails because of

Less screening + Larger coupling

Failure of screening inside a source

The Vainshtein mechanism can be *partially* broken.

$$\frac{\mathrm{d}\tilde{\Phi}}{\mathrm{d}r} = G_N \left(\frac{M}{r^2} - \underline{\epsilon M''} \right)$$

Gravity is modified ***inside*** a source but not outside.

The full analysis in GLPV theories [Kobayashi+, 2015]

No deviations on the earth or in the space,

BUT possible large impacts on **the stellar structure**.

Impact on the stellar structure

R. Saito, D. Yamauchi, S. Mizuno, J. Gleyzes and D. Langlois, [arXiv: 1503.01448]

Model for stellar interiors

Static, spherically symmetric, polytropic model

(Realistic stars [Koyama & Sakstein 2015])

Basic equations

Force balance

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\rho\frac{\mathrm{d}\tilde{\Phi}}{\mathrm{d}r}$$

Only the gravitational law is modified!

Poisson equation

$$\frac{\mathrm{d}\tilde{\Phi}}{\mathrm{d}r} = G_N\left(\frac{M}{r^2} - \epsilon M''\right)$$

Equation of state

$$P = K\rho^{1+\frac{1}{n}}$$ (n=1: neutron star, n=3: sun)

Modified Lane-Emden equation

Closed equation for the density

$$\boxed{\frac{1}{\xi^2}\frac{\mathrm{d}}{\mathrm{d}\xi}\left[\xi^2\frac{\mathrm{d}}{\mathrm{d}\xi}\left(\chi-\epsilon\xi^2\chi^n\right)\right]=-\chi^n}$$

The variables were made dimensionless through

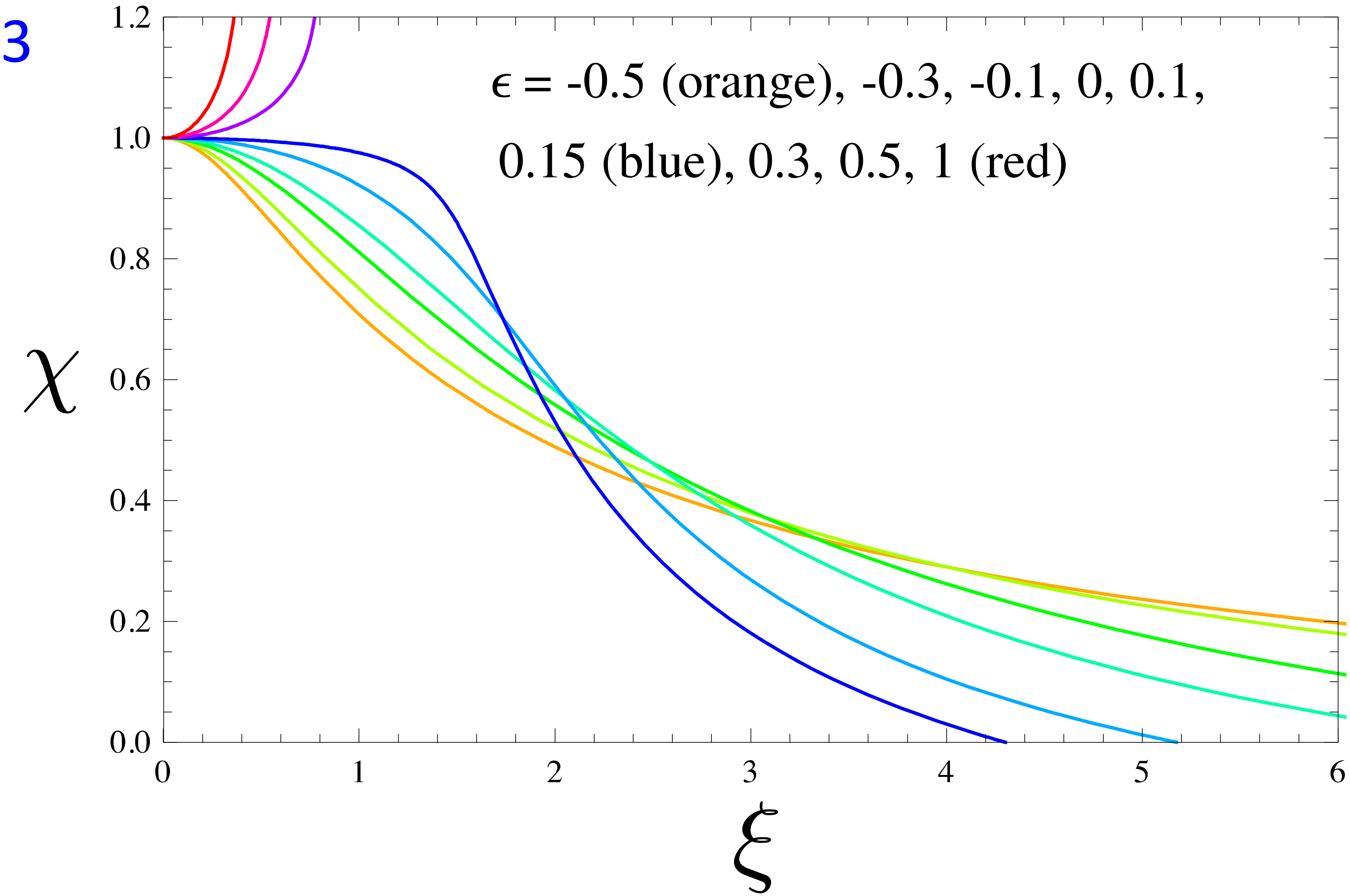
$$\xi\equiv r\sqrt{\frac{4\pi G_{\mathrm{N}}}{(n+1)K\rho_c^{-1+\frac{1}{n}}}},\qquad \rho=\rho_c\chi^n$$

It reduces to the standard Lane-Emden equation when $\epsilon\longrightarrow 0$

Solutions

We have numerically solved the modified LE equation.

n=3



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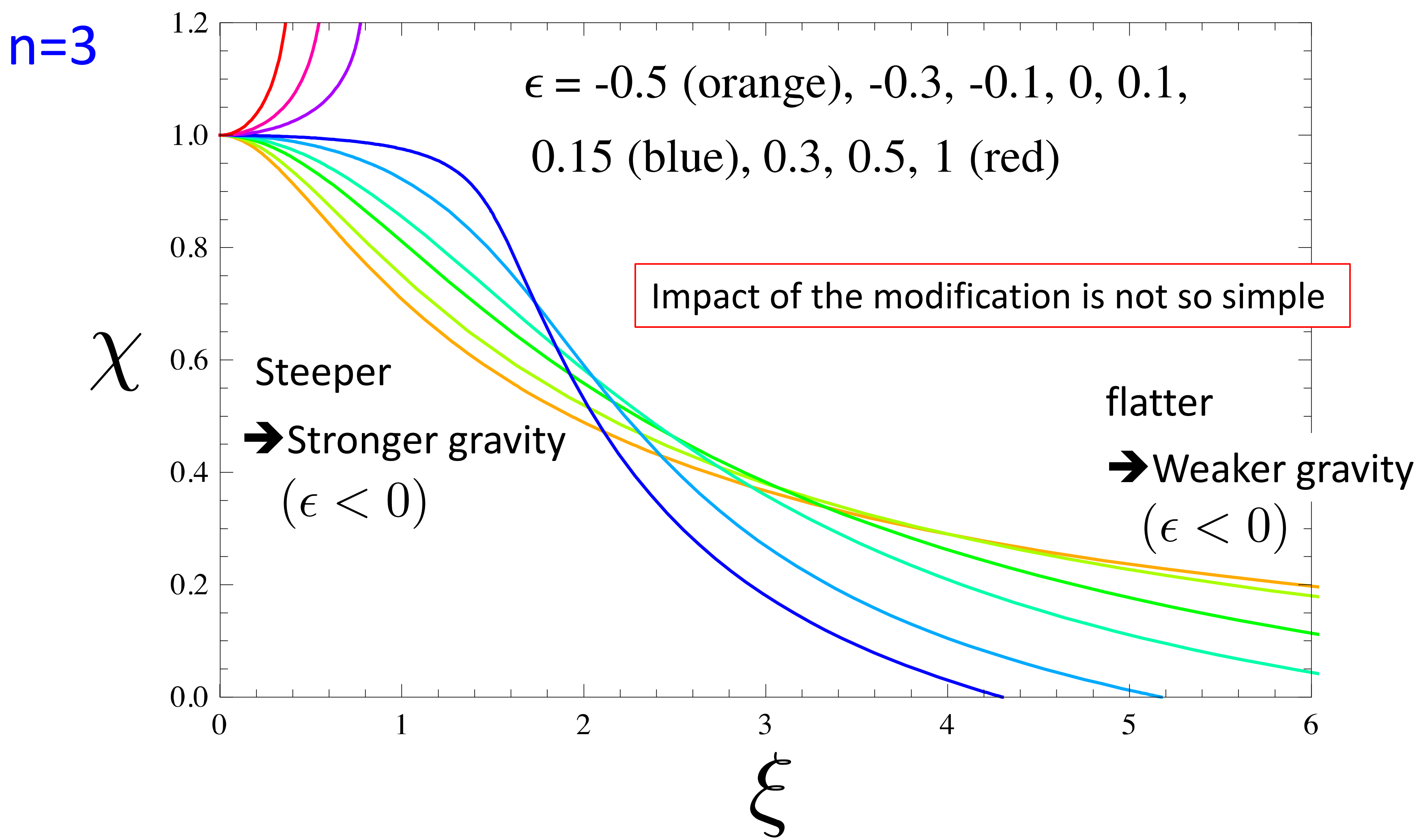
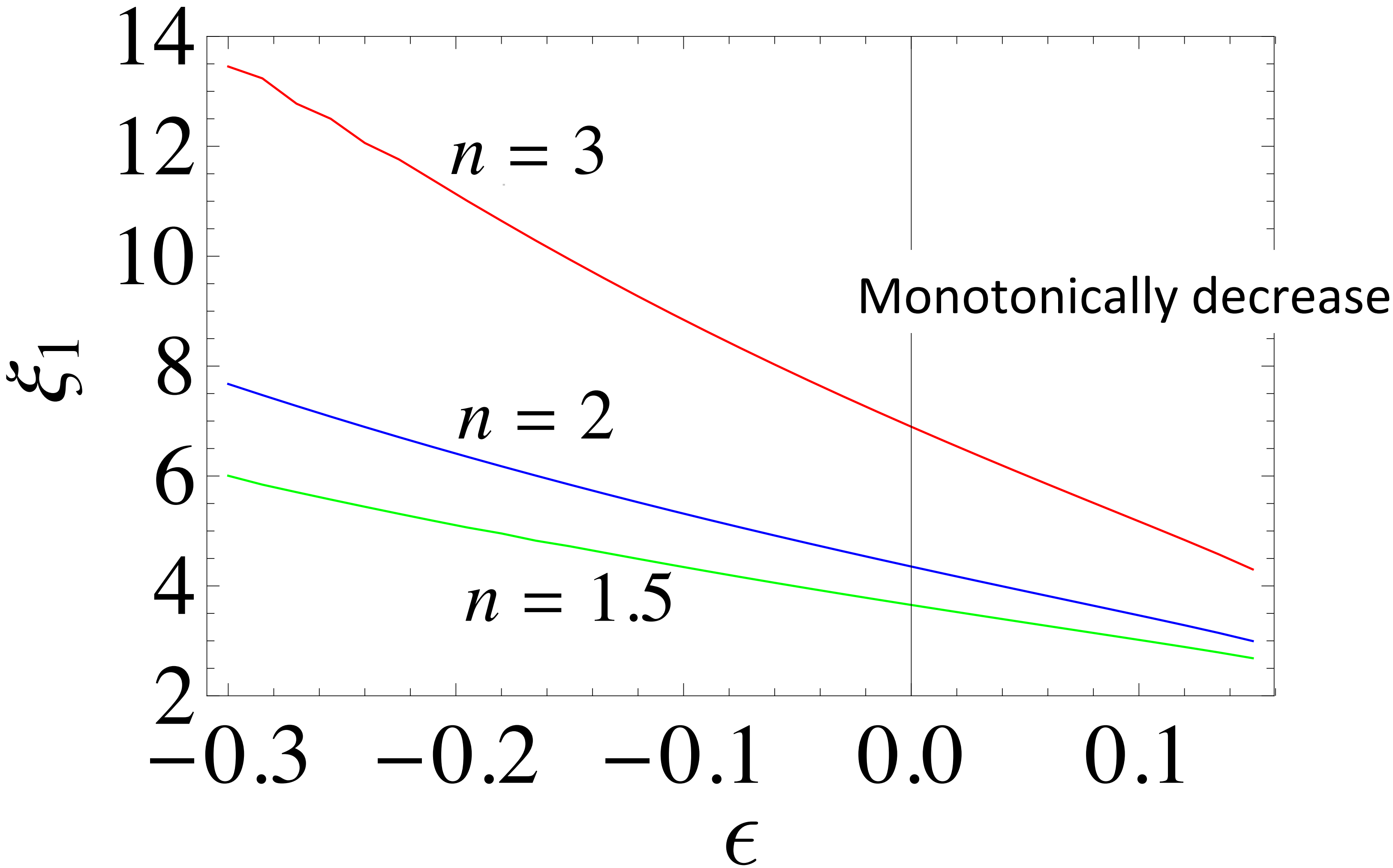
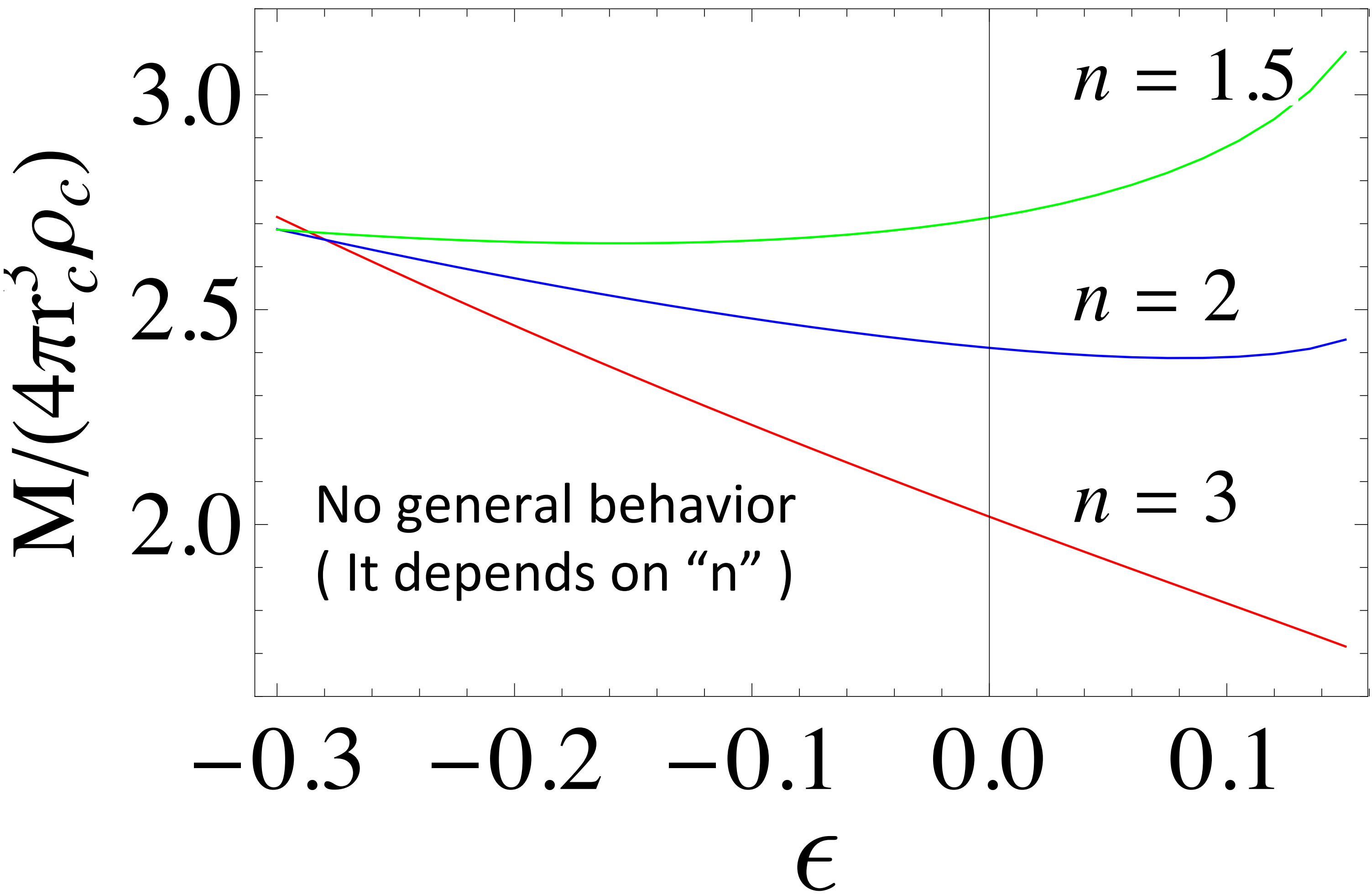


Radius



More sensitive to ε for softer equation of state.

Mass



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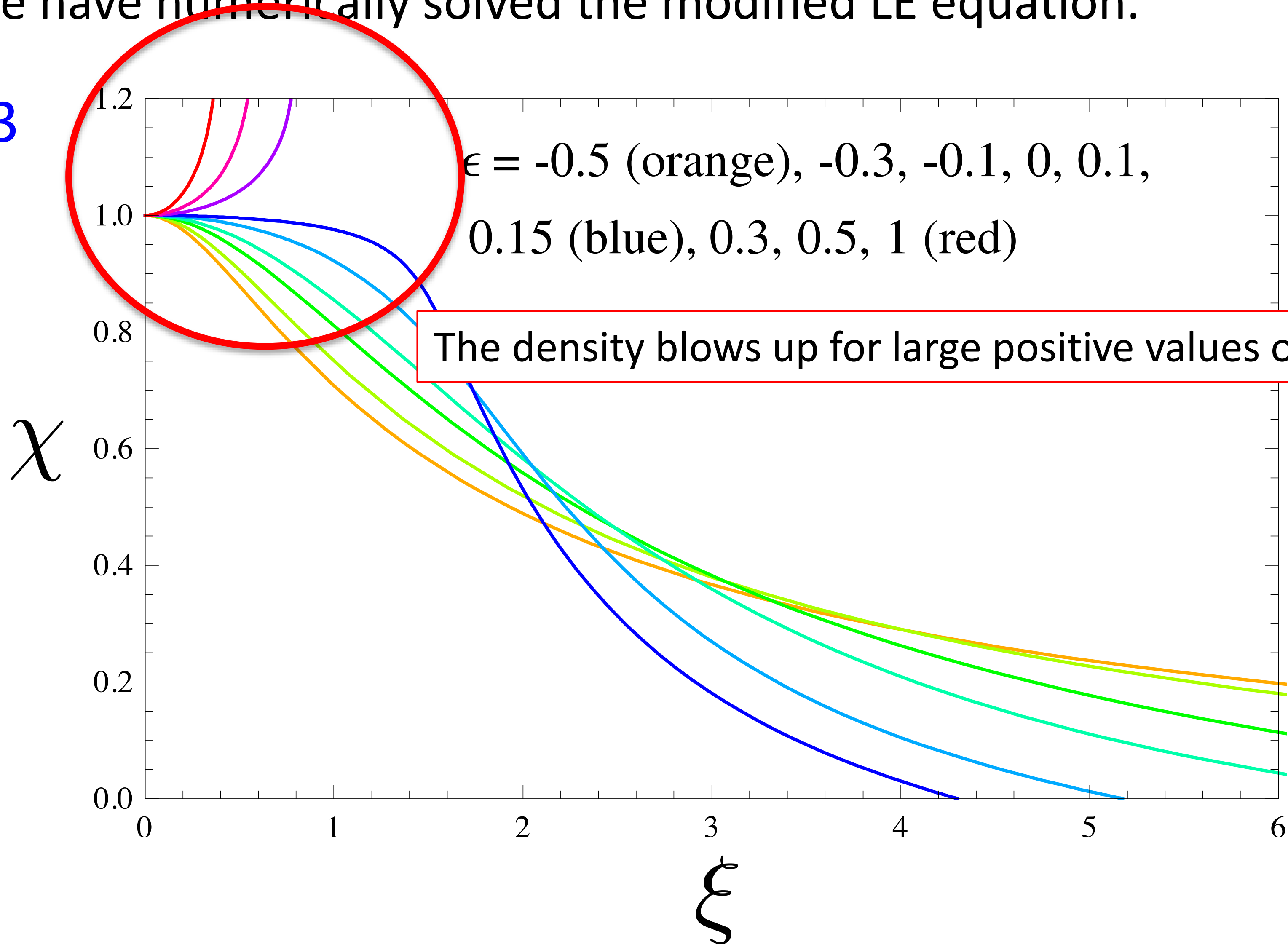


The density blows up for large positive values of epsilon.

Universal bound on the modification

Near the center,

$$\rho = \rho_c + \frac{1}{2}\rho_2\left(\frac{r}{R}\right)^2 + \cdots \quad \Rightarrow \quad M = \frac{4\pi\rho_c r^3}{3} + \mathcal{O}(r^5)$$

Gravity becomes repulsive for $\epsilon > \dfrac{1}{6}$.

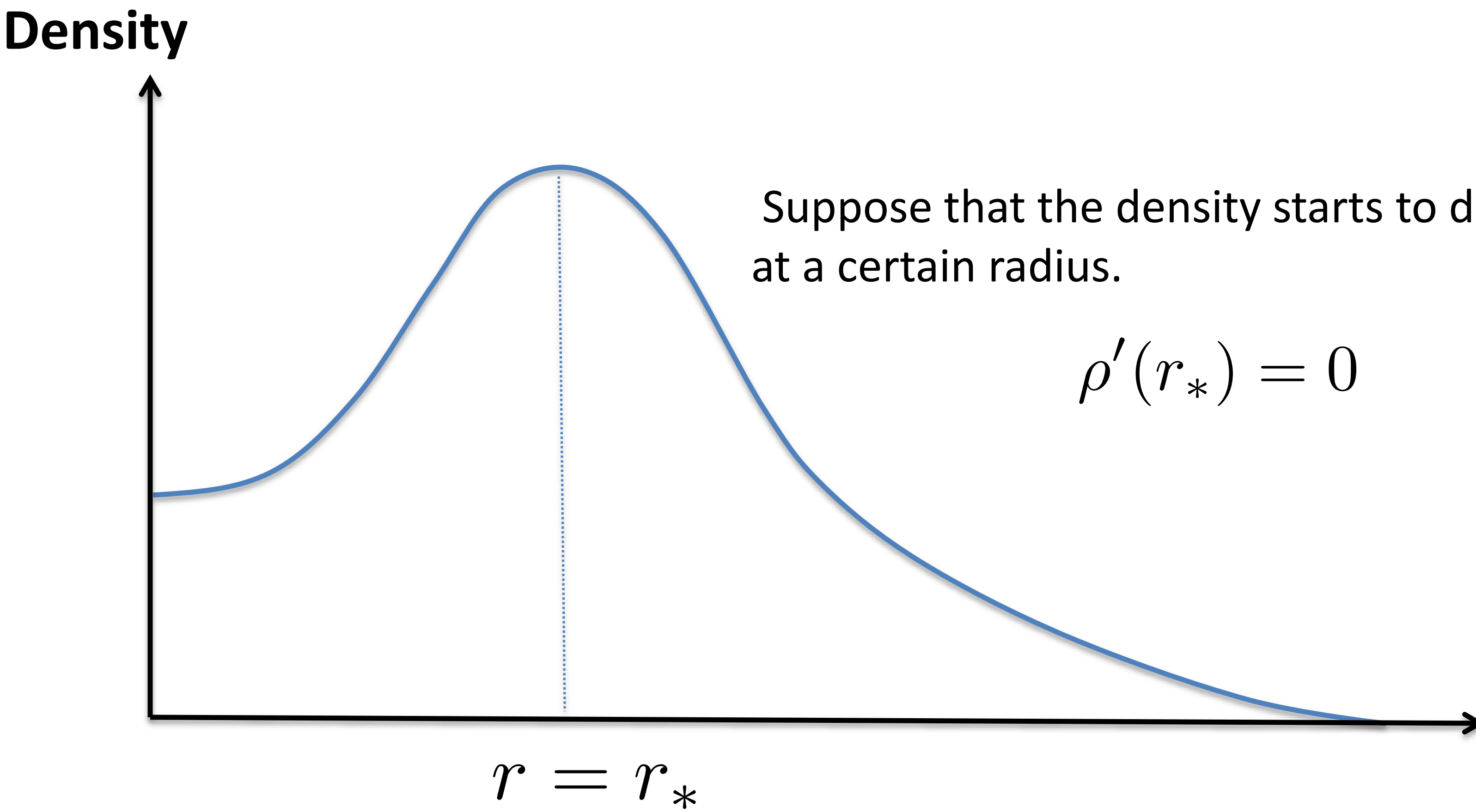
$$\frac{\mathrm{d}\tilde{\Phi}}{\mathrm{d}r} = G_N\left(\frac{M}{r^2} - \epsilon M''\right)$$

$$\simeq (1-6\epsilon)G_N\frac{M}{r^2}$$

The pressure has to increase $\longrightarrow$ The density has to increase

Universal bound on the modification

This property continues for higher radii.



Suppose that the density starts to decrease at a certain radius.

$$\rho'(r_*) = 0$$

$$M(r_*) = 4\pi \int_0^{r_*} r^2 \mathrm{d}r \rho(r) < \frac{4\pi}{3}\rho(r_*) r_*^3$$

& $$M''(r_*) = 4\pi[2\rho(r_*)r_* + \rho'(r_*)r_*^2] = 8\pi\rho(r_*)r_*$$

$$\Rightarrow \quad \left.\frac{\mathrm{d}P}{\mathrm{d}r}\right|_{r=r_*} = -G_N \rho \left(\frac{M(r_*)}{r_*^2} - \epsilon M''(r_*) \right) > 0$$

This contradicts the assumption $\rho'(r_*) = 0$

→ The density has to continue to increase for higher radii when it increases in the core.

No physically sensible profile for any reasonable EoS when $\epsilon > \dfrac{1}{6}$

Summary

- The disformal (nonlinear derivative) coupling causes **partial breaking of the Vainshtein screening mechanism**:

  A deviation from general relativity *inside* a source but not outside.

- **Large changes** in the stellar structure:

  Objects with a softer EoS will be a better probe of the modification.

- **A universal bound** on the amplitude of the modification can be obtained , independently of the details of the equation of state.