Existence and disappearance of conical singularities in GLPV theories

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2nd Mini-Workshop on Gravity and Cosmology

Paris, Oct 7, 2015

[with prof. Tsujikawa, prof. Kase, prof. Koyama]

Introduction - going beyond GR

- Inflation: geometric effect/scalar field or beyond SM
- Dark Energy (and dark matter): large distance modification of gravity?
- Theory of quantum gravity [Boulware, Deser: PRD 1972]

Dark energy models

- Models have been proposed to discuss dark energy
- f(R) models [Capozziello, 2002]
- Extra dimensionals models [Dvali, Gabadadze, Porrati 2000]
- Massive gravity/bigravity [de Rham, Gabadadze, Tolley 2010]
- Quintessence
- Extended scalar field Lagrangians [Horndeski 1974; Deffayet et al 2011]

Horndeski theories

- Most general scalar-tensor theories with 2nd order eoms
- It avoids Ostrogradski instability
- Only one single scalar is added
- Avoids unwanted/unstable degrees of freedom
- It includes gravitational theories: BD, f(R), Galileons, etc.
- Beyond Horndeski?

ADM decomposition of space-time

- Based on 3+1 decomposition of space-time [Arnowitt et al, 1959]
- Having a timelike scalar field we can choose uniform field gauge $\varphi=\varphi(t)$
- ADM splitting of 4D into 1+ 3,

$$ds^2=-N^2dt^2+h_{ij}(dx^i+N^i dt)(dx^j+N^j dt)$$

- Introducing ${}^{(3)}R, G_{ij}$ for the 3D metric
- Introducing $$K_{ij}=\frac{1}{2N}(\partial_t h_{ij}-D_i N_j-D_j N_i)$$

Beyond Horndeski: GLPV theory

[Gleyzes, Langlois, Piazza, Vernizzi 2014]

- Introduce the action $S = \int d^3x\, dt\, N \sqrt{h} \sum_{n=2}^{4} L_n$
- Build up ADM

$$L_2 = A_2(t, N),$$
$$L_3 = A_3(t, N) K,$$
$$L_4 = A_4(t, N)(K^2 - K^i{}_j K^j{}_i) + B_4(t, N)\,{}^{(3)}R, \quad L_5$$

- Horndeski theory as a sub-theory:

$$A_4 = 2X B_{4,X} - B_{4,} \qquad \alpha_H \equiv \frac{2X B_{4,X} - B_4 - A_4}{A_4}$$

Phenomenology

- Hamiltonian analysis (3 dof, no extra ghosts)
  [Gleyzes et al 2014; Lin et al 2014; Deffayet et al 2015]
- Cosmology: existence of late-time acc. solutions
- $G_{eff} < G_N$ [Tsujikawa 2015]
- Matter fields get modified speed of prop.
  [ADF, Koyama, Tsujikawa 2015]
- Perturbation could be massive [ADF, Koyama, Tsujikawa 2015]

Solar system constraints?

- The action originally found initially for time-like field
- Find the action in covariant form, undoing gauge
- Extend it to generic scalar field dynamics, e.g.

$$\mathcal{L}_2 = A_2(\varphi, X), \quad X = \nabla_\alpha \varphi \nabla^\alpha \varphi,$$

$$\mathcal{L}_3 = (C_3 + 2 X C_{3,X}) \nabla^2 \varphi + X C_{3,\varphi},$$

$$A_3 = 2|X|^{3/2} \left(C_{3,X} + \frac{B_{4,\varphi}}{X} \right),$$

$$\mathcal{L}_4 = \mathcal{L}_4(A_{4,} B_{4,} R, X, \nabla_\mu \varphi)$$

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Spherically symmetric space-time

[ADF, Kase, Tsujikawa 2015]

- Assume now spherical symmetry
- General functions $A_4 = A_4(\varphi, X), \;\; B_4 = B_4(\varphi, X)$
- Fix background gauge,

$$\varphi = \varphi(r),$$

$$ds^2 = -e^{2\Psi(r)} dt^2 + e^{2\Phi(r)} dr^2 + r^2 d\Omega^2$$

- Solve the eoms, look for bounds

Vacuum solution

- Consider vacuum solution and exact solutions $\varphi' = 0$
- Solutions

$$\exp(2\Psi) = \exp(-2\Phi) = 1 + \alpha_H + \frac{\Lambda}{6A_4} r^2 - \frac{2GM}{r}$$

- Ricci scalar

$$R = -\frac{2\Lambda}{A_4} - \frac{2\alpha_H}{r^2}$$

- Singularity is present even in the limit M = 0

Nature of the singularity

- Exact solution is strong hint to prove the existence of a singularity
- What kind of singularity is it? Consider M = 0, Λ = 0

$$ds^2 = -(1+\alpha_H)\,dt^2 + \frac{dr^2}{1+\alpha_H} + r^2\,d\Omega^2$$

- Conical singularity, unless $\alpha_H = 0$
- With matter field?

Interior Schwarzschild solution

- Consider matter $\rho=\rho_c$ for $r\leqslant r_0$, 0 otherwise
- For exact solutions consider $\varphi'=0$
- Solve the eom

$$e^{-2\Phi}=1+\alpha_H+\frac{\Lambda+\rho_c}{6A_4}r^2$$

- Singularity remains

$$R\rightarrow-\frac{2\alpha_H}{r^2}$$

General behaviour?

- Consider now general setup
- Assume analytic behaviour for the fields
- Expand around the origin

$$\rho=\rho_0+\frac{\rho_2}{2}r^2,\quad \varphi=\varphi_0+\frac{\varphi_2}{2}r^2,\Phi=\Phi_0+\frac{\Phi_2}{2}r^2,\quad \Psi=\Psi_0+\frac{\Psi_2}{2}r^{2,}$$

- 0-0 eom around the origin → conical singularity remains:

$$\frac{2}{r^2}\left[B_4(\varphi_0,0)+e^{-2\Phi_0}A_4(\varphi_0,0)\right]+O(r^0)=0,\qquad e^{-2\Phi_0}=1+\alpha_H(\varphi_0,0)$$

Trying to excape the singularity

- Let us try to avoid the singularity
- We need then $\alpha_H = 0$
- For X → 0, then $\alpha_H = \alpha(\varphi_c, 0) + O(X)$
- 1) Solve $\alpha(\bar{\varphi}_c, 0) = 0$ for some values of $\bar{\varphi}_c$, if exist
- 2) Theories which are Horndeski for X = 0

First option

- $\alpha(\bar{\varphi}_c, 0) = 0$ implies choosing two BCs at r = 0 for the field

$$\varphi'(r=0) = 0, \;\; \varphi(r=0) = \bar{\varphi}_c$$

- No degrees of freedom left for the field at infinity
- Second order ODEs on the background
- The system is over-constrained

Second option

- In the Taylor expansion of α_H in X the first term vanishes
- This condition is not protected by symmetries
- Need a particular functional form for Lagrangian
- Protected against quantum corrections?
- If present, would they spoil the compact objs solutions?

Classical way-out

- Considering classical regime
- Toy model

$$A_2 = -X/2, \;\; C_3 = 0,$$

$$A_4 = -\frac{1}{2} M_P^2 F_1(\varphi) + f_1(X),$$

$$B_4 = \frac{1}{2} M_P^2 F_1(\varphi) + f_2(X),$$

$$f_1(X) = a_4 X^m, \;\; f_2(X) = b_4 X^n$$

- Possible to find Vainshtein mechanism

Conclusions

- General functional form of GLPV does not allow static spherically symmetric compact objects
- Possible to find classical action which avoid the problem
- Stable against quantum corrections?
- Possible to implement the Vainshtein mechanism
- Cosmology? Heavy perturbation oscilations still existing?