Is bimetric gravity cosmologically viable?

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arXiv: [1412.5979], arXiv: [1505.01091] and arXiv: coming out soon

Struggling to give the graviton a mass

Building a consistent theory of massive gravity is a non-trivial problem: some historical steps in this process...

Good achievements

- linear Fierz-Pauli MG (1939)
- Vainshtein screening (1970)
- dRGT potential (deRham et al. 2011)
- Hassan-Rosen bigravity! (2012)

Problems

- vDVZ discontinuity (van Dam et al. 1970)
- Boulware-Deser ghost (1972)
- no FRW solutions (D'Amico et al. 2011)
- cosmologically ok? we will see...

Massive bigravity theory: two interacting gravitons Hassan, Rosen [1111.2070]

$$S = -\int d^4x\sqrt{-g}\left[\frac{M_g^2}{2}(R(g) - 2m^2 V(g,f)) + \mathcal{L}_m(g,\Phi)\right] - \int d^4x\sqrt{-f}\frac{M_f^2}{2}R(f)\,,$$

$$V(g,f) = \sum_{n=0}^{4}\beta_n e_n(X)\,, \qquad X = \sqrt{g^{-1}f}\,,$$

where

$$e_0 = \mathbb{I}, \quad e_1 = [X], \quad e_2 = \frac{1}{2}([X]^2 - [X^2])\,, \quad e_3 = \frac{1}{6}([X]^3 - 3[X][X^2] + 2[X^3]\,,$$
$$e_4 = \frac{1}{24}([X]^4 - 6[X]^2[X^2] + 8[X][X^3] + 3[X^2]^2 - 6[X^4]) = \det X\,.$$

- 2+5 dofs around every backgrounds $\rightsquigarrow$ good candidate for ghost-free MG!
- Dynamical dark energy density $\rightsquigarrow$ cosmological interest!

Background solutions (I)

- Homogeneous and isotropic background solutions Comelli et al. [1111.1983]

$$ds_g^2 = a^2(\tau)\left(-d\tau^2 + dx_i dx^i\right) , \quad ds_f^2 = b^2(\tau)\left(-c^2(\tau)d\tau^2 + dx_i dx^i\right) ,$$

$$H = \frac{\mathcal{H}}{a} = \frac{a'}{a^2} , \qquad H_f = \frac{\mathcal{H}_f}{b} = \frac{b'}{b^2\, c} , \qquad r = \frac{b}{a} .$$

- Energy-momentum tensor of a perfect fluid coupled with $g_{\mu\nu}$
- Friedmann equation for g

$$H^2 = \frac{8\pi G}{3}\,(\rho + \rho_g) , \qquad \rho_g = \frac{m^2}{8\pi G}\left(\beta_3\, r^3 + 3\beta_2\, r^2 + 3\beta_1\, r + \beta_0\right) .$$

- Bianchi constraint can be realized in two ways: two branches

$$m^2\left(\beta_3 r^2 + 2\beta_2 r + \beta_1\right)(\mathcal{H} - \mathcal{H}_f) = 0$$

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background: GR with effective Λ
perturbations: GW evolve differently than in ΛCDM [G.C. et al. in progress]

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dynamical dark energy density

Background solutions (II)

- Following set of independent equations for the background

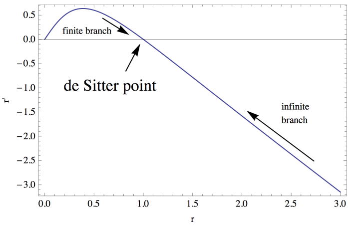
$$c = \frac{\mathcal{H} r + r'}{\mathcal{H} r}\,,$$

$$\rho_m = M_p^2 m^2 \left(\frac{\beta_1}{r} - 3\beta_1 r + \beta_4 r^2 \right) - \rho_r\,,$$

$$\frac{r'}{r} = \frac{-9\beta_1 r^2 + 3\beta_1 + 3\beta_4 r^3 + r M_p^{-2} m^{-2} \rho_r}{3\beta_1 r^2 + \beta_1 - 2\beta_4 r^3} \mathcal{H}\,,$$

$$\mathcal{H}^2 = a^2 m^2 \frac{\beta_1 + \beta_4 r^3}{3r}$$

$\rightsquigarrow$ we can extract value $r(\tau_0)$



finite branch: gradient instabilities in the scalar sector $\forall \beta_i$

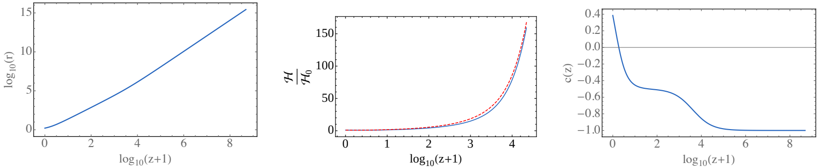
infinite branch: no exponential instabilities in the scalar sector for $\beta_1\beta_4$ model

Koennig et al. [1407.4331]

From a first analysis $\beta_1\beta_4$ submodel seems to be cosmologically promising ...

- Viable cosmological background evolution   Koennig et al. [1407.4331]
- Scalar perturbations free of (exponential) instabilities

...further investigations/tests needed!



G.C et al. [1412.5979]

Is the evolution of tensor perturbations cosmologically viable?

Tensor perturbations in $\beta_1\beta_4$ submodel

$$(1) \qquad h_g'' + 2\mathcal{H}\, h_g' + k^2 h_g + m^2 a^2 r\, \beta_1\, (h_g - h_f) = 0$$

$$(2) \qquad h_f'' + \left[2\left(\mathcal{H} + \frac{r'}{r}\right) - \frac{c'}{c}\right] h_f' + c^2 k^2\, h_f - m^2 \beta_1 \frac{c\, a^2}{r}\, (h_g - h_f) = 0$$

In the radiation dominated Universe

- $r \gg 1$, $r \propto a^{-2}$
- $c \simeq -1 \simeq cnst$
- coupling term in eq. (2) suppressed by a factor $1/r^2$ wrt the one in eq. (1)

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$$(2) \qquad h_f'' - 2\mathcal{H}\, h_f' + 1\cdot k^2\, h_f - \cancel{m^2 \beta_1 \frac{c\, a^2}{r} (h_g - h_f)} = 0 \qquad \textit{decoupled!}$$

In the radiation dominated Universe

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- $c \simeq -1 \simeq cnst$
- coupling term in eq. (2) suppressed by a factor $1/r^2$ wrt the one in eq. (1)
- $K^2 = m^2 \beta_1 a^2 r \simeq (0.05\mathcal{H}_0)^2 = cnst < k^2$

$$h_f = c_3 (k\tau)^2 y_1(ck\tau) - 3c_4 \frac{(k\tau)^2}{(k\tau_{\rm in})^3} j_1(ck\tau)$$

Tensor perturbations in $\beta_1\beta_4$ submodel

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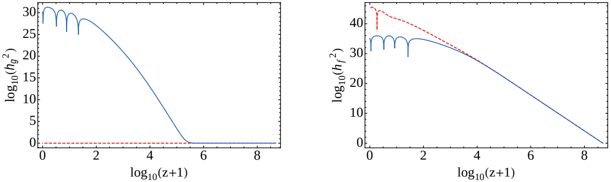
Hence, in the radiation era, on super-Hubble scales

$$h_g = c_1 + c_2 \left(\frac{\tau_{\text{in}}}{\tau}\right) \simeq A$$

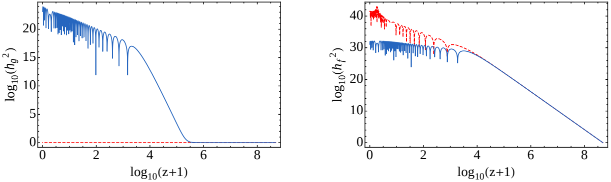
$$h_f = c_3 (k\tau)^2 y_1(ck\tau) - 3c_4 \frac{(k\tau)^2}{(k\tau_{\text{in}})^3} j_1(ck\tau) \simeq c_3 + c_4 \left(\frac{\tau}{\tau_{\text{in}}}\right)^3 \simeq B \left(\frac{\tau}{\tau_{\text{in}}}\right)^3$$

Initial conditions for the numerical evolution $A \simeq B$

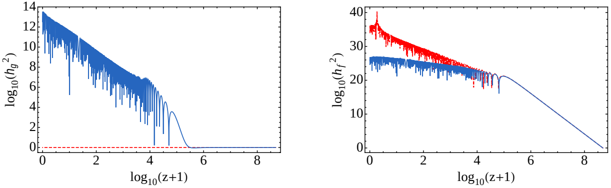
$A \simeq B \simeq 1$, $k = 10\mathcal{H}_0$



$A \simeq B \simeq 1$, $k = 100\mathcal{H}_0$



$A \simeq B \simeq 1$, $k = 2000\mathcal{H}_0$



G.C et al. [1412.5979]

Which is the origin of the instability?

Violation of the (generalized) Higuchi bound in the tensor sector!

Kinetic action for tensor modes

$$S_{\rm kin}^{(\pm 2)} \propto M_g^2 \int d^4x\, a^2 \left((h_g')^2 + \frac{r^2}{c} (h_f')^2 \right)$$

$$\downarrow$$

kinetic term for the tensor mode of f is positive-definite only if $c \geq 0$

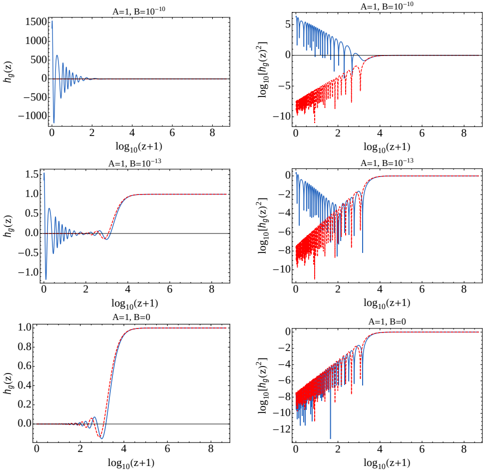
- h_f ghost-like along the entire cosmological evolution, until $z_c \simeq 0.9$
- instability transferred to the physical g-sector through the coupling
- instability is power-low, not exponential (FRW background)

The Higuchi bound is violated also in the scalar sector (saturated in late dS)

see De Felice et al. [1404.0008] for the general Higuchi condition in bigravity

see Lagos et al. [1410.0207] and G.C et al. [1412.5979] for $\beta_1 \beta_4$ submodel

Presence of instability $\leftrightarrow$ initial conditions for the tensor modes after inflation



instability does not show-up in the physical sector if $h_f(\tau_0) \ll h_g(\tau_0)$

$\downarrow$

stability condition quantified in term of fine-tuning B/A

see G.C et al. [1412.5979]

Which are physical values for B/A? $\leftrightarrow$ we need to study inflation in this model!

Embedding bigravity in inflation

- Single scalar field inflation with quadratic potential G.C. et al. [1505.01091]
- Toy model of reheating
- Evolution of primordial GW studied both analytically and numerically

In deep inflation $r_I \simeq H_I/H_0$, $c \simeq 1$. Canonically normalized variables

$$Q_g = M_p \, a \, h_g \, , \qquad Q_f = M_p \, b \, h_f \, .$$

$$Q_g'' + \left(k^2 - \frac{2}{\tau^2}\right) Q_g + \left(\frac{H_0}{H_I}\right) \frac{1}{\tau^2} Q_g = 0 \, ,$$
$$Q_f'' + \left(k^2 - \frac{2}{\tau^2}\right) Q_f - \left(\frac{H_0}{H_I}\right)^2 \frac{1}{\tau^2} Q_g = 0 \, .$$

$$Q_g = \frac{1}{\sqrt{2\,k}} \exp\left(-i\,k\tau\right) , \qquad Q_f = \frac{1}{\sqrt{2\,k}} \exp\left(-i\,k\tau\right) , \quad \text{for} \quad |k\tau| \gg 1 \, .$$

$$P_{h_g}(z,k) \simeq \left(\frac{H_I}{M_p}\right)^2 \simeq r_I^2 \, P_{h_f}(z,k) \, , \quad |k\,\tau| \ll 1 \quad \text{huge suppression } \left(H_I/H_0\right)^2$$

What can we say about the instability?

Let us examine the behavior of the dangerous unstable mode h_f

End of inflation

$$h_f = \frac{1}{r_I}\frac{H_I}{M_p}\left[1+\left(\frac{k}{H_I}\right)^2(1+z_{end})^2\right]$$

$$\left|\frac{\text{decaying}}{\text{constant}}\right| \simeq \left(\frac{k}{H_0}\right)^2\frac{H_0}{H_I}\,,$$

Radiation domination

$$h_f = c_3 + c_4\frac{1}{(1+z)^3}$$

growing mode effectively not excited!

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$$\left|\frac{\text{decaying}}{\text{constant}}\right| \simeq \left(\frac{k}{H_0}\right)^2 \frac{H_0}{H_I},$$

growing mode effectively not excited!

$$\left(\frac{h_f}{h_g}\right)(\tau_0) \leq r_I^{-1}\left(\frac{H_0}{H_I}\right)^{-1/2} \simeq \left(\frac{H_0}{H_I}\right)^{1/2} \ll 1$$

no meaningful bound is obtained!

$\rightsquigarrow$ The "dark-mode" h_f never influences the physical one
$\rightsquigarrow$ Evolution of the tensor sector is ΛCDM-like!

In agreement with Johnson, Terrana [1503.05560]

Is $\beta_1\beta_4$ viable?

No constraints are coming from the tensor sector!

Results of our analysis:

- Evolution of GW is ΛCDM-like for every inflation scale
- Mild constraint on the inflation scale coming from the vector sector
- Higuchi ghost in the scalar sector, even during de Sitter inflation!

What to do to make this branch viable?

Fix the problem of the "primordial" scalar ghost ...

- in absence of a mechanism to modify the scalar sector in the UV the branch is ruled out...

Is this the end of bigravity?

Some ideas on the market...

- Saving the infinite branch $\beta_1\beta_4$ (from scalar Higuchi ghost)
  - Higuchi ghost: modify scalar sector in the UV (new couplings? β_i functions of time?)

- Saving the finite branch (from gradient instabilities)
  - cure instabilities non-linearly (Vainshtein screening) Mortsell et al. [1506.04977]
  - pushing instability at unobservable scales with hierarchy between the two Planck masses Akrami et al. [1503.07521]

- Alternative approaches:
  - Non-FRW background Nersisyan et al. [1502.03988]
  - doubly coupled bigravity (but there are problems ...) Akrami et al. [1306.0004], Gumrukcuoglu et al. [1501.02790]
  - other modifications: varying mass, Lorentz violation ...

Thank you!

Quantifying the fine-tuning of initial conditions

Standard inflation:

tensor to scalar ratio (for typical amplitude of GW, $A \simeq H_{in}/M_p$)

$$r = \frac{A^2}{A_s^2} = 16\epsilon$$

Bigravity:

super-horizon modes are not anymore constant in radiation and matter

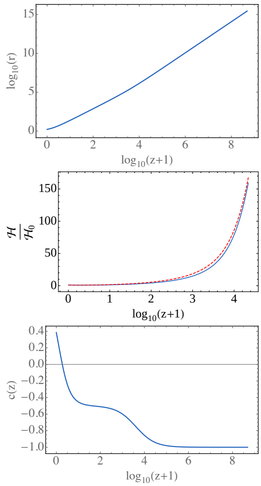
$$r \simeq 16\epsilon \left(\frac{T_{in}}{T_{eq}}\right)^6 \left(\frac{T_{eq}}{T_0}\right)^3 \frac{B^2}{A^2}$$
$$\simeq 0.3 \left(\frac{T_{\rm in}}{1 \rm GeV}\right)^{10} \left(\frac{B}{A}\right)^2$$

$\rightsquigarrow$ requiring $r \simeq 0.1$ we get an upper bound for $B/A = h_f(\tau_{end})/h_g(\tau_{end})$

$\rightsquigarrow$ we need to embed the model in inflation to find the realistic values for A and B

Apparent singularity in $\beta_1\beta_4$ submodel

- Viable cosmological background evolution
- Viable scalar perturbations in matter- and dark energy- dominated eras



- Evolution of the Hubble $\mathcal{H}$ is ΛCDM-like
- The lapse c changes sign at $z_c \simeq 0.9$

singularity in the f-metric?

Yes, but for the Bianchi constraint, $\mathcal{H}_f = \mathcal{H}$ remains finite

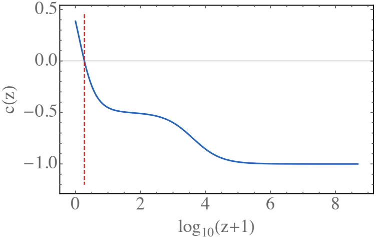
$\downarrow$

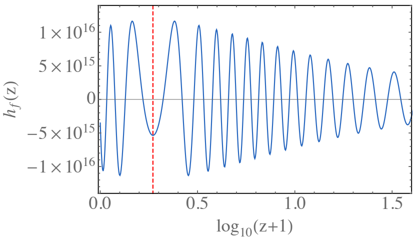
no physical observable diverges!

G.C et al. [1412.5979]

Physical interpretation for $c < 0$ (I)

$k = 100\mathcal{H}_0\,, \quad A \simeq B = 1$





Physical interpretation $c < 0$

- time for the f-metric sector goes in the opposite direction wrt the one of g
- scale factor b is decreasing when a is increasing since for the Bianchi constraint $\mathcal{H}_f = b'/bc = \mathcal{H} = a'/a$
- $\rightsquigarrow$ amplitude of tensor perturbations for the f-metric are growing in time

Physical interpretation for $c < 0$ (II)

Kinetic Lagrangian

$$S_{\text{kin}}^{(\pm 2)} \propto M_g^2 \int d^4x\, a^2 \left((h_g')^2 + r^2 \frac{\sqrt{c^2}}{c^2} (h_f')^2 \right)$$

- $\sqrt{c^2}$ comes from the square root of the determinant of the f-metric
- we can choose either c or $-c$ for $\sqrt{c^2}$
- ... but we can not choose $|c|$ in order to have a differentiable action
- to reproduce a viable phenomomenology $\rightsquigarrow$ positive square root

$$\downarrow$$

$$S_{\text{kin}}^{(\pm 2)} \propto M_g^2 \int d^4x\, a^2 \left((h_g')^2 + r^2 \frac{1}{c} (h_f')^2 \right)$$

the kinetic term for the f-sector is positive definite only if

$$c \geq 0$$