Bound states in dark-matter physics

Kallia Petraki
Université Pierre et Marie Curie, LPTHE, Paris

GDR, Grenoble 2015

Long-range interactions

mediated by

a light or massless particle

Bound states

Long-range interactions

- **Iconoclastic-ism:**
  Most DM research has focused on contact interactions.
  Prototypical WIMP scenario: $m_{DM} \sim m_{mediators} \sim 100$ GeV.
  What happens when $m_{DM} \gg m_{mediators}$?

- **Long-range interactions appear in a variety of DM theories:**
  - Self-interacting DM
  - Asymmetric DM
  - Dissipative DM
  - DM explanations of galactic positrons
  - DM explanations of IceCube PeV neutrinos
  - WIMP DM with m_{DM} > few TeV ! [Hisano et al. 2002]

Hidden sector DM

Minimal DM [Cirelli et al.]
LHC implications for SUSY
Direct/Indirect detection bounds

Long-range interactions

Complications

- **Large logarithmic corrections:**

  $$\delta\sigma \sim \ln(m_{DM} / m_{mediator}), \quad \ln\ln(m_{DM} / m_{mediator})$$

  → resummation techniques etc. [Baumgart et al. (2014)]

- **Non-perturbative effects:**
  Sommerfeld enhancement in the non-relativistic regime.
  Usually invoked for DM annihilation into radiation, but in fact affects *all* processes with same initial state.

- **More processes:**
  Radiative formation of bound states [Sommerfeld enhanced]

Bound states — Phenomenological implications

- **Asymmetric DM → stable bound states**
  - Kinetic decoupling of DM from radiation, in the early universe
  - DM self-scattering in haloes (screening)
  - Indirect detection signals (radiative level transitions)
  - Direct detection signals (screening, inelastic scattering)

- **Symmetric DM → unstable bound states**
  formation + decay = extra annihilation channel
  - Relic abundance [von Harling, KP (2014); Ellis et al. (2015)]
  - Indirect detection

Relic density of symmetric DM

[von Harling, KP (2014)]

Toy model: **Dirac fermions (X, $\bar{X}$) of mass *m*,**
coupled to a massless dark photon γ, with dark fine-structure constant α.

Very important parameter: $\zeta = \alpha / v_{rel}$

Annihilation

$$X + \bar{X} \rightarrow \gamma + \gamma$$

$$\sigma_{ann} v_{rel} = \sigma_0 S_{ann}(\zeta)$$

$$\sigma_0 = \pi \alpha^2 / m^2$$

$$S_{ann}(\zeta) = \frac{2\pi\zeta}{1 - e^{-2\pi\zeta}}$$

$$S_{ann}(\zeta \ll 1) \simeq 1$$

$$S_{ann}(\zeta \gtrsim 1) \simeq 2\pi\zeta$$

Bound state formation and decay

$$X + \bar{X} \rightarrow (X \bar{X})_{bound} + \gamma$$

$$(X \bar{X})_{bound} \rightarrow 2\gamma \quad or \quad 3\gamma$$

$$\sigma_{BSF} v_{rel} = \sigma_0 S_{BSF}(\zeta)$$

$$\sigma_0 = \pi \alpha^2 / m^2$$

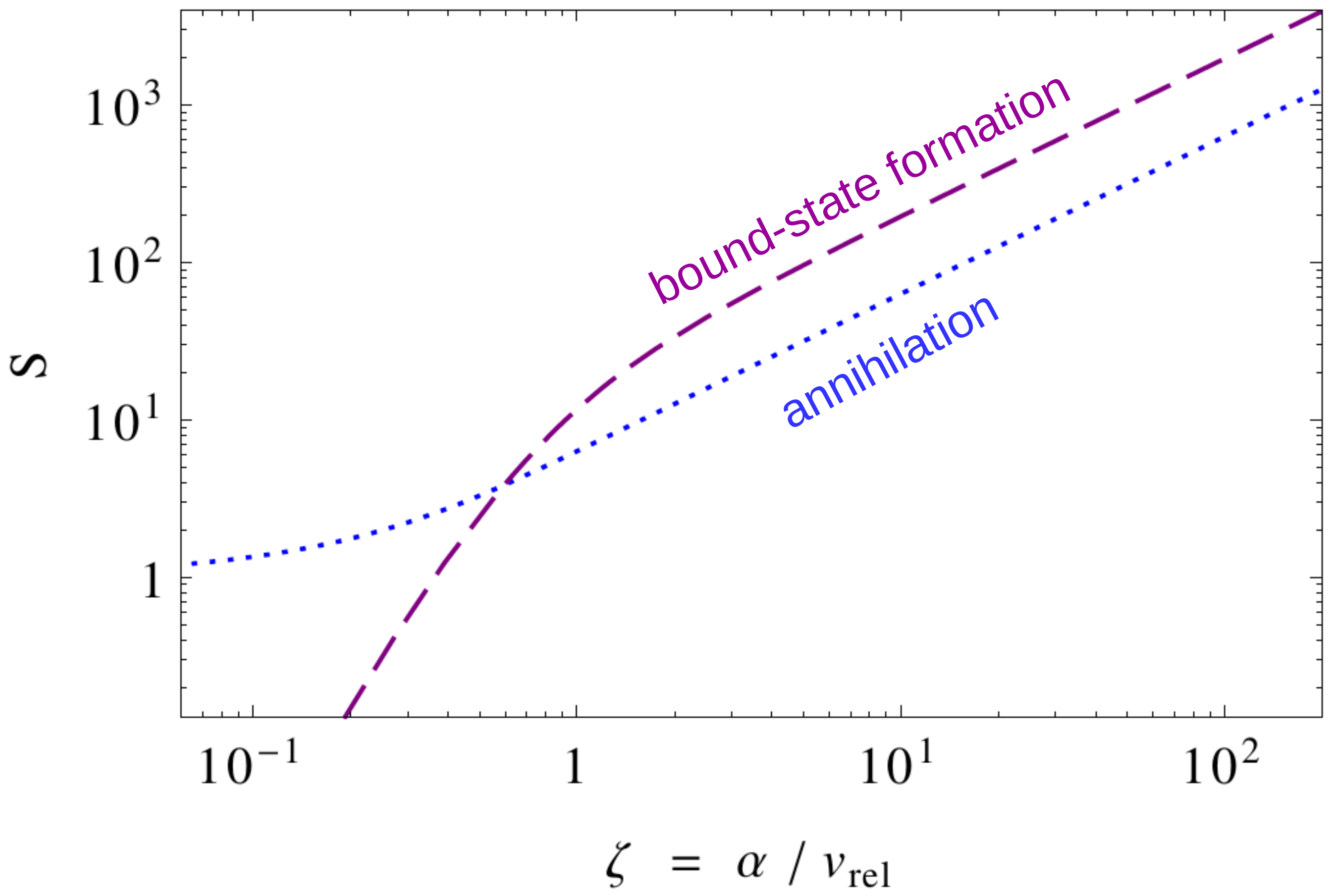
$$S_{BSF}(\zeta) = \left[\frac{2^9}{3 e^{4\zeta \operatorname{arccot}(\zeta)}} \frac{\zeta^4}{(1+\zeta^2)^2} \right] \frac{2\pi\zeta}{1 - e^{-2\pi\zeta}}$$

$$S_{BSF}(\zeta \ll 1) \simeq \frac{2^9 \zeta^4}{3} \ll 1$$

$$S_{BSF}(\zeta \gtrsim 1) \simeq \frac{2^9}{3 e^4} \times 2\pi\zeta \simeq \mathbf{3.13 \times S_{ann}}$$

Relic density of symmetric DM

[von Harling, KP (2014)]



BSF dominates over annihilation everywhere the Sommerfeld effect is important ($\zeta > 1$) !

Relic density of symmetric DM

[von Harling, KP (2014)]

$$\frac{dn_X}{dt} + 3Hn_X = -\left(n_X^2 - n_X^{eq\ 2}\right)\langle\sigma_{ann} v_{rel}\rangle - n_X^2\langle\sigma_{BSF} v_{rel}\rangle + \left(n_{\uparrow\downarrow} + n_{\uparrow\uparrow}\right)\Gamma_{ion}$$

$$\frac{dn_{\uparrow\downarrow}}{dt} + 3Hn_{\uparrow\downarrow} = +\frac{1}{4}n_X^2\langle\sigma_{BSF} v_{rel}\rangle - n_{\uparrow\downarrow}\left(\Gamma_{ion} + \Gamma_{decay,\uparrow\downarrow}\right)$$

$$\frac{dn_{\uparrow\uparrow}}{dt} + 3Hn_{\uparrow\uparrow} = +\frac{3}{4}n_X^2\langle\sigma_{BSF} v_{rel}\rangle - n_{\uparrow\uparrow}\left(\Gamma_{ion} + \Gamma_{decay,\uparrow\uparrow}\right)$$

$(X\bar{X})_{\uparrow\downarrow} \rightarrow 2\gamma$: $\quad \Gamma_{decay,\uparrow\downarrow} = \alpha^5 (m/2)$

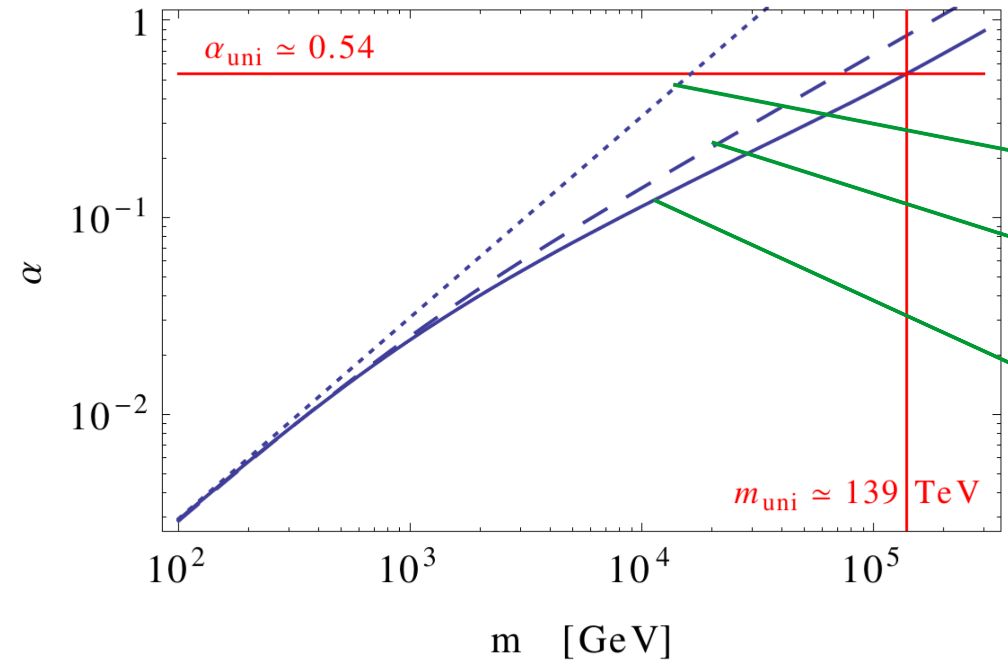
$(X\bar{X})_{\uparrow\uparrow} \rightarrow 3\gamma$: $\quad \Gamma_{decay,\uparrow\uparrow} = \frac{4(\pi^2-9)}{9\pi}\alpha^6(m/2)$

$(X\bar{X})_{\uparrow\downarrow \text{ or } \uparrow\uparrow} + \gamma \rightarrow X + \bar{X}$: $\quad \Gamma_{ion}(T) = \frac{2}{(2\pi)^3} 4\pi \int_0^\infty d\omega \frac{\omega^2}{e^{\omega/T}-1}\sigma_{ion}(\omega)$

related to σ_{BSF}

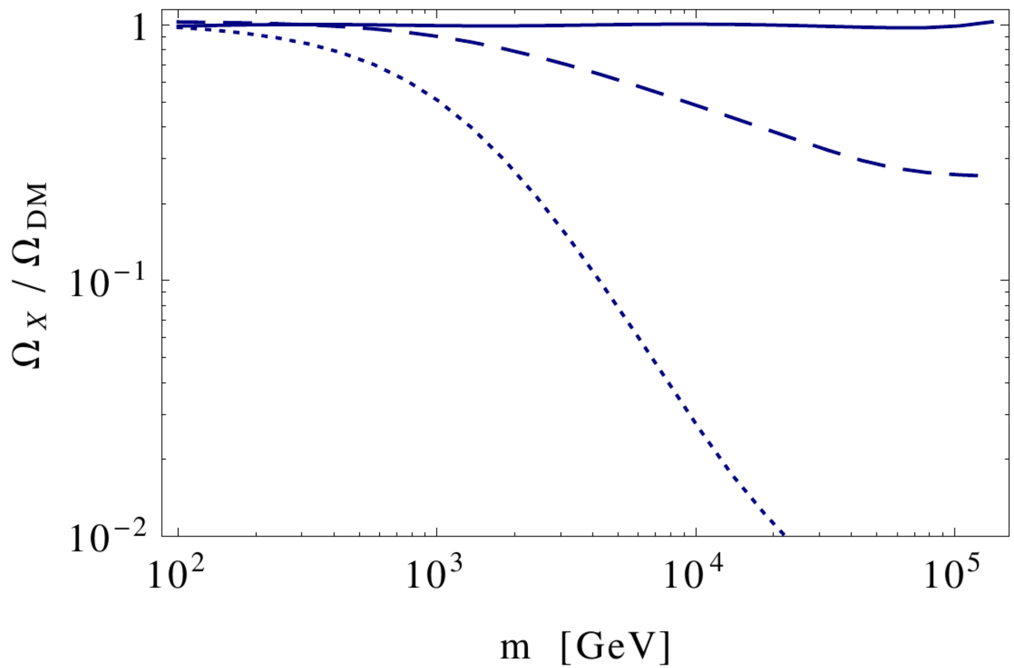
BSF important only when $\Gamma_{decay} > \Gamma_{ion}(T)$

Relic density of symmetric DM
[von Harling, KP (2014)]



Determination of α

- Perturbative annihilation
- Sommerfeld-enhanced annihilation
- Sommerfeld-enhanced annihilation and BSF



Effect on the relic density

As large as a factor of 3 >> 1% !

Long-range interactions

(Partial-wave) Unitarity

$$\sigma_{inel,J}\, v_{rel} \leqslant \frac{(2J+1)\,4\pi}{m^2\, v_{rel}}$$

feature of long-range inelastic processes

- Implies upper limit on mass of thermal relic DM. [Griest, Kamionkowski (1990)]

- Can be realised only if DM possesses long-range interactions: [von Harling, KP (2014)]
  At $\alpha >> v_{rel}$: $\sigma_{ann} v_{rel}$ and $\sigma_{BSF} v_{rel}$ ~ # $\alpha^3 / (m^2 v_{rel})$. Unitarity realised (perturbatively) for α ~ 0.5, i.e. well below the perturbativity limit (α ~ π or 4π).
  Non-self-conjugate DM: $m_{UNI} < 83$ TeV → 139 TeV for ***s*-wave** processes.

- Close to the unitarity limit, all partial waves must have the same v-dependence.
  Confirmed by explicit calculations for long-range interactions. [Cassel (2009)]
  For annihilation, higher *J* → higher powers of α.

- For BSF, higher partial waves give significant or dominant contribution, [KP, Postma, Wiechers (2015)]
  e.g. BSF with vector boson emission: $\mathcal{M} \sim \sin\theta$ → *J*=0: 62%, *J*=2: 24% ...
  → Unitarity limit on m_{DM} even higher?

Indirect detection of symmetric DM

BSF implies:

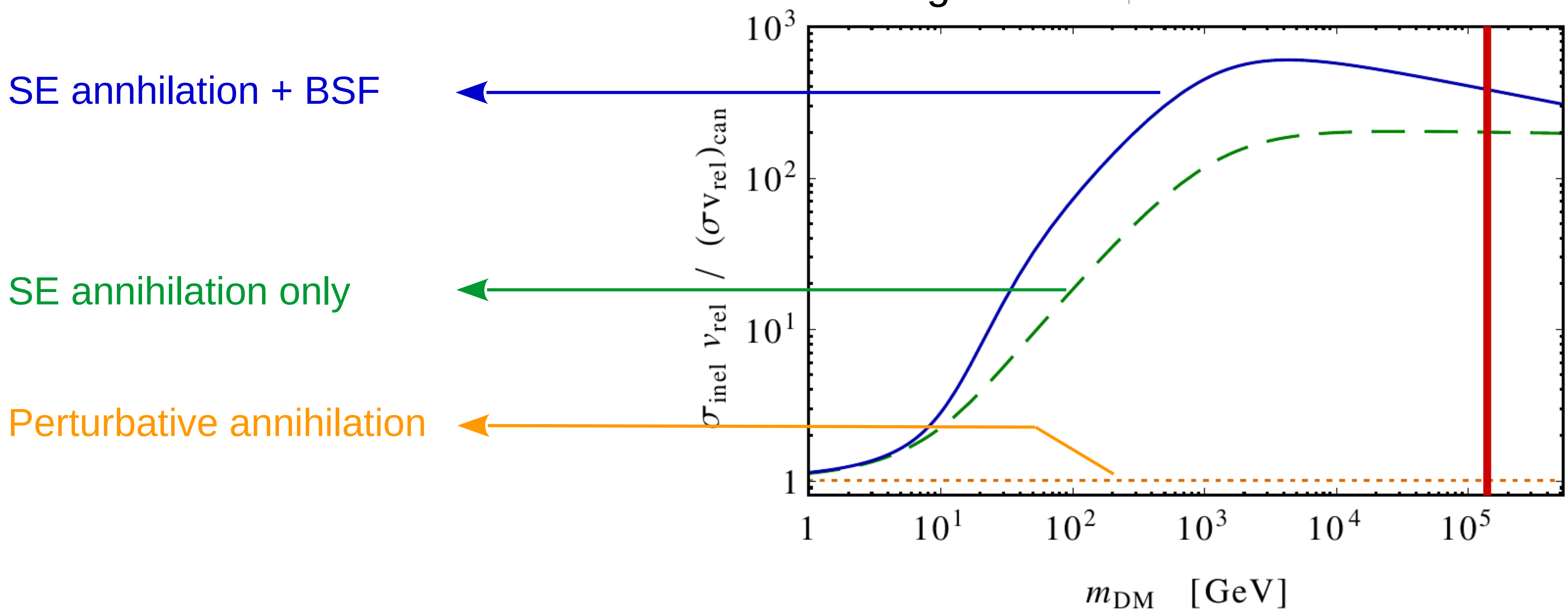
- Enhanced signal rate, $\sigma_{BSF} > \sigma_{ann}$.
- Features in the spectrum:

$X + \bar{X} \rightarrow (X\bar{X})_{bound} + \gamma$ **[$= \alpha^2 m_{DM}/4$]**

$(X\bar{X})_{bound,\uparrow\downarrow} \rightarrow 2\gamma$ **[= 2 x m_{DM}]**

$(X\bar{X})_{bound,\uparrow\uparrow} \rightarrow 3\gamma$ **[= 3 x (2/3)m_{DM}]**

Milky Way: $v_{rel} \sim 10^{-3}$.
Large Sommerfeld enhancement



Bound states

Generalisations needed

- Massive mediator – Yukawa potential
- Different interactions (e.g. scalar mediator)
- Non-confining non-Abelian theories,
  e.g. electroweak interactions

Bound states

Reform and calculate

[KP, Postma, Wiechers (2015)]

Develop QFT formalism instead of QM, even though we care about the non-relativistic and weak-coupling regime:

- Can accommodate non-Abelian interactions.
- Allows systematic inclusion of higher-order corrections in the coupling strength and in the momentum transfer (particularly important when leading order terms cancel).

Radiative capture, in QFT

[KP, Postma, Wiechers (2015)]

$$\chi_1 + \chi_2 \rightarrow \mathcal{B} + \phi$$

- Why radiative?
  Need to get rid of the kinetic + binding energies.
  (Similar to internal bremsstrahlung, where kinetic energy only is radiated.)

- How do we describe the incoming (scattering) state?
  Long-range interaction → Two-particle state (branch cut) ≠ Two plane waves,
  → *Sommerfeld effect* (in the non-relativistic regime) Sommerfeld 1931

- How do we describe the bound state?
  One-particle state (pole in scattering amplitude), Bethe & Salpeter 1957
  with the quantum charges of χ_1 & χ_2.

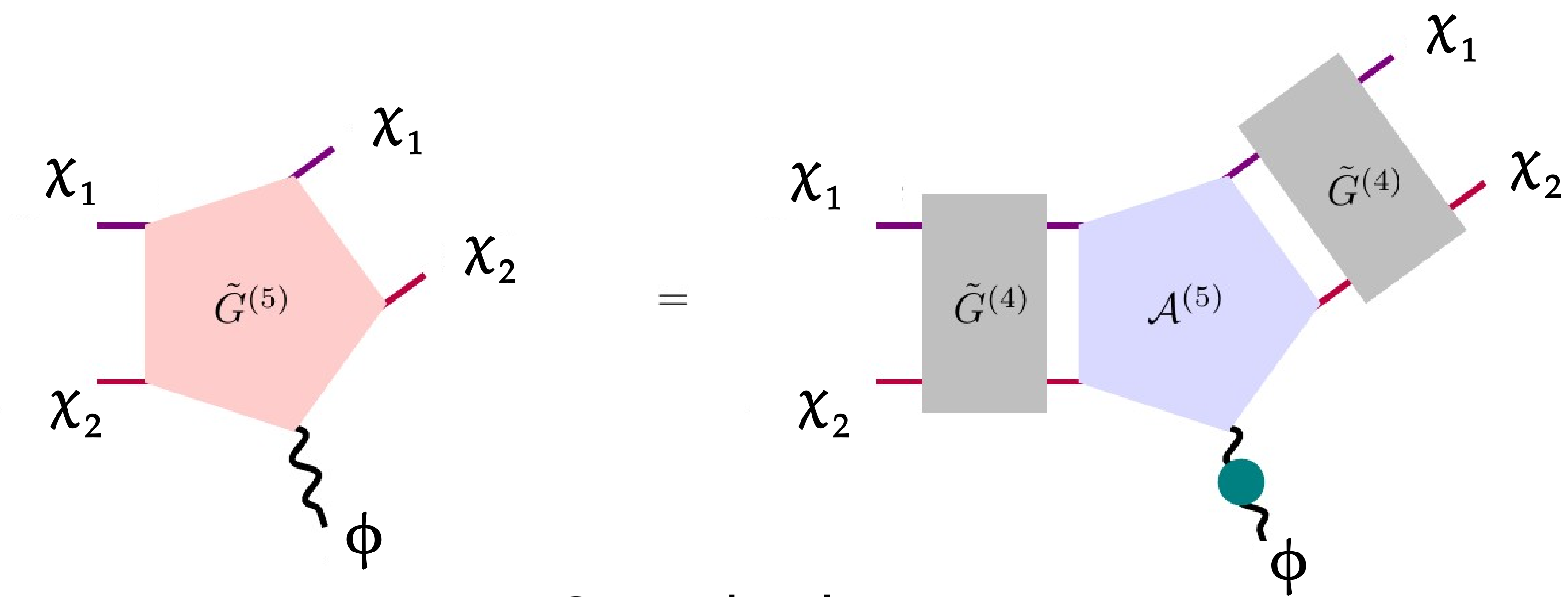
- How does the radiative vertex get into the picture?
  Perturbative part of the amplitude. Feynman diagrams

- **How do we put everything together to calculate the amplitude?** LSZ reduction

Radiative capture, in QFT

[KP, Postma, Wiechers (2015)]

$$\chi_1 + \chi_2 \rightarrow \mathcal{B} + \phi$$



LSZ reduction

5-point function ⟺ BSF amplitude

branch-cut & pole structure

Bound states

The 4-point function: **Decomposition**

$$G^{(4)} = \langle \Omega | T \chi_1(x_1) \chi_2(x_2) \chi_1^\dagger(y_1) \chi_2^\dagger(y_2) | \Omega \rangle$$

$$1 \sim \sum_n \int d^3Q \, |\mathcal{B}_{Q,n}\rangle \langle \mathcal{B}_{Q,n}| + \int d^3q \, d^3Q \, |\mathcal{U}_{Q,q}\rangle \langle \mathcal{U}_{Q,q}|$$

bound states

scattering (unbound) states

expectation value of relative momentum

$$G^{(4)}(Q) \sim \sum_n \frac{i\Psi_{Q,n}\Psi^*_{Q,n}}{Q^0 - \omega_{Q,n} + i\epsilon} + \int d^3q \, \frac{i\Phi_{Q,q}\Phi^*_{Q,q}}{Q^0 - \omega_{Q,q} + i\epsilon}$$

where

$$\Psi_{Q,n}(x_1, x_2) \equiv \langle \Omega | T \chi_1(x_1) \chi_2(x_2) | \mathcal{B}_{Q,n} \rangle$$

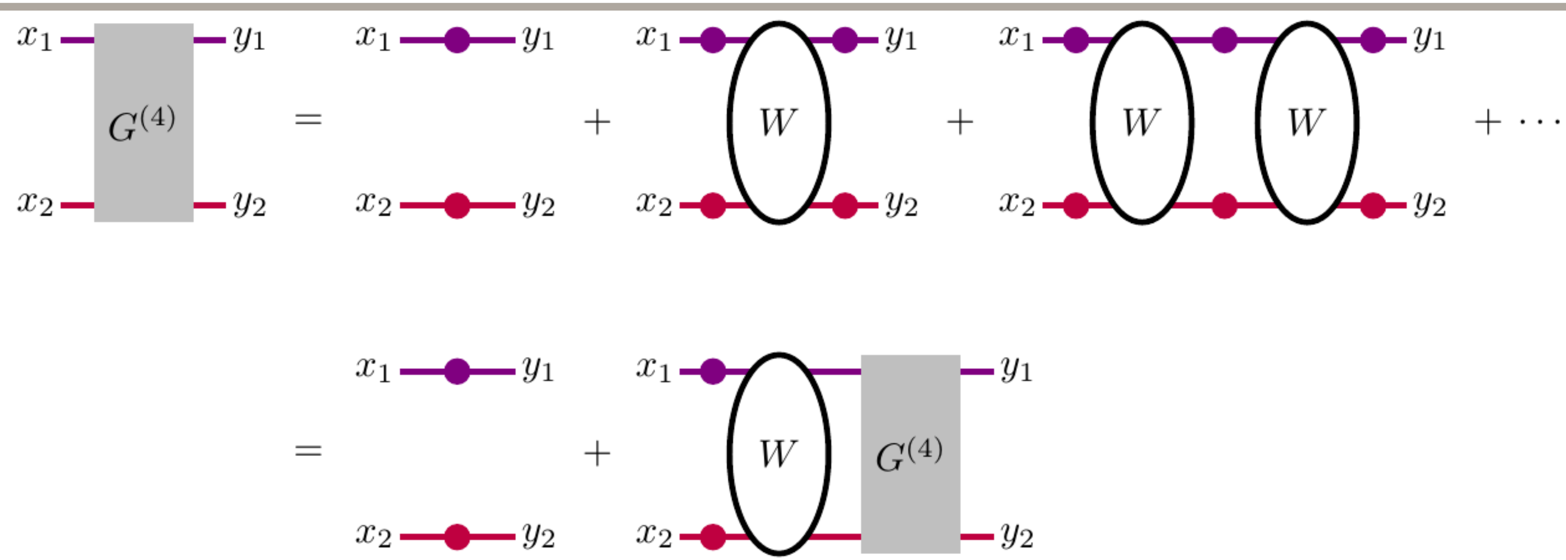
$$\Phi_{Q,q}(x_1, x_2) \equiv \langle \Omega | T \chi_1(x_1) \chi_2(x_2) | \mathcal{U}_{Q,q} \rangle$$

Bethe-Salpeter wavefunctions

By fixing the energy Q^0, we pick out a state.

Bound states

The 4-point function: **Dyson-Schwinger equation**



+

$G^{(4)}$ decomposition into bound & scattering-state contributions

=

Bethe – Salpeter equations for bound & scattering state WFs

$$\tilde{\Psi}_{\mathbf{Q},n}(p) = S(p;Q) \int \frac{d^4k}{(2\pi)^4}\, \tilde{W}(p,k;Q)\, \tilde{\Psi}_{\mathbf{Q},n}(k)$$

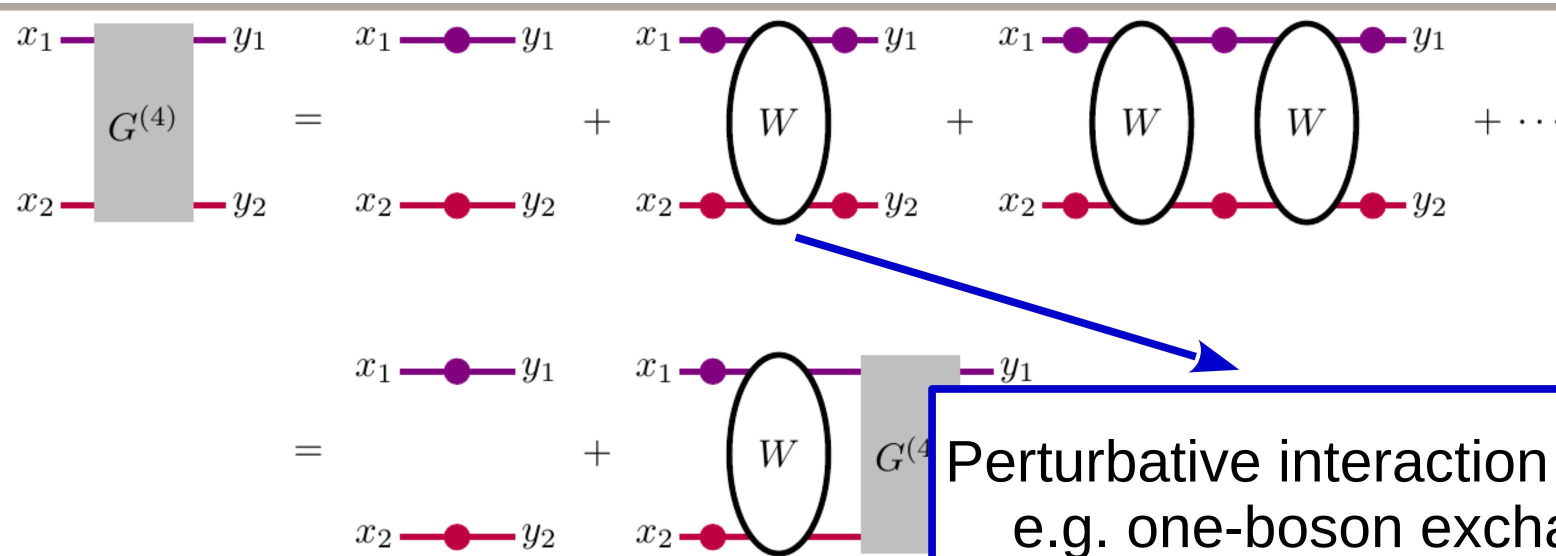
$$\tilde{\Phi}_{\mathbf{Q},\mathbf{q}}(p) = S(p;Q) \int \frac{d^4k}{(2\pi)^4}\, \tilde{W}(p,k;Q)\, \tilde{\Phi}_{\mathbf{Q},\mathbf{q}}(k)$$

Sommerfeld effect!

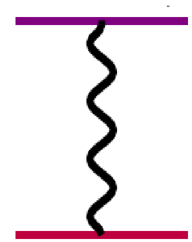
Reduce to Schrodinger eq in non-relativistic regime

Bound states

The 4-point function: **Dyson-Schwinger equation**



Perturbative interaction kernel, e.g. one-boson exchange



+

$G^{(4)}$ decomposition into bound & scat

=

Bethe – Salpeter equations for bound & scattering state WFs

$$\tilde{\Psi}_{\mathbf{Q},n}(p) = S(p;Q) \int \frac{d^4k}{(2\pi)^4}\ \tilde{W}(p,k;Q)\ \tilde{\Psi}_{\mathbf{Q},n}(k)$$

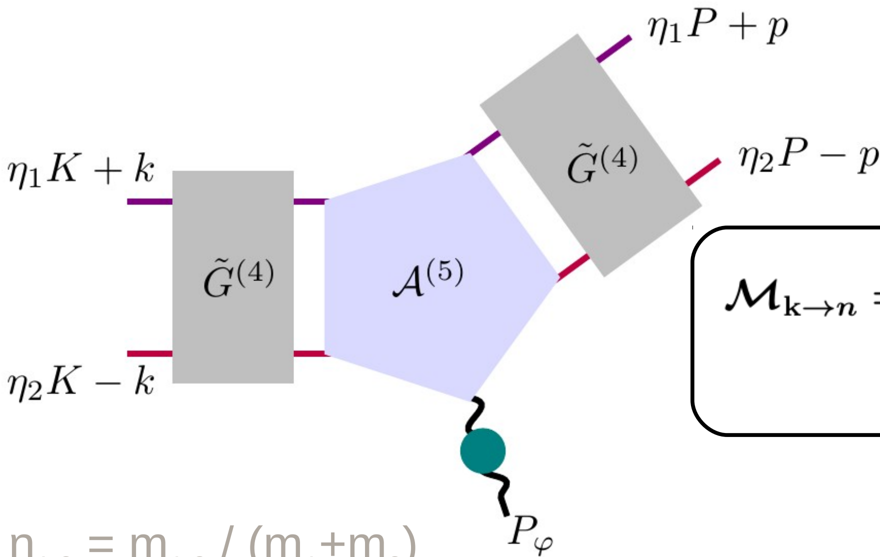
$$\tilde{\Phi}_{\mathbf{Q},\mathbf{q}}(p) = S(p;Q) \int \frac{d^4k}{(2\pi)^4}\ \tilde{W}(p,k;Q)\ \tilde{\Phi}_{\mathbf{Q},\mathbf{q}}(k)$$

Sommerfeld effect!

Reduce to Schrodinger eq in non-relativistic regime

Bound states

LSZ reduction → Amplitude



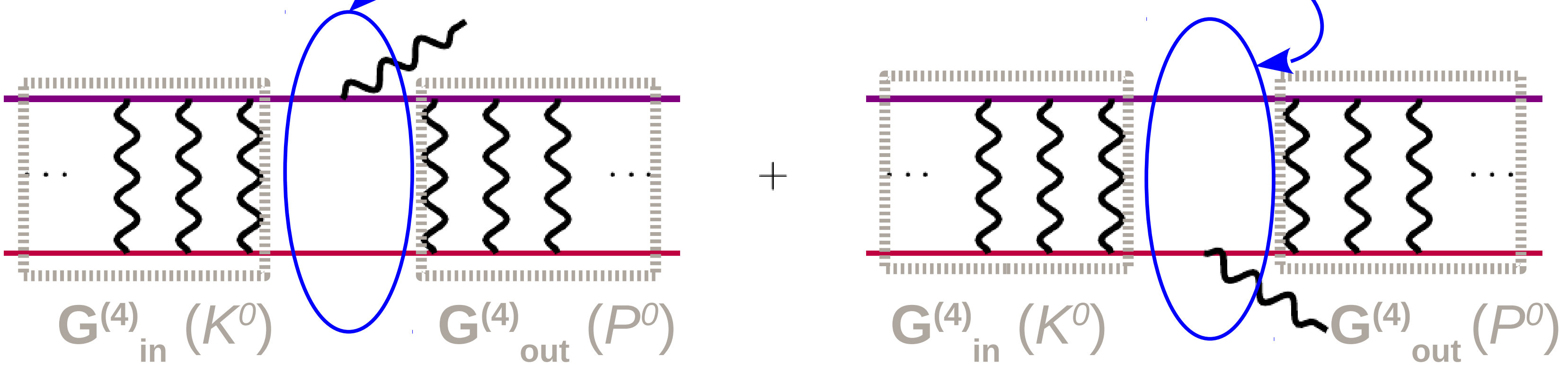
$\eta_{1,2} = m_{1,2} / (m_1+m_2)$

non-perturbative physics

$$\mathcal{M}_{k\to n} = \int \frac{d^4p}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}\ \tilde{\Psi}^*_{P,n}(p)\ \tilde{\Phi}_{K,k}(q)\times$$
$$\mathcal{A}^{(5)}(P_\varphi, \eta_1 P + p, \eta_2 P - p;\ \eta_1 K + q, \eta_2 K - q)$$

Perturbative *off-shell* transition amplitude

One-boson-exchange kernel:



Setup generalisable to multiple interacting quartetos $\chi_i + \chi_j \rightarrow \chi_k + \chi_l$
with interaction kernels $\mathbf{W}_{ijkl}$, described by $\mathbf{G}^{(4)}_{ijkl}$.

Conclusion

- **Long-range interactions imply bound states.**

  If they exist, they likely form!
  If they form, we need to predict it.

- **Need tools to calculate bound-state related processes.**

  Non-relativistic regime relevant for DM phenomenology.
  Nevertheless, QFT formalism preferable or even necessary.

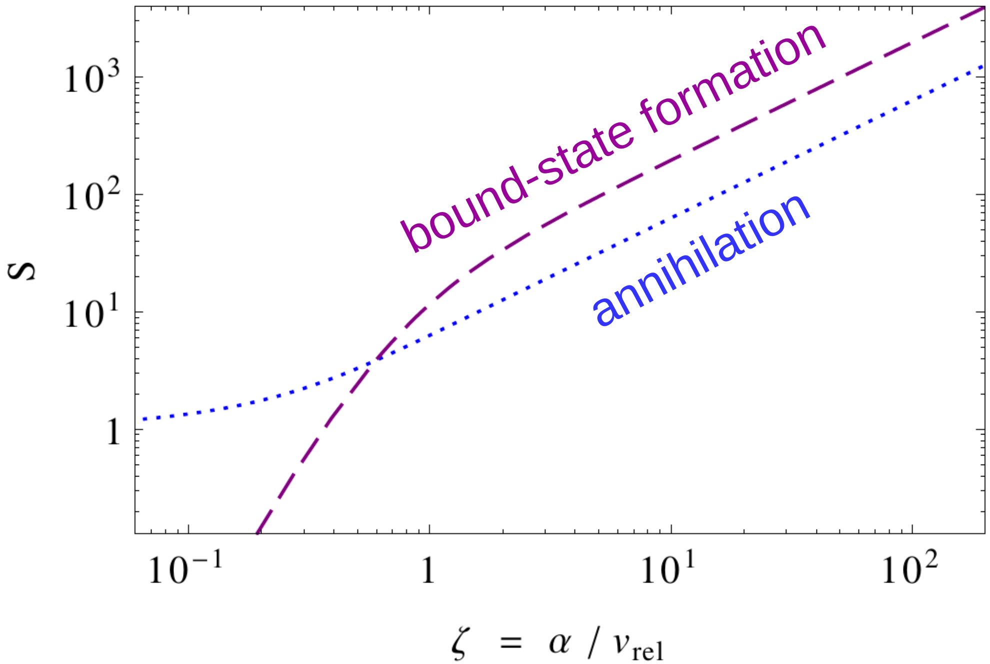
  Can accommodate DM residing in multiplets, and multiple interaction kernels. Can reliably yield higher-order corrections.

extra slides

Relic density of symmetric DM

[von Harling, KP (2014)]

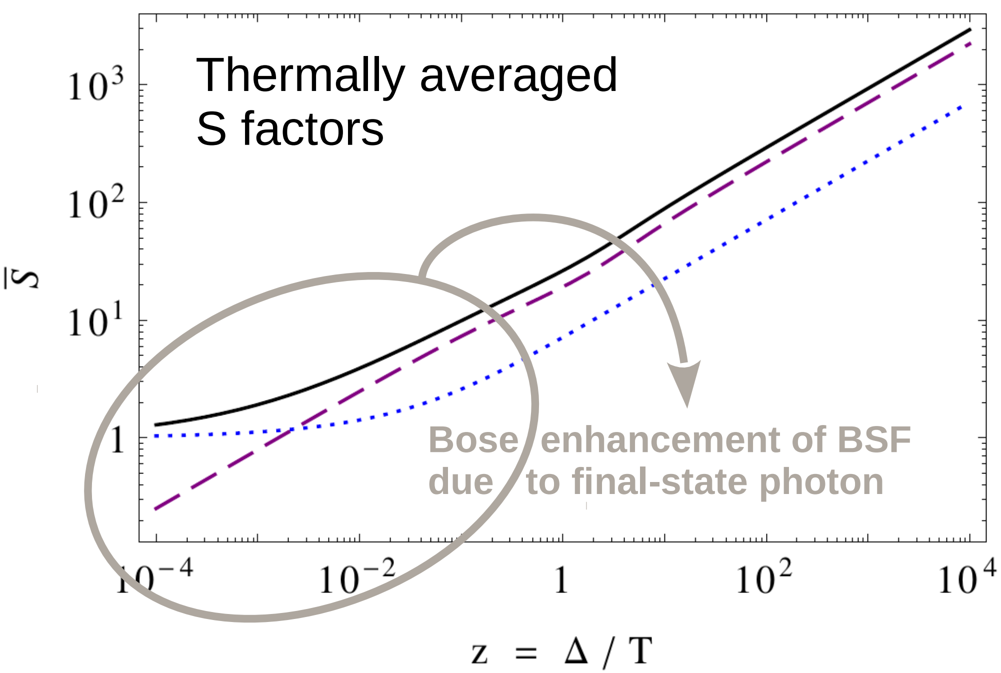
Rates



reduced mass $\mu = m/2$

$$\zeta \equiv \frac{\textit{Bohr momentum}}{\textit{relative momentum}} = \frac{\mu\alpha}{\mu v_{rel}}$$

BSF dominates over annihilation everywhere the Sommerfeld effect is important (ζ > 1) !



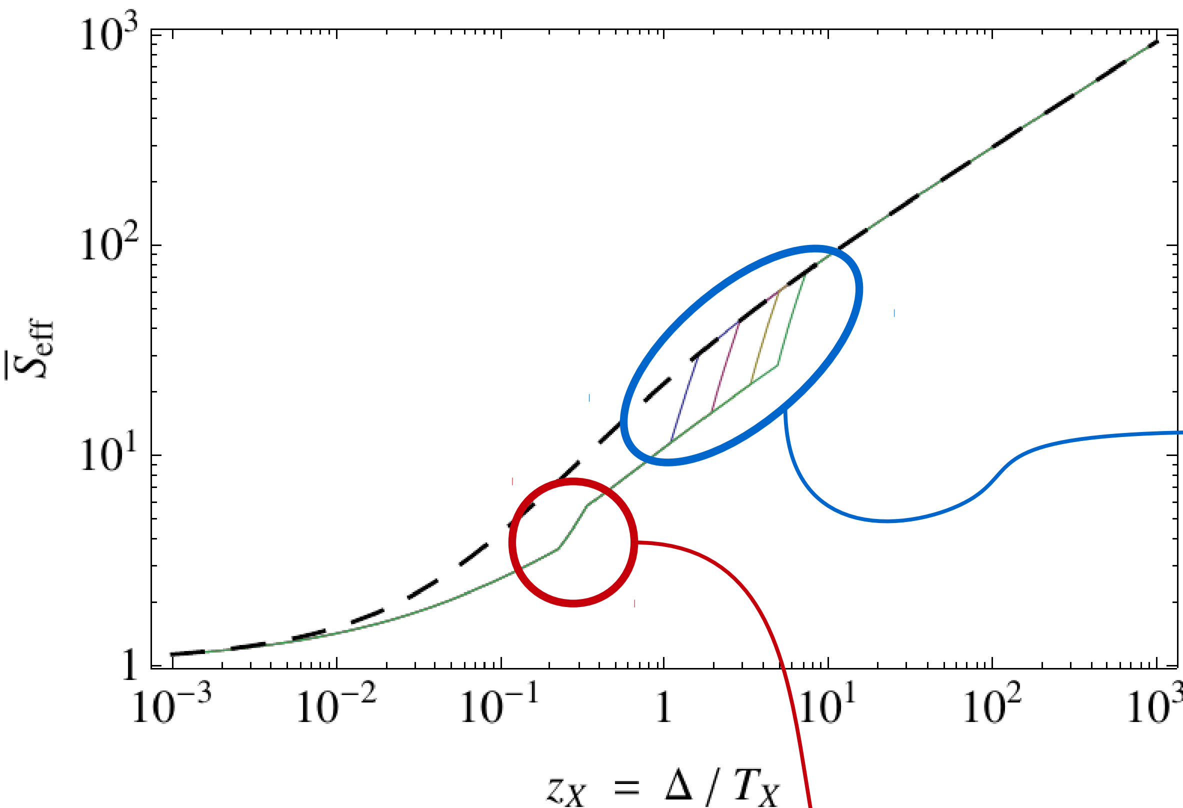
$$z = \frac{\textit{binding energy}\,[\Delta]}{T} \sim \frac{(1/2)\mu\alpha^2}{(1/6)\mu\langle v_{rel}^2\rangle} \sim 3\langle\zeta^2\rangle$$

BSF can annihilate DM at z >1, when disassociation of bound states becomes unimportant.

Relic density of symmetric DM

[von Harling, KP (2014)]

“Effective” enhancement



Decay of ↑↑ bound state faster than ionisation

(depends on α; from left to right: α = 0.5, 0.1, 0.01, 0.001)

$S_{eff} = S_{ann} + (1/4)S_{BSF} + (3/4)S_{BSF}$

Decay of ↑↓ bound state faster than ionisation

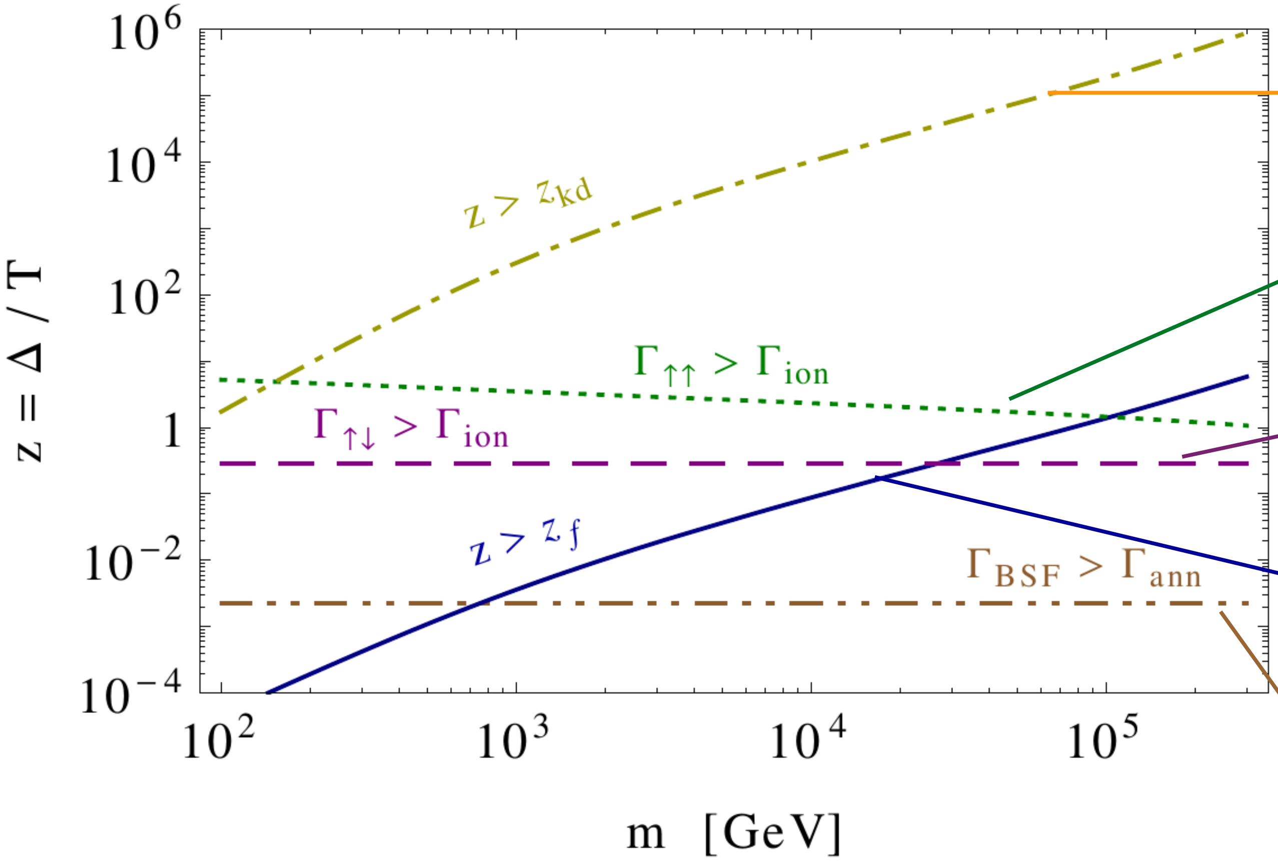
(independent of α)

$S_{eff} = S_{ann} + (1/4)S_{BSF}$

Relic density of symmetric DM

[von Harling, KP (2014)]

α(m) fixed from relic abundance (see results)



Kinetic decoupling: Sommerfeld enhancement unimportant

Ortho-positronium decay faster than ionisation.

Para-positronium decay faster than ionisation.

Freeze-out: DM density differs from equilibrium value by O(1) factor [standard definition]

BSF becomes faster than annihilation