

Singlet-like Higgs bosons at present and future colliders

Filippo Sala

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mainly based on Buttazzo S Tesi 1505.05488

GDR Terascale, Grenoble, 24 Nov 2015

◇ Can new scalar singlets be the lightest new particles around?

◇ How to look for them?

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Extra **Singlet** scalars are ubiquitous, for example in

- Twin Higgs
- Supersymmetry
- Electroweak Baryogenesis (independent of naturalness)

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Why TH interesting? Solves little hierarchy, without coloured top partners

If nothing new at the LHC14, TH models still quite natural!

→ Add a Z_2 -symmetric copy of the SM

[only copy of top strictly necessary see e.g. J Serra @ MIAPP 2015]

→ 8 “Higgs” degrees of freedom - vs 4 in the SM

7 are massless Goldstone bosons

one, σ = radial mode of $\mathcal{G} \rightarrow \mathcal{H}$

$\langle \sigma \rangle = f$, $m_\sigma \sim f$ conceivable if UV completion is weakly coupled

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Other particles? Either $M \gtrsim 4\pi f$ or very weakly coupled

Singlet scalars in Supersymmetry

Could the singlet-like scalar be the first new particle seen?

Singlet scalars in Supersymmetry

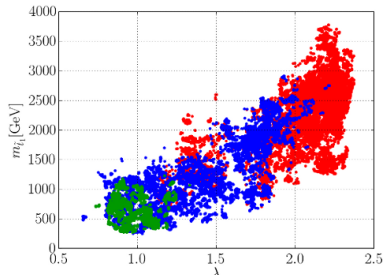
Could the singlet-like scalar be the first new particle seen?

NMSSM = MSSM + singlet S

$$W = W_{\text{MSSM}} + \lambda S H_u H_d + f(S)$$

Fine tuning better than 5%

[green points, $\tan \beta \lesssim 5$, $\Lambda = 20$ TeV]



Gherghetta et al. 1212.5243

see also Gherghetta et al. 1401.8291

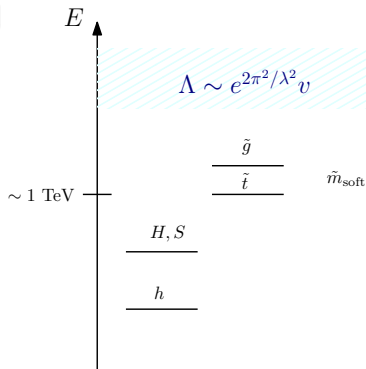
Cao et al. 1409.8431

Given a fixed tuning, \tilde{t} and \tilde{g} heavier by $\sim \lambda/g$ than in MSSM

NMSSM spectrum

NMSSM with $\lambda \sim 1$ and heavy stops & gluinos

[$\lambda \gtrsim 0.7$ needs a completion before GUT scale]



NMSSM spectrum

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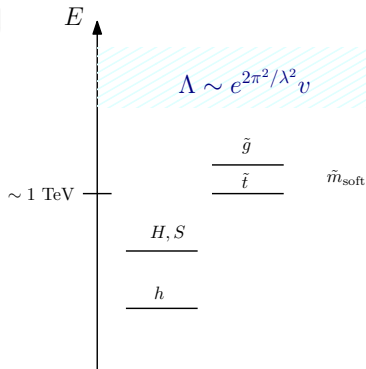
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The scalars are:

CP-even h, h_3, ϕ (from h_ν, H, S)

CP-odd A, A_s

H^\pm



NMSSM spectrum

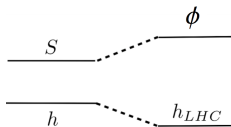
NMSSM with $\lambda \sim 1$ and heavy stops & gluinos

[$\lambda \gtrsim 0.7$ needs a completion before GUT scale]

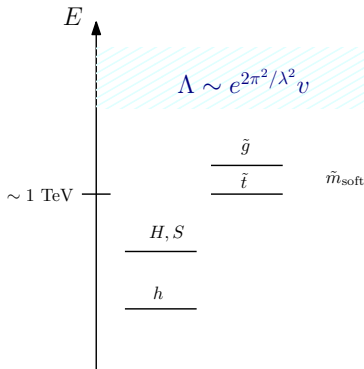
The scalars are:

CP-even h, h_3, ϕ (from h_ν, H, S)

$$\mathcal{H}_{\text{ph}} \equiv \begin{pmatrix} h_3 \\ h \\ \phi \end{pmatrix} = R^T \begin{pmatrix} H \\ h_\nu \\ S \end{pmatrix}, \quad R = \begin{matrix} R_\delta^{12} R_\gamma^{23} R_\sigma^{13} \end{matrix}$$



$\gamma = h_\nu - S$ mixing



A motivated limiting case

$m_{h_3} \gg m_{h,\phi}$ and $\sigma, \delta \rightarrow 0$

Bottom-up motivation for a Singlet: Higgs couplings fit

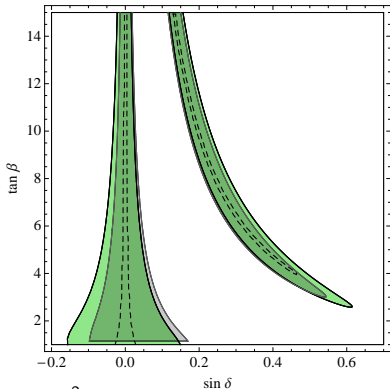
Bottom-up motivation for a Singlet: Higgs couplings fit

$$h_{\text{LHC}} = h = c_\gamma(c_\delta h_\nu - s_\delta H) + s_\gamma S$$

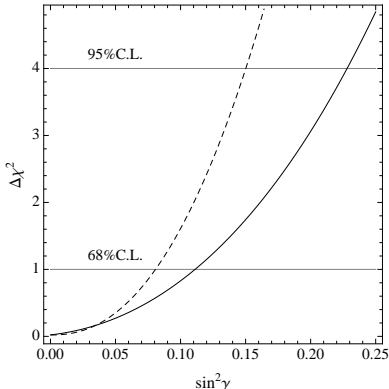
$$\left[\frac{g_{htt}}{g_{htt}^{\text{SM}}} = c_\gamma \left(c_\delta + \frac{s_\delta}{\tan \beta} \right), \quad \frac{g_{hbb}}{g_{hbb}^{\text{SM}}} = c_\gamma (c_\delta - s_\delta \tan \beta), \quad \frac{g_{hVV}}{g_{hVV}^{\text{SM}}} = c_\gamma c_\delta \right]$$

cont: LHC8 status

dashed: LHC14 projections (300 fb⁻¹)



$$s_\gamma^2 = 0, 0.15$$



✓ Can new Higgses be the lightest new particles around?

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Generic singlet

$$\sin^2 \gamma = \frac{M_{hh}^2 - m_h^2}{m_\phi^2 - m_h^2}$$

Master formula, valid for **any** model

2 free parameters control all pheno!

+ $\text{BR}_{\phi \rightarrow hh}$ (= $\text{BR}_{\phi \rightarrow ZZ}$ at $m_\phi \gg m_W$)

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h : signal strengths $\mu = c_\gamma^2 \times \mu_{\text{SM}}$ ϕ : $\mu(m_\phi) = s_\gamma^2 \times \mu_{\text{SM}}(m_\phi)$ [barring $\phi \rightarrow hh$]

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What does one learn from the potential $f(S)$?

$$\text{BR}_{\phi \rightarrow hh} = \frac{1}{4} - \frac{3}{4} \frac{v}{v_s} \frac{\sqrt{M_{hh}^2 - m_h^2}}{m_\phi} + \mathcal{O}\left(\frac{v^2}{m_\phi^2}\right)$$

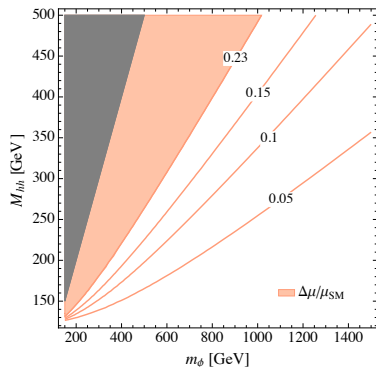
$$\frac{g_{h^3}}{g_{h^3}^{\text{SM}}} = 1 + \frac{2}{3} \frac{v}{v_s} \frac{\sqrt{M_{hh}^2 - m_h^2}}{m_\phi} \left(\frac{M_{hh}^2}{m_h^2} - 1 \right) + \mathcal{O}\left(\frac{v^2}{m_\phi^2}\right)$$

Valid for **any** potential!! v_s leading new parameter

Generic singlet: Higgs couplings

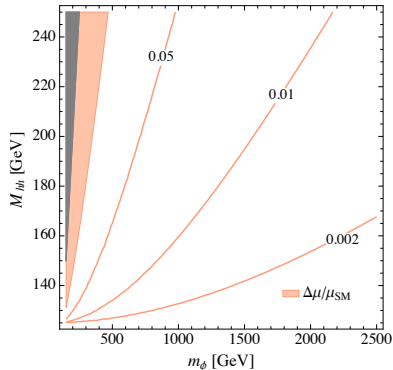
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1σ reach in	s_γ^2	$ 1 - \frac{g_{hh\phi}}{g_{hh}^{SM}} $
LHC8	0.2	-
LHC14	0.08-0.12	-
HL-LHC	$4-8 \times 10^{-2}$	0.5
HE-LHC	-	0.2
FCC-hh	-	0.08
ILC	2×10^{-2}	0.21-0.83
ILC-up	4×10^{-3}	0.13-0.46
CLIC	$2-3 \times 10^{-3}$	0.1-0.21
CEPC	2×10^{-3}	-
FCC-ee	1×10^{-3}	-

Snowmass 2013

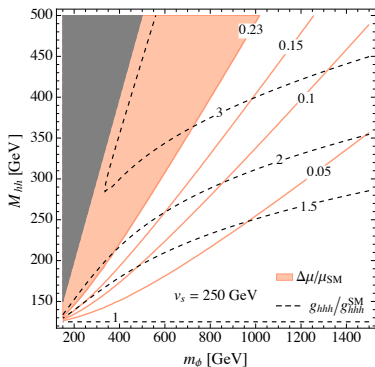


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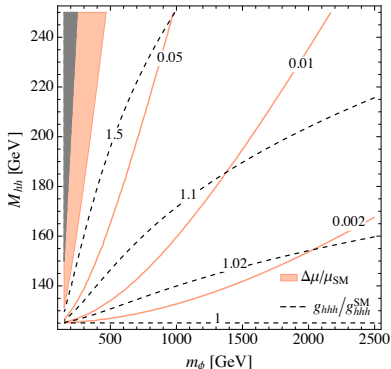
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Add g_{hhh} : could be first deviation seen!



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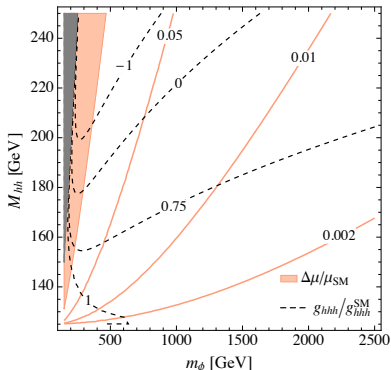
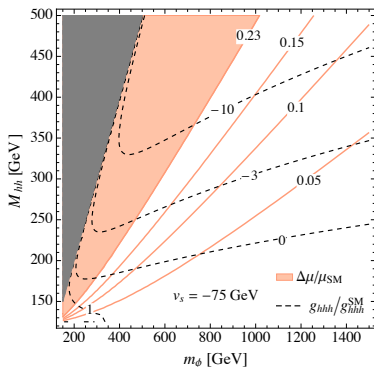


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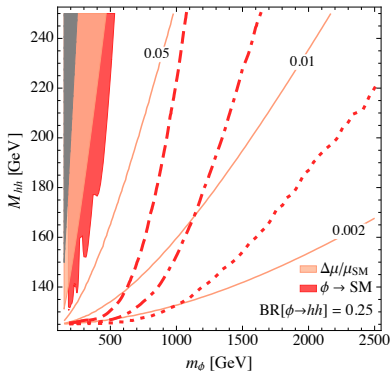
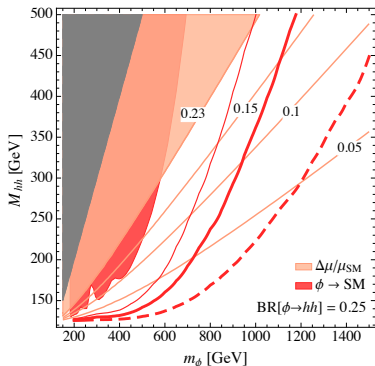
Generic singlet: Higgs couplings vs direct searches

$$\sin^2 \gamma = \frac{M_{hh}^2 - m_h^2}{m_\phi^2 - m_h^2}$$

$[\phi \rightarrow VV$ dominates over $\phi \rightarrow hh$, unless $v_s < 0$ and small]

h : signal strengths $\mu = c_\gamma^2 \times \mu_{SM}$

ϕ : $\mu(m_\phi) = s_\gamma^2 \times \mu_{SM}(m_\phi)$ [barring $\phi \rightarrow hh$]



Twin Higgs and the NMSSM

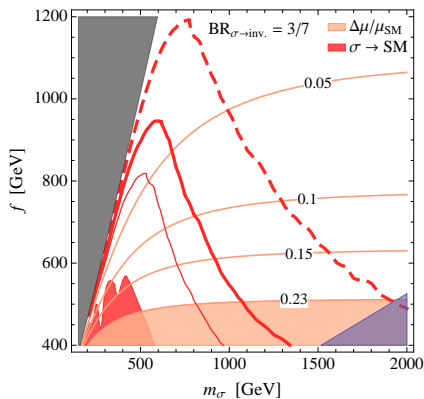
$$\sin^2 \gamma = \frac{M_{hh}^2 - m_h^2}{m_\phi^2 - m_h^2}$$

$$\text{Twin Higgs: } M_{hh}^2 = (m_h^2 + m_\phi^2) v^2 / f^2$$

Take-home messages

Twin Higgs:

- Signal strengths μ_h more effective than direct ϕ searches, unless $m_\phi \sim f$
- no significant deviations in g_{hhh}



...more in back up slides

Twin Higgs and the NMSSM

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$$\text{Twin Higgs: } M_{hh}^2 = (m_h^2 + m_\phi^2) v^2 / f^2$$

$$\text{NMSSM: } M_{hh}^2 = m_Z^2 c_{2\beta}^2 + \lambda^2 v^2 s_{2\beta}^2 + \Delta^2$$

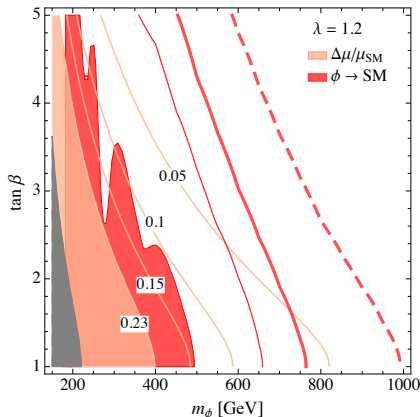
Take-home messages

Twin Higgs:

- Signal strengths μ_h more effective than direct ϕ searches, unless $m_\phi \sim f$
- no significant deviations in g_{hhh}

NMSSM:

- For μ_h to do better than direct ϕ searches, per-mille precision needed
- g_{hhh} could show significant deviations

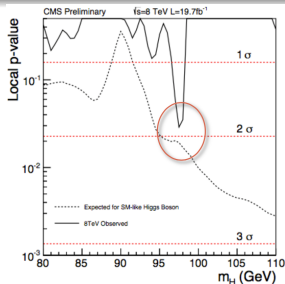


...more in back up slides

Fully mixed case and a $\gamma\gamma$ signal

Singlet-like state lighter than 125 GeV

Hard to see, could it explain this hint?

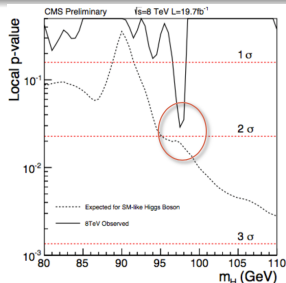


Singlet-like state lighter than 125 GeV

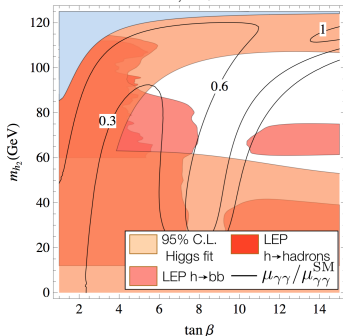
Hard to see, could it explain this hint?

[see also [Badziak et al. 1304.5437](#),...]

$$[m_{h_3} = 500 \text{ GeV}, s_\sigma^2 = 10^{-3}, v_s = v]$$



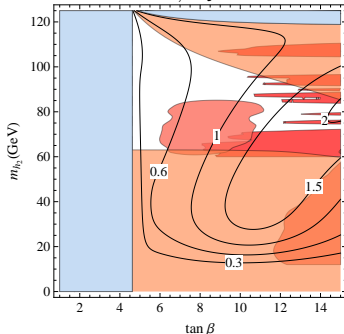
$$\lambda = 0.1, \Delta_t = 85 \text{ GeV}$$



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$$\lambda = 0.8, \Delta_t = 75 \text{ GeV}$$



Singlets at present and future colliders

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A (necessary) input to plan future HEP strategy

$\phi \rightarrow VV, hh, h$ couplings, g_{hhh}, \dots

Very relevant for natural theories

NMSSM with $\lambda \sim 1$

singlet-like ϕ could show up before \tilde{t}, \tilde{g}

Twin Higgs (weakly coupled)

radial mode could be first particle seen

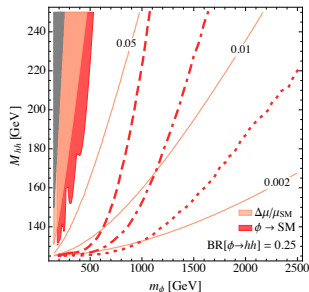
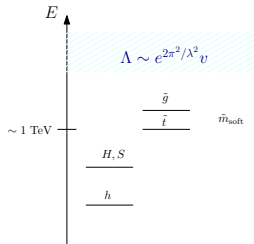
What Next?

more on extra doublet, e.g. $f\bar{f}$ prospects

CP-odd scalars

UV model for Twin Higgs with light σ

for extra Higgses see also [F.S. 1509.08655](#)

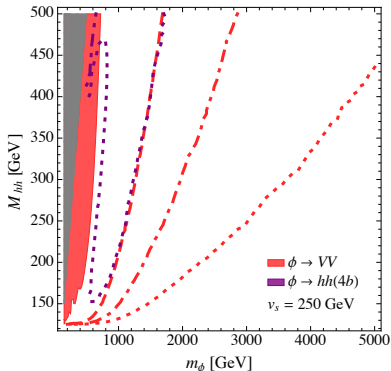
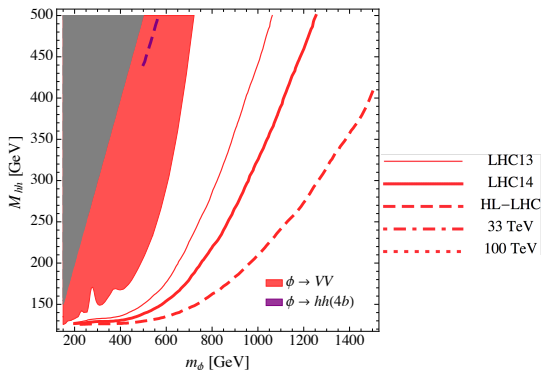


Back up

Generic singlet: direct searches

$$\sin^2 \gamma = \frac{M_{hh}^2 - m_h^2}{m_\phi^2 - m_h^2}$$

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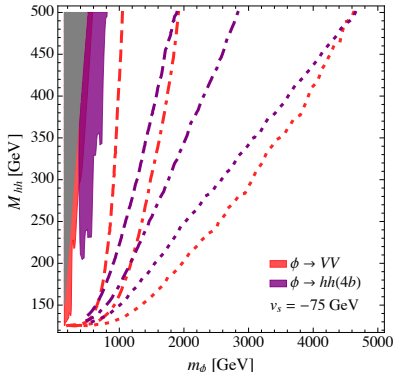
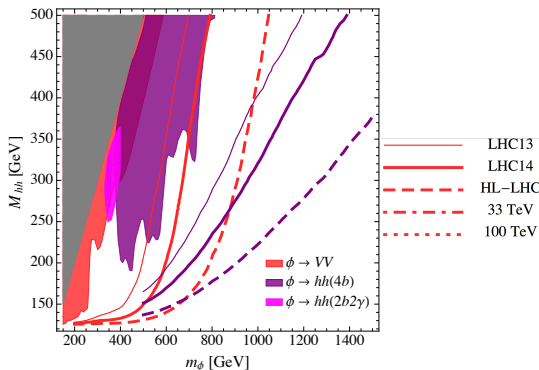


Generic singlet: direct searches

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Large $\text{BR}_{\phi \rightarrow hh}$ easier for $v_s < 0$

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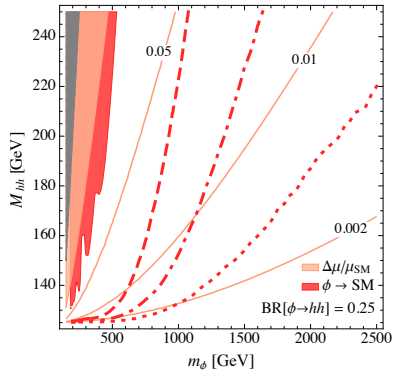
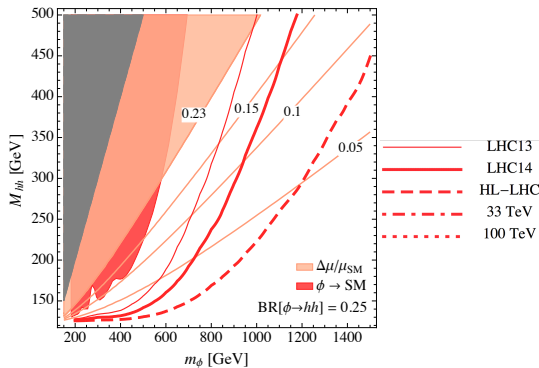


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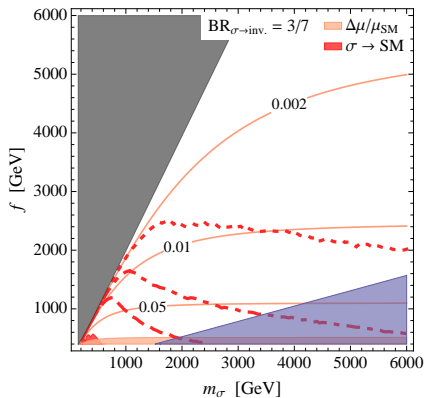
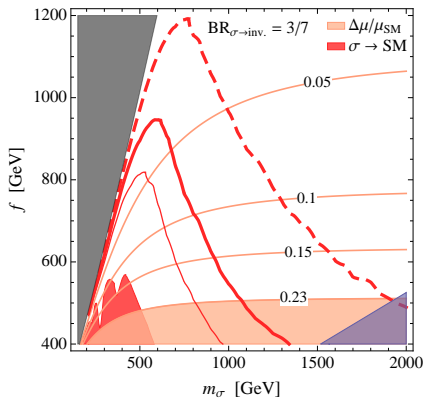
Higgs as a PNG boson: Twin Higgs

$$\sin^2 \gamma = \frac{M_{hh}^2 - m_h^2}{m_\sigma^2 - m_h^2}$$

$$M_{hh}^2 = (m_h^2 + m_\sigma^2) v^2 / f^2$$

Only two free parameters f and $m_\sigma \Rightarrow \text{BR}_{\sigma \rightarrow hh}$ fixed everywhere

Twin SM $\Rightarrow \text{BR}_{\sigma \rightarrow \text{inv.}} \neq 0$ [equivalence theorem: $\text{BR}_{\sigma \rightarrow \text{inv.}} \rightarrow 3/7$ for $m_\sigma > m_Z \times f/v$]



The NMSSM

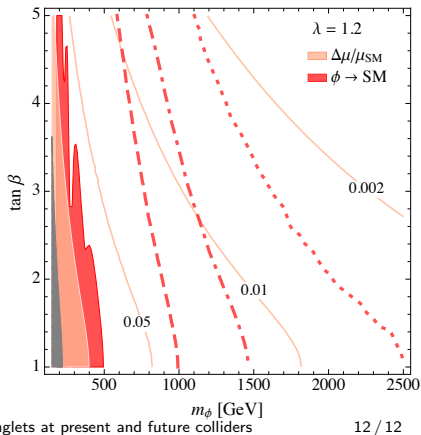
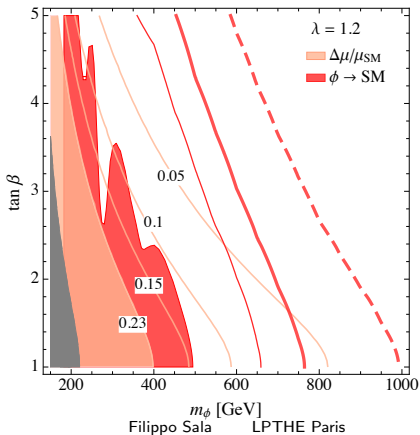
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$$M_{hh}^2 = m_Z^2 c_{2\beta}^2 + \lambda^2 v^2 s_{2\beta}^2 + \Delta^2$$

Δ = all loop effects, e.g. top-stop

Here $\lambda = 1.2$ $\Delta = 70$ GeV

$\tan \beta$ “small” otherwise EWPT



The NMSSM

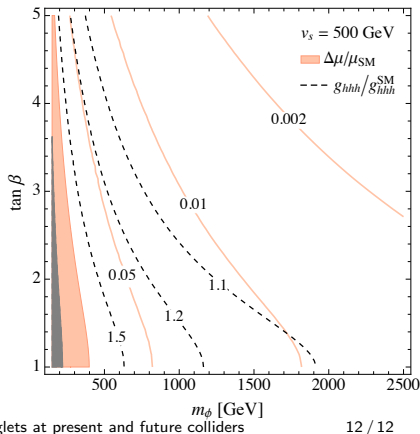
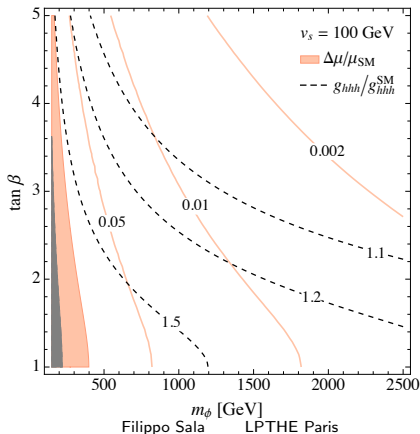
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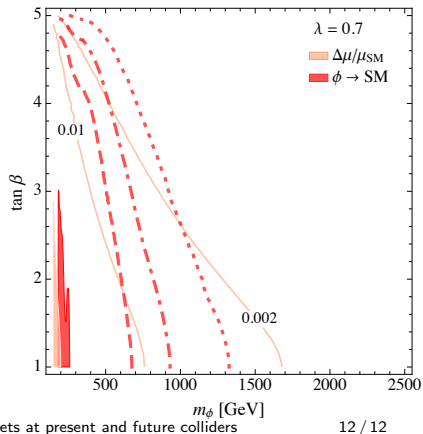
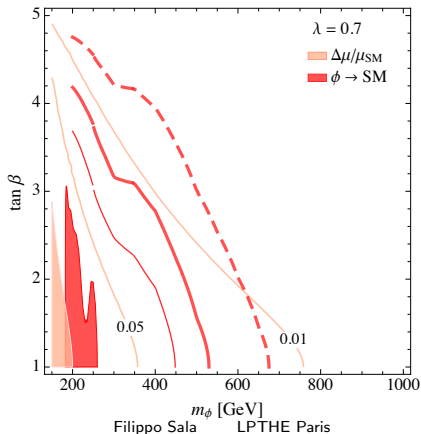
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Δ = all loop effects, e.g. top-stop

Here $\lambda = 0.7$ $\Delta = 80$ GeV

$\tan \beta$ “small” otherwise EWPT



Extrapolation of direct searches I

We started from

i) Collider Reach (β) Salam Weiler 2014 ii) Thamm Torre Wulzer 1502.01701

m_0 excluded at LHC8, obtain m_1 at future collider via $B(s_1, L_1, m_1) = B(s_0, L_0, m_0)$

$$B(s, L, m) \propto L \times \int d\hat{s} \frac{1}{\hat{s}} \hat{s} \hat{\sigma}(\hat{s}) \frac{d\mathcal{L}}{d\hat{s}}(s)$$

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$$B(s, L, m) \propto L \times \frac{\Delta \hat{s}}{m^2} c \left. \frac{d\mathcal{L}}{d\hat{s}}(s) \right|_{\hat{s}=m^2} \quad \hat{s} \hat{\sigma}(\hat{s}) = c \Rightarrow \frac{d\mathcal{L}}{d\hat{s}} \text{ drives the reach}$$

Extrapolation of direct searches

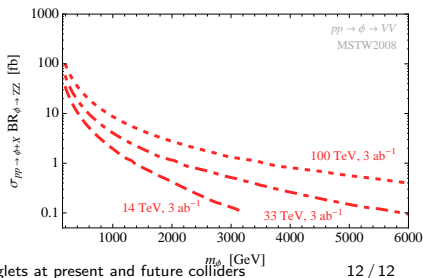
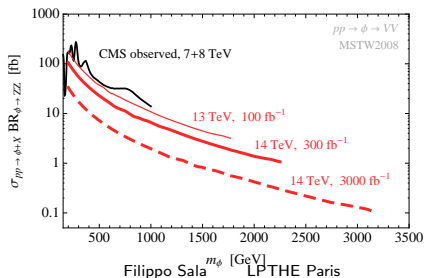
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$$B(s, L, m) \propto L \times \frac{\Delta \hat{s}}{m^2} c \frac{d\mathcal{L}}{d\hat{s}}(s) \Big|_{\hat{s}=m^2} \quad \hat{s}\hat{\sigma}(\hat{s}) = c \Rightarrow \frac{d\mathcal{L}}{d\hat{s}} \text{ drives the reach}$$

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Extrapolation of direct searches

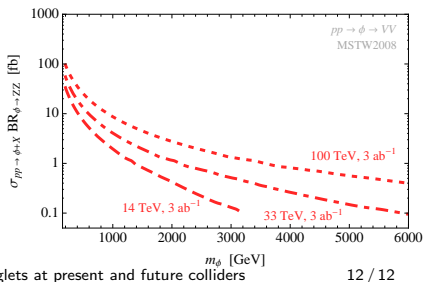
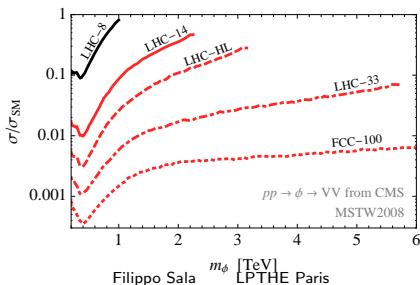
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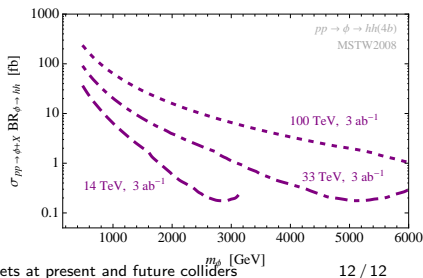
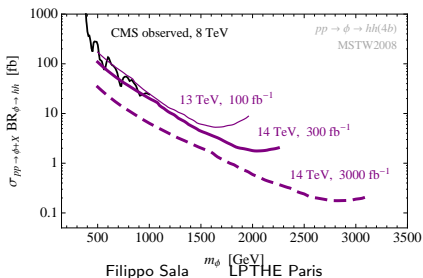
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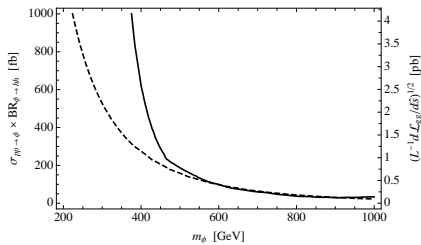
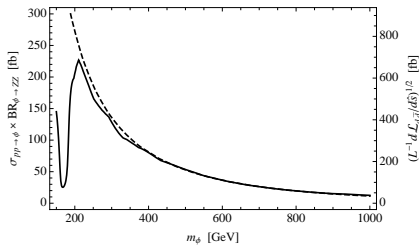
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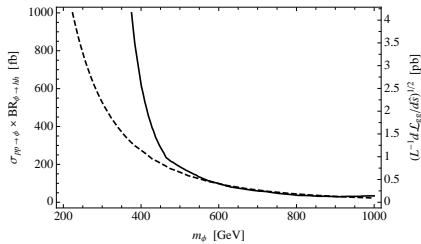
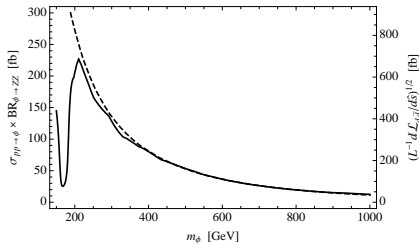
Extrapolation of direct searches II



Assumptions/limitations

→ Not valid if systematics dominate and change significantly from s_0 to s_1

Extrapolation of direct searches II



Assumptions/limitations

- Not valid if systematics dominate and change significantly from s_0 to s_1
- $\hat{s} \gg m_{\text{bkg}}$ [i.e. not valid at $\hat{s} \sim 2m_t$ for $\phi \rightarrow hh(4b)$]
- $\frac{\Delta\hat{s}}{m^2} \ll 1$ i.e. not valid if analysis depends a lot on shape far from peak

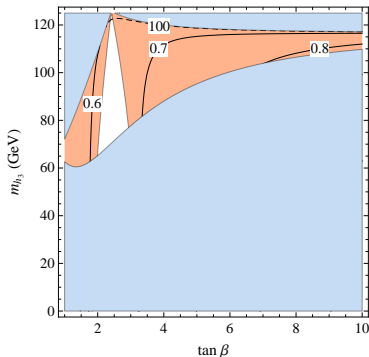
An extra doublet-like state H

$$[\gamma, \sigma = 0, \quad m_{h_2} \gg m_{h_1}, m_{h_3}]$$

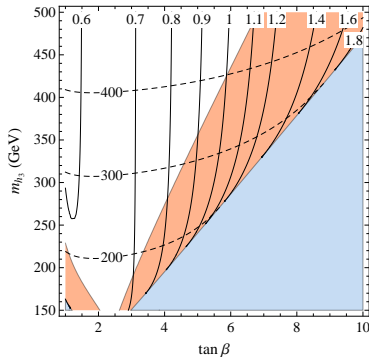
Barbieri Buttazzo Kannike Sala Tesi 1304.3670, 1307.4937

$$\frac{g_{h_3 tt}}{g_{htt}^{\text{SM}}} = s_\delta - \frac{c_\delta}{t_\beta} \quad \frac{g_{h_3 bb}}{g_{hbb}^{\text{SM}}} = s_\delta + t_\beta c_\delta \quad \frac{g_{h_3 VV}}{g_{hVV}^{\text{SM}}} = s_\delta \quad [\Delta_t = 75 \text{ GeV}]$$

Status fit LHC8:



dashed: m_{H^\pm} cont: λ



$m_{H^\pm} > 480 \text{ GeV}$ from $B \rightarrow X_s \gamma$!

$$[\widetilde{\mathcal{M}}_{12}^2(t_\beta, \dots) = 0 \rightarrow \delta = 0]$$

h_3 phenomenology: more similar to MSSM

see e.g. [Craig et al. 1504.04630](#)

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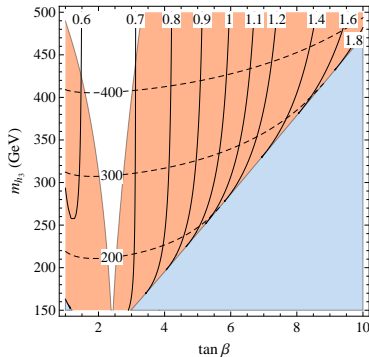
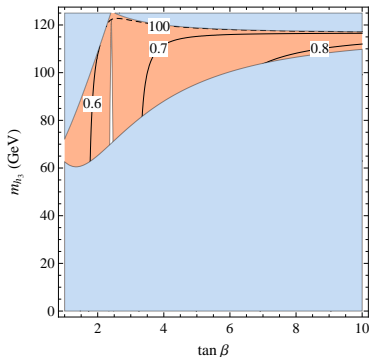
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Projections fit LHC14 (300 fb^{-1}):

dashed: m_{H^\pm} cont: λ



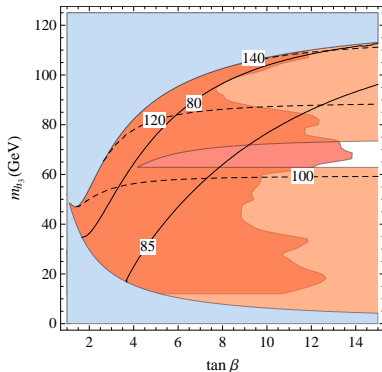
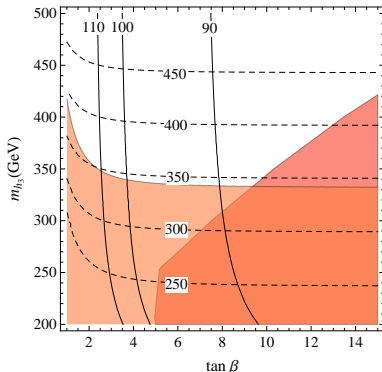
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Status fit LHC8:

[dashed: m_{H^\pm} cont: Δ_t]

Red regions excluded by direct searches at LEP and CMS

Projections fit LHC14: above regions completely excluded

[if $\frac{\mu A_t}{m_t^2}$ very large, conclusions could change...]