

LagSHT: 3D analysis

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LagSHT

- Laguerre & Spherical Harmonic 3D Transform
- Original work done by Boris Leistedt and Jason D. McEwen (IEEE TRANSACTIONS ON SIGNAL PROCESSING, VOL. 60, NO 12 Dec 2012.)
- C++ library, author: J.E.C; contributions from M. Reinecke (libsharp). Improve the speed and validity domain of B.L & J.D.M
- <https://gitlab.in2p3.fr/campagne/LagSHT>
- Performs from $f_{ijk} = f(\theta_i, \phi_j, r_k)$ with r_k the Laguerre roots to f_{lmn} the cplx coefficients (and vice-versa)
- Usefull by product: a_{lm} cplx Ylm-coefficients on each shell r_k

Analysis :

$$f_{ijk} \xrightarrow{SHT} a_{lmk} = a_{lm}(r_k) \xrightarrow{\text{Lag.Trans}} f_{lmn}$$

Synthesis :

$$f_{lmn} \xrightarrow{\text{Inv.Lag.Trans}} a_{lmk} = a_{lm}(r_k) \xrightarrow{\text{Inv.SHT}} f_{ijk}$$

Some basics (1/2)

$$f(r, \Omega_r) = \sum_{n=0}^{\infty} \sum_{l=0}^{\infty} \sum_{m=-l}^l f_{lmn} Y_{lm}(\Omega_r) \mathcal{L}_n(r)$$

$$f_{lmn} = \iiint dr d\Omega_r r^2 f(r, \Omega_r) Y_{lm}^*(\Omega_r) \mathcal{L}_n(r)$$

$$Y_{l,m}(\Omega) = \lambda_{lm}(\theta) e^{im\phi} = \sqrt{\frac{2l+1}{4\pi}} \frac{(l-m)!}{(l+m)!} P_l^m(\cos \theta) e^{im\phi}$$

$$\mathcal{L}_n(r) = \sqrt{\frac{n!}{(n+\alpha)!}} e^{-r/2} L_n^{(\alpha)}(r)$$

Laguerre Func.

$$\int_{\mathbb{R}^+} dx x^2 \mathcal{L}_n(x) \mathcal{L}_m(x) = \delta_{nm}$$

$$\int_{\theta=0}^{\pi} \int_{\varphi=0}^{2\pi} Y_{\ell}^m Y_{\ell'}^{m'*} d\Omega = \delta_{\ell\ell'} \delta_{mm'}$$

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Some basics (2/2)

- Radial part (Gauss-Laguerre quadrature*)

$$f(r) = \sum_{n=0}^{N-1} f_n \mathcal{L}_n^{(\alpha)}(r)$$

$$f_n = \sum_{i=0}^{N-1} w_i f(r_i) \mathcal{L}_n^{(\alpha)}(r_i)$$

r_i : the roots of generalized Laguerre polynomials

w_i : the associated Gauss quadrature weights

- The (θ, ϕ) part uses Spherical Harmonic decomposition library based on some usual spherical pixelizations

Exercices

Stage de M2 A&A Ph. Bacon

- Generate $\delta\rho/\rho$ on cube of 512x512x512 cells each 8x8x8 Mpc³ using SimLSS (parametrization of P(k) by Eisenstein et Hu)

- Fill N = 128 sub-shells with the radii :

$$R_k = R_{max} \times \frac{r_k}{r_{N-1}}$$

- Perform LagSHT Analysis to get the a_{lmk} and the f_{lmn}

- Computes “with BAO” or NoOsc & ratio BAO/NoOsc

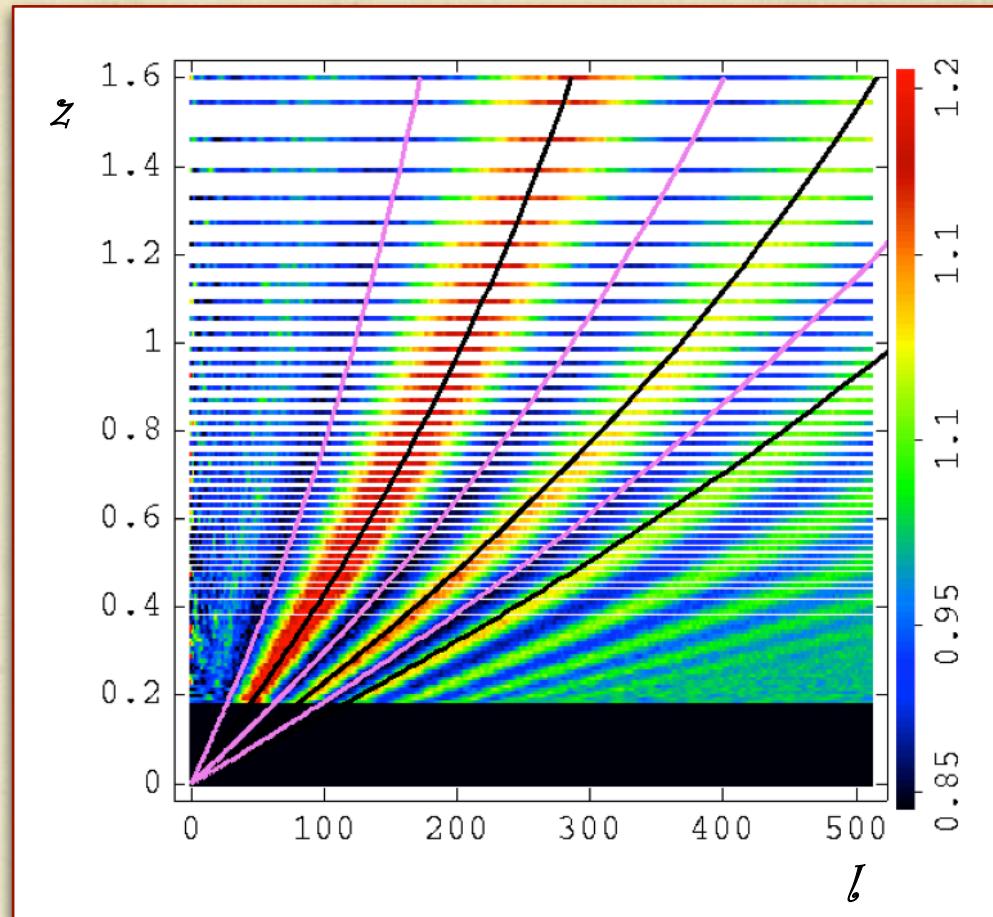
- Change (Ω_Λ , Ω_b/Ω_m)

$$C_l(k) = \frac{1}{2l+1} \sum_{m=-l}^l |a_{lm}|^2$$

$$C_l(n) = \frac{1}{2l+1} \sum_{m=-l}^l |f_{lmn}|^2$$

a_{lmk}

z_k obtained from Light of Sight distance corresponding to R_k



The (black/pink) curves give the locations of peak/valey using the simple formula

$$\left(\frac{4n \pm 1}{2} \right) \frac{\pi D_{com}(z)}{r_s}$$

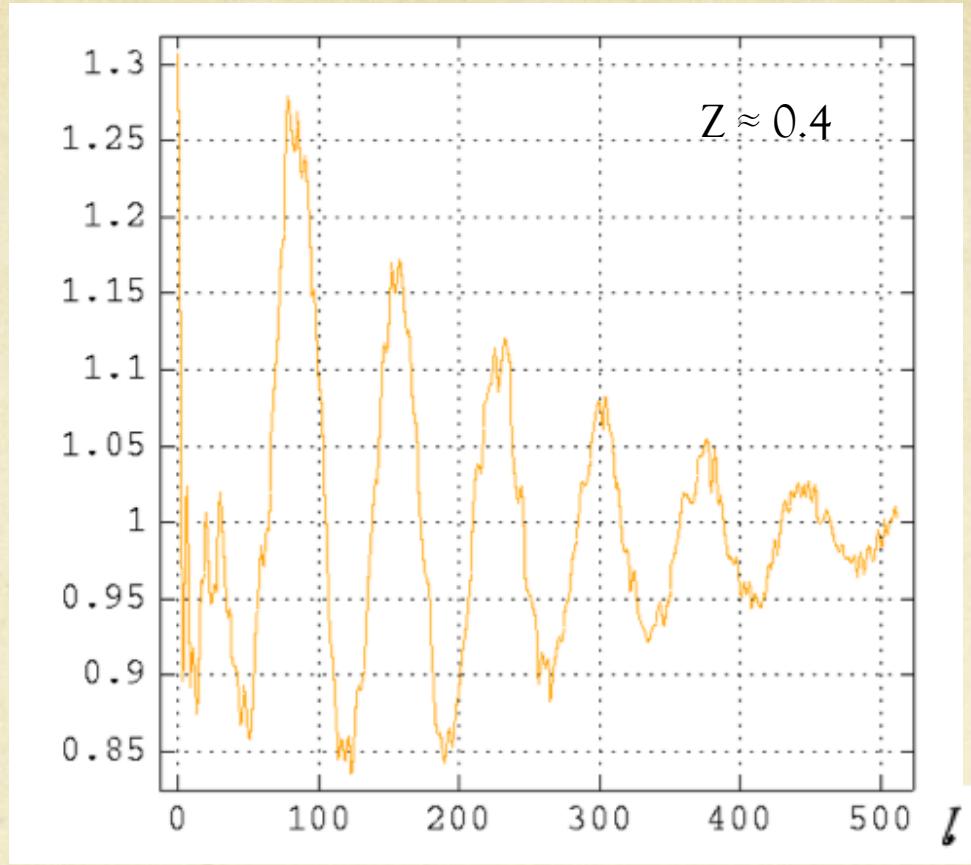
Cosmology param.

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$$\Omega_m = 0.218, \Omega_b = 0.131, \Omega_\Lambda = 0.779$$

r_s : sound horizon @ drag time

$$Cl(\ell, 0.4)_{BAO}/Cl(\ell, 0.4)_{NoOsc}$$

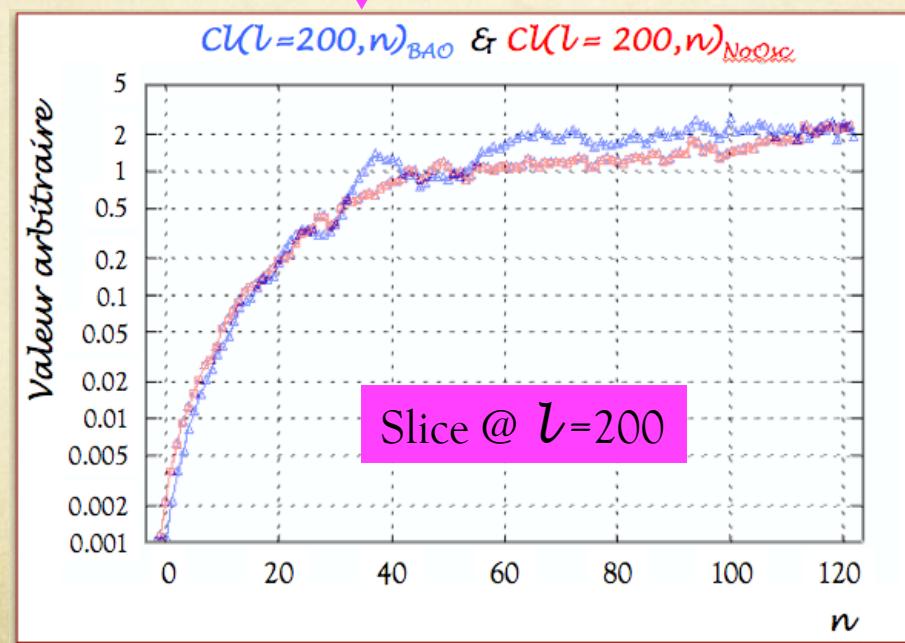
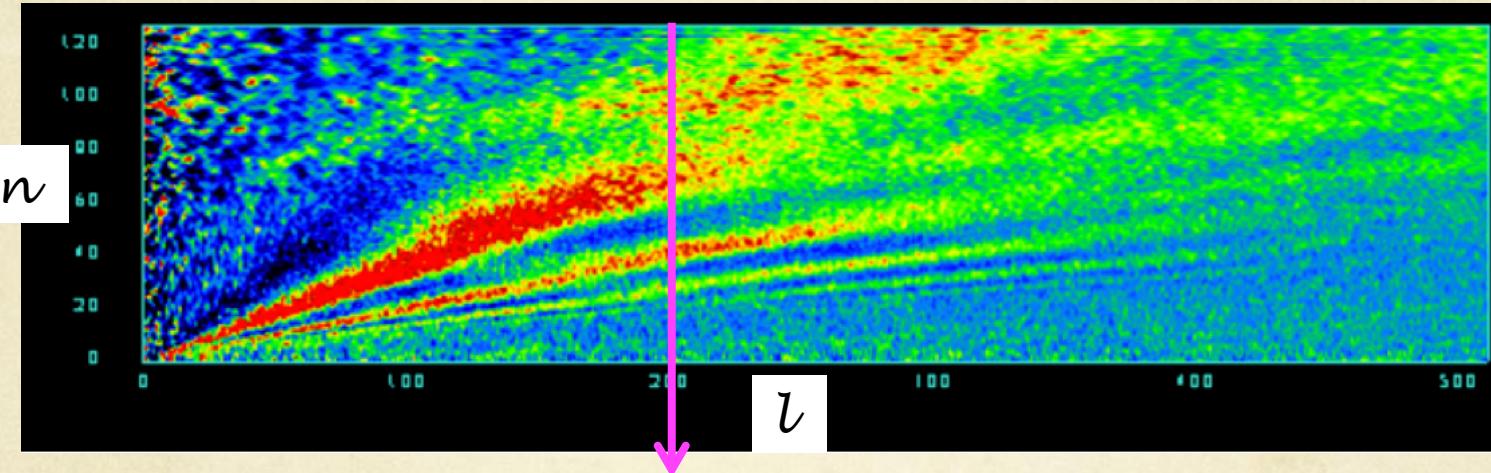


ℓ	1er creux	1er pic	2eme creux	2eme pic
ℓ mesuré	47	80	120	155
ℓ formule	50	84	117	150
$\Delta\ell/\ell$	5 %	6,3 %	2,5 %	3,2 %

Compatible with Eisenstein et Hu parametrization accuracy

f_{lmn}

$$C_\ell(\ell, n)_{\text{BAO}} / C_\ell(\ell, n)_{\text{NoBAO}}$$



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Summary

- LagSHT is publicly available well tested by M.R & me on the numerical/speed side
- Simple analysis seems to give satisfactory results
- Possible future studies: « Fisher matrix »

$$C_l(n, n') = \frac{1}{2l+1} \sum_m f_{lmn} f_{lmn'}^* = \mathbb{C}_l$$
$$F_{\alpha\beta} \propto \sum_l \frac{2l+1}{2} \text{Tr} \left[\mathbb{C}_l^{-1} \frac{\partial \mathbb{C}_l}{\partial \Theta_\alpha} \mathbb{C}_l^{-1} \frac{\partial \mathbb{C}_l}{\partial \Theta_\beta} \right]$$
$$\Theta = (\Omega_m, \Omega_b, w_0, w_a, \dots)$$

- Link « Laguerre \rightarrow Bessel » in progress (Maths Ok, Numerical pbs currently investigated)