Signal reconstruction in $B_s^0 \to \tau^+\tau^-$

A. Mordá, J. Charles

CPPM & CPT - Marseille

Novel aspects of $b \to s$ transitions

Marseille - Campus S. Charles

6^{th} October 2015

Overview

- Motivations
- Challenging aspects of $B^0_{(s)} \to \tau^+\tau^-$
- Signal reconstruction in $B^0_{(s)} \to \tau^+\tau^-$
  - Kinematic equations
  - Discriminating variables
  - Latest developments
- Conclusions and Prospects

Measured values of $\mathcal{BR}$ of $B_{(s)}^0 \to \mu^+\mu^-$ rule out large New Physics effects

Still room for New Physics in processes involving 3rd generation

- Third family quite peculiar, required for CP violation
- Anomalous like-sign dimuon charge asymmetry
- Correlations with $\Delta\Gamma_s/\Gamma_s$
  [Dighe *et al.*PRD82 031502,(2010)]

} **Can be related to $(\bar{b}\mathcal{O}s)(\bar{\tau}\mathcal{O}\tau)$ operators**

- Hints of LFU violation: $\boldsymbol{R_K} \equiv \frac{\mathcal{BR}(B^+ \to K^+\mu^+\mu^-)}{\mathcal{BR}(B^+ \to K^+e^+e^-)}$, $\boldsymbol{R_{D^{(\star)}}} \equiv \frac{\mathcal{BR}(\bar{B} \to D^{(\star)}\tau\nu_\tau)}{\mathcal{BR}(\bar{B} \to D^{(\star)}\ell\nu_\ell)}$
- Anomalous excess in $h \to \tau\mu$ channel

Search for $B_{(s)}^0 \to \tau^+\tau^-$

Standard Model predictions and current status

In the Standard Model (SM) [Bobeth *et al.*, arXiv:1311.0903v1]:

$$\mathcal{BR}(B_s^0 \to \tau^+\tau^-) = (7.73 \pm 0.49) \times 10^{-7}$$
$$\mathcal{BR}(B_d^0 \to \tau^+\tau^-) = (2.22 \pm 0.04) \times 10^{-8}$$

- respecting all the constraints on other B_s^0 decays it might be as large as **15%**
- in models with a flavor depending Z' coupling it might be up to 5%
- in models with scalar Leptoquark it might be up to 0.3%

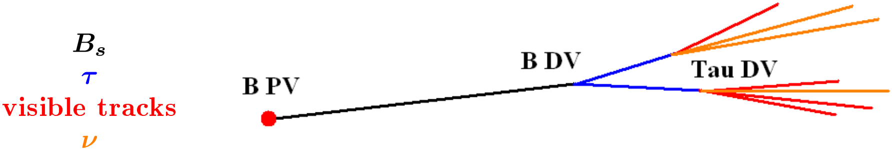
Model independent analysis [R. Alonso *et al.*, arXiv:1505.05164v1] allows for an enhancement of $\sim 10^3$ in $\mathcal{BR}$ of modes involving $(\bar{b}s)(\bar{\tau}\tau)$ operators ($i.e.$ $B^0_{(s)} \to \tau\tau$, $B^0 \to K^\star\tau\tau$)

- Current status:
  - **$\mathcal{BR}(B_d^0 \to \tau^+\tau^-) < 4 \cdot 10^{-3}$ @ 90% CL by BaBar [PRL 96 (2006) 241802]**
  - **$\mathcal{BR}(B_s^0 \to \tau^+\tau^-)$ has not yet been constrained**

Challenging issues of $B^0_{(s)} \to \tau^+\tau^-$

τ have a very short lifetime $\Longrightarrow$ we must reconstruct them from their daughter particles

At least one neutrino for each τ decay (1 for hadronic or 2 for leptonic channels) $\Rightarrow$ at least **2 undetectable neutrinos**



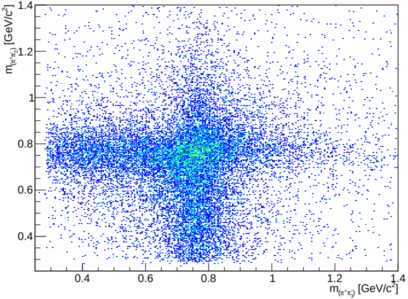
The $\tau \to 3\pi\,\nu_\tau$ decay final state

- **Only 2 neutrinos**
- 2 3-prong vertexes
- **Reconstruction of the decay plane**: kinematic constraints & partial neutrino momentum reconstruction
- Needs 6-charged tracks in the detector acceptance
- $\mathcal{BR}(\tau \to 3\pi\,\nu_\tau) = 9.31\%$
- $\mathcal{BR}^{\mathbf{SM}}(\mathbf{B_s} \to \mathbf{6}\pi\nu\bar{\nu}) \simeq \mathbf{6.7 \times 10^{-9}}$

The $\tau \to 3\pi\,\nu_\tau$ decay chain proceeds through the a_1 and ρ resonances, *i.e.*

$$\tau^- \to a_1^- \nu$$
$$a_1^- \to \rho^0\pi^- \to \pi^+\pi_1^-\pi_2^-$$

a_1 Dalitz plane



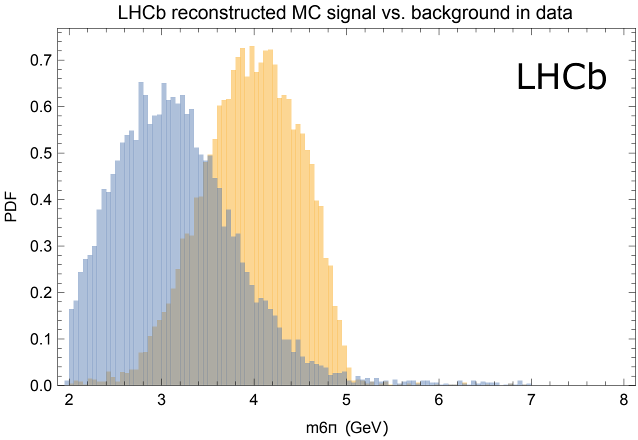
Sources of background

- $\boldsymbol{\tau \to 3\pi\nu}$ backgrounds:
  - **combinatorial $\boldsymbol{\tau}$**: 3 random π
  - **true particles**: $D_{(s)}^{(\star)} \to 3\pi X$

	Mass (MeV)	τ (10^{-15} s)	Spin
τ	1776	290	1/2
D	1869	1040	0^-
D_s	1968	500	0^-

- $\boldsymbol{B_s \to \tau\tau \to (3\pi\nu)(3\pi\nu)}$ backgrounds:
  - **2 combinatorial $\boldsymbol{\tau}$**
  - from **two $\boldsymbol{b}$-hadron decays**
  - from **one $\boldsymbol{b}$-hadron decay**:
    - hadronic or semileptonic decay of a b-hadron with ≥ 6 charged tracks

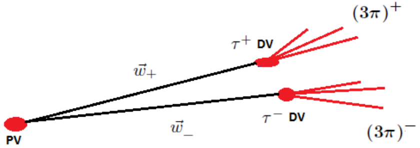
What we start with: $M_{6\pi}$



Signal reconstruction for $B_s \to \tau\tau \to (3\pi\nu)(3\pi\nu)$

In the reconstructed events the following quantities are known:

- B origin vertex
- 3D sides of triangle $\vec{w}_\pm$
- 4-momenta $p^\mu_\pm$ of $(3\pi)_\pm$ system

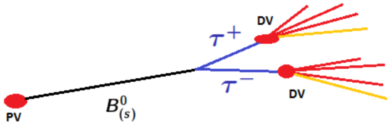


Can we reconstruct the two τ candidate momenta $p_\pm$?

Signal reconstruction for $B_s \to \tau\tau \to (3\pi\nu)(3\pi\nu)$

In the reconstructed events the following quantities are known:

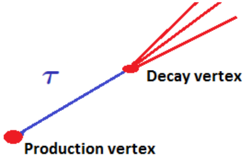
- B origin vertex
- 3D sides of triangle $\vec{w}_\pm$
- 4-momenta $p^\mu_\pm$ of $(3\pi)_\pm$ system



Can we reconstruct the two τ candidate momenta $p_\pm$?

Signal reconstruction for $B_s \to \tau\tau \to (3\pi\nu)(3\pi\nu)$

Example. 1D case:



- τ production and decay vertexes are known: $\vec{w}_\tau$
- 4 unknowns: the p_τ^μ momentum components
- General relation between 4-vectors:

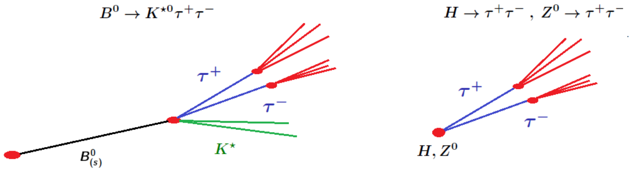
$$\boldsymbol{w_\tau = h p_\tau} \quad , \quad h \equiv \frac{\tau_\tau}{m_\tau}$$

- Constraints on $\vec{p}_\tau \propto \vec{w}_\tau$, $p_\tau^2 = m_\tau$ and $p_\tau \cdot p_{3\pi}$ (with m_ν fixed)
- The norm of the τ momentum is defined up to a twofold ambiguity

Can we reduce our problem to 2 $\otimes$ (1D-case)?

Signal reconstruction for $B_s \to \tau\tau \to (3\pi\nu)(3\pi\nu)$

Example. 1D case:



- τ production and decay vertexes are known: $\vec{w}_\tau$
- 4 unknowns: the p^μ_τ momentum components
- General relation between 4-vectors:

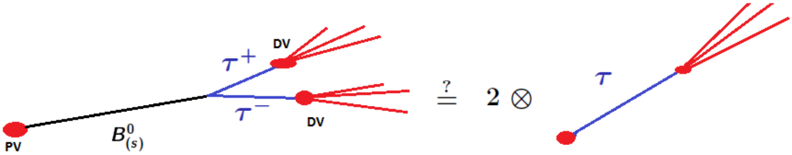
$$\boldsymbol{w_\tau = h p_\tau} \quad , \quad h \equiv \frac{\tau_\tau}{m_\tau}$$

- Constraints on $\vec{p}_\tau \propto \vec{w}_\tau$, $p^2_\tau = m_\tau$ and $p_\tau \cdot p_{3\pi}$ (with m_ν fixed)
- The norm of the τ momentum is defined up to a twofold ambiguity

Can we reduce our problem to $2 \otimes$ (1D-case)?

Signal reconstruction for $B_s \to \tau\tau \to (3\pi\nu)(3\pi\nu)$

Example. 1D case:



- τ production and decay vertexes are known: $\vec{w}_\tau$
- 4 unknowns: the p^μ_τ momentum components
- General relation between 4-vectors:

$$\boldsymbol{w_\tau = h p_\tau} \quad , \quad h \equiv \frac{\tau_\tau}{m_\tau}$$

- Constraints on $\vec{p}_\tau \propto \vec{w}_\tau$, $p^2_\tau = m_\tau$ and $p_\tau \cdot p_{3\pi}$ (with m_ν fixed)
- The norm of the τ momentum is defined up to a twofold ambiguity

Can we reduce our problem to 2 $\otimes$ (1D-case)?

Signal reconstruction for $B_s \to \tau\tau \to (3\pi\nu)(3\pi\nu)$

The answer is "**yes**" (at least formally) if

- **manifest Lorentz-covariance** is kept
- **particular choice of unknown** momenta are made

Let's define:

$$w_\pm \equiv (\boldsymbol{w^0_\pm}, \vec{w}_\pm)$$

and

$$W \equiv (w_+, w_-) \quad , \quad P \equiv (p_+, p_-)$$

$$\boldsymbol{W = H \cdot P}$$

$\longleftarrow$ **Formally equal to the 1D case**

with ($\hat{\tau}_i \equiv \frac{\tau_i}{m_i}$)

$$\boldsymbol{H} \equiv \begin{pmatrix} \hat{\tau}_B + \hat{\tau}_+ & \boldsymbol{\hat{\tau}_B} \\ \boldsymbol{\hat{\tau}_B} & \hat{\tau}_B + \hat{\tau}_- \end{pmatrix}$$

Note: if $\tau_B \to 0$, 2⊗1D-case trivially

Signal reconstruction for $B_s \to \tau\tau \to (3\pi\nu)(3\pi\nu)$

H is a real 2×2 symmetric matrix. Diagonalizing:

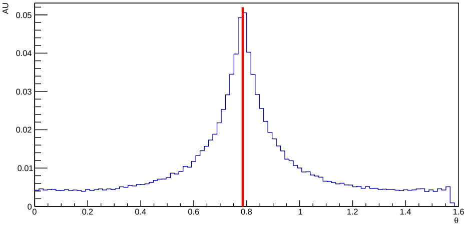
$$H = \begin{pmatrix} \hat{\tau}_B + \hat{\tau}_+ & \hat{\tau}_B \\ \hat{\tau}_B & \hat{\tau}_B + \hat{\tau}_- \end{pmatrix} = R(\theta) \cdot \begin{pmatrix} h_+ & 0 \\ 0 & h_- \end{pmatrix} \cdot R^{-1}(\theta)$$

$$(\hat{\tau}_B, \hat{\tau}_\pm) \leftrightarrow (\theta, h_\pm)$$

$$\cos(\theta) \simeq \cos(\frac{\pi}{4}) + \mathcal{O}(\hat{\tau}_+ - \hat{\tau}_-)$$

θ sensitive to the asymmetry of the triangle in the proper time dimension of each τ

θ-distribution for Signal MC



Signal reconstruction for $B_s \to \tau\tau \to (3\pi\nu)(3\pi\nu)$

Using the "rotated" sides & momenta

$$\begin{pmatrix}\tilde{w}_+\\ \tilde{w}_-\end{pmatrix} \equiv R^{-1}(\theta)\cdot\begin{pmatrix}w_+\\ w_-\end{pmatrix} \quad , \quad \begin{pmatrix}\tilde{p}_+\\ \tilde{p}_-\end{pmatrix} \equiv R^{-1}(\theta)\cdot\begin{pmatrix}p_+\\ p_-\end{pmatrix}$$

the following relation holds:

$$\begin{pmatrix}\tilde{w}_+\\ \tilde{w}_-\end{pmatrix} = \begin{pmatrix}h_+ & 0\\ 0 & h_-\end{pmatrix}\cdot\begin{pmatrix}\tilde{p}_+\\ \tilde{p}_-\end{pmatrix}$$

Formally equal to 2 $\otimes$ 1D cases

Signal reconstruction for $B_s \to \tau\tau \to (3\pi\nu)(3\pi\nu)$

Constraints:

- **"mass-shell" condition for $\tilde{p}_\pm^2 = m_\tau^2 \pm p_+ p_- \sin(2\theta)$**
- **direction of $\vec{\tilde{p}}_\pm = \sigma_\pm \vec{\tilde{w}}_\pm$**, with $\sigma_\pm \equiv h_\pm^{-1}$
- $p_\pm \cdot p^{(3\pi)^\pm} = \frac{m_\tau^2 + m_{3\pi^\pm}^2}{2}$

> **These constraints give 2 equations of 2^{nd} degree in $\sigma_\pm$,**
> $\mathcal{P}_\pm^{(2)}(\sigma_\pm, \theta) = 0$,
> **which linearly depend on $p_+ p_-$**

An additional constraint (trivially linear in $p_+ p_-$) can be imposed on

$$p_+ p_- = \frac{M_B^2 - 2m_\tau^2}{2} = 11.24 \frac{GeV^2}{c^4}$$

In total 3 equations for 3 unknowns ($\sigma_\pm$, θ)

What has been done so far...

Signal reconstruction for $B_s \to \tau\tau \to (3\pi\nu)(3\pi\nu)$

- Fix p_+p_- to its signal value, approximate θ (see next slide) and solve

$$\mathcal{P}_{\pm}^{(2)}(\sigma_{\pm}, \theta) = 0$$

- This system geometrically represents **the intersection of two hyperbola**

- Solutions $\sigma_{\pm}$ are expressed in term of the root of a 4^{th} degree polynomial:

$$P(\xi) = \sum_{i=0}^{4} a^{(i)}(\theta)\xi^i = 0$$

with $a^{(i)}(\theta)$ **functions of measurable quantities and the θ angle**

There exist analytical solutions

- For signal events **there must exist real solutions ξ with $\sigma_{\pm} \geq 0$**

Signal reconstruction for $B_s \to \tau\tau \to (3\pi\nu)(3\pi\nu)$

Some options to deal with θ-dependence:

- **substitute θ with its average value $\overline{\theta} = \frac{\pi}{4}$**
  - it's much better for signal than for exclusive backgrounds (with D and τ)
- **approximate θ by using measurable quantities**
  - θ as a (Lorentz invariant) function of $\vec{w}_+$ and $\vec{w}_-$

Due to

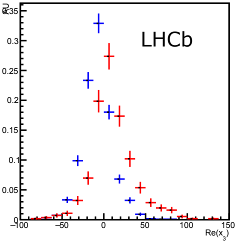
- **approximation of θ**
- **detector resolution**
- **radiation emission from $\pi^\pm$**

imaginary solutions of $P(\xi) = 0$ will appear also for signal events

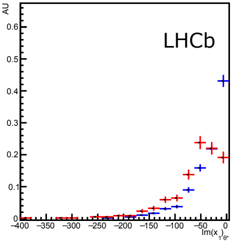
Discriminating variables

- **In principle**: only the two reconstructed eigenvalues $h_\pm$ of the H matrix ($m_{\tau,\nu,B}$ used as external constraints)
- **In practice**: also other functions of measurable quantities (*e.g.* the four complex ξ solutions) can be used to discriminate signal against background

Re(ξ)



Im(ξ)



Signal MC, **Background**

- The most discriminating ones are used as input of a Decision Tree whose output is used for the signal search

Latest developments

Signal reconstruction for $B_s \to \tau\tau \to (3\pi\nu)(3\pi\nu)$

Explore other ways to exploit the available information

- use in a different way the kinematic equations
  - in total 3 equations, can be re-arranged into
  - two p_+p_- independent equations: $\mathcal{S}_\pm(\sigma_\pm, \theta) = 0$
  - one p_+p_- linearly dependent equations: $p_+p_- = \mathcal{S}_0(\sigma_\pm, \theta)$
  - $\mathcal{S}_{\pm,0}(\sigma_\pm, \theta) = 0$ equations of hyperbola
- improve the approximation of θ

Signal reconstruction for $B_s \to \tau\tau \to (3\pi\nu)(3\pi\nu)$

$$\begin{cases} \mathcal{S}_\pm(\sigma_\pm, \theta) = 0 \\ p_+ p_- = \mathcal{S}_0(\sigma_\pm, \theta) \end{cases}$$

Two possible strategies:

- **full reconstruction**: use $\mathcal{S}_\pm(\sigma_\pm, \theta) = 0$ to find $\sigma_\pm(\theta)$ and fix θ with $\mathcal{S}_0(\sigma_\pm, \theta)$
  - only the $\tau_{B,\tau^\pm}$ as discriminating variables
  - use a Dirac peak (p_+p_-) to approximate a non-trivial function (θ)
- **p_+p_- as a function of θ**: find $\sigma_\pm(\theta)$ with $\mathcal{S}_\pm(\sigma_\pm, \theta) = 0$, use $\mathcal{S}_0(\sigma_\pm, \theta)$ to approximate p_+p-
  - p_+p_- is more discriminating that decay time
  - use a non-trivial distribution to approximate a Dirac peak
  - needs an approximation of θ

Signal reconstruction for $B_s \to \tau\tau \to (3\pi\nu)(3\pi\nu)$

$$\begin{cases} \mathcal{S}_\pm(\sigma_\pm, \theta) = 0 \\ p_+p_- = \mathcal{S}_0(\sigma_\pm, \theta) \end{cases}$$

Two possible strategies:

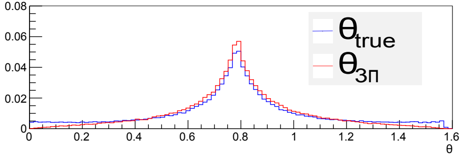
- **full reconstruction**: use $\mathcal{S}_\pm(\sigma_\pm, \theta) = 0$ to find $\sigma_\pm(\theta)$ and fix θ with $\mathcal{S}_0(\sigma_\pm, \theta)$
  - only the $\tau_{B,\tau^\pm}$ as discriminating variables
  - use a Dirac peak (p_+p_-) to approximate a non-trivial function (θ)
- **p_+p_- as a function of θ**: find $\sigma_\pm(\theta)$ with $\mathcal{S}_\pm(\sigma_\pm, \theta) = 0$, use $\mathcal{S}_0(\sigma_\pm, \theta)$ to approximate p_+p-
  - p_+p_- is more discriminating that decay time
  - use a non-trivial distribution to approximate a Dirac peak
  - needs an approximation of θ

θ approximation and estimation of p_+p_-

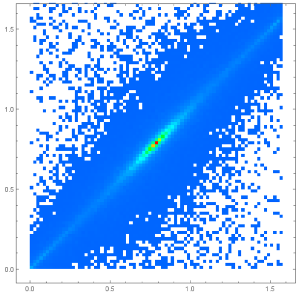
Use the relation $W = HP$ in 3D with $P = P_{3\pi}$:

$$\boldsymbol{H_{3\pi} = (P_{3\pi}P_{3\pi}^T)^{-1}[P_{3\pi}P_{3\pi}^T W W^T]^{\frac{1}{2}}}$$

Use $\boldsymbol{\theta_{3\pi}}$ which diagonalizes $\boldsymbol{H_{3\pi}}$ as approximation of the true θ



$\boldsymbol{\theta}$ vs $\boldsymbol{\theta_{3\pi}}$

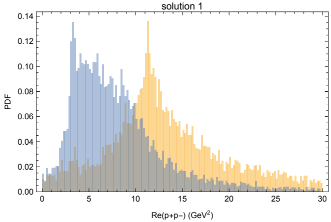


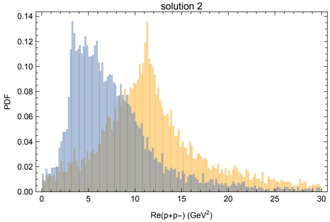
Find $\boldsymbol{\sigma_\pm(\theta_{3\pi})}$ using $\boldsymbol{\mathcal{S}_\pm(\sigma_\pm, \theta_{3\pi}) = 0}$

Replace $\boldsymbol{\sigma_\pm(\theta_{3\pi})}$ and $\boldsymbol{\theta_{3\pi}}$ in $\boldsymbol{\mathcal{S}_0(\sigma_\pm, \theta)}$ to estimate $\boldsymbol{p_+p_-}$

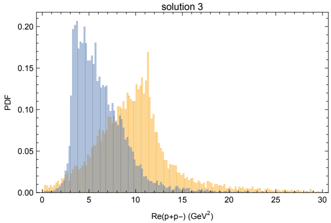
p_+p_- distribution: multiple solutions in the ideal case

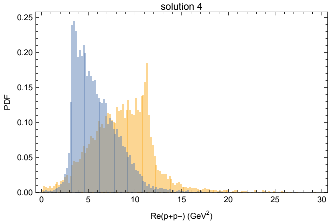
4 pairs (σ_+, σ_-) are solutions of $\boldsymbol{\mathcal{S}_\pm(\sigma_\pm, \theta_{3\pi}) = 0}$





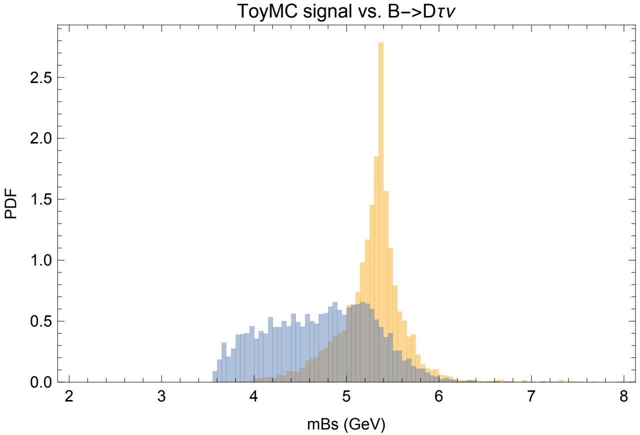
Distribution of p_+p_- for Signal MC (68% of events with real solutions), $B \to D\tau\nu$ background (44% of events with real solutions)





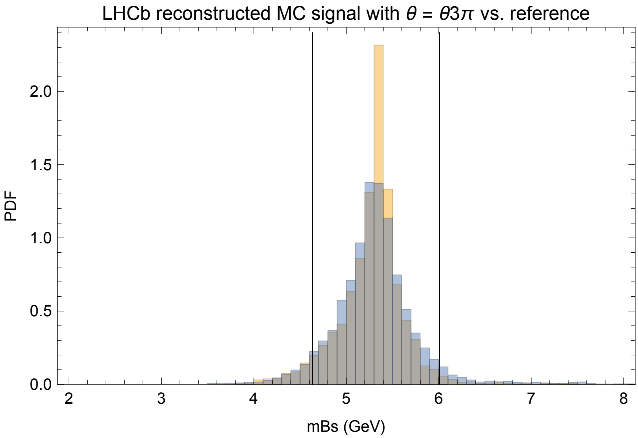
$M_{B_s^0}$ distribution: choice of the best solution in the ideal case

Distribution of M_{B_s} for Signal MC, $B \to D\tau\nu$ background for events with real solutions and choosing p_+p_- closest to the exact signal value ($11.24\frac{GeV^2}{c^4}$)



This choice does not create a false peak in the signal region for background!

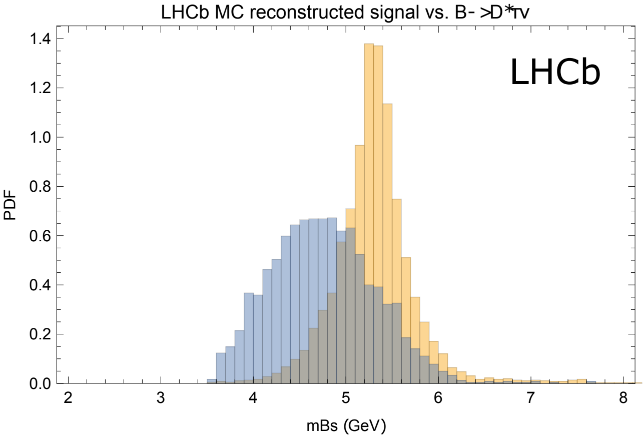
$M_{B_s^0}$ distribution: detector resolution effect



Distribution of M_{B_s} for Signal MC, Signal MC with detector resolution

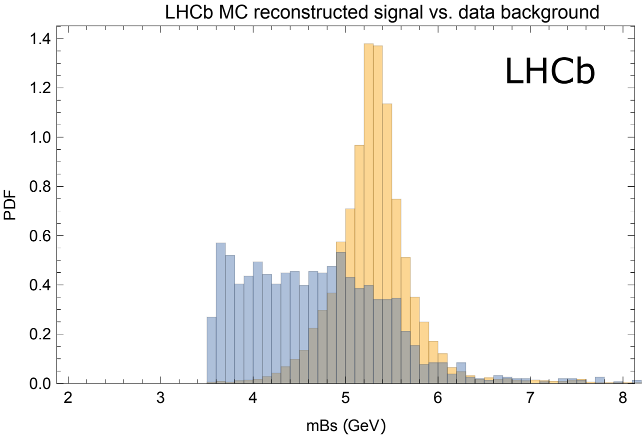
Only small $M_{B_s^0}$ resolution degradation due to detector resolution is observed

$M_{B_s^0}$ distribution: $B \to D^\star\tau\nu_\tau$



Distribution of M_{B_s} for Signal MC (36% of events with real solutions), $B^0 \to D^\star\tau\nu_\tau$ background (24% of events with real solutions)

$M_{B_s^0}$ distribution: generic background from data



Distribution of M_{B_s} for Signal MC (36% of events with real solutions), background in data (16% of events with real solutions)

Conclusions & Prospects - $B_{(s)}^0 \to \tau^+\tau^-$

$B_{(s)}^0 \to \tau^+\tau^-$ **is a window to test several New Physics scenarios**

- **Pioneer analysis**:
  - **missing energy makes signal reconstruction very challenging**
  - **a method for the $M_{B_s^0}$ has been presented**
  - **some discriminating variables already used in the experimental search**
  - **new strategies under study to leave $M_{B_s^0}$ a free variable**
  - **issues about multiple and complex solutions and ways to estimate θ**
- **Still to study how to use these variables from the latest development in the experimental search**

Backup

4^{th} degree equation

$$ax^4 + bx^3 + cx^2 + dx + e = 0$$

Solutions:

$$x_{1,2} = -\frac{b}{4a} - S \pm \frac{1}{2}\sqrt{-4S^2 - sp + \frac{q}{S}}$$
$$x_{3,4} = -\frac{b}{4a} + S \pm \frac{1}{2}\sqrt{-4S^2 - sp + \frac{q}{S}}$$

with

- $p = \frac{8ac-3b^2}{8a^2}$
- $q = \frac{b^3-4abc+8a^2d}{8a^3}$
- $S = \frac{1}{2}\sqrt{-\frac{2}{3}p + \frac{1}{3a}\left(Q + \frac{\Delta_0}{Q}\right)}$
- $Q = \sqrt{\frac{\Delta_1+\sqrt{\Delta_1^2-4\Delta_0^3}}{2}}$
- $\Delta_0 = c^2 - 3bd + 12ae$
- $\Delta_1 = 2c^3 - 9bcd + 27b^2e + 27ad^2 - 72ace$

Distribution of the 4 solutions - $\theta = \theta_{true}$

Distributions of the real part of the 4 solutions

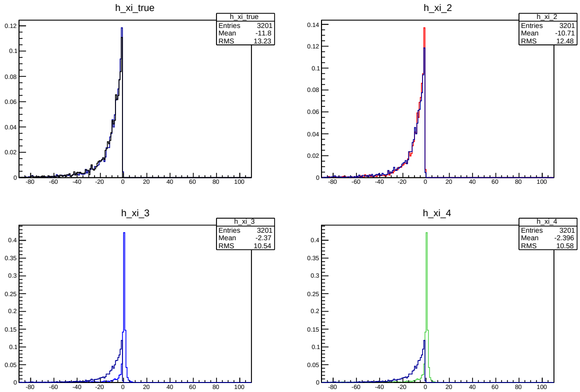


Figure: Distribution of $Re(\xi_i)$ and of ξ_{true} superimposed

Distribution of the 4 solutions - $\theta = \theta_{true}$

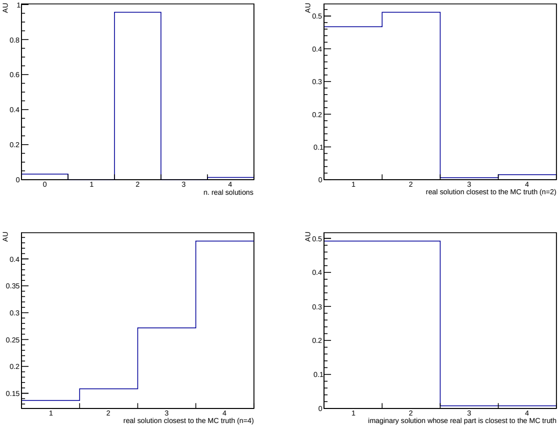


Figure: Behavior of the solutions for $\theta = \theta_{true}$

θ approximation

Define a matrix $\overline{H}$ function of measurables quantities with the following structure

$$\overline{H} = \begin{pmatrix} 1+\bar{t}_+ & 1 \\ 1 & 1+\bar{t}_- \end{pmatrix}$$

with $\bar{t}_\pm$ functions of $(\vec{w}_+, \vec{w}_-)$

Use $\bar{\theta}$ (which diagonalizes $\overline{H}$) to approximate θ

- For analogy with the H matrix :
  - $\overline{H}$ must trasform as H for exchange $+ \leftrightarrow -$ ($i.e.$ $\overline{H}_{11} \leftrightarrow \overline{H}_{22}$),
  - the functions $\bar{t}_\pm(\vec{w}_+, \vec{w}_-)$ must be adimensional and
  - Lorentz invariant
- The functions

$$\bar{t}_\pm \equiv \frac{|\vec{w}_\pm|}{|\vec{w}_+ + \vec{w}_-|}$$

have the required properties

- An **improved approximation $\theta_{\mathbf{imp}}$** has been found using a **recursive method** with zeroth order $\bar{\theta}$.

Distribution of the 4 solutions - $\theta = \bar{\theta}$

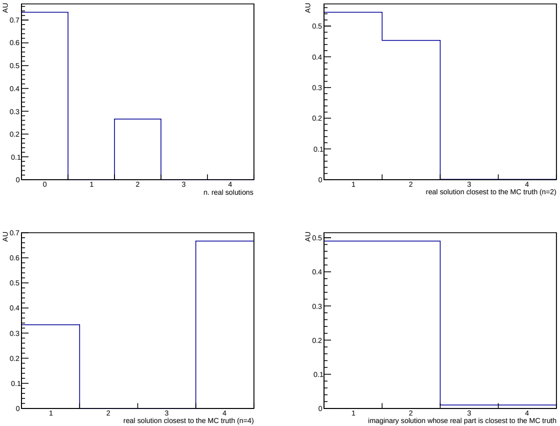


Figure: Behavior of the solutions for $\theta = \bar{\theta}$

τ-reconstruction

To fix θ the general relation between $\tilde{s}$ and s:

$$\tilde{s}(\theta) = s\cos 2\theta \tag{1}$$

can be exploited.
By developing both sides of the previous equation around $\theta^\star$ the following relation is found:

$$\tilde{s}(\theta^\star + \delta\theta) = \tilde{s}(\theta^\star) + \delta\theta \frac{d\tilde{s}}{d\theta}\|_{\theta^\star} = s(\cos^2\theta^\star - \sin^2\theta^\star - 4\delta\theta\cos\theta^\star\sin\theta^\star). \tag{2}$$

where the quantities on the l.h.s. are meant to be computed by solving the main equation.

- The value of $\frac{d\tilde{s}}{d\theta}\|_{\theta^\star}$ must be evaluated numerically, by solving the fundamental equation with $\theta = \theta^\star \pm d\theta$ and for each of these two values computing the value $\tilde{s}(\theta^\star \pm d\theta)$ and then the incremental ratio $\frac{\tilde{s}(\theta^\star + d\theta) - \tilde{s}(\theta^\star - d\theta)}{2d\theta}$

τ-reconstruction

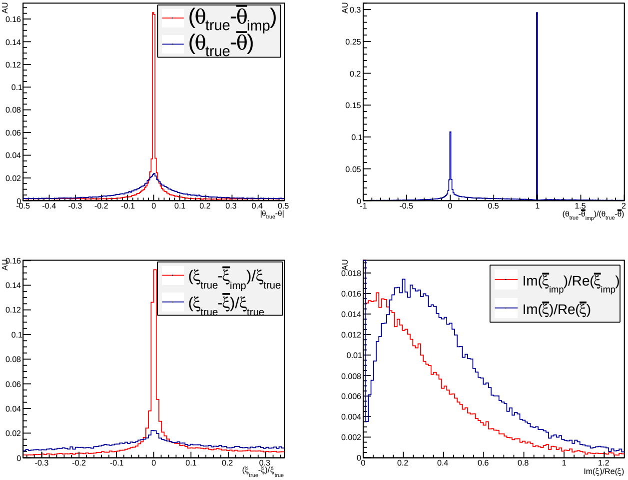
From the previous equation follows follows that

$$\delta\theta = \frac{s(\cos^2\theta^\star - \sin^2\theta^\star) - \tilde{s}(\theta^\star)}{4s\cos\theta^\star\sin\theta^\star + \frac{d\tilde{s}}{d\theta}\|_{\theta^\star}} \quad (3)$$

By iterating the process 10 times, with $d\theta_n = \frac{0.0075}{2^n}$, stopping the process as soon:

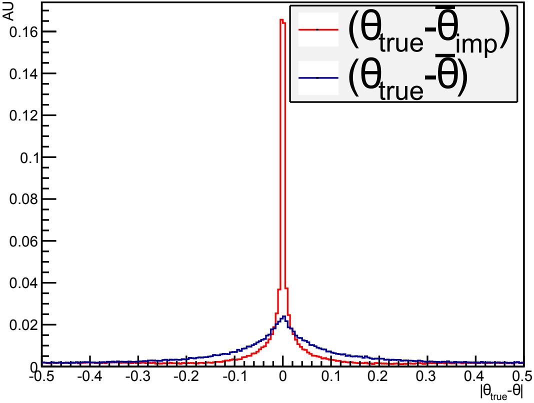
- $|\delta\theta_{n+1}| > |\bar{\theta}_n|$
- $|\delta\theta_{n+1}| > |d\theta_n|$
- $|\delta\theta| < 10^{-4}$

τ-reconstruction



Efficiency to find a real solution ($Im(\xi)/Re(\xi) < 5\%$): $\sim 53\%$ on 10^5 events.

θ approximation



Fraction of events with a real solution ε

θ approx	ε
$\pi/4$	~ 21 %
$\bar{\theta}$	~ 27 %
$\bar{\theta}_{imp}$	~ 50 %

Distribution of the 4 solutions - $\theta = \bar{\theta}_{recursion}$

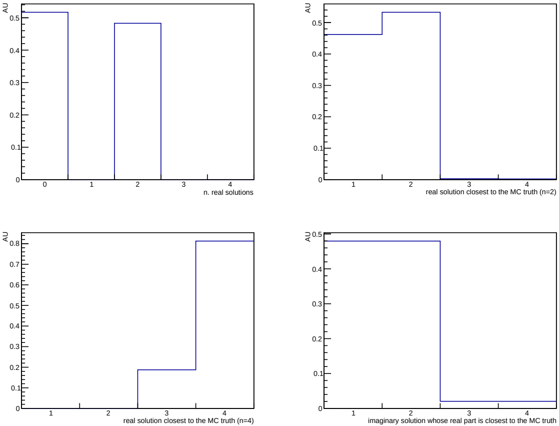
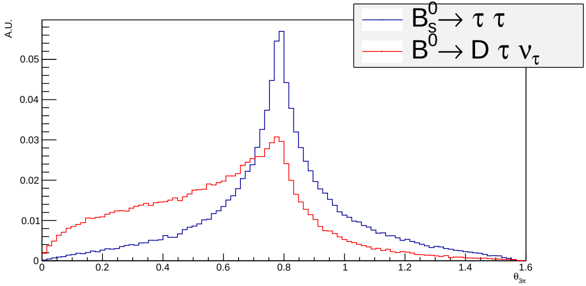


Figure: Behavior of the solutions for $\theta = \bar{\theta}_{recursion}$

$\theta_{3\pi}$ distribution for signal an $B^0 \to D\tau\nu_\tau$



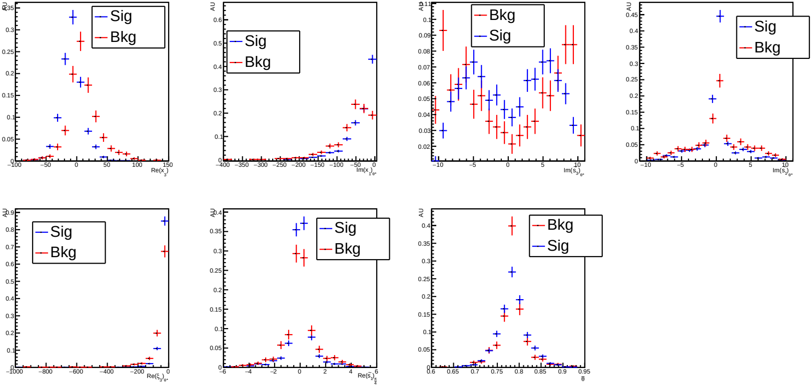
Reconstructed $M_{B_s^0}$ resolution in $(\frac{MeV}{c^2})$

θ	Mean $M_{B_s^0}$	$\sigma(M_{B_s^0})$	Median $M_{B_s^0}$	Median deviation
θ_{true}	5368.23	92.74	5366.27	1.94
$\theta_{3\pi}$	5294.95	428.66	5326.86	167.37
$\theta_{3\pi} \oplus$ resolution	5330.68	530.639	5302.48	210.56

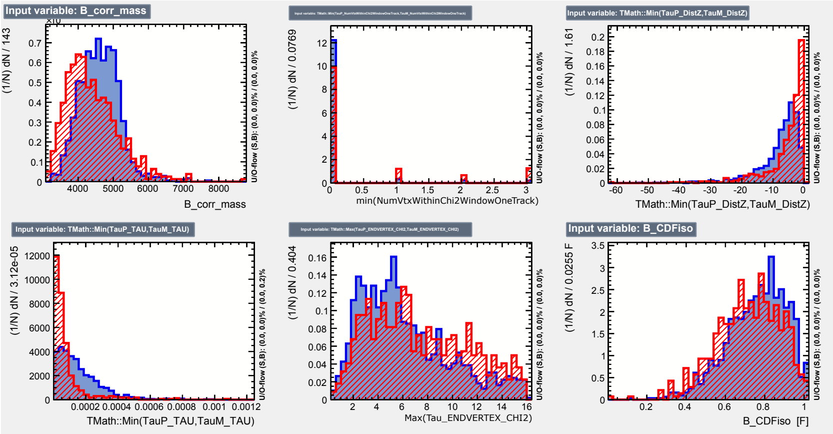
BDT input variables

- The following set of 13 variables has been used to train a new BDT:
  - Re_x_3
  - Max(Tau_DistZ)
  - Min(Tau_DistZ)
  - B_corr_mass
  - Im_x_1_ar
  - Im_stildepm_1_ar
  - Im_stildepm_3_ar
  - Min(Tau_NumVtxWithinChi2WindowOneTrack)
  - Re_stildepm_1_Pi4
  - Max(Tau_ENDVERTEX_CHI2)
  - Re_xi_1_ar
  - theta_bar_W
  - B_CDFiso

BDT input variables



BDT input variables



BDT input variables