Angular analysis prospects in $b \to s\mu\mu$

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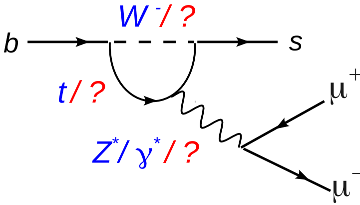
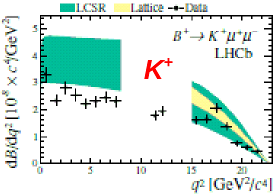
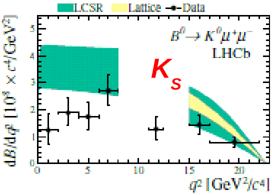
The $b \to s\ell^+\ell^-$ "industry" at the LHC

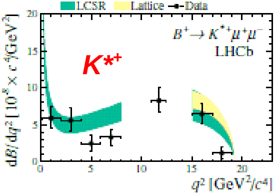
- Flavor-changing-neutral-current (FCNC).
- No tree-level diagram in the SM. Many ways where NP can enter.
- Several ways to explore this:
  - $B_s \to \mu^+\mu^-$ BF @ LHCb/CMS
  - $B \to K^{*J}\ \gamma_{\rm pol}$ @ LHCb
  - $B_d \to K^{(*)}\ell^-\ell^+$ @ LHCb/CMS/ATLAS
  - $B_s \to \phi\mu^+\mu^-$, $\Lambda_b \to \Lambda^{(*)}\mu^+\mu^-$ ...



Suite of anomalies...







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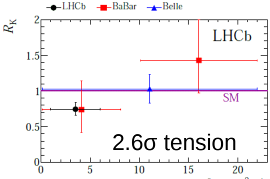

PRL 112, 161801 (2014)

$B_s \to \phi\mu\mu$ (3.3 σ)

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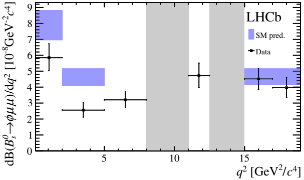

$\Lambda_b \to \Lambda\mu\mu$

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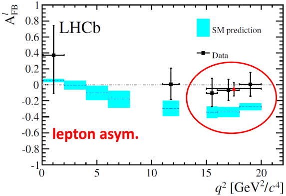
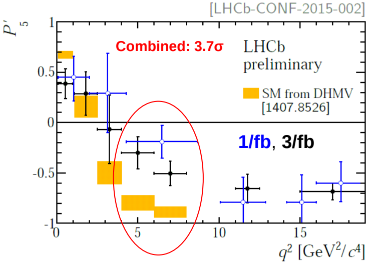


Motivation for angular analyses

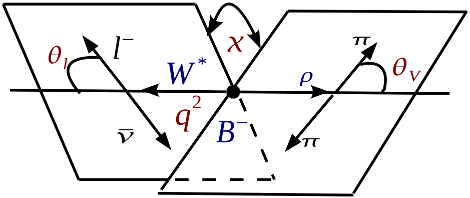
- Lesson from P_5': anomalies can show up in hitherto unexpected places.



- Angular observables being interference terms have more sensitivity than rates. Good bet for NP hunting.

$B_d \to K\pi\mu\mu$ AND FRIENDS

- $b \to X(\to h_1 h_2)\ell_1\ell_2$ topology: $\Phi \in \{m_X, q^2, \theta_\ell, \theta_V, \chi\}$



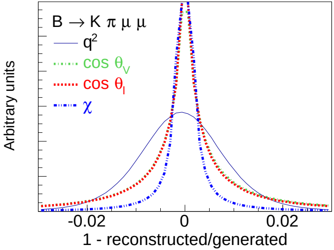
- Focus of this talk: wider window in the m_X spectrum.
- Higher waves in the dihadron system $\Rightarrow$ more observables to play around with.

The golden modes

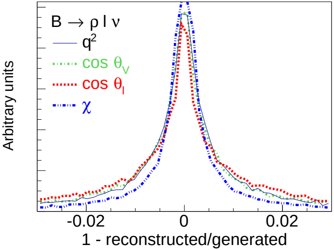
- $\bar{B} \to K^- \pi^+ \mu\mu$
- Flavor-tagged, but unpolarized.
- After LHCb Run II $\sim$ 12000 K^*.
- Everybody's favorite EWP!

- $\bar{B} \to \{\pi\pi, D^*\}\ell^- \bar{\nu}_\ell$, hadronic tagged
- Fully flavor-tagged and polarized.
- Belle II $\rho \sim$ 50000. D^*: $\times$20.
- RH currents, incl./excl. $|V_{ub}|/|V_{cb}|$ puzzle.

Prelim. LHCb MC:



Prelim. BABAR MC:



- Experimentally, easiest/cleanest. Percent-level resolutions in both.

The not-so optimal

- $B_s \to KK\mu\mu$: flavor-averaging is a dampener. Also, not polarized. After LHCb Run II, ~ 2500 KK $(\phi + f'(1525) + ...)$ events.
- For every two B_d's in LHCb acceptance, we produce one Λ_b. Richer spinor angular structure than the pseudoscalars in $B_{d/s}$.
  - $\Lambda_b \to pK\mu\mu$: flavor-tagged, but unpolarized. Around 1500 signal, after Run II. Very poorly known $m(pK)$ spectrum.
  - $\Lambda_b \to \Lambda\mu\mu$: flavor-tagged, but unpolarized. Self-analyzing $\Lambda \to p\pi$ helps. Downstream tracks (no VeLo) have lower efficiency. Roughly 1500 Λ's expected after Run II.

Setup for $B \to X^J(\to h_1 h_2)\ell_1\ell_2$

- $X^J \in \{\pi\pi, K\pi, KK, D\pi\}$ is in spin-J.
- Helicity amplitudes $H_\lambda^{J,\eta}$ tagged by J, $\eta \equiv (\lambda_{\ell_1} - \lambda_{\ell_2}) = \pm 1$, and $\lambda \in \{0, \pm 1\}$.
- Amplitude squared reads:

$$|\overline{\mathcal{M}}|^2 = \sum_{\eta=\pm 1} \left| \sum_{\lambda\in\{0,\pm 1\}} \sum_J \sqrt{2J+1}\mathcal{H}_\lambda^{J,\eta} d_{\lambda,0}^J(\theta_V) d_{\lambda,\eta}^1(\theta_\ell) e^{i\lambda\chi} \right|^2$$

Difference between SL and EWP modes

- For SL decays, the LH(RH) $\nu_L(\overline{\nu}_R)$ tags the polarization of both outgoing spinors.
- This is a "complete" measurement. Observables uniquely determine the underlying amplitudes.
- For EWP case, outgoing muon spins not known. Dilution of information due to incoherent summation over $\eta = \pm 1$.
- This is an "*in*complete" measurement. Observables do *not* uniquely determine the amplitudes.

The two-fold ambiguity and bilinears

- Rate is invariant under the symmetry $\mathcal{H}_{\lambda}^{\eta,J} \to \left(\mathcal{H}_{-\lambda}^{-\eta,J}\right)^*$.
- Generalization of the same "two-fold ambiguty" in determination of $\sin 2\beta_{(s)}$ from $B_d \to J/\psi K^*$ and $B_s \to J/\psi \phi$.
- Consider the complex matrices: $n_{\lambda}^{J} \equiv \begin{pmatrix} \mathcal{H}_{\lambda}^{\eta,J} \\ \left(\mathcal{H}_{-\lambda}^{-\eta,J}\right)^* \end{pmatrix}$
- All observables occur as bilinears $\Gamma \sim n_i^{\dagger} n_j$, respecting this symmetry.

Example: *SPD* waves in the $[K\pi]$ system

- 7 two-component matrices that produce 56 real bilinears:

$$s = \begin{pmatrix} S^L \\ S^{R*} \end{pmatrix}$$

$$h_{\parallel} = \begin{pmatrix} H_{\parallel}^L \\ H_{\parallel}^{R*} \end{pmatrix} \quad h_{\perp} = \begin{pmatrix} H_{\perp}^L \\ -H_{\perp}^{R*} \end{pmatrix} \quad h_0 = \begin{pmatrix} H_0^L \\ H_0^{R*} \end{pmatrix}$$

$$d_{\parallel} = \begin{pmatrix} D_{\parallel}^L \\ D_{\parallel}^{R*} \end{pmatrix} \quad d_{\perp} = \begin{pmatrix} D_{\perp}^L \\ -D_{\perp}^{R*} \end{pmatrix} \quad d_0 = \begin{pmatrix} D_0^L \\ D_0^{R*} \end{pmatrix}$$

- Rate comprises 41 angular observables, like:

i	$f_i(\Omega)$	$\Gamma_i(q^2)$
1	$P_0^0 Y_0^0$	$\left[\lvert s\rvert^2 + \lvert h_0\rvert^2 + \lvert h_{\parallel}\rvert^2 + \lvert h_{\perp}\rvert^2 + \lvert d_0\rvert^2 + \lvert d_{\parallel}\rvert^2 + \lvert d_{\perp}\rvert^2\right]$
27	$P_3^0 \sqrt{2}\, Im(Y_2^2)$	$-\frac{3}{5}\sqrt{\frac{3}{7}}\, Im(h_{\parallel}^{\dagger} d_{\perp} + d_{\parallel}^{\dagger} h_{\perp})$
41	$P_4^1 \sqrt{2}\, Im(Y_1^1)$	$-\frac{3}{7}\sqrt{10}\, Im(d_0^{\dagger} d_{\parallel})$

Relations amongst the observables

- I can sandwich a unitary matrix inside the product as: $n_i^\dagger U^\dagger U n_j$.
- 3 generators + 1 phase, so $n_{\mathrm{gen}} = 4$ symmetry relations.
- 14 complex amplitudes mean $n_{\mathrm{obs}} = 2 \times 14 - 4 =$ 24 independent observables.
- This means, 17 relations amongst the 41 Γ_i.
- Some are simple: $\Gamma_{25} = -\sqrt{\frac{7}{3}}\Gamma_{27}$. Some are messy:

$$
\begin{aligned}
0 = &\left[\left(\sqrt{\frac{5}{3}} f_{23} + \frac{\sqrt{5} f_{10} + f_5}{3}\right)\left(f_{14}^2 + \frac{f_{41}^2}{5}\right) - \left(\frac{f_5/2 - \sqrt{5} f_{10}}{54}\right)\left((f_{29} + \sqrt{5} f_{31})^2 + 5(f_{24} + \sqrt{5} f_{24})^2\right)\right. \\
&+ \frac{2}{3\sqrt{15}}\left[(f_{37} f_{14} + f_{18} f_{41})\left(f_{24} + \sqrt{5} f_{31}\right) + (f_{37} f_{41} - 5 f_{18} f_{14})\left(f_{24} + \sqrt{5} f_{24}\right)\right] \\
&- \left(\sqrt{\frac{5}{3}} f_{23} - \frac{\sqrt{5} f_{10} + f_5}{3}\right)\left[\left(\frac{f_5/2 - \sqrt{5} f_{10}}{2}\right)\left(\sqrt{\frac{5}{3}} f_{23} + \frac{\sqrt{5} f_{10} + f_5}{3}\right) + \left(\frac{f_{37}^2}{5} + f_{18}^2\right)\right]
\end{aligned}
$$

Relations amongst the observables (cntd.)

- The pure P-wave case is special. 6 real bilinears, directly solvable from the observables.
- Aside from the two-fold ambiguity, things are determined.
- "Almost" true for SP-wave case (see Matias). $Im(s^\dagger h_0)$ absent in the observables, but all relations known.
- For SPD waves (and higher), several problems:
  - Unlike, SP case, the pure S, P, D-waves do *not* decouple. Eg.: $(|S|^2 + |H_0|^2)$ occurs together, so we might not have F_S.
  - 56 real bilinears, but only 41 observables. Many missing.
  - Deriving the relations between observables yet unsolved.

The Moments Method: introduction

- De facto reference: `arXiv:1505.02873`
- Recap: higher waves and full 4-D fit difficult because we don't know the minimal set of independent observables.
- Constrained fits could be one way: take FF predictions and float a few Wilson coeffcients.
- Other way is to bypass doing a fit at all: the moments method.
- Rewrite $|\mathcal{M}|^2 = \sum_i \Gamma_i(q^2)\, f_i(\theta_\ell, \theta_V, \chi)$ in an orthonormal f_i basis.
- Orthonormality guarantees $\langle f_i \rangle \equiv \overline{\Gamma}_i$.
- Convenient basis: products of $Y_l^m \equiv Y_l^m(\theta_\ell, \chi)$ and $P_l^m \equiv \sqrt{2\pi} Y_l^m(\theta_V, 0)$.

Moments: what results we will provide

- For $B_d \to K\pi\mu\mu$ *SPD* analysis, we will provide the 40 normalized Γ_i's and 40×40 cov. matrix in "some" $\{q^2, m(K\pi)\}$ binning.
- Straightforward to compare to these to theory $\Rightarrow$ core results.
- Some specific components extractable:

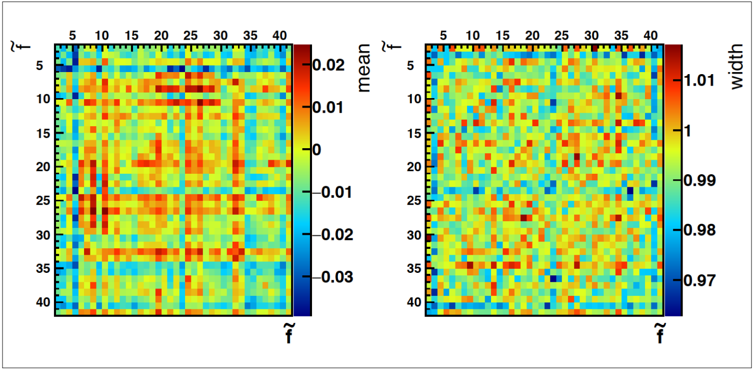
$$|d_0|^2 = \frac{7}{9}\left(\frac{f_5}{2} - \sqrt{5}f_{10}\right)$$

$$|d_{\parallel}|^2 = \frac{7}{4}\left(\sqrt{\frac{5}{3}}f_{23} - \frac{1}{3}\left(\sqrt{5}f_{10} + f_5\right)\right)$$

$$|d_{\perp}|^2 = \frac{7}{4}\left(-\sqrt{\frac{5}{3}}f_{23} - \frac{1}{3}\left(\sqrt{5}f_{10} + f_5\right)\right)$$

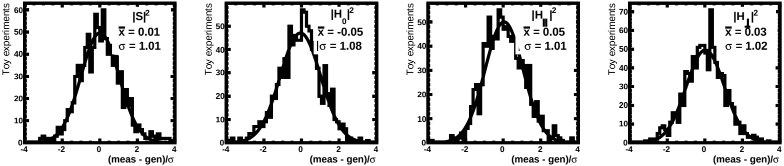
Moments: toy studies

- Many toy studies done, both LHCb and *BABAR*. Method works beautifully.
- Covariance matrix element between Γ_i and Γ_j checked by looking at pulls in $(\Gamma_i + \Gamma_j)$.



More advertisement

- Not having to do a fit *is* a big deal! Just count.
- $B^- \to \pi^+\pi^-\ell^-\overline{\nu}_\ell$ in *BABAR* with $S+P$ waves under the ρ.
- *Highly* statistics limited, yet toys seem to perform very well. Full moments paper in the pipeline.



- Without the moments method, pulling out F_S is semi-impossible.

THE $\Lambda_b \to \{pK, \Lambda\}\mu\mu$ CASE

- In addition to the muons, we now have a proton whose polarization is being averaged over.
- Exacerbates "incompleteness" of the measurement.
- $\Lambda_b \to \Lambda\mu\mu$: BF, $A_{FB}^{\ell,h}$ published.
- $\Lambda_b \to pK\mu\mu$: BF and $A_{FB}^{\ell,h}$ in the pipeline.

$\Lambda_b \to \Lambda\mu\mu$ MOMENTS ANALYSIS

- Go beyond $A_{FB}^{\ell,h}$. Assuming Λ_b almost unpolarized, full 4-D rate in the `Korner` paper.
- 12 complex amplitudes: $H^{V,A}_{\lambda_\Lambda,\lambda}$, where λ is the "usual" helicity of the spin-1 dimuon.
- Very preliminary calculation gives 10 moments.

i	$f_i(\Omega)$	$\Gamma_i(q^2)$
1	$P_0^0 Y_0^0$	$\left[2\sqrt{2\pi}(U+L) + 2\sqrt{2\pi}(U-L)/3\right]$
5	$P_0^0 Y_2^0$	$\left[4\sqrt{2\pi}(U-L)/5\right]$
10	$\sqrt{2}P_1^1 Im(Y_1^1)$	$[-4\sqrt{\pi}/3I_3]$

- As before, the moments are not independent, with possible very complicated inter-relations.

SUMMARY: $B_d \to K\pi\mu\mu$

- Run I analysis till $m(K\pi) = 1530$ MeV, including *SPD*-waves making good progress.
- $\{q^2, m(K\pi)\}$ binning under discussion.
- Run II will include *F* and *G*-waves as well. Moments calculations in progress.
- Same tools for $B_s \to KK\mu\mu$ in Run II.
- We already have reasonable statistics. Need theory predictions.

Summary: $\Lambda_b \to \Lambda\mu\mu$ and $\Lambda_b \to pK\mu\mu$

- Extend the Run I paper with the full set of moments.
- From MC studies, expect ~ 400 [pK] events in Run I.
- Large suite of poorly known Λ^*'s makes the moments derivation complicated.
- Thrust is to retain the full set of available moments and come up with a global χ^2 against theory.

More ideas...

- Charm-loop effects:
  - can we go closer to the $c\bar{c}$ resonances?
  - any specific observable sensitive to the non-factorizable part?
  - can we measure the relative phase between pen. and $c\bar{c}$?
- *ee* analyses (lots of ongoing effort):
  - $R(K^*)$, $R(\phi)$ and $R(K\pi)$. $R(K)$ in $q^2 > 1$ GeV^2.
  - joint *ee* and $\mu\mu$ angular analyses? Bin-migration.
- I assumed $4\times$ more statistics after Run II. Of course, this can increase as well.

Backup: non spin-1 dilepton states

- The Zwicky paper considers non-P-wave in the dilepton system as well.
- For the τ this is well known (see Ligeti), or even at $q^2 \to m_\mu^2$.
- For the massless case $\eta \equiv (\lambda_{\ell_1} - \lambda_{\ell_2}) = \pm 1$ for spin-1, and $\eta = 0$ for spin-0.
- In addition to helicity suppression, the $\eta = 0$ component can add only incoherently to $\eta = \pm 1$. No interference means NLO effect.