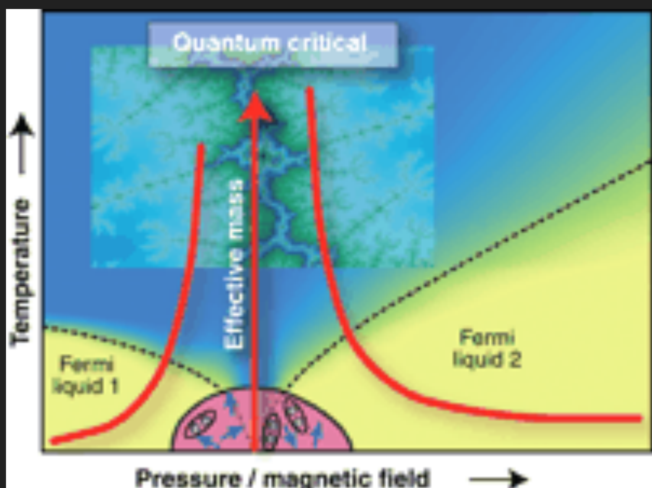
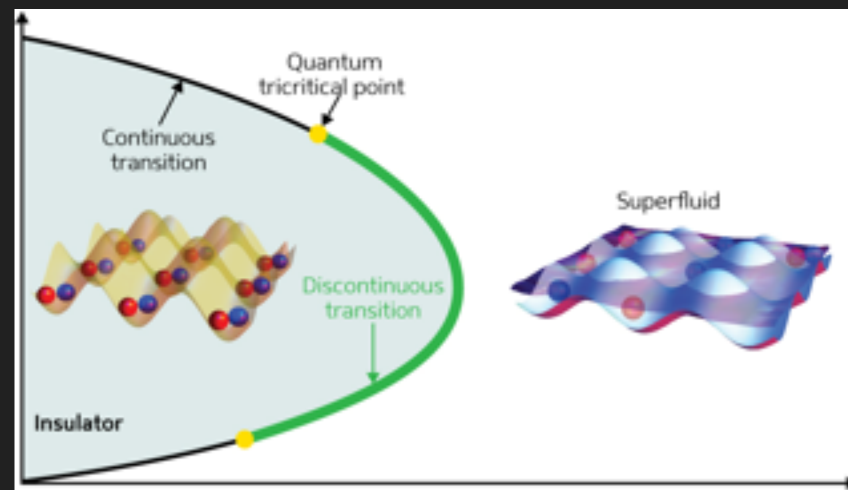
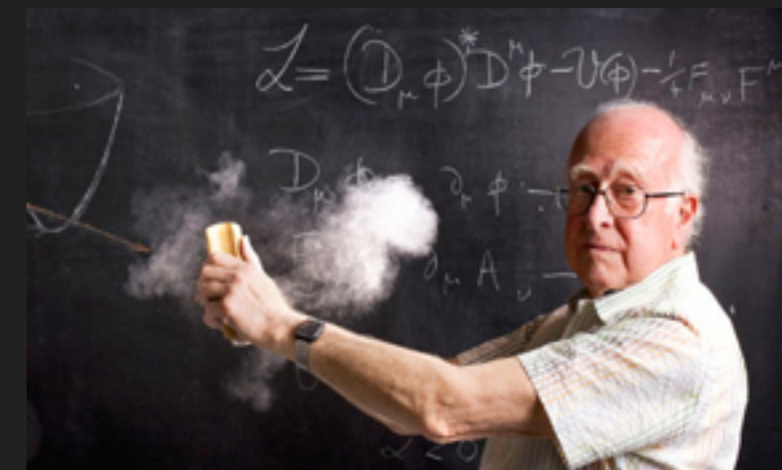
LIO international conference on Flavour, Composite models and Dark matter







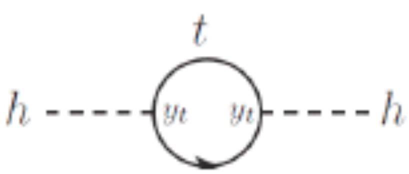
The Quantum Critical Higgs

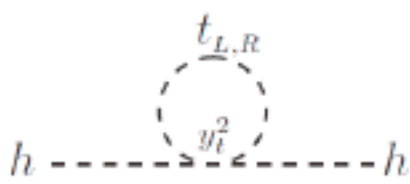
Seung J. Lee
Korea University

With B. Bellazzini, C. Csaki, J. Hubisz, J. Serra, J. Terning
to appear this week

Run 2 is now running: will NP keep hiding or finally show up?

Naturalness => New Physics at ~TeV scales (EW scales)





$$\frac{\delta m_h^2}{m_h^2} \sim \left(\frac{\tilde{m}_t}{400\,\mathrm{GeV}} \right)^2$$

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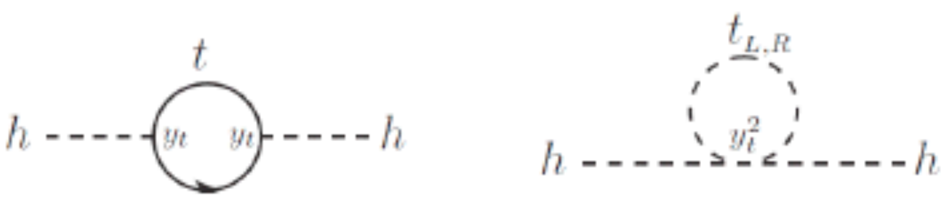

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2 leading frameworks
of naturalness

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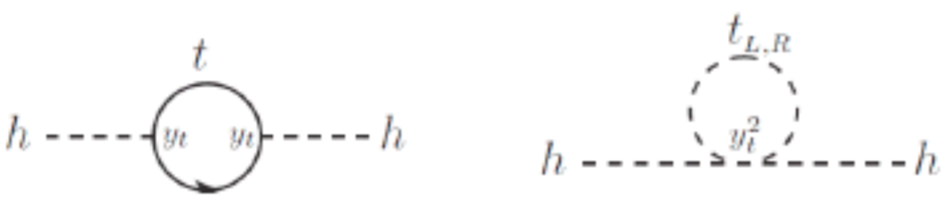

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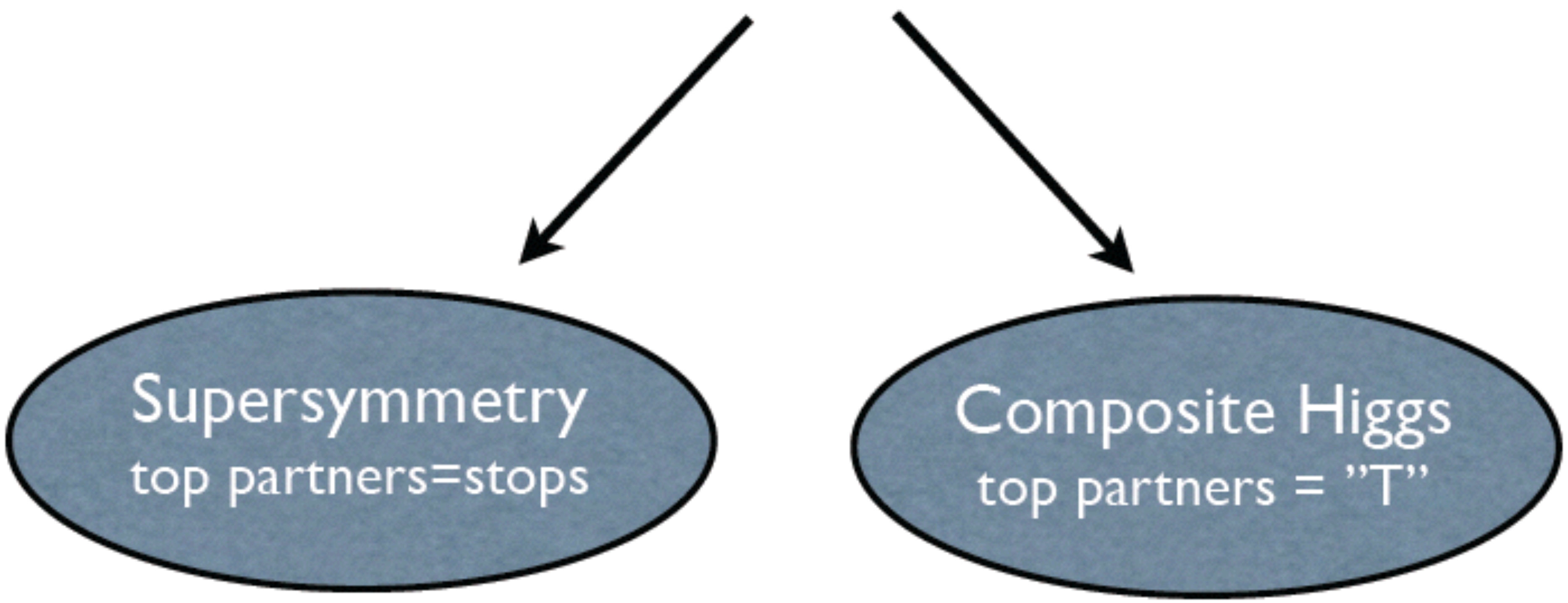

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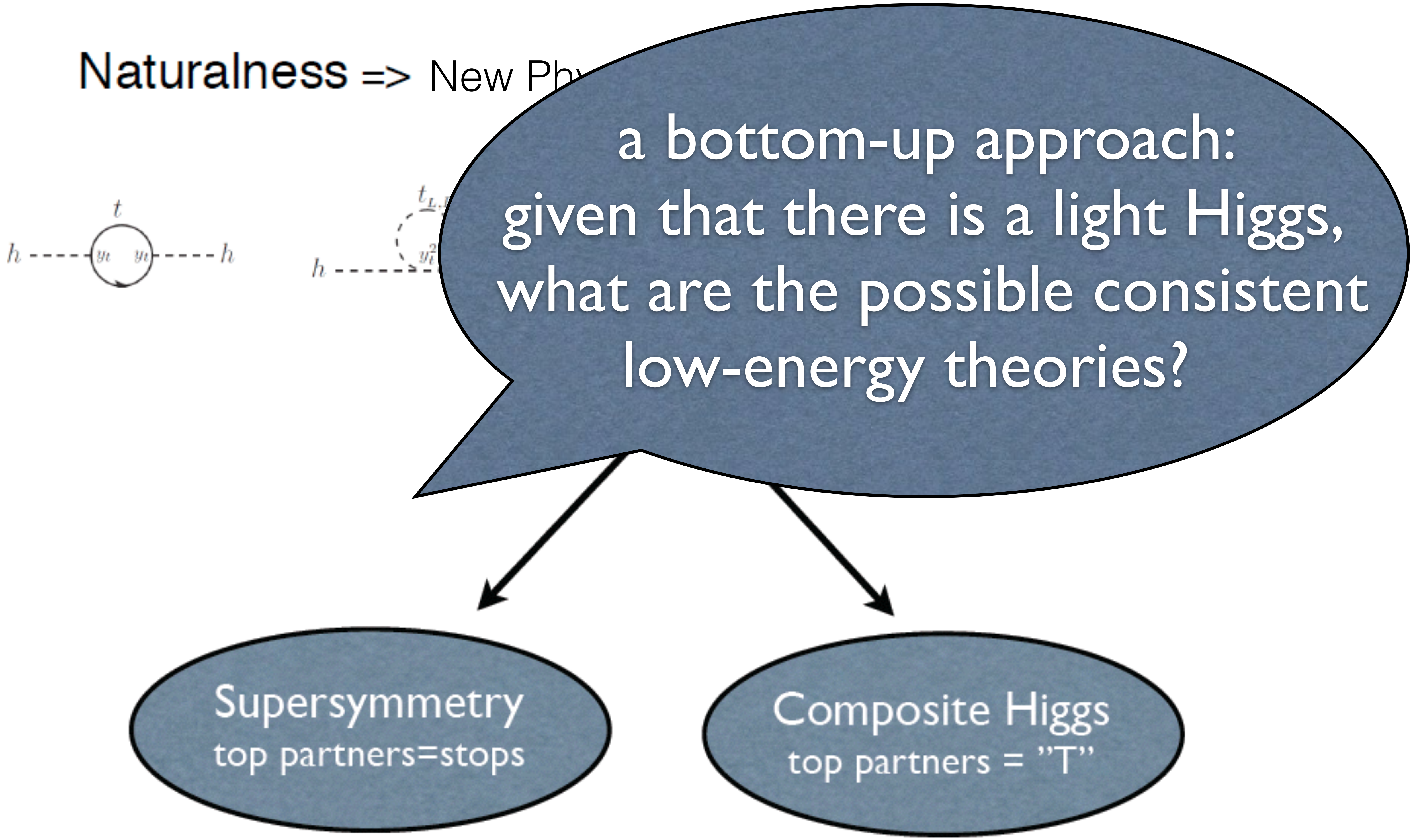


Ising Model

$$H = -J \sum s(x) s(x+n)$$

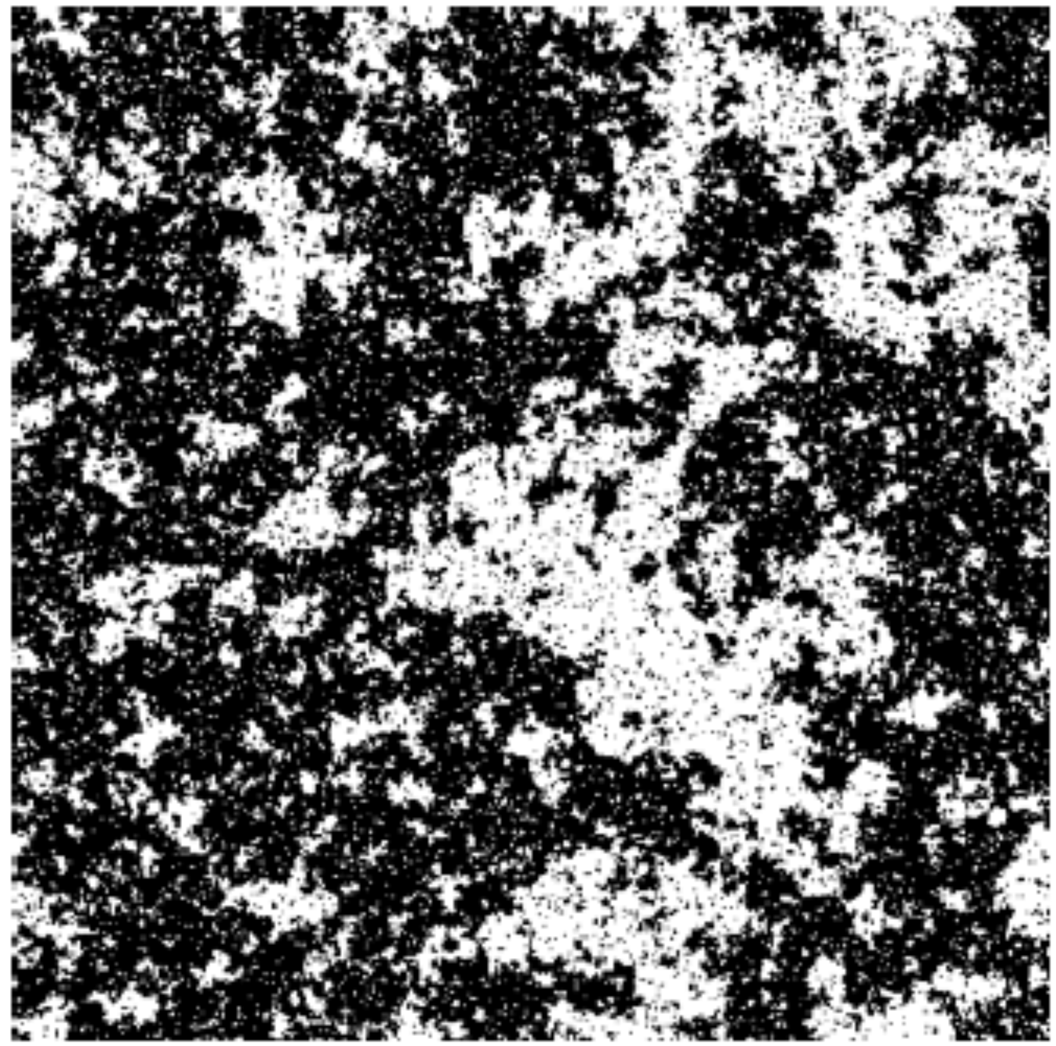
$$s(x) = \pm 1$$

$$\langle s(0) s(x) \rangle = e^{-|x|/\xi}$$

at $T = T_c$ $\quad \xi \to \infty$

Courtesy of J. Terning

Critical Ising Model is Scale Invariant



http://bit.ly/2Dcrit

at $T=T_c$ $$\langle s(0)s(x)\rangle \propto \frac{1}{|x|^{2\Delta-1}}$$

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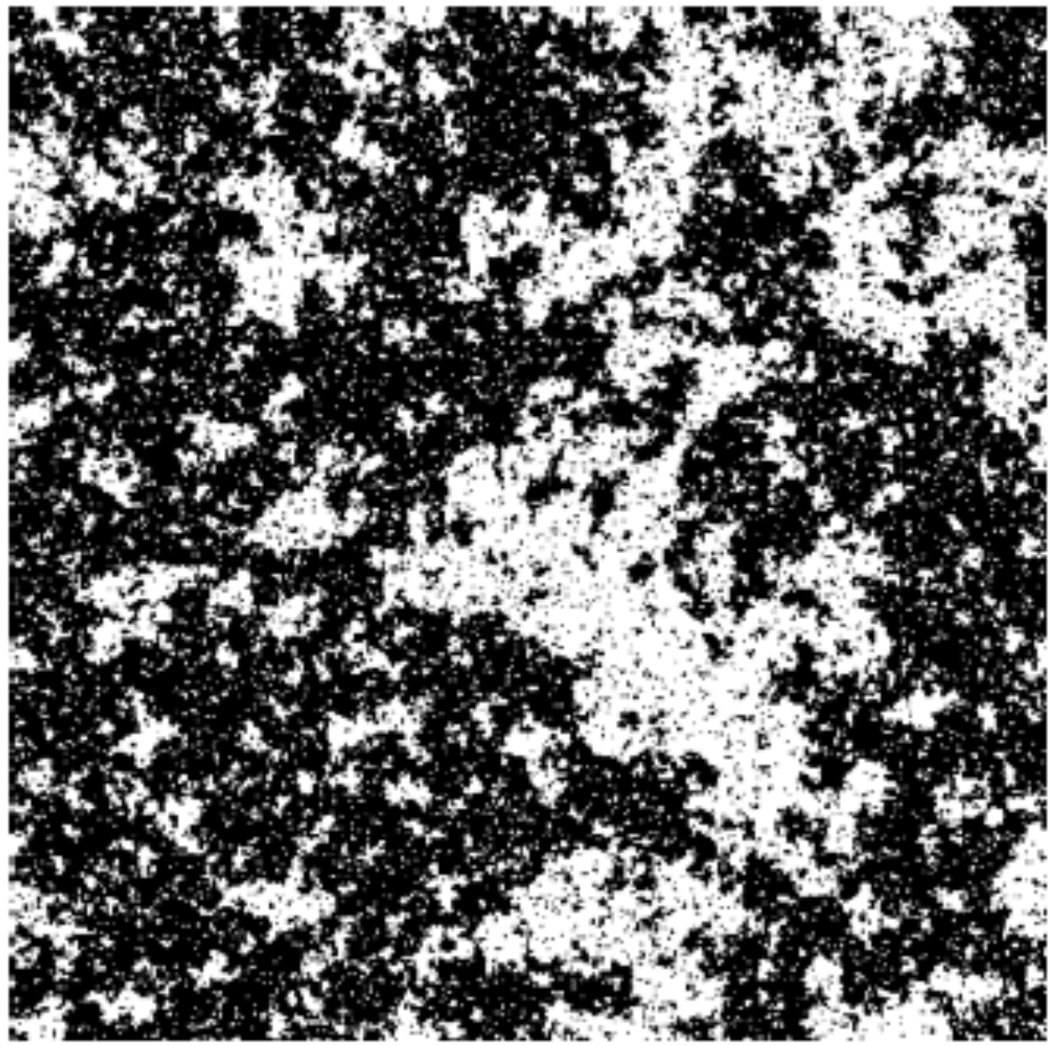
Critical Ising Model is Scale Invariant



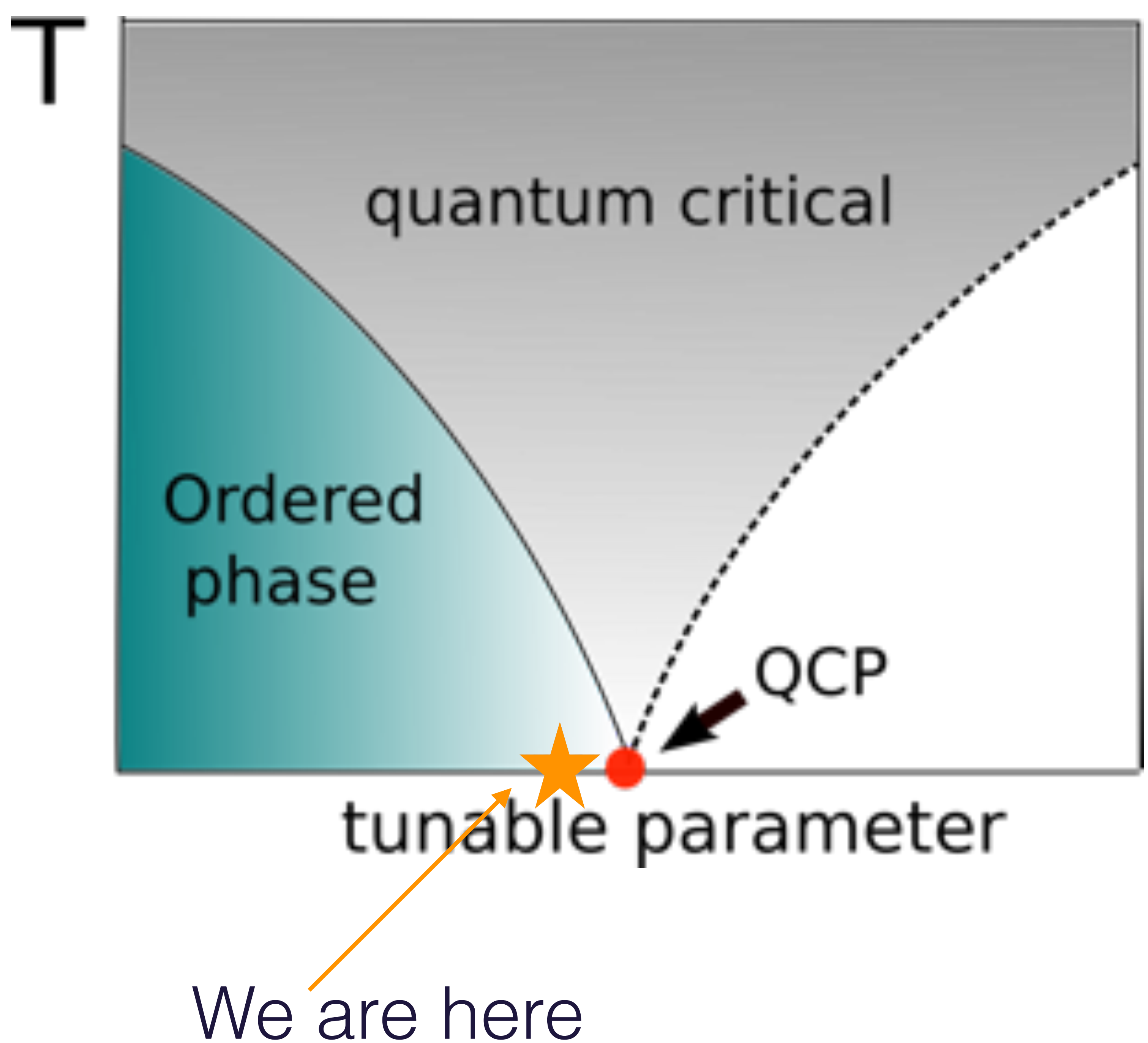
http://bit.ly/2Dcrit

at T=T$_c$ $\langle s(0)s(x)\rangle \propto \frac{1}{|x|^{2\Delta-1}} = \int d^3p\, \frac{e^{ip\cdot x}}{|p|^{4-2\Delta}}$

critical exponent

Courtesy of J. Terning

Quantum Phase Transition



Quantum Critical Higgs

- At a QPT the approximate scale invariant theory is characterized by the scaling dimension Δ of the gauge invariant operators.

  SM: $\Delta = 1 + \mathcal{O}(\alpha/4\pi)$.

- We want to present a general class of theories describing a Higgs eld near a non-mean-eld QPT.

- In such theories, in addition to the pole corresponding to the recently discovered Higgs boson, there can also be a Higgs continuum, which could potentially start not too far above the Higgs mass, representing additional states associated with the dynamics underlying the QPT

- One result of the presence of the continuum will be the appearance of form factors in couplings of the Higgs to the SM particles.

Form factors for QPT Higgs

- We are interested in formulating a general low-energy EFT consistent with a QPT and no new massless particles

- We consider a QPT Higgs scenario where Higgs is (partially) imbedded into a strongly coupled sector, which is approximately conformal at scale well above the EW scale.

  => Higgs pick up a significant anomalous dimension, and there is a large mixing with the continuum

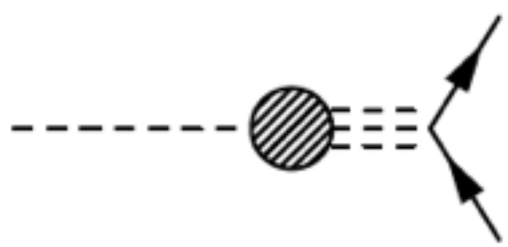
- The effects of Higgs emerging from the quantum critical point can be parametrized in terms of form factors in a model independent way.

- We assume that the SM fermions, the massless gauge bosons, and the transverse parts of the W and Z are external to the CFT, that is elementary, while the Higgs (Z_{long}, W_{long}) originates from or is mixed with the strong sector, corresponding to a theory with spontaneously or explicitly broken conformal symmetry.

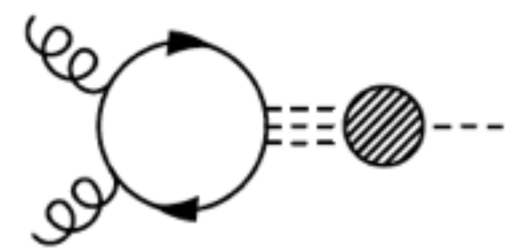
  => this strong sector is characterized by its n-point functions

Form factors for QPT Higgs

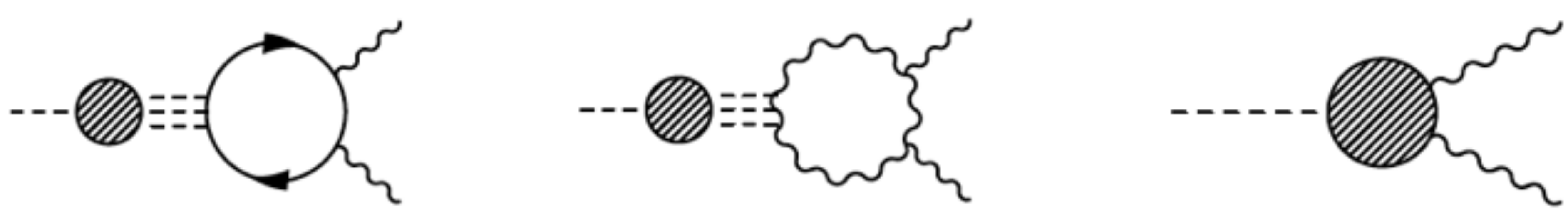
On-shell behavior: constant form factors (form factor reduces to a constant),)



$$\mathcal{M}_{\bar{f}fh} = \bar{u}_1^a F_{hff}(p_1 \cdot p_2; \mu) v_{2a} \;,$$



$$\mathcal{M}_{ggh} = \left[(\epsilon_1 \cdot p_2)(\epsilon_2 \cdot p_1) - \frac{m_h^2}{2} (\epsilon_1 \cdot \epsilon_2) \right] F_{ggh}\left(p_1 \cdot p_2 = m_h^2/2; \mu\right)$$



$$\mathcal{M}_{\gamma\gamma h} = \left[(\bar{\epsilon}_1 \cdot p_2)(\bar{\epsilon}_2 \cdot p_1) - \frac{m_h^2}{2} (\bar{\epsilon}_1 \cdot \bar{\epsilon}_2) \right] F_{\gamma\gamma h}\left(p_1 \cdot p_2 = m_h^2/2; \mu\right)$$

Form factors for QPT Higgs

Effects of the strong sector leading to the QPT are added via its n-point functions, leading to form factors

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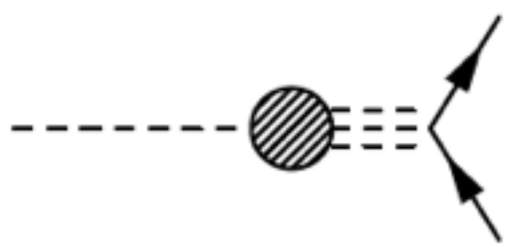

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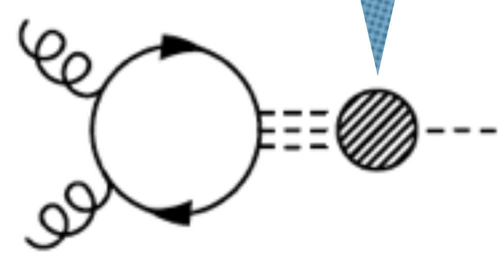

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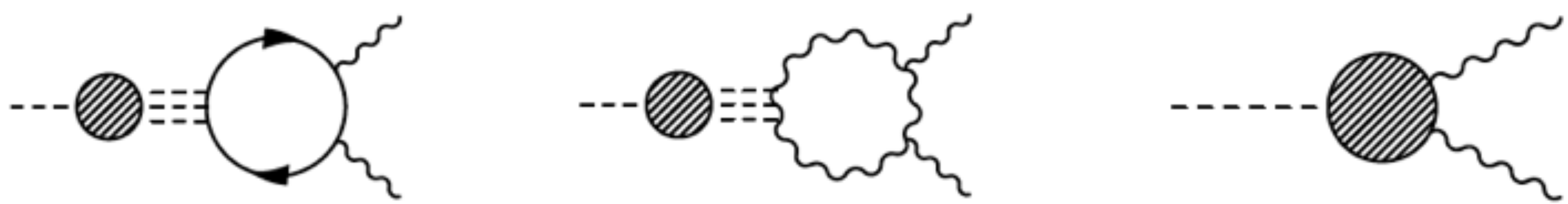

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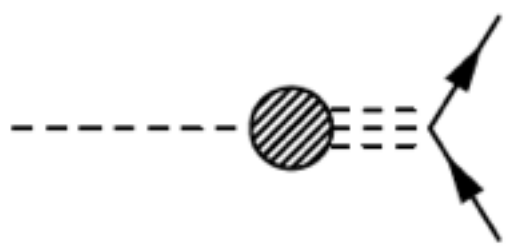

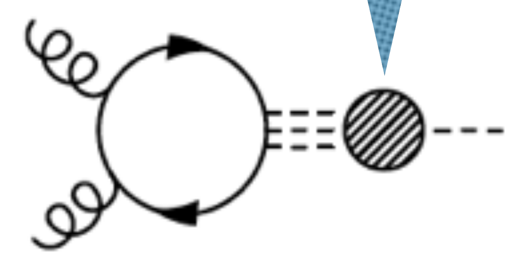

$p_1 \cdot p_2 = m_h^2/2$

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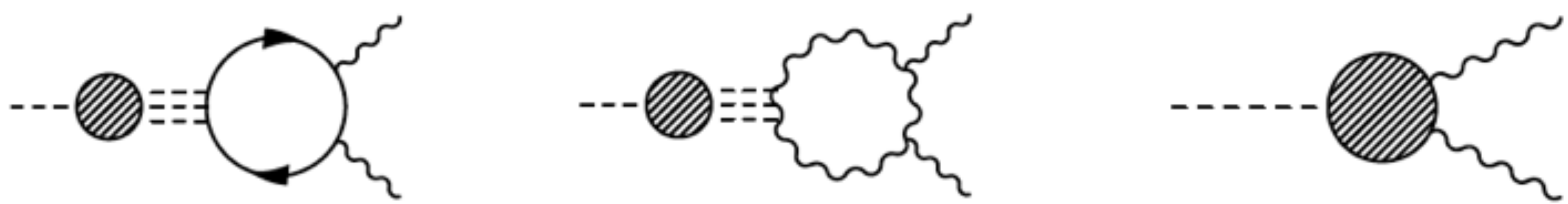

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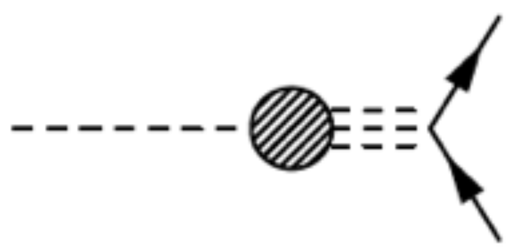

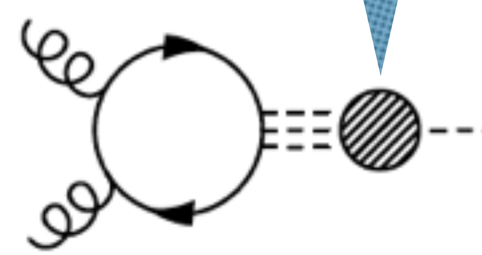

$p_1 \cdot p_2 = m_h^2/2$

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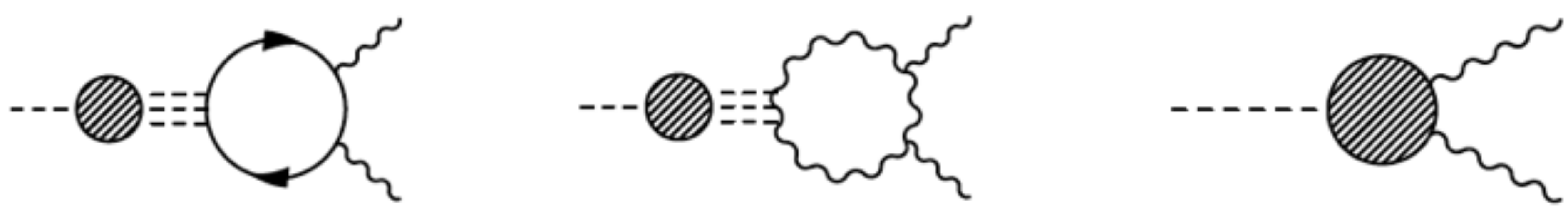
represents the parametric dependence on the scale of conformal symmetry breaking,



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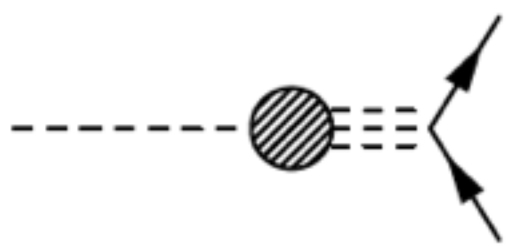

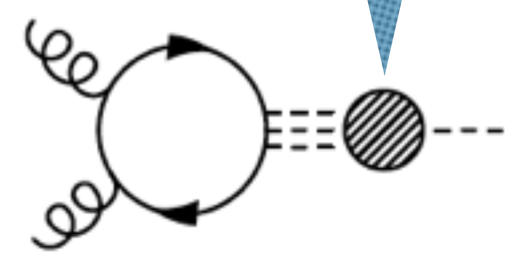

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represents the parametric de...

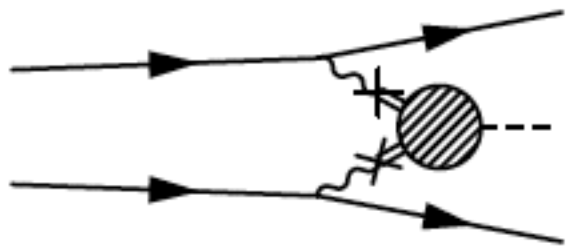
restriction to on-shell states reduces the on-shell form factor to an effective coupling constant.

$$\mathcal{M}_{\gamma\gamma h} = \left[(\bar{\epsilon}_1 \cdot p_2)(\bar{\epsilon}_2 \cdot p_1) - \frac{m_h^2}{2} (\bar{\epsilon}_1 \cdot \bar{\epsilon}_2) \right] F_{\gamma\gamma h} \left(p_1 \cdot p_2 = m_h^2/2; \mu \right)$$

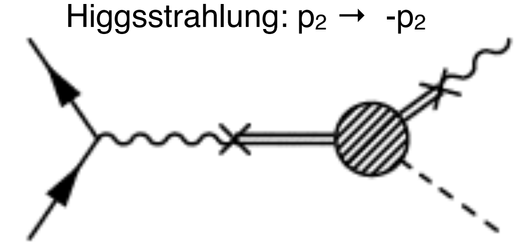
Off-shell form factors for QPT Higgs

Off-shell behavior: nontrivial momentum dependent form factors

VBF



Higgsstrahlung: $p_2 \to -p_2$



$$\mathcal{M}_{VBF} \;=\; J_1^\alpha G^V_{\alpha\mu}(p_1)\; J_2^\beta G^V_{\nu\beta}(p_2) F^{\mu\nu}_{\mathrm{VVh}}(p_i;\mu)\, N_V$$

$$\mathcal{M}_{q\bar{q}\to Vh} \;=\; J_I^\alpha G^V_{\alpha\mu}(p_1)\, \bar{\epsilon}_{2\nu} F^{\mu\nu}_{\mathrm{VVh}}(p_1,-p_2;\mu)\, N_V$$

$$F^{\mu\nu}_{VVh}(p_i;\mu) = g^{\mu\nu}\,\Gamma_1 + (g^{\mu\nu} p_1\cdot p_2 - p_2^\mu p_1^\nu)\,\Gamma_2 + (p_1^\mu p_1^\nu + p_2^\mu p_2^\nu)\,\Gamma_3 + (p_1^\mu p_1^\nu - p_2^\mu p_2^\nu)\,\Gamma_4 + p_1^\mu p_2^\nu\,\Gamma_5$$

$$\Gamma_i = \Gamma_i(p_1^2, p_2^2, p_1\cdot p_2)$$

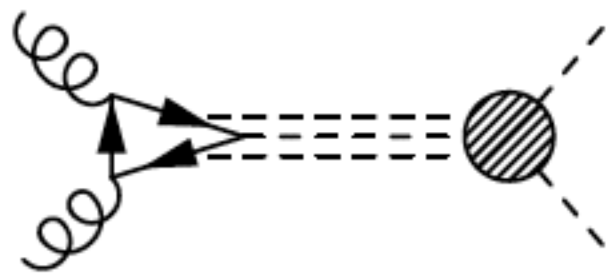
$$\Gamma_1^{(\mathrm{SM})} = 1 \text{ and } \Gamma_{i\neq 1}^{(\mathrm{SM})} = 0.$$

etc…

Off-shell form factors for QPT Higgs

Off-shell behavior: nontrivial momentum dependent form factors

$p_1 \cdot p_2 = s/2$



$p_1 \cdot p_3 = (m_h^2 - t)/2$

$$\mathcal{M}_{gghh} = \left[(\epsilon_1 \cdot p_2)(\epsilon_2 \cdot p_1) - (p_1 \cdot p_2)(\epsilon_1 \cdot \epsilon_2) \right] \Xi_1 (p_1 \cdot p_2, p_1 \cdot p_3; \mu)$$
$$+ \epsilon_2 \cdot [(p_1 \cdot p_2) p_3 - (p_2 \cdot p_3) p_1] \, \epsilon_1 \cdot [(p_1 \cdot p_2) p_3 - (p_1 \cdot p_3) p_2] \, \Xi_2 (p_1 \cdot p_2, p_1 \cdot p_3; \mu)$$

Bose Symmetry: $\Xi_i (p_1 \cdot p_2, p_1 \cdot p_3; \mu) = \Xi_i (p_1 \cdot p_2, p_2 \cdot p_3; \mu)$

Off-shell form factors for QPT Higgs

Off-shell behavior: nontrivial momentum dependent form factors

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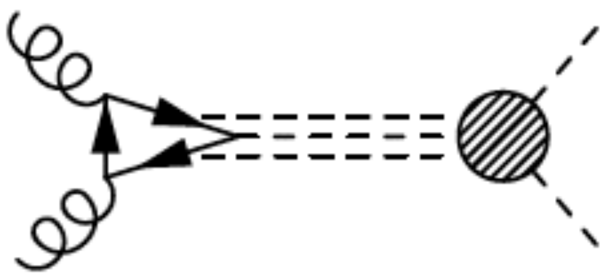

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suppressed in the large top mass limit in the SM

Bose Symmetry: $\Xi_i (p_1 \cdot p_2, p_1 \cdot p_3; \mu) = \Xi_i (p_1 \cdot p_2, p_2 \cdot p_3; \mu)$

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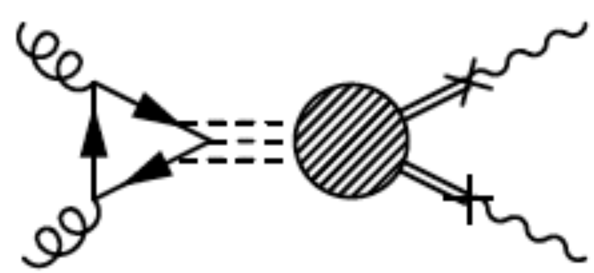
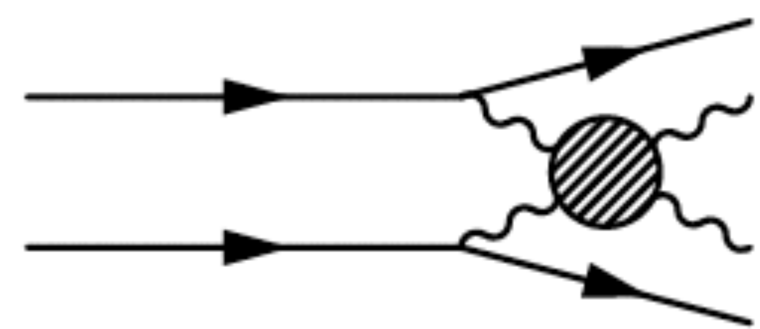
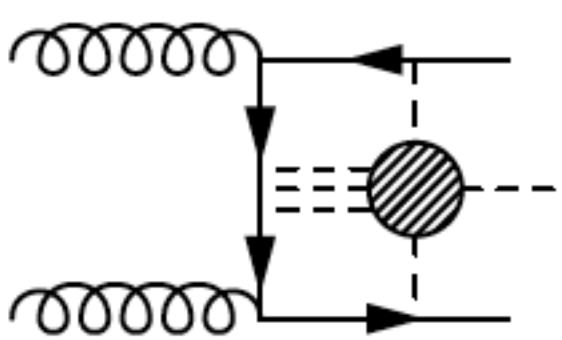
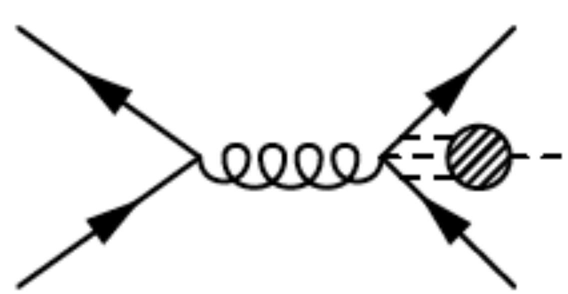




$$\mathcal{M}_{ggVV} = \epsilon_{1\,\mu}\epsilon_{2\,\nu}\left[F^{\mu\nu\rho\sigma}_{ggVV}(p_i;\mu) + \widehat{F}^{\mu\nu\rho\sigma}_{ggVV}(p_i;\mu)\right]\bar{\epsilon}_{3\,\rho}\bar{\epsilon}_{4\,\sigma}$$

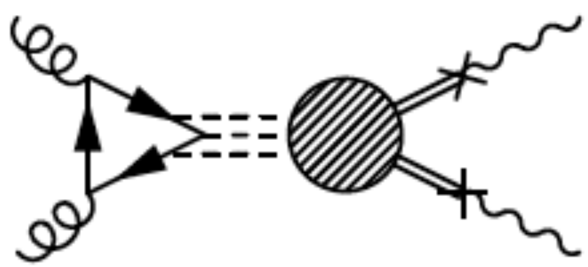
$$\begin{aligned}
F^{\mu\nu\rho\sigma}_{ggVV}(p_i;\mu) = &\left[g^{\mu\nu}(p_1\cdot p_2) - p_1^\nu p_2^\mu\right]\left(g^{\rho\sigma}\Theta_1 + p_3^\rho p_3^\sigma\Theta_2 + p_3^\rho p_3^\sigma\Theta_3\right)\\
&+\left[g^{\mu\rho}g^{\nu\sigma}(p_1\cdot p_2) + g^{\mu\sigma}p_1^\nu p_2^\rho - g^{\nu\sigma}p_1^\rho p_2^\mu - g^{\rho\sigma}p_2^\mu p_1^\nu\right]\Theta_4\\
&+g^{\mu\nu}\left[g^{\rho\sigma}(p_1\cdot p_3)(p_2\cdot p_3) - p_3^\rho p_3^\sigma(p_1\cdot p_2) + p_3^\rho p_1^\sigma(p_2\cdot p_3) + p_2^\rho p_3^\sigma(p_1\cdot p_3)\right]\Theta_5\\
&+p_3^\mu\left[g^{\nu\sigma}p_2^\rho(p_1\cdot p_3) + g^{\nu\rho}p_3^\sigma(p_1\cdot p_2) - g^{\nu\rho}p_2^\sigma(p_1\cdot p_3) - p_3^\rho p_1^\nu p_2^\sigma\right]\Theta_6\\
&+\left[g^{\mu\sigma}p_3^\nu p_1^\rho(p_2\cdot p_3) - g^{\mu\rho}p_3^\nu p_1^\sigma(p_2\cdot p_3) + g^{\mu\sigma}p_1^\nu p_3^\rho(p_1\cdot p_2) - g^{\mu\rho}p_1^\nu p_3^\sigma(p_1\cdot p_2)\right]\Theta_7\\
&+\left[g^{\nu\sigma}p_3^\mu p_2^\rho(p_1\cdot p_3) - g^{\nu\rho}p_3^\mu p_2^\sigma(p_1\cdot p_3) + g^{\nu\sigma}p_2^\mu p_3^\rho(p_1\cdot p_2) - g^{\nu\rho}p_2^\mu p_3^\sigma(p_1\cdot p_2)\right]\Theta_8\\
\widehat{F}^{\mu\nu\rho\sigma}_{ggVV}(p_i;\mu) = &\, p_{1\,\alpha}p_{2\,\beta}p_{i\,\gamma}p_{j\,\delta}\Big(\epsilon^{\mu\nu\alpha\beta}\epsilon^{\rho\sigma\gamma\delta}\widehat{\Theta}_1^{ij} + \epsilon^{\mu\rho\alpha\gamma}\epsilon^{\nu\sigma\beta\delta}\widehat{\Theta}_2^{ij} + \epsilon^{\mu\sigma\alpha\gamma}\epsilon^{\nu\rho\beta\delta}\widehat{\Theta}_3^{ij}\\
&+\delta_1^i\delta_2^j\epsilon^{\mu\nu\alpha\sigma}\epsilon^{\rho\beta\gamma\delta}\widehat{\Theta}_4 + \delta_2^i\delta_3^j\epsilon^{\nu\beta\rho\sigma}\epsilon^{\mu\alpha\gamma\delta}\widehat{\Theta}_5\Big)\,,\\
&\Theta_k = \Theta_k(p_1\cdot p_2, p_1\cdot p_3)\,,\quad \widehat{\Theta}_k^{ij} = \widehat{\Theta}_k^{ij}(p_1\cdot p_2, p_1\cdot p_3)
\end{aligned}$$

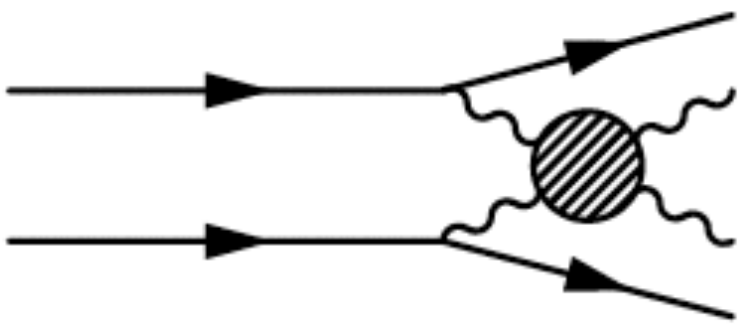




Off-shell form factors for QPT Higgs

Off-shell behavior: nontrivial momentum dependent form factors

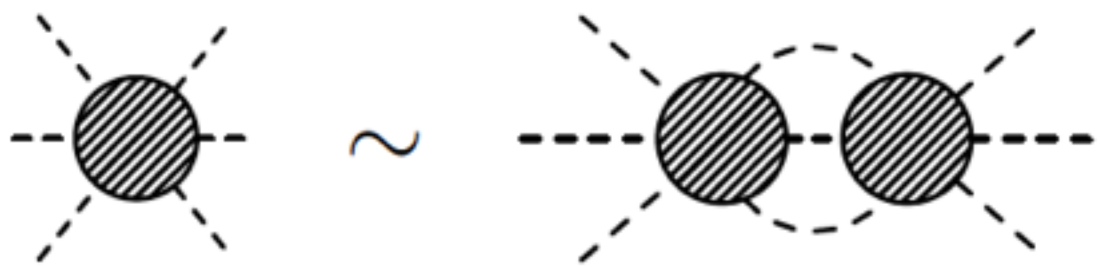




On can estimate from an effective field theory perspective, where the only light degree of freedom surviving from the strongly coupled sector is the Higgs particle =>can estimate the size of the N-point Higgs correlator by considering the effect of loops on its renormalization…

Off-shell form factors for QPT Higgs

In NDA, loop corrections ~ original n-point function. e.g. n = 6:



$$\mathcal{L} = \frac{\alpha_n}{\Lambda^{n-4}}\phi^n$$

loop contribution with two insertions of this operator (that contributes to the same n-point) would be one in which each vertex has n/2 external lines, and n/2 propagators exchanged in n/2 -1 loops.

quantum corrections: $$\alpha_n \to \alpha_n \left(1 + \frac{\alpha_n}{(16\pi^2)^{n/2-1}}\right)$$

$$\Rightarrow \quad \alpha_n \sim (16\pi^2)^{n/2-1}$$

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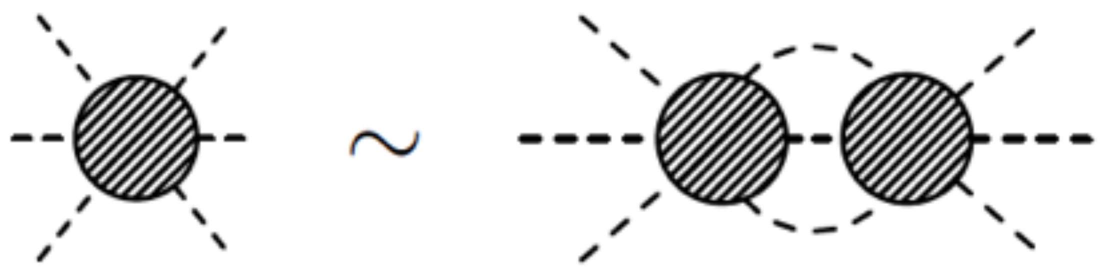

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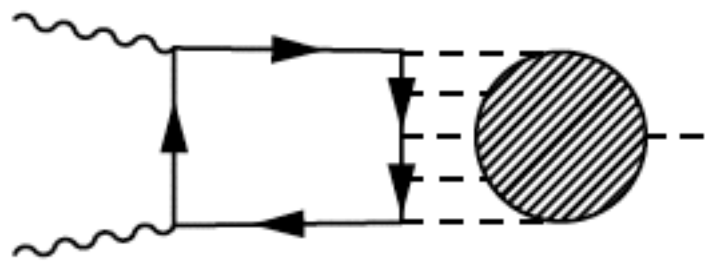
n/2 propagators exchange[...]

quantum correction[...]

e.g. with this, we can see that the n-point contribution to the gluon fusion process is suppressed by insertions of the perturbative coupling of the top-quark to the strongly coupled sector, along with a loop factor that is only partially cancelled by the large coefficient

→ $\alpha_n \sim (16\pi^2)^{n/2-1}$

Off-shell form factors for QPT Higgs



If the shaded region corresponds to the n-point function, there are n - 1 insertions of the top Yukawa, and n - 2 loops. There are n - 1 scalar propagators and n - 2 fermionic propagators running in these loops.

estimate for the contribution of the n-point correlator to the $h\bar{t}t$ coupling:

$$g_n^{tth} \sim 4\pi \left(\frac{\lambda_t}{4\pi}\right)^{n-1}$$

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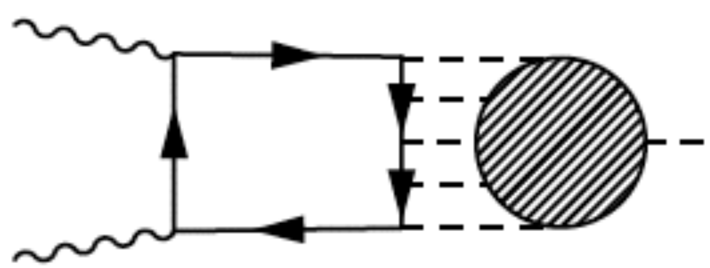

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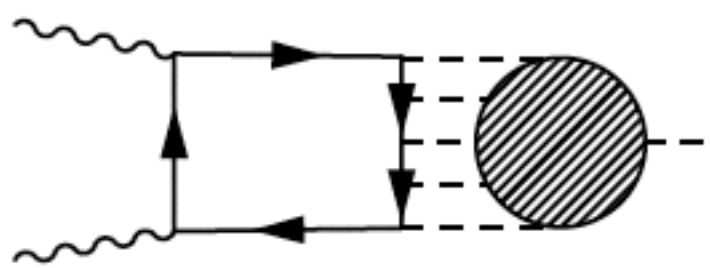

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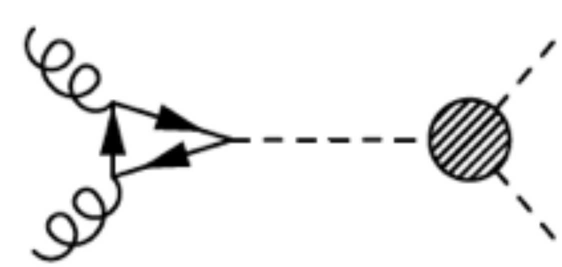
e.g. double Higgs production through gluon fusion would be dominated by



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Modeling the Quantum Critical Higgs

- The upshot is that there is a CFT (QPT), and it has non-trivial dynamics that will have a non-trivial 4-point function H^4 which will have momentum dependence, and the pole (physical Higgs) arises as a composite bound state of CFT similar to composite Higgs models

- there is no requirement that strongly coupled dynamics produces a strongly coupled effective theory at the low energy. Counter examples: Seiberg duality, rho meson at large N, and the AdS/CFT correspondence

The infinite N limit in AdS/CFT yields a subclass of models that generalize to a broader category of strongly coupled theories: it is possible that the strongly coupled sector is completely specified by the two-point functions of that theory, with or without a large N expansion. Such theories are referred to as models of **generalized free fields**. Polyakov, early '70s- skeleton expansions

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=>Weakly coupling a fundamental light Higgs to such a theory would produce the type of dynamics we are interested in

Quantum Critical Higgs

- The assumption that the theory is close to a QPT implies that the Higgs field should be characterized by its scaling dimension Δ, where $1 \leq \Delta < 2$. In general, the two point function of a scalar with scaling dimension Δ in a CFT is

$$G_{\rm CFT}(p) = -\frac{i}{(-p^2 + i\epsilon)^{2-\Delta}}$$

SM: $\Delta = 1 + \mathcal{O}(\alpha/4\pi)$

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- This however gives a large contribution to the potential in the p ➔ 0 limit, removing all light degrees of freedom. A light pole can be reintroduced (while leaving the cut starting at μ by subtracting the mass term:

$$-\mathcal{H}^\dagger \left[D^2 + \mu^2\right]^{2-\Delta} \mathcal{H} + \mu^{4-2\Delta} \mathcal{H}^\dagger \mathcal{H}$$

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Higgs is mixing with the states of the conformal matter corresponding to the physics of the quantum phase transition

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AdS/CFT

$$\left\langle e^{\int d^4x \phi_0(x)\mathcal{O}(x)} \right\rangle_{\mathrm{CFT}} \approx e^{-S_{\mathrm{5Dgravity}}[\phi(x,z)|_{z=0}=\phi_0(x)]}$$

$$ds^2 = \frac{R^2}{z^2}\left(dx_\mu^2 - dz^2\right)$$

$\mathcal{O} \subset \mathrm{CFT} \leftrightarrow \phi$ AdS_5 field

AdS/CFT

$$ds^2 = \frac{R^2}{z^2}\left(dx_\mu^2 - dz^2\right)$$

$$z > \epsilon$$

$$S_{bulk} = \frac{1}{2}\int d^4x dz \sqrt{g}(g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi + m^2\phi^2)$$

$$\phi(p,z) = az^2 J_\nu(pz) + bz^2 J_{-\nu}(pz)$$

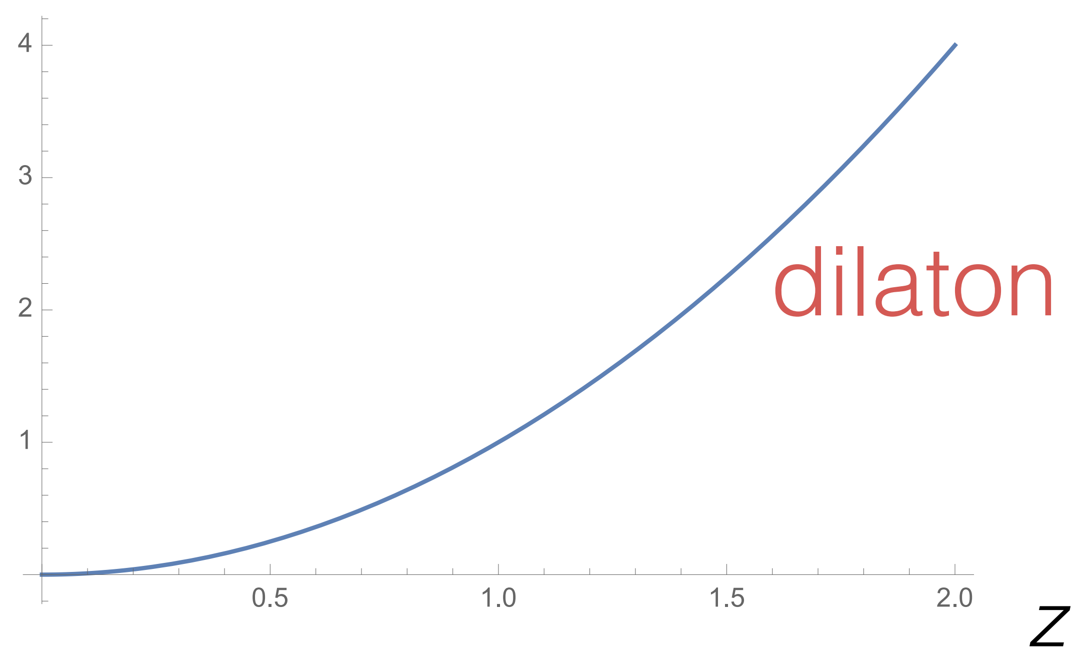
$$\Delta[\mathcal{O}] = 2 \pm \nu = 2 \pm \sqrt{4 + m^2 R^2}$$

$$< \mathcal{O}(p)\mathcal{O}(p) > \propto \frac{\delta^{(4)}(p+p')}{(2\pi)^2}(p^2)^{\Delta-2}$$

Witten, Klebanov 99'

broken CFT

- Randall Sundum 2 (only UV brane and bulk): cuts from 0 (CFT)
- RS1: putting IR cutoff at TeV
- New type of IR cutoff (soft wall) gives rise to a different phenomenology



Karch, Katz, Son, Stephaniv 06`

broken CFT by IR cutoff

$$S_{\rm int} = \frac{1}{2}\int d^4x dz \sqrt{g}\phi \mathcal{H}^\dagger \mathcal{H}$$

$$\phi = \mu z^2$$

$$z^5 \partial_z \left(\frac{1}{z^3}\partial_z \mathcal{H}\right) - z^2(p^2 - \mu^2)\mathcal{H} - m^2 R^2 \mathcal{H} = 0$$

$$< \mathcal{O}(p)\mathcal{O}(p) > \propto \frac{\delta^{(4)}(p+p')}{(2\pi)^2}(p^2 - \mu^2)^{\Delta - 2}$$

$$[\partial^2 - \mu^2]^{2-\Delta}\delta(x-y)$$

Non-local operator

$$\mathcal{L}_{\mathcal{H}} = -\mathcal{H}^\dagger \left[D^2 + \mu^2\right]^{2-\Delta} \mathcal{H} + \mu^{4-2\Delta} \mathcal{H}^\dagger \mathcal{H} - V(|\mathcal{H}|)$$

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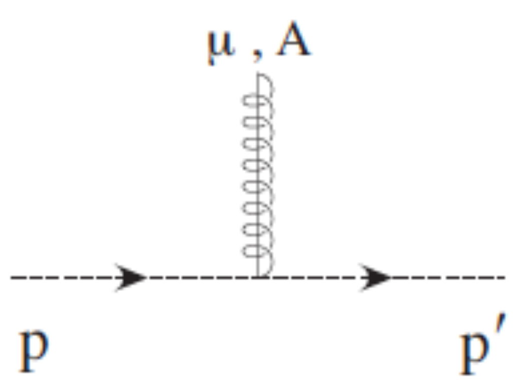

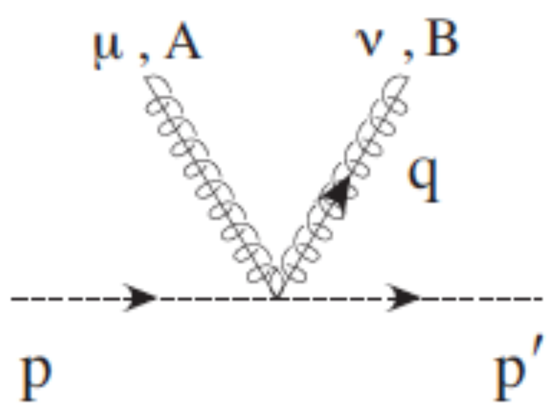

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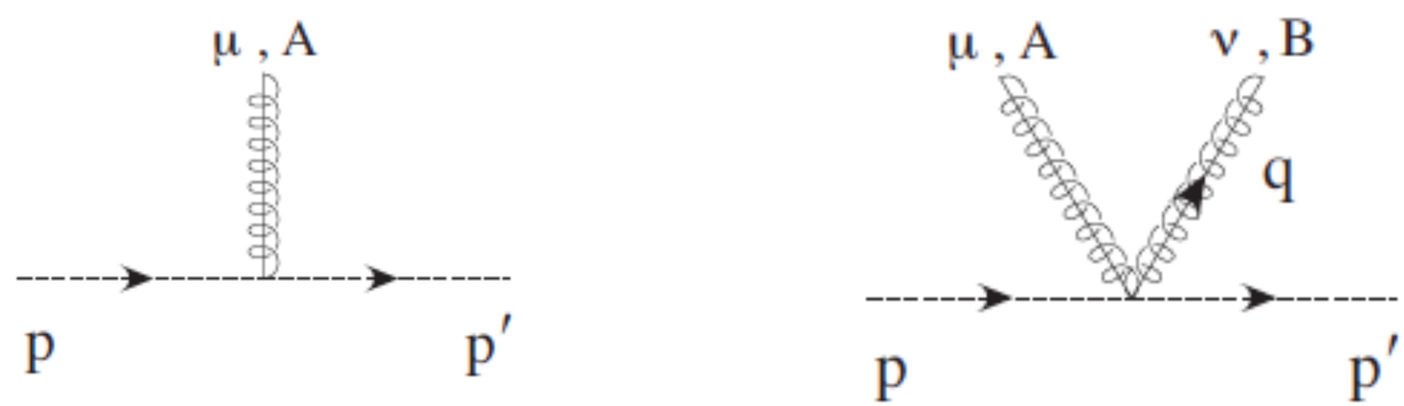

- e.g. for the trilinear interaction in momentum space: $\mathcal{H}^\dagger(p+q)A_\mu^a(q)\mathcal{H}(p)\Gamma^{\mu,a}(p,q)$

$$\Gamma^{\mu,a}(p,q) = gT^a\left(2p^\mu+q^\mu\right)F(p,q)\ ,$$

$$F(p,q) = -\frac{\left(\mu^2-(p+q)^2\right)^{2-\Delta}-\left(\mu^2-p^2\right)^{2-\Delta}}{2p\cdot q+q^2}$$

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Modeling the Quantum Critical Higgs: Generalized Free Fields

- A general two point function with a pole and a cut:

$$G_h(p^2) = \frac{i}{p^2 - m_h^2} + \int_{\mu^2}^{\infty} dM^2 \frac{\rho(M^2)}{p^2 - M^2}$$

- A Lagrangian give rise to the above two point function:

$$\mathcal{L}_{\text{quadratic}} = -\frac{1}{Z_h}\mathcal{H}^\dagger \left[\partial^2 + \mu^2\right]^{2-\Delta} \mathcal{H} + \frac{1}{Z_h}(\mu^2 - m_h^2)^{2-\Delta}\mathcal{H}^\dagger\mathcal{H}$$

Assuming $\mathcal{H}$ to be weakly coupled, the scaling dimension of $\mathcal{H}^\dagger\mathcal{H}$ is 2Δ

The momentum space propagator for the physical Higgs scalar

$$G_h(p) = -\frac{i\,Z_h}{(\mu^2 - p^2 + i\epsilon)^{2-\Delta} - (\mu^2 - m_h^2)^{2-\Delta}}\,, \qquad Z_h = \frac{(2-\Delta)}{(\mu^2 - m_h^2)^{\Delta - 1}}$$

Generalized Free Fields and AdS/CFT

- SO(4) global symmetry is gauged in the 5D bulk

Cacciapaglia, Marandella and Terning 08'
Falkowski and Perez-Victoria 08'

$$S = \int d^4x dz \sqrt{g} \left[|D_M H|^2 - \frac{1}{4g_4^2} W^{a\,2}_{MN} - \phi(z) |H|^2 + \mathcal{L}_{\rm int}(H) \right] + \int d^4x \, \mathcal{L}_{\rm perturbative}.$$

$$ds^2 = a(z)^2 \left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right) \qquad a(z) = \frac{R}{z} e^{-\frac{2}{3}\mu(z-R)}$$

$$G_h(R,R,p^2) = i\tilde{Z}_h \left[\frac{\mu K_{1-\nu}(\mu R)}{R K_\nu(\mu R)} - \frac{\sqrt{\mu^2 - p^2} K_{1-\nu}(\sqrt{\mu^2 - p^2} R)}{R K_\nu(\sqrt{\mu^2 - p^2} R)} - M_0^2 \right]^{-1}$$

The bulk to brane propagator is then given by $G_h(R,z,p^2) = a^{-\frac{3}{2}}(z)(z/R)^{\frac{1}{2}} \frac{K_\nu(\sqrt{\mu^2 - p^2} z)}{K_\nu(\sqrt{\mu^2 - p^2} R)}$

=> reduce to the previous propagator in the limit pR <<1 :

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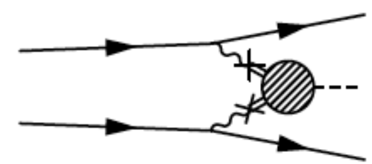
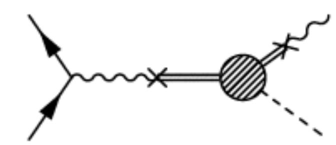
obtain such propagator from a calculable model of this sort based on a Banks-Zaks fixed point in a supersymmetric QCD theory: Csaki, SL, Shirmanm, Parolini (in preparation)

Generalized Free Fields and AdS/CFT

- Form factors



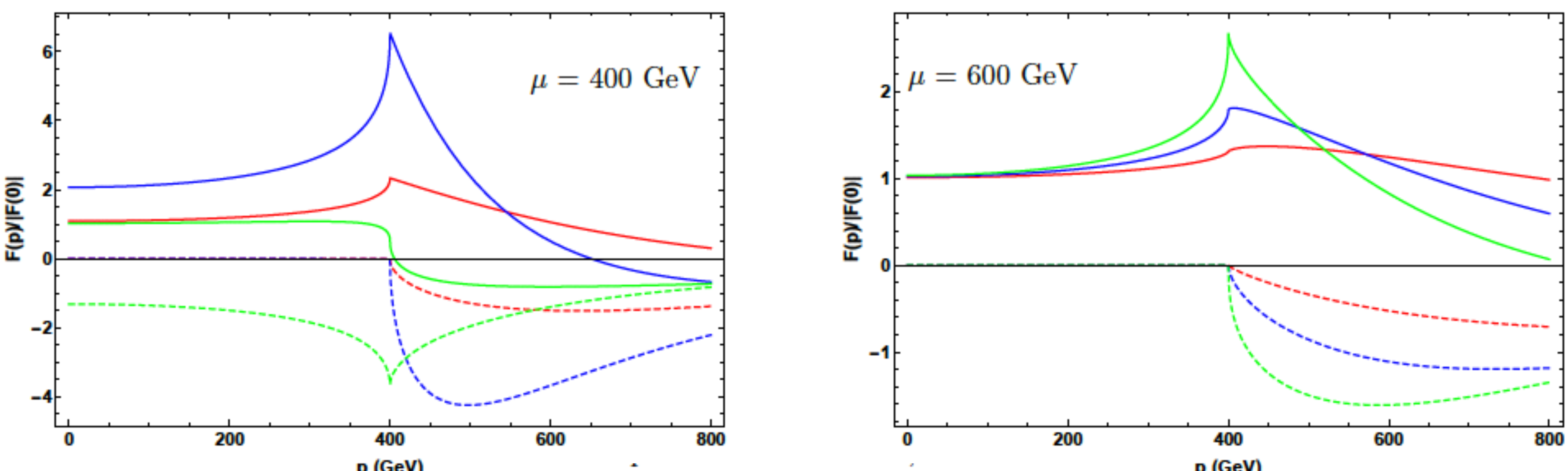


$$\mathcal{M}_{VBF} = J_1^\alpha G^V_{\alpha\mu}(p_1)\; J_2^\beta G^V_{\nu\beta}(p_2) F^{\mu\nu}_{\rm VVh}(p_i;\mu)\, N_V$$

$$\mathcal{M}_{qq\to Vh} = J_I^\alpha G^V_{\alpha\mu}(p_1)\, \bar{\epsilon}_{2\nu} F^{\mu\nu}_{\rm VVh}(p_1,-p_2;\mu)\, N_V$$

$$\mathcal{H} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \mathcal{V}+h \end{pmatrix}$$

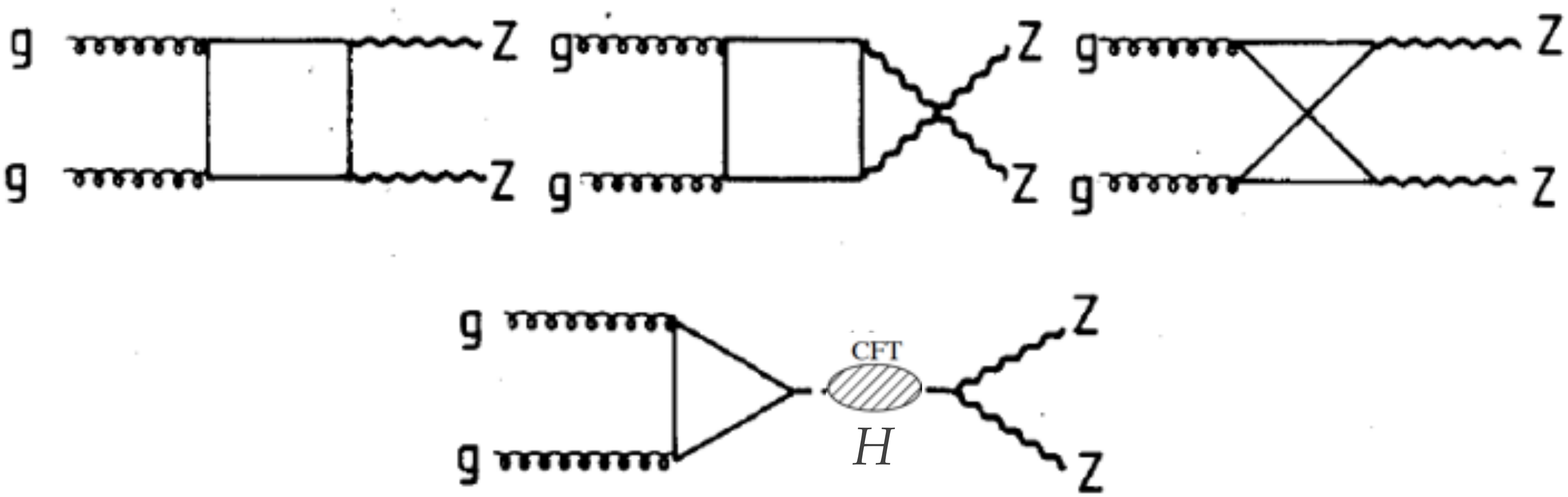
$$F^{ab}_{VVh} = 2\frac{\mathcal{V}}{L\,M^2}\int_R^\infty dz\, a^2\left(\frac{z}{R}\right)\frac{K_{2-\Delta}(\sqrt{\mu^2-(p_1+p_2)^2}\,z)K_{2-\Delta}(\mu\,z)}{K_{2-\Delta}(\sqrt{\mu^2-(p_1+p_2)^2}\,R)K_{2-\Delta}(\mu\,R)}$$



$\Delta = 1.2$ (Red), 1.4 (Blue), and 1.6 (Green)

Direct Signals

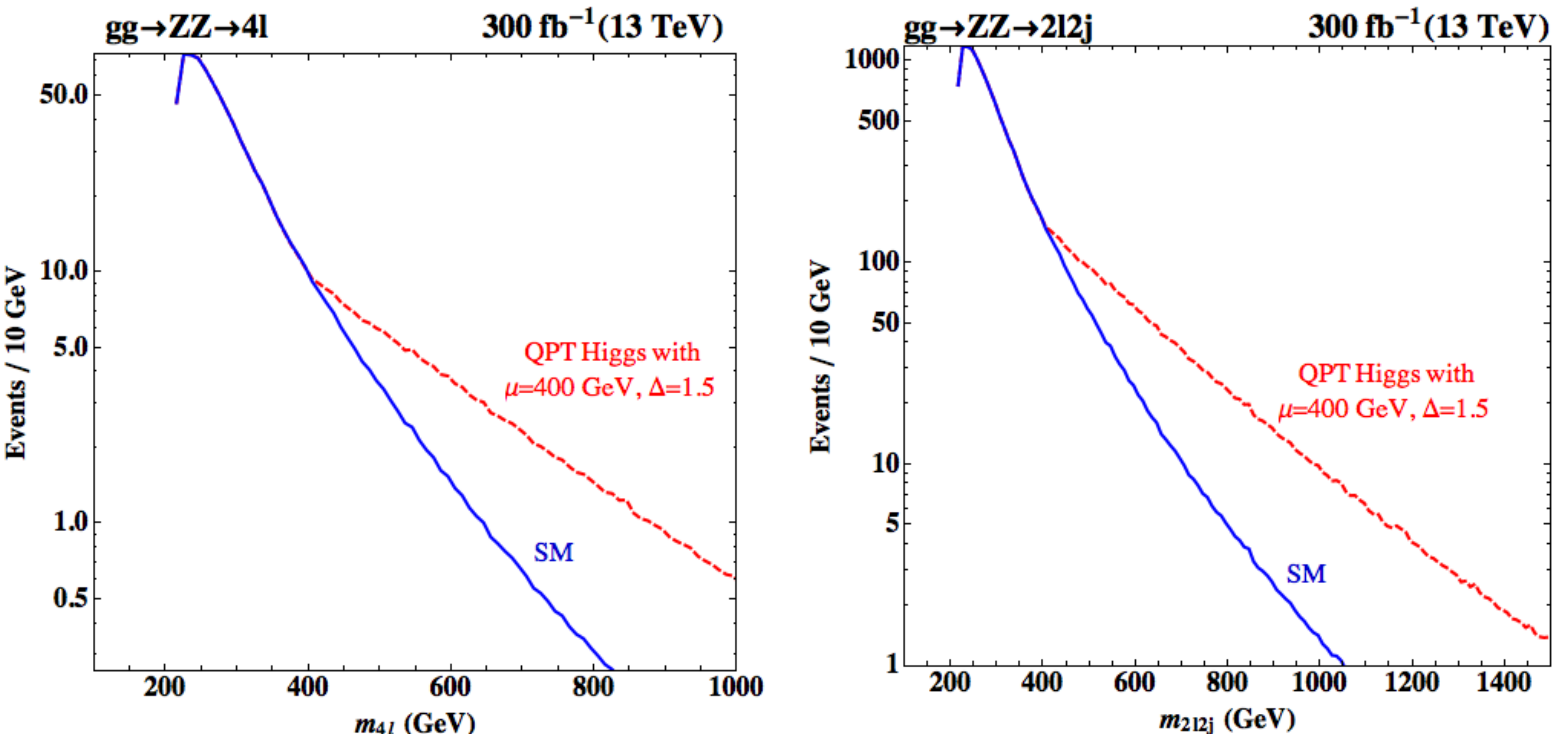
- Off-shell Higgs can be tested via interference.



sensitive to the
modications of the Higgs two-point function

Direct Signals

- Single Higgs production: Production of the cut modifies Higgs cross sections for energies above μ => modifies any cross sections that involve the (tree-level) exchange of the components of Higgs



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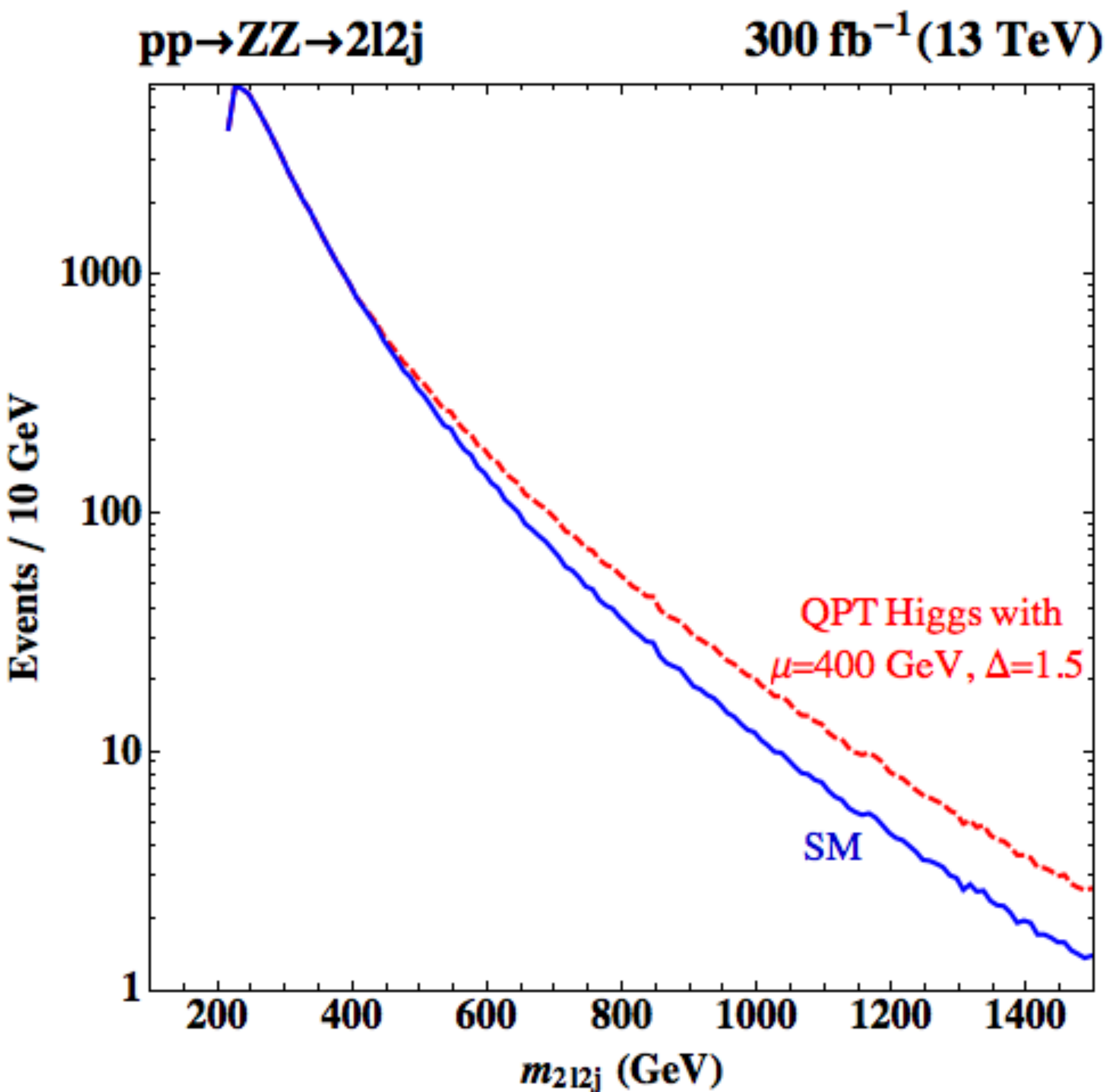

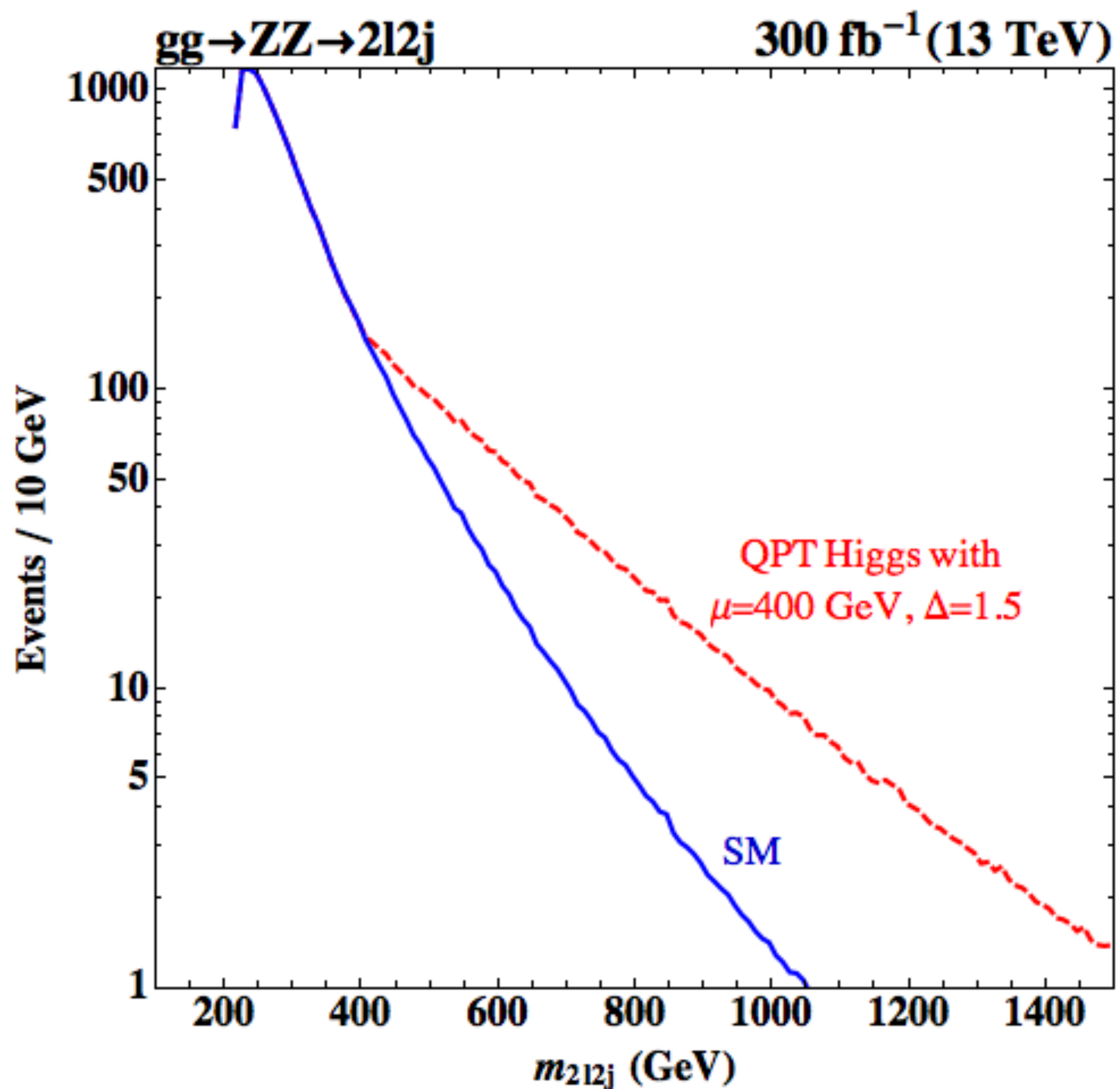

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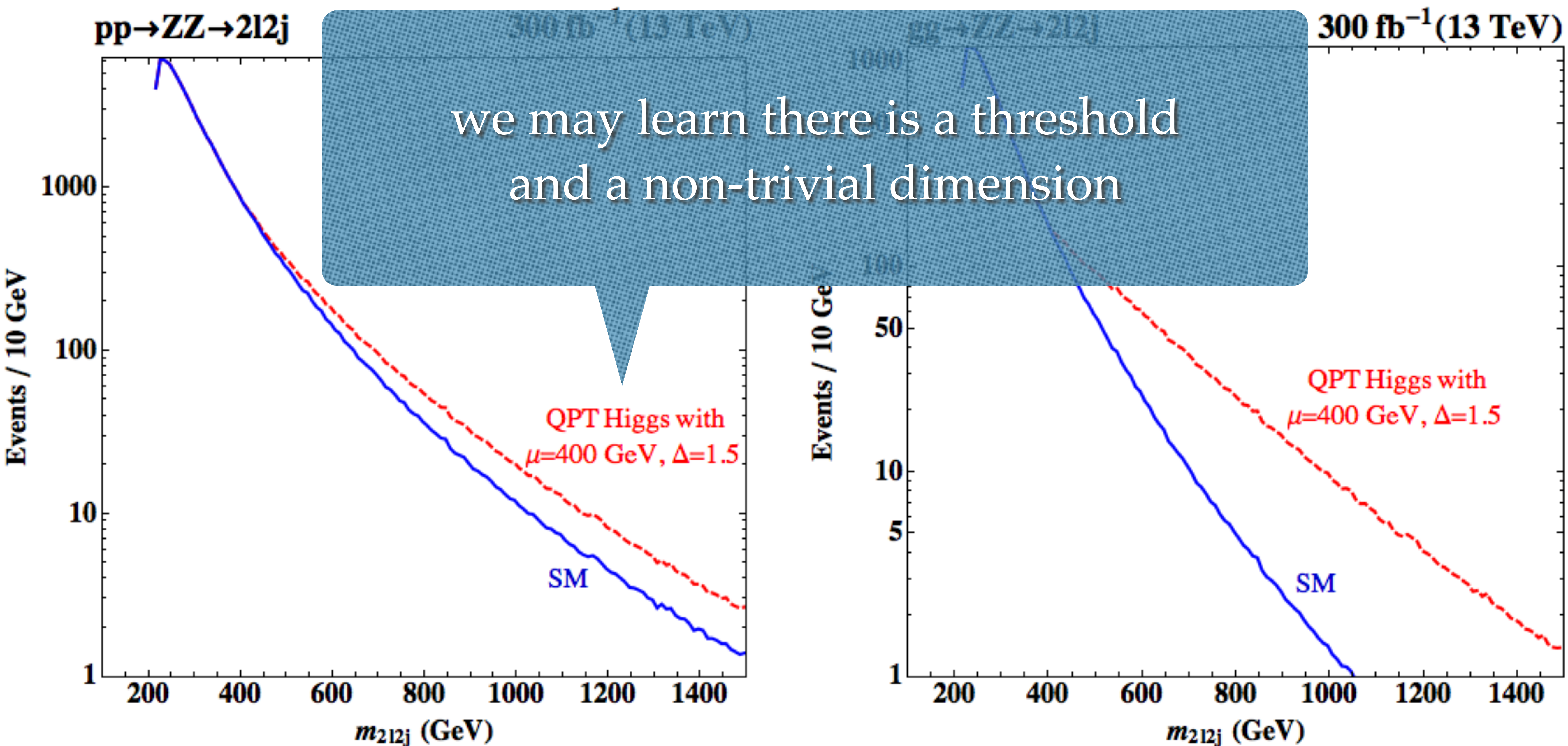
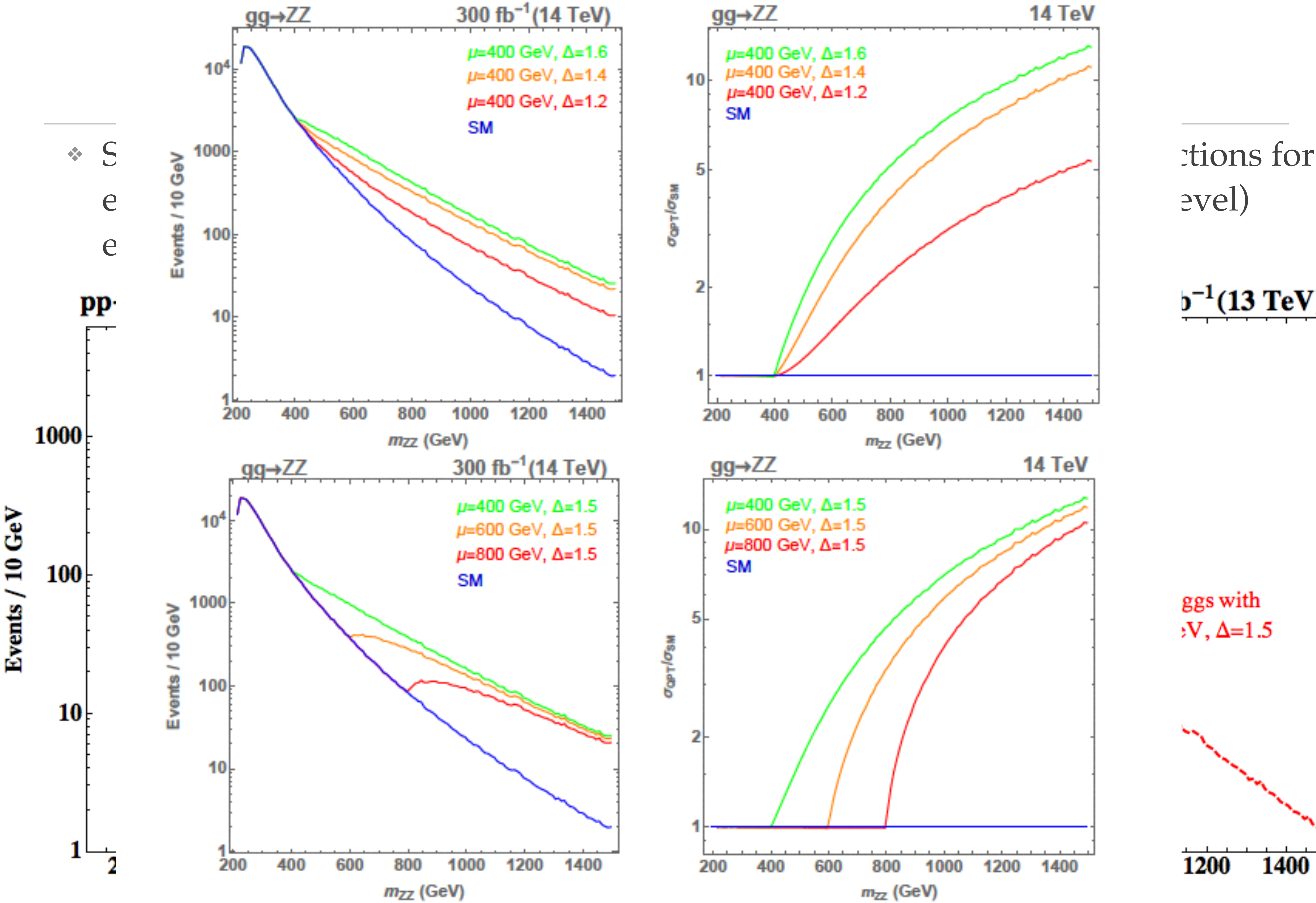
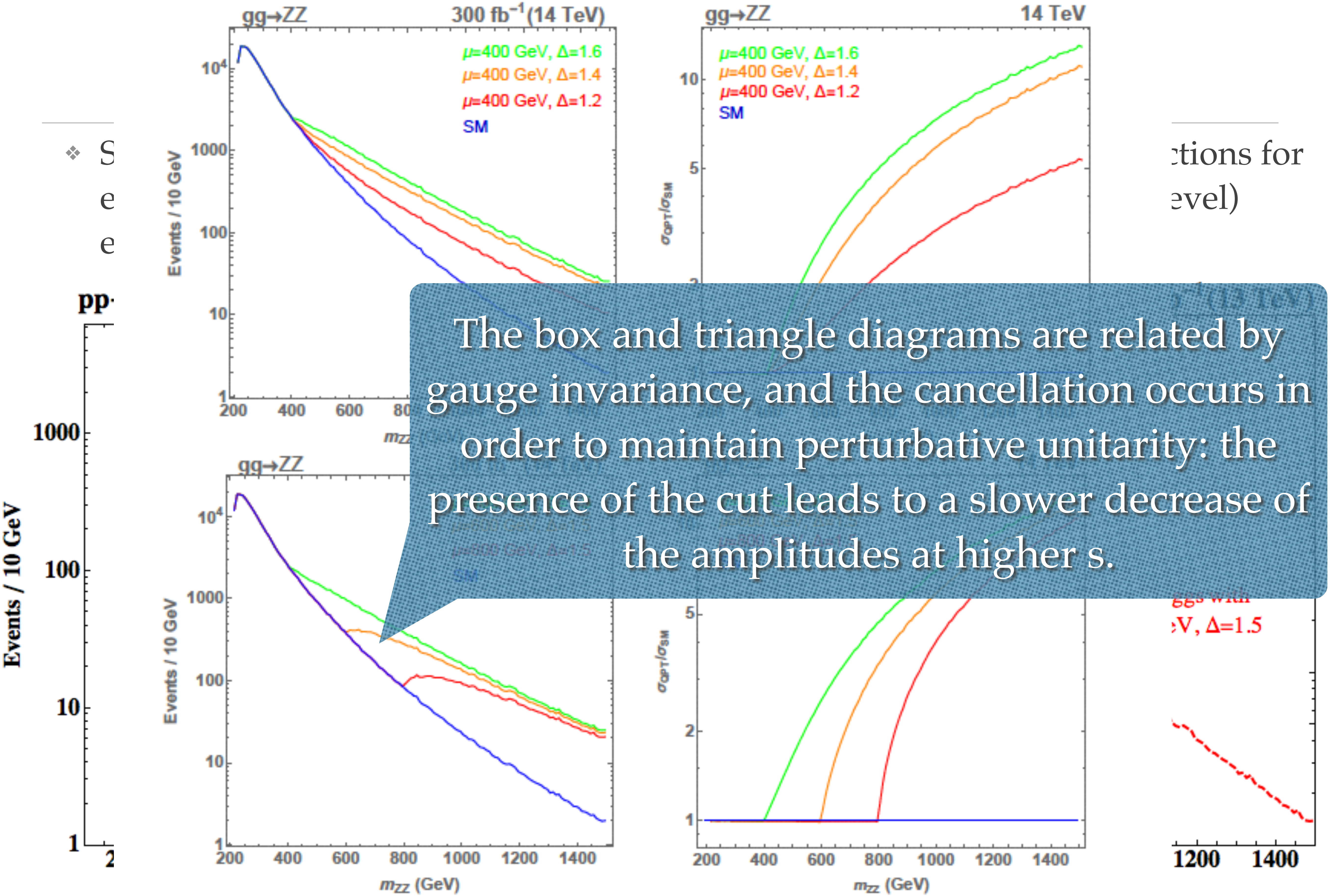


- S … ctions for e … evel) e





The box and triangle diagrams are related by gauge invariance, and the cancellation occurs in order to maintain perturbative unitarity: the presence of the cut leads to a slower decrease of the amplitudes at higher s.

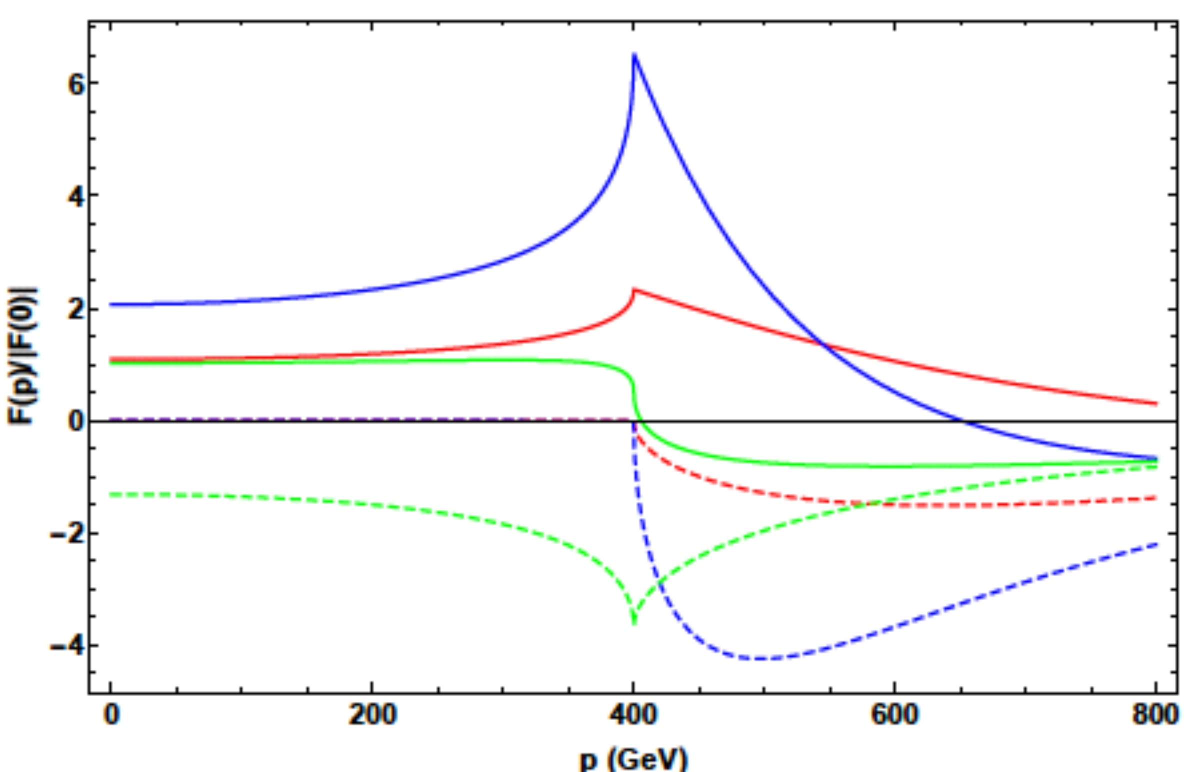
Generalized Free Fields and AdS/CFT

- Form factors for trilinear Higgs self coupling $\lambda_5 (H^\dagger H)^2$

$$F_{hhh} = \frac{\lambda_5}{L^2} \mathcal{V} \int_R^\infty dz\, \frac{1}{a} \left(\frac{z}{R}\right)^2 \frac{K_{2-\Delta}(\mu\, z)}{K_{2-\Delta}(\mu\, R)} \prod_{i=1}^{3} \frac{K_{2-\Delta}(\sqrt{\mu^2 - p_i^2}\, z)}{K_{2-\Delta}(\sqrt{\mu^2 - p_i^2} R)}$$

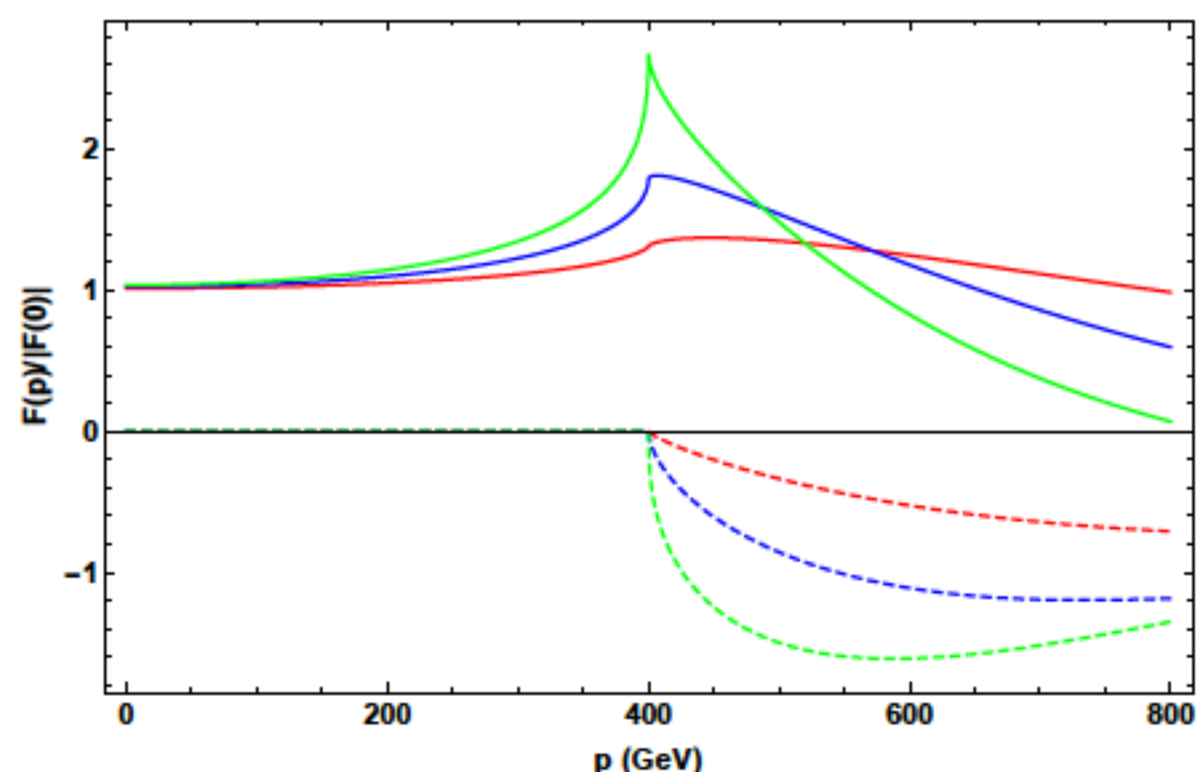
$\mu = 400$, $\Delta = 1.5$,

Higgs momentum: 200 GeV (Red), 400 GeV (Blue), and 600 GeV (Green)



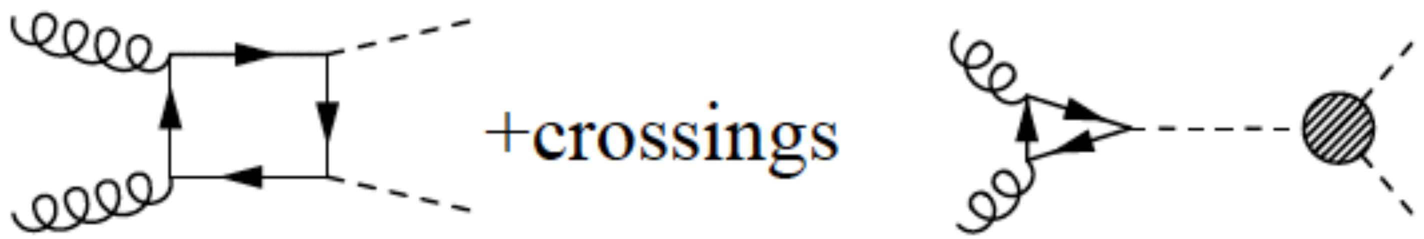
$\mu = 400$,

$\Delta = 1.2$ (Red) 1.4 (Blue), and 1.6 (Green).



Direct Signals

- Double Higgs production



$$\frac{d\hat{\sigma}}{d\hat{t}} = \frac{\alpha_w^2 \alpha_s^2}{2^{15}\pi M_W^4 \hat{s}^2}\left(|\text{gauge1}|^2 + |\text{gauge2}|^2\right)$$

gauge1 = box + triangle (negative interference)

gauge2 = box (largest contribution)

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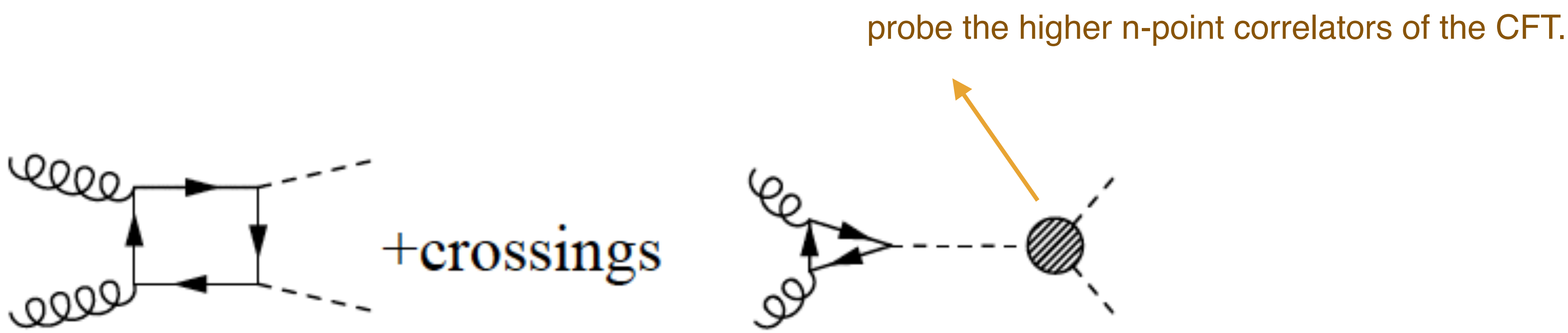

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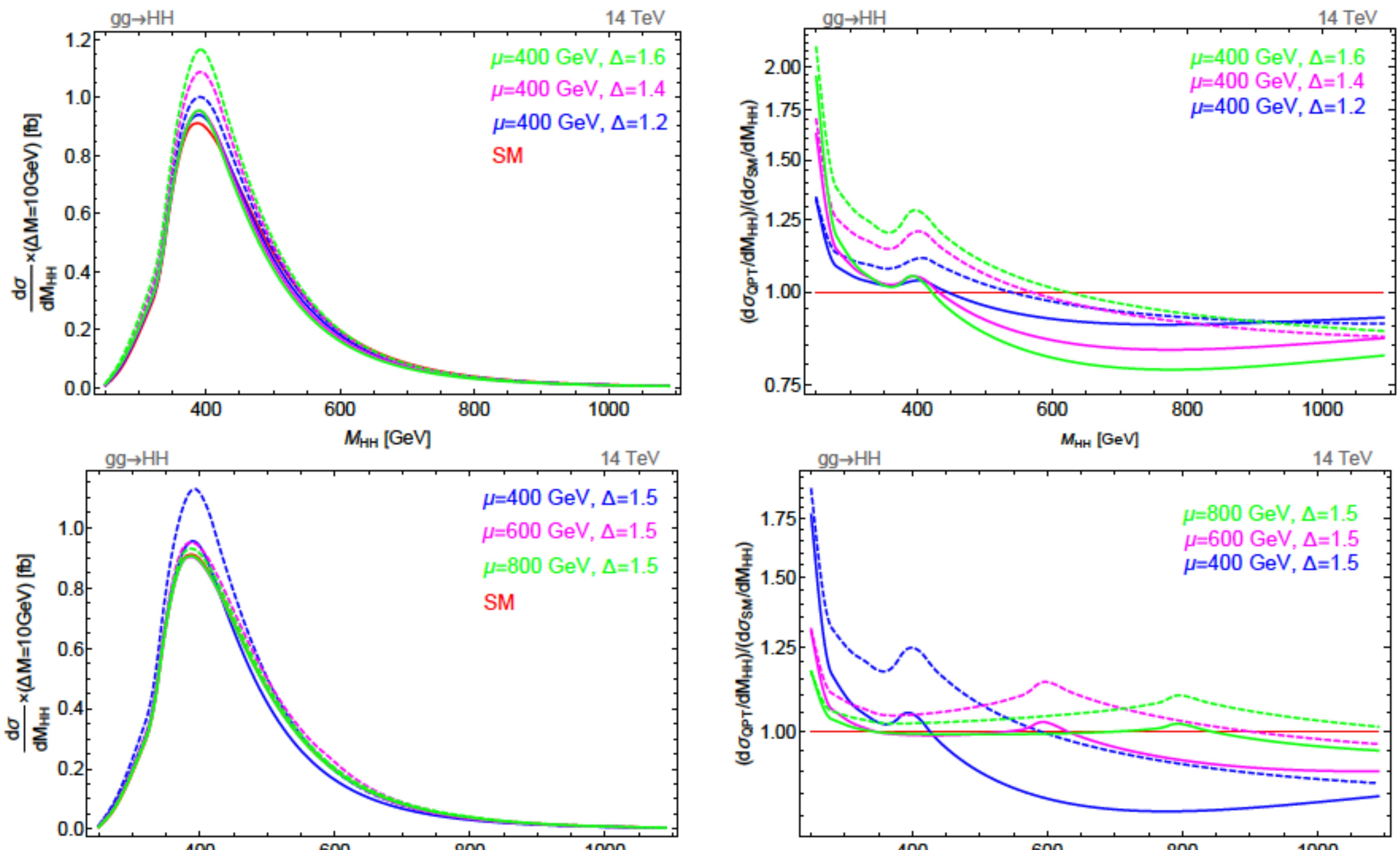
Direct Signals

- Double Higgs production

dashed lines correspond to the case where only the Higgs two-point function has non-trivial behavior inherited from a sector with strong dynamics.



Summary

- So far nothing but Higgs
- It's interesting whether the Higgs sector is close to a quantum critical point with non-mean-field behavior, that is with non-trivial critical exponents and scaling dimensions
- Low-energy effective theory for such a quantum critical Higgs shows that critical exponents can be extracted from the LHC measurements (and future colliders…)
- Nontrivial momentum-dependent form factors for Higgs physics interesting for the future measurement