

# Global analysis of $b \rightarrow sll$ anomalies

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*In collaboration with:* **S. Descotes-Genon, L. Hofer and J. Virto**

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JM'12 **PRD86 (2012) 094024**, HM'15 **JHEP 1509(2015)104**, DHMV'15 **1510.04239**

**All originated in Frank Krueger, J.M., Phys. Rev. D71 (2005) 094009**

November 25, 2015

This talk will try to answer the following questions:

- What does the global fit on  $b \rightarrow s\ell\ell$  tell us about Wilson coefficients?
  - Which Wilson coefficients/scenarios receive a dominant NP contribution?
  - What does other approaches using different observables and methodology obtain?
- Are the alternative explanations (factorizable power corrections and charm) raised to explain (**some**) anomaly on the fit really robust?
  - Where those "explanations" fail in front of a possible New Physics explanation?
- What can we expect in the near future?

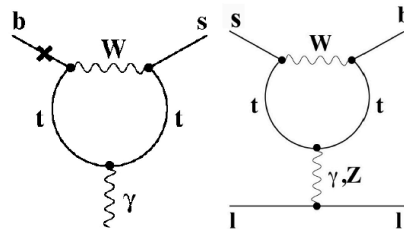
Since long time ago...

$\Rightarrow b \rightarrow s\gamma$  and  $b \rightarrow sll$  **Flavour Changing Neutral Currents** have been used as **our portal** to explore the fundamental theory beyond SM.

Analysis in a model-independent approach effective Hamiltonian:

$$b \rightarrow s\gamma(^*) : \mathcal{H}_{\Delta F=1}^{SM} \propto \sum_{i=1}^{10} V_{ts}^* V_{tb} C_i \mathcal{O}_i + \dots$$

- $\mathcal{O}_7 = \frac{e}{g^2} m_b \bar{s} \sigma^{\mu\nu} (1 + \gamma_5) F_{\mu\nu} b$
- $\mathcal{O}_9 = \frac{e^2}{g^2} \bar{s} \gamma_\mu (1 - \gamma_5) b \bar{l} \gamma_\mu l$
- $\mathcal{O}_{10} = \frac{e^2}{g^2} \bar{s} \gamma_\mu (1 - \gamma_5) b \bar{l} \gamma_\mu \gamma_5 l$



- **SM** Wilson coefficients up to NNLO + e.m. corrections at  $\mu_{ref} = 4.8$  GeV [[Misiak et al.](#)]:

$$C_7^{SM} = -0.29, C_9^{SM} = 4.1, C_{10}^{SM} = -4.3$$

- **NP** changes short distance  $C_i - C_i^{SM} = C_i^{NP}$  and induce new operators, like  $\mathcal{O}'_{7,9,10} = \mathcal{O}_{7,9,10} (\gamma_5 \leftrightarrow -\gamma_5)$

**Our Aim:** To disentangle hadronic effects from New Physics effects.

**Our Tool:** A global analysis of  $b \rightarrow sll$ ,  $b \rightarrow s\gamma$  will allow to test these Wilson coefficients with an unprecedented precision.

# THE OBSERVABLES

- Inclusive

- $B \rightarrow X_s \gamma$  ( $BR$ ) .....  $c_7^{(\prime)}$
- $B \rightarrow X_s \ell^+ \ell^-$  ( $dBR/dq^2$ ) .....  $c_7^{(\prime)}, c_9^{(\prime)}, c_{10}^{(\prime)}$

- Exclusive leptonic

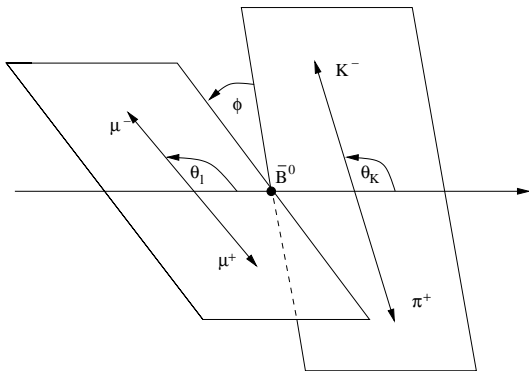
- $B_s \rightarrow \ell^+ \ell^-$  ( $BR$ ) .....  $c_{10}^{(\prime)}$

- Exclusive radiative/semileptonic

- $B \rightarrow K^* \gamma$  ( $BR, S, A_I$ ) .....  $c_7^{(\prime)}$
- $B \rightarrow K \ell^+ \ell^-$  ( $dBR/dq^2$ ) .....  $c_7^{(\prime)}, c_9^{(\prime)}, c_{10}^{(\prime)}$
- **$B \rightarrow K^* \ell^+ \ell^-$**  ( $dBR/dq^2$ , **Optimized Angular Obs.**) ..  $c_7^{(\prime)}, c_9^{(\prime)}, c_{10}^{(\prime)}$
- $B_s \rightarrow \phi \ell^+ \ell^-$  ( $dBR/dq^2$ , Angular Observables) .....  $c_7^{(\prime)}, c_9^{(\prime)}, c_{10}^{(\prime)}$
- $\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$  (None so far)
- etc.

The optimized observables  $P_i^{(\prime)}$  come from the angular distribution  $\bar{B}_d \rightarrow \bar{K}^{*0}(\rightarrow K^- \pi^+) l^+ l^-$  with the  $K^{*0}$  on the mass shell. It is described by  $\mathbf{s} = \mathbf{q}^2$  and three angles  $\theta_\ell$ ,  $\theta_K$  and  $\phi$

$$\frac{d^4\Gamma(\bar{B}_d)}{dq^2 d\cos\theta_\ell d\cos\theta_K d\phi} = \frac{9}{32\pi} \mathbf{J}(\mathbf{q}^2, \theta_\ell, \theta_K, \phi) = \sum_i J_i(q^2) f_i(\theta_\ell, \theta_K, \phi)$$



$\theta_\ell$ : Angle of emission between  $\bar{K}^{*0}$  and  $\mu^-$  in di-lepton rest frame.

$\theta_K$ : Angle of emission between  $\bar{K}^{*0}$  and  $K^-$  in di-meson rest frame.

$\phi$ : Angle between the two planes.

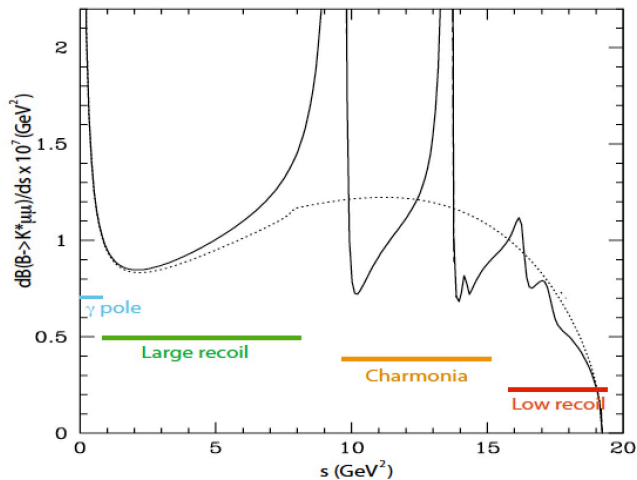
$q^2$ : dilepton invariant mass square.

$J_i(q^2)$  are function of transversity amplitudes of  $K^*$ :  $A_{\perp,\parallel,0}^{L,R}$  and they depend on FF and Wilson coefficients.

Notice LHCb uses  $\theta_\ell^{LHCb} = \pi - \theta_\ell^{us}$ .

Ongoing discussion on  $\phi^{LHCb}$  versus  $\phi^{theory}$  irrelevant for the fit (checked explicitly) (sign of  $S_{7,8}$  or  $P'_{6,8}$ ). (Zwicky)

# Four regions in $q^2$



Four regions in  $q^2$ :

- **very large  $K^*$ -recoil** ( $4m_\ell^2 < q^2 < 1 \text{ GeV}^2$ ):  $\gamma$  almost real.
- **large  $K^*$ -recoil/low- $q^2$** :  $E_{K^*} \gg \Lambda_{QCD}$  or  $4m_\ell^2 \leq q^2 < 9 \text{ GeV}^2$ : LCSR-FF
- **charmonium region** ( $q^2 = m_{J/\psi}^2, \dots$ ) between  $9 < q^2 < 14 \text{ GeV}^2$ .
- **low  $K^*$ -recoil/large- $q^2$** :  $E_{K^*} \sim \Lambda_{QCD}$  or  $14 < q^2 \leq (m_B - m_{K^*})^2$ : LQCD-FF

The distribution (massless case) including the **S-wave** and normalized to  $\Gamma'_{full}$ :

$$\begin{aligned} \frac{1}{\Gamma'_{full}} \frac{d^4\Gamma}{dq^2 d\cos\theta_K d\cos\theta_l d\phi} = & \frac{9}{32\pi} \left[ \frac{3}{4} \mathbf{F}_T \sin^2\theta_K + \mathbf{F}_L \cos^2\theta_K \right. \\ & + \left( \frac{1}{4} \mathbf{F}_T \sin^2\theta_K - \mathbf{F}_L \cos^2\theta_K \right) \cos 2\theta_l + \frac{1}{2} \mathbf{P}'_1 \mathbf{F}_T \sin^2\theta_K \sin^2\theta_l \cos 2\phi \\ & + \sqrt{\mathbf{F}_T \mathbf{F}_L} \left( \frac{1}{2} \mathbf{P}'_4 \sin 2\theta_K \sin 2\theta_l \cos \phi + \mathbf{P}'_5 \sin 2\theta_K \sin \theta_l \cos \phi \right) \\ & - \sqrt{\mathbf{F}_T \mathbf{F}_L} \left( \mathbf{P}'_6 \sin 2\theta_K \sin \theta_l \sin \phi - \frac{1}{2} \mathbf{P}'_8 \sin 2\theta_K \sin 2\theta_l \sin \phi \right) \\ & \left. + 2\mathbf{P}'_2 \mathbf{F}_T \sin^2\theta_K \cos \theta_l - \mathbf{P}'_3 \mathbf{F}_T \sin^2\theta_K \sin^2\theta_l \sin 2\phi \right] (1 - \mathbf{F}_S) + \frac{1}{\Gamma'_{full}} \mathbf{W}_S \end{aligned}$$

- in **blue** the set of relevant observables  $\mathbf{P}'_{1,2}, \mathbf{P}'_{4,5}$ .
- the S-wave terms are (see discussion [HM'15]) not all free observables:

$$\begin{aligned} \frac{\mathbf{W}_S}{\Gamma'_{full}} = & \frac{3}{16\pi} \left[ \mathbf{F}_S \sin^2\theta_\ell + \mathbf{A}_S \sin^2\theta_\ell \cos\theta_K + \mathbf{A}_S^4 \sin\theta_K \sin 2\theta_\ell \cos\phi \right. \\ & \left. + \mathbf{A}_S^5 \sin\theta_K \sin\theta_\ell \cos\phi + \mathbf{A}_S^7 \sin\theta_K \sin\theta_\ell \sin\phi + \mathbf{A}_S^8 \sin\theta_K \sin 2\theta_\ell \sin\phi \right] \end{aligned}$$

Basis (massless):

$\{\Gamma'_{K^*}, A_{FB} \text{ or } F_L, P_1, P_2, P_3, P'_4, P'_5, P'_6\}$  and only 4 of  $\{F_S, A_S, A_S^4, A_S^5, A_S^7, A_S^8\}$  are independent.



There are basically two theory approaches:

**1. Improved-QCDF approach:** QCDF+exploit symmetry relations at large-recoil (limit) among FF:

$$\frac{m_B}{m_B+m_{K^*}} V(q^2) = \frac{m_B+m_{K^*}}{2E} A_1(q^2) = T_1(q^2) = \frac{m_B}{2E} T_2(q^2) = \xi_{\perp}(E)$$

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- factorizable power corrections (using a systematic procedure for each FFp, see later)
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## 2. Full FF approach: Compute correlations from a specific LCSR computation.

- ⇒ Parametric correlations easy, but Borel parameters choice delicate. Use of EOM.
- ⇒ Factorizable  $\mathcal{O}(\alpha_s)$  and factorizable p.c. included in a particular LCSR parametrization.
- ⇒ **Less general**, attached to a single FF parametrization with all inner choices included.
- ⇒ Extra pieces that **need to be added**:
  - known  $\alpha_s$  non-factorizable corrections from QCDF.
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Usually applied to  $S_i = (J_i + \bar{J}_i)/(d\Gamma + \bar{d}\Gamma)$  **highly dependent on FF-error estimate**.

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  - They fit for the amplitudes after fixing 3 of them to zero by means of the symmetries.
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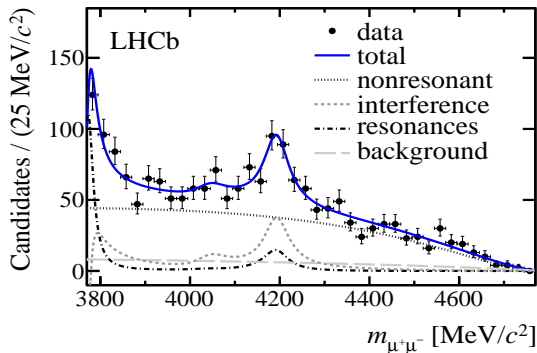
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# Theoretical description of $B \rightarrow K^* \ell^+ \ell^-$ : large- $q^2$

- It corresponds to **large**  $\sqrt{q^2} \sim \mathcal{O}(m_b)$  above  $\Psi'$  mass, i.e.,  $E_K$  is around GeV or below.
- OPE in  $E_K/\sqrt{q^2}$  or  $\Lambda_{QCD}/\sqrt{q^2}$  (Buchalla et al)
- **NLO QCD corrections** to the OPE coeffs (Greub et al)
- **Lattice QCD form factors with correlations** (Horgan et al proceeding update)
- $\pm 10\%$  on angular observables to account for possible Duality Violations.  
⇒ Estimates on BR from GP (5%) and BBF (2%) using Shifman's model.

Existence of  $c\bar{c}$  **resonances** in this region (clearly seen  $\psi(4160)$  in  $B^- \rightarrow K^- \mu^+ \mu^-$ ),  
⇒ require to take a long bin.

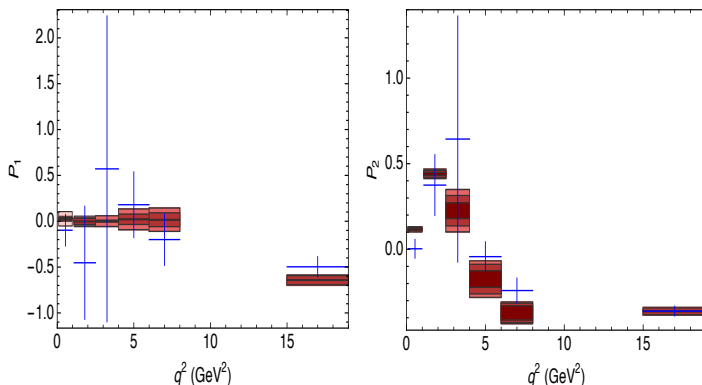


# A few properties of the relevant observables $P_{1,2}$

The idea of **exact cancellation of the poorly known soft form factors at LO** at the zero of  $A_{FB}$  was incorporated in the construction of the transverse asymmetry (this is the meaning of the word “**clean**”)

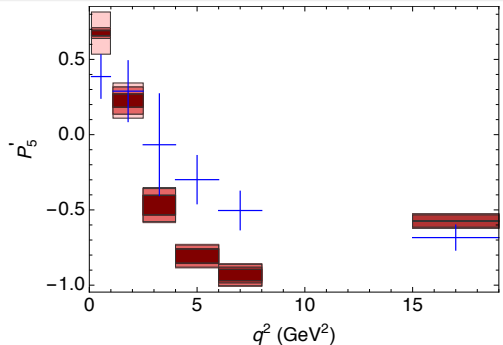
## $P_1$ and $P_2$ observables function of $A_{\perp}$ and $A_{\parallel}$ amplitudes

- $P_1$ : Proportional to  $|A_{\perp}|^2 - |A_{\parallel}|^2$ 
  - Test the LH structure of SM and/or existence of RH currents that breaks  $A_{\perp} \sim -A_{\parallel}$
- $P_2$ : Proportional to  $\text{Re}(A_i A_j)$ 
  - Zero of  $P_2$  at the same position as the zero of  $A_{FB}$
  - $P_2$  is the clean version of  $A_{FB}$ . Their different normalizations offer different sensitivities.



- $P_3$  and  $P'_{6,8}$  are proportional to  $\text{Im}A_i A_j$  and small if there are no large phases. All are  $< 0.1$ .
- $P_i^{CP}$  are all negligibly small if there is no New Physics in weak phases.

# Brief Discussion on: $P'_5$ and $P'_4$



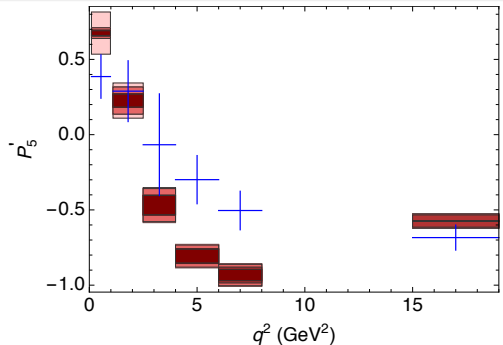
$P'_5$  was proposed for the first time in [DMRV, JHEP 1301\(2013\)048](#)

$$P'_5 = \sqrt{2} \frac{\text{Re}(A_0^L A_{\perp}^{L*} - A_0^R A_{\perp}^{R*})}{\sqrt{|A_0|^2(|A_{\perp}|^2 + |A_{\parallel}|^2)}} = \sqrt{2} \frac{\text{Re}[n_0 n_{\perp}^{\dagger}]}{\sqrt{|n_0|^2(|n_{\perp}|^2 + |n_{\parallel}|^2)}}.$$

with  $n_0 = (A_0^L, A_0^{R*})$ ,  $n_{\perp} = (A_{\perp}^L, -A_{\perp}^{R*})$  and  $n_{\parallel} = (A_{\parallel}^L, A_{\parallel}^{R*})$

- If no-RHC  $|n_{\perp}| \simeq |n_{\parallel}|$  ( $H_{+1} \simeq 0$ )  $\Rightarrow P'_5 \propto \cos \theta_{0,\perp}(\mathbf{q}^2)$

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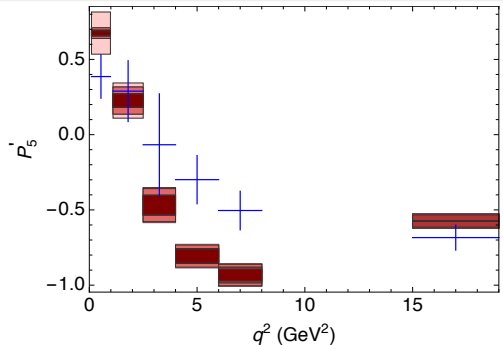
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In the large-recoil limit with no RHC

$$A_{\perp,\parallel}^L \propto (1, -1) \left[ C_9^{\text{eff}} - C_{10} + \frac{2\hat{m}_b}{\hat{s}} C_7^{\text{eff}} \right] \xi_{\perp}(E_{K^*}) \quad A_{\perp,\parallel}^R \propto (1, -1) \left[ C_9^{\text{eff}} + C_{10} + \frac{2\hat{m}_b}{\hat{s}} C_7^{\text{eff}} \right] \xi_{\perp}(E_{K^*})$$

$$A_0^L \propto - \left[ C_9^{\text{eff}} - C_{10} + 2\hat{m}_b C_7^{\text{eff}} \right] \xi_{\parallel}(E_{K^*}) \quad A_0^R \propto - \left[ C_9^{\text{eff}} + C_{10} + 2\hat{m}_b C_7^{\text{eff}} \right] \xi_{\parallel}(E_{K^*})$$

- In SM  $C_9^{SM} + C_{10}^{SM} \simeq 0 \rightarrow |A_{\perp,\parallel}^R| \ll |A_{\perp,\parallel}^L|$
- In  $P'_5$ : If  $C_9^{NP} < 0$  then  $A_{0,\parallel}^R \uparrow$ ,  $A_{\perp}^R \uparrow$  and  $|A_{0,\parallel}^L| \downarrow$ ,  $A_{\perp}^L \downarrow$  and due to  $-$ ,  $|P'_5|$  gets **strongly** reduced.



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with  $n_0 = (A_0^L, A_0^{R*})$ ,  $n_{\perp} = (A_{\perp}^L, -A_{\perp}^{R*})$  and  $n_{\parallel} = (A_{\parallel}^L, A_{\parallel}^{R*})$

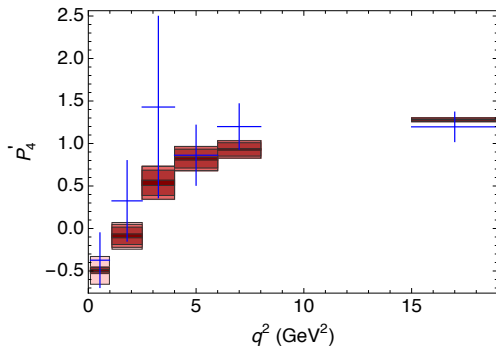
- If no-RHC  $|n_{\perp}| \simeq |n_{\parallel}|$  ( $H_{+1} \simeq 0$ )  $\Rightarrow P'_5 \propto \cos \theta_{0,\perp}(\mathbf{q}^2)$

In the large-recoil limit with no RHC

$$A_{\perp,\parallel}^L \propto (1, -1) \left[ C_9^{\text{eff}} - C_{10} + \frac{2\hat{m}_b}{\hat{s}} C_7^{\text{eff}} \right] \xi_{\perp}(E_{K^*}) \quad A_{\perp,\parallel}^R \propto (1, -1) \left[ C_9^{\text{eff}} + C_{10} + \frac{2\hat{m}_b}{\hat{s}} C_7^{\text{eff}} \right] \xi_{\perp}(E_{K^*})$$

$$A_0^L \propto - \left[ C_9^{\text{eff}} - C_{10} + 2\hat{m}_b C_7^{\text{eff}} \right] \xi_{\parallel}(E_{K^*}) \quad A_0^R \propto - \left[ C_9^{\text{eff}} + C_{10} + 2\hat{m}_b C_7^{\text{eff}} \right] \xi_{\parallel}(E_{K^*})$$

- In SM  $C_9^{SM} + C_{10}^{SM} \simeq 0 \rightarrow |A_{\perp,\parallel}^R| \ll |A_{\perp,\parallel}^L|$
- In  $P'_5$ : If  $C_9^{NP} < 0$  then  $A_{0,\parallel}^R \uparrow$ ,  $A_{\perp}^R \uparrow$  and  $A_{0,\parallel}^L \downarrow$ ,  $A_{\perp}^L \downarrow$  and due to  $-$ ,  $|P'_5|$  gets **strongly** reduced.



$P'_4$  was proposed for the first time in **DMRV, JHEP 1301(2013)048**

$$P'_4 = \sqrt{2} \frac{\text{Re}(A_0^L A_{\parallel}^{L*} + A_0^R A_{\parallel}^{R*})}{\sqrt{|A_0|^2(|A_{\perp}|^2 + |A_{\parallel}|^2)}} = \sqrt{2} \frac{\text{Re}[n_0 n_{\parallel}^{\dagger}]}{\sqrt{|n_0|^2(|n_{\perp}|^2 + |n_{\parallel}|^2)}}.$$

with  $n_0 = (A_0^L, A_0^{R*})$ ,  $n_{\perp} = (A_{\perp}^L, -A_{\perp}^{R*})$  and  $n_{\parallel} = (A_{\parallel}^L, A_{\parallel}^{R*})$

- If no-RHC  $|n_{\perp}| \simeq |n_{\parallel}|$  ( $H_{+1} \simeq 0$ )  $\Rightarrow P'_4 \propto \cos \theta_{0,\parallel}(q^2)$

In the large-recoil limit with no RHC

$$A_{\perp,\parallel}^L \propto (1, -1) \left[ C_9^{\text{eff}} - C_{10} + \frac{2\hat{m}_b}{\hat{s}} C_7^{\text{eff}} \right] \xi_{\perp}(E_{K^*}) \quad A_{\perp,\parallel}^R \propto (1, -1) \left[ C_9^{\text{eff}} + C_{10} + \frac{2\hat{m}_b}{\hat{s}} C_7^{\text{eff}} \right] \xi_{\perp}(E_{K^*})$$

$$A_0^L \propto - \left[ C_9^{\text{eff}} - C_{10} + 2\hat{m}_b C_7^{\text{eff}} \right] \xi_{\parallel}(E_{K^*}) \quad A_0^R \propto - \left[ C_9^{\text{eff}} + C_{10} + 2\hat{m}_b C_7^{\text{eff}} \right] \xi_{\parallel}(E_{K^*})$$

- In SM  $C_9^{SM} + C_{10}^{SM} \simeq 0 \rightarrow |A_{\perp,\parallel}^R| \ll |A_{\perp,\parallel}^L|$
- In  $P'_4$ : If  $C_9^{NP} < 0$  then  $A_{0,\parallel}^R \uparrow$ ,  $|A_{\perp}^R| \uparrow$  and  $|A_{0,\parallel}^L| \downarrow$ ,  $A_{\perp}^L \downarrow$  due to + what L loses R gains (little change).

From recent Bobeth's talk at cern

year	arXiv:	group/programs	authors	method
2010	1006.5013	EOS	CB/Hiller/van Dyk	$\Delta\chi^2$
2011	1104.3342	DGMR	Descotes-Genon/Ghosh/JM/Ramon	$\Delta\chi^2$
	1105.0376	EOS	CB/Hiller/van Dyk	$\Delta\chi^2$
	1111.1257	APS	Altmannshofer/Paradisi/Straub	$\Delta\chi^2$
	1111.2558	EOS	CB/Hiller/van Dyk/Wacker	$\Delta\chi^2$
2012	1205.1838	EOS	Beaujean/CB/van Dyk/Wacker	bayesian
	1206.0273	AS	Altmannshofer/Straub	$\Delta\chi^2$
	1207.0688	SuperISO	Hurth/Mahmoudi	$\Delta\chi^2$
	1207.2753	DMRV	Descotes-Genon/JM/Ramon/Virto	$\Delta\chi^2$

$\Delta\chi^2$  means frequentist.

- **global fits**: combination of observables governed by  $b \rightarrow s\ell\ell$  and  $b \rightarrow s\gamma$
- Public software: **EOS**, **SuperIso**, **HEPfit** and private codes **DHMV**, **AS**,...

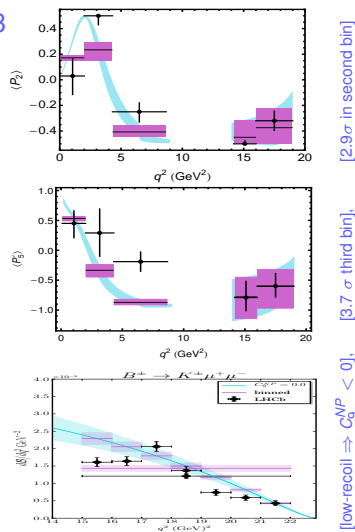
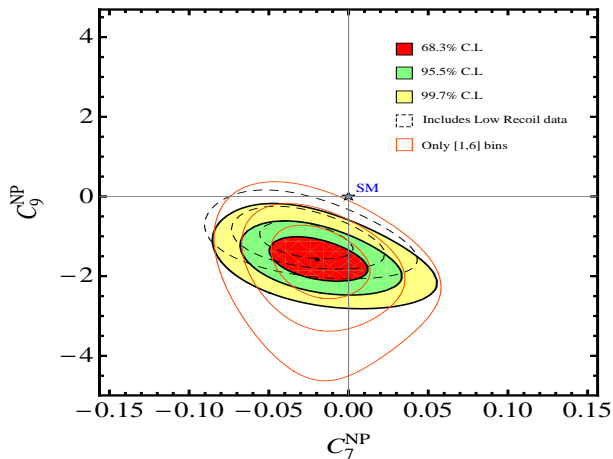
# History of global $b \rightarrow s\ell\ell$ fits (II)

year	arXiv:	group/programs	authors	method
2013	1307.5683	DMV	Descotes-Genon/JM/Virto	$\Delta\chi^2$ <b>Anomaly</b>
	1308.1501	AS	Altmannshofer/Straub	$\Delta\chi^2$
	1310.2478	EOS	Beaujean/CB/van Dyk	bayesian
	1310.3887	HLMW	Horgan/Liu/Meinel/Wingate	$\Delta\chi^2$
	1312.5267	SuperISO	Hurth/Mahmoudi	$\Delta\chi^2$
2014	1408.4097	GNR	Ghosh/Nardecchia/Renner	bayesian
	1410.4545	SuperIso	Hurth/Mahmoudi/Neshatpour	$\Delta\chi^2$
	1411.3161/1503.06199	AS	Altmannshofer/Straub	$\Delta\chi^2$
2015	1508.01526	EOS	Beaujean/CB/Jahn	bayesian
	$\Rightarrow$ 1510.04239	DHMV	Descotes-Genon/Hofer/Matias/Virto	$\Delta\chi^2$

- 1/fb dataset from LHCb first analysis done using **optimized observables**  
 $\Rightarrow$  **the " $B \rightarrow K^*\mu^+\mu^-$  anomaly** is described in 1307.5683
- 3/fb dataset from LHCb **the anomaly is confirmed.**



Situation in 2013: Descotes-Genon, Matias, Virto 1307.5683



Our statement in July 2013 DMV'13:

“We found that the Standard Model hypothesis  $C_7^{\text{NP}} = 0$ ,  $C_9^{\text{NP}} = 0$  has a pull of  $4.5\sigma$ ”.

Other groups later on confirmed the relevance of  $C_9$  using FFD-observables (Altmannshofer, Straub 1308.1501), low-recoil (Horgan et al. 1310.3887), Bayesian approach (Beaujean, Bobethm Van Dyk 1310.2478).

# FIT 2015

## see talk J. Virto

- $BR(B \rightarrow X_s \gamma)$ 
  - New theory update:  $\mathcal{B}_{s\gamma}^{\text{SM}} = (3.36 \pm 0.23) \cdot 10^{-4}$  (Misiak et al 2015)
  - +6.4% shift in central value w.r.t 2006 → excellent agreement with WA
- $BR(B_s \rightarrow \mu^+ \mu^-)$ 
  - New theory update (Bobeth et al 2013), New LHCb+CMS average (2014)
- $BR(B \rightarrow X_s \mu^+ \mu^-)$ 
  - New theory update (Huber et al 2015)
- $BR(B \rightarrow K \mu^+ \mu^-)$  :
  - LHCb 2014 + Lattice form factors at large  $q^2$  (Bouchard et al 2013, 2015)
- $B_{(s)} \rightarrow (K^*, \phi) \mu^+ \mu^-$  : BRs & Angular Observables
  - LHCb 2015 + Lattice form factors at large  $q^2$  (Horgan et al 2013)
- $BR(B \rightarrow K e^+ e^-)_{[1,6]}$  (or  $R_K$ ) and  $B \rightarrow K^* e^+ e^-$  at very low  $q^2$ 
  - LHCb 2014, 2015

Frequentist approach:

$$\chi^2(C_i) = [O_{\text{exp}} - O_{\text{th}}(C_i)]_j [\text{Cov}^{-1}]_{jk} [O_{\text{exp}} - O_{\text{th}}(C_i)]_k$$

- **Cov** = **Cov**<sup>exp</sup> + **Cov**<sup>th</sup>. We have  $\text{Cov}^{\text{exp}}$  for the first time
- Calculate  $\text{Cov}^{\text{th}}$ : correlated multigaussian scan over all nuisance parameters
- $\text{Cov}^{\text{th}}$  depends on  $C_i$ : Must check this dependence

For the Fit:

- Minimise  $\chi^2 \rightarrow \chi_{\text{min}}^2 = \chi^2(C_i^0)$  (Best Fit Point =  $C_i^0$ )
- Confidence level regions:  $\chi^2(C_i) - \chi_{\text{min}}^2 < \Delta\chi_{\sigma,n}$

**In a model with a single free param.  $C_9$  the fit result  $\Rightarrow$  measurement of  $C_9$  (confidence interval).  $\text{Pull}_{SM}$  tells you how much in this model the measured value of  $C_9$  is in tension with  $C_9 = C_9^{SM}$**

## NO SINGLE MEASUREMENT

HAVE A PULL LARGER THAN  $3.3\sigma$  (several deviations  $2-3\sigma$ )

**GLOBAL ANALYSIS** TELLS YOU

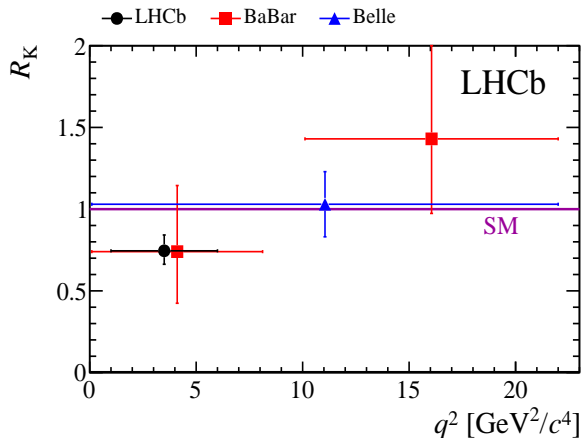
**HOW MUCH**  $C_i^{\text{NP}} = 0$  (SM) (i=9 for instance) IS

**DISFAVOURED COMPARED** TO THE **BEST FIT POINT**



THIS OBVIOUSLY CAN BE **LARGER THAN  $3\sigma$**  if deviations are consistent

# SM predictions and Pulls 2015: $B \rightarrow K\mu\mu$ : What's new in 2015?



$$R_K = \frac{\text{Br}(B^+ \rightarrow K^+ \mu^+ \mu^-)}{\text{Br}(B^+ \rightarrow K^+ e^+ e^-)} = 0.745_{-0.074}^{+0.090} \pm 0.036$$

- It deviates  $2.6\sigma$  from SM.
- Data on  $B^+ \rightarrow K^+ \mu^+ \mu^-$  is below SM in **all bins** at large and low-recoil.

Also neutral mode:

$10^7 \times BR(B^0 \rightarrow K^0 \mu^+ \mu^-)$	Standard Model	Experiment	Pull
[0.1, 2]	$0.63 \pm 0.18$	$0.23 \pm 0.11$	<b>+1.9</b>
[2, 4]	$0.65 \pm 0.21$	$0.37 \pm 0.11$	<b>+1.2</b>
[4, 6]	$0.64 \pm 0.22$	$0.35 \pm 0.10$	<b>+1.2</b>
[6, 8]	$0.64 \pm 0.24$	$0.54 \pm 0.12$	+0.4
[15, 19]	$0.90 \pm 0.13$	$0.67 \pm 0.12$	<b>+1.4</b>

THERE ARE COMMON NEW PHYSICS  
MECHANISMS ABLE TO EXPLAIN

$$P'_5 \text{ and } R_K$$

$10^7 \times BR(B^0 \rightarrow K^{*0} \mu^+ \mu^-)$	Standard Model	Experiment	Pull
[0.1, 2]	$1.26 \pm 1.03$	$1.14 \pm 0.18$	+0.1
[2, 4.3]	$0.84 \pm 0.59$	$0.69 \pm 0.12$	+0.2
[4.3, 8.68]	$2.52 \pm 2.09$	$2.15 \pm 0.31$	+0.2
[16, 19]	$1.66 \pm 0.15$	$1.23 \pm 0.20$	<b>+1.7</b>
$10^7 \times BR(B^+ \rightarrow K^{*+} \mu^+ \mu^-)$	Standard Model	Experiment	Pull
[0.1, 2]	$1.31 \pm 1.08$	$1.12 \pm 0.27$	+0.2
[2, 4]	$0.79 \pm 0.55$	$1.12 \pm 0.32$	-0.5
[4, 6]	$0.94 \pm 0.71$	$0.50 \pm 0.20$	+0.6
[6, 8]	$1.15 \pm 0.95$	$0.66 \pm 0.22$	+0.5
[15, 19]	$2.59 \pm 0.24$	$1.60 \pm 0.32$	<b>+2.5</b>
$10^7 \times BR(B_s \rightarrow \phi \mu^+ \mu^-)$	Standard Model	Experiment	Pull
[0.1, 2.]	$1.81 \pm 0.36$	$1.11 \pm 0.16$	<b>+1.8</b>
[2., 5.]	$1.88 \pm 0.31$	$0.77 \pm 0.14$	<b>+3.3</b>
[5., 8.]	$2.25 \pm 0.41$	$0.96 \pm 0.15$	<b>+3.0</b>
[15, 18.8]	$2.20 \pm 0.16$	$1.62 \pm 0.20$	<b>+2.3</b>



New  $3\text{fb}^{-1}$  dataset confirms the anomaly in  $P'_5$  in bins [4,6] and [6,8]

$P_1(B \rightarrow K^* \mu^+ \mu^-)$	Standard Model	Experiment	Pull
[15, 19]	$-0.64 \pm 0.05$	$-0.50 \pm 0.11$	<b>-1.2</b>
$P_2(B \rightarrow K^* \mu^+ \mu^-)$	Standard Model	Experiment	Pull
[0.1, 0.98]	$0.12 \pm 0.02$	$0.00 \pm 0.05$	<b>+2.0</b>
[6, 8]	$-0.38 \pm 0.08$	$-0.24 \pm 0.07$	<b>-1.2</b>
$P'_5(B \rightarrow K^* \mu^+ \mu^-)$	Standard Model	Experiment	Pull
[0.1, 0.98]	$0.67 \pm 0.14$	$0.39 \pm 0.14$	<b>+1.4</b>
[2.5, 4]	$-0.49 \pm 0.13$	$-0.07 \pm 0.34$	<b>-1.2</b>
[4, 6]	$-0.82 \pm 0.08$	$-0.30 \pm 0.16$	<b>-2.9</b>
[6, 8]	$-0.94 \pm 0.08$	$-0.50 \pm 0.13$	<b>-2.9</b>
[15, 19]	$-0.57 \pm 0.05$	$-0.68 \pm 0.08$	<b>+1.2</b>
$P_1(B_s \rightarrow \phi \mu^+ \mu^-)$	Standard Model	Experiment	Pull
[15, 18.8]	$-0.69 \pm 0.03$	$-0.25 \pm 0.34$	<b>-1.3</b>
$P'_4(B_s \rightarrow \phi \mu^+ \mu^-)$	Standard Model	Experiment	Pull
[15, 18.8]	$1.30 \pm 0.01$	$0.62 \pm 0.49$	<b>+1.4</b>

# Result of the fit with 1D Wilson coefficient 2015

This is the first analysis: - using the basis of **optimized observables** ( $B \rightarrow K^* \mu \mu$  and  $B_s \rightarrow \phi \mu \mu$ )  
 - using the **full dataset** of  $3\text{fb}^{-1}$ :

Coefficient	Best fit	$1\sigma$	$3\sigma$	Pull <sub>SM</sub>
$C_7^{\text{NP}}$	-0.02	[-0.04, -0.00]	[-0.07, 0.04]	1.1
$C_9^{\text{NP}}$	-1.11	[-1.32, -0.89]	[-1.71, -0.40]	<b>4.5</b> $\Leftarrow$
$C_{10}^{\text{NP}}$	0.58	[0.34, 0.84]	[-0.11, 1.41]	2.5
$C_{7'}^{\text{NP}}$	0.02	[-0.01, 0.04]	[-0.05, 0.09]	0.7
$C_{9'}^{\text{NP}}$	0.49	[0.21, 0.77]	[-0.33, 1.35]	1.8
$C_{10'}^{\text{NP}}$	-0.27	[-0.46, -0.08]	[-0.84, 0.28]	1.4
$C_9^{\text{NP}} = C_{10}^{\text{NP}}$	-0.21	[-0.40, 0.00]	[-0.74, 0.55]	1.0
$C_9^{\text{NP}} = -C_{10}^{\text{NP}}$	-0.69	[-0.88, -0.51]	[-1.27, -0.18]	<b>4.1</b> $\Leftarrow$
$C_9^{\text{NP}} = -C_{9'}^{\text{NP}}$	-1.09	[-1.28, -0.88]	[-1.62, -0.42]	<b>4.8</b> $\Leftarrow$ (no $R_K$ )
$C_9^{\text{NP}} = -C_{10}^{\text{NP}}$ $= -C_{9'}^{\text{NP}} = -C_{10'}^{\text{NP}}$	-0.68	[-0.49, -0.49]	[-1.36, -0.15]	3.9

# Result of the fit with 2D Wilson coefficient constrained and unconstrained

Coefficient	Best Fit Point	Pull <sub>SM</sub>	p-value (%)
$(C_7^{NP}, C_9^{NP})$	$(-0.00, -1.11)$	<b>4.1</b>	60.0
$(C_9^{NP}, C_{10}^{NP})$	$(-1.16, 0.35)$	<b>4.3</b>	67.0
$(C_9^{NP}, C_{7'}^{NP})$	$(-1.16, 0.02)$	<b>4.2</b>	63.0
$(C_9^{NP}, C_{9'}^{NP})$	$(-1.15, 0.77)$	<b>4.5</b>	71.0
$(C_9^{NP}, C_{10'}^{NP})$	$(-1.23, -0.38)$	<b>4.5</b>	72.0
$(C_9^{NP} = -C_{9'}^{NP}, C_{10}^{NP} = C_{10'}^{NP})$	$(-1.17, 0.26)$	<b>4.6</b>	73.0
$(C_9^{NP} = -C_{9'}^{NP}, C_{10}^{NP} = -C_{10'}^{NP})$	$(-1.14, 0.04)$	<b>4.5</b>	69.0
$(C_9^{NP} = C_{9'}^{NP}, C_{10}^{NP} = C_{10'}^{NP})$	$(-0.68, -0.26)$	3.8	54.0
$(C_9^{NP} = -C_{10}^{NP}, C_{9'}^{NP} = C_{10'}^{NP})$	$(-0.74, 0.26)$	3.7	52.0

- $C_9^{NP}$  always play a dominant role
- All 2D scenarios above  $4\sigma$  are quite indistinguishable. We have done a systematic work to check what are the most relevant Wilson Coefficients to explain all deviations, by allowing progressively different WC to get NP contributions and compare the pulls.

# QUESTION 1: Branching Ratios versus Angular Observables $P_i$ ?

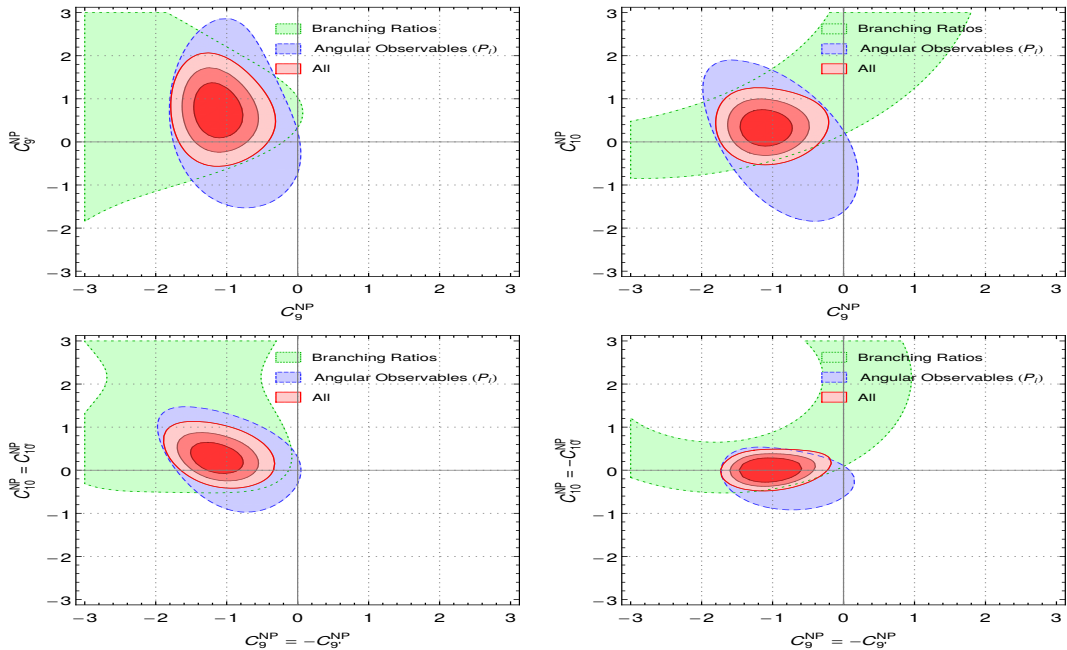


Figure: Angular observables (FFI at LO  $P_i$ ) dominates clearly over Branching ratios

# QUESTION 2: $B \rightarrow K^* \mu \mu$ , $B \rightarrow K \mu \mu$ and $B_s \rightarrow \phi \mu \mu$ ?

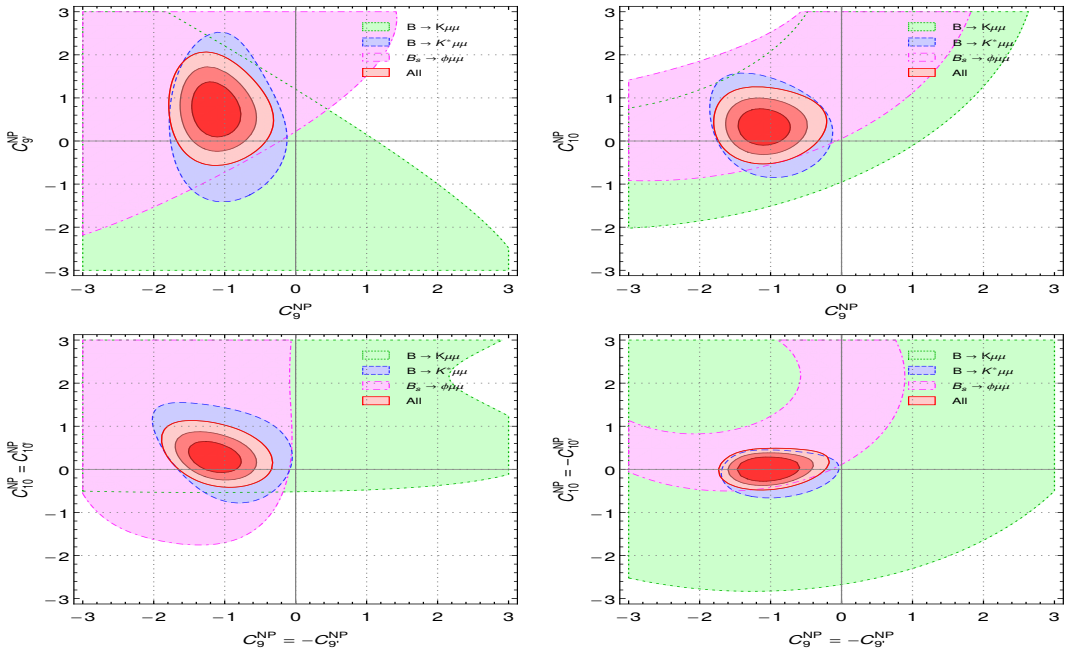


Figure: The hierarchy of importance for the fit:  $B \rightarrow K^* \mu \mu$ ,  $B_s \rightarrow \phi \mu \mu$  and  $B \rightarrow K \mu \mu$

# QUESTION 3: Which information and constraints provide each region?

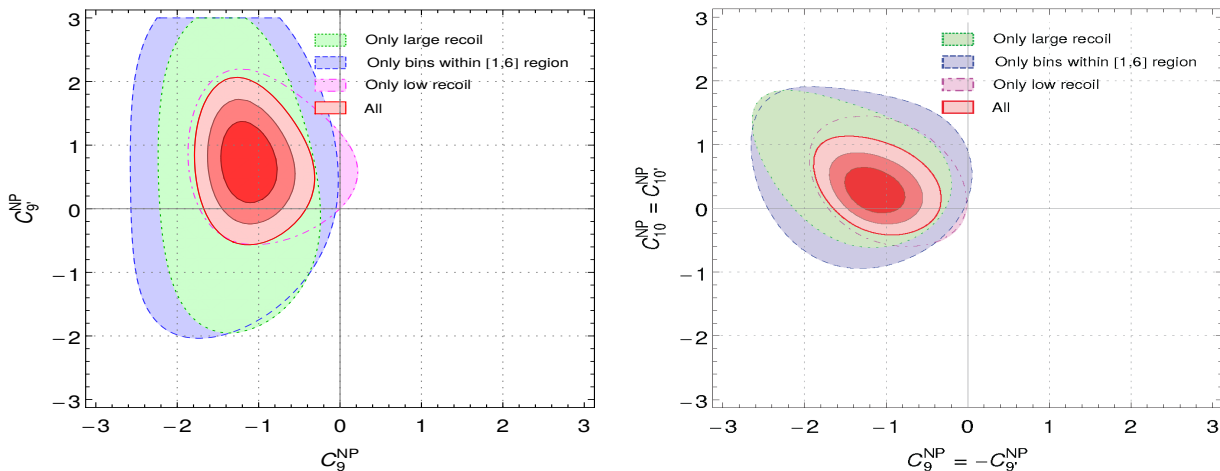


Figure: We show the  $3\sigma$  regions allowed by large-recoil only (dashed green), by bins in the [1-6] range (long-dashed blue), by low recoil (dot-dashed purple) and by considering all data (red, with 1,2,3  $\sigma$  contours).

- Low-recoil is strongly constraining! Important implications for power corrections and charm.
- Bins [1,6] are perfectly coherent with the full large-recoil.

Impact of  $B \rightarrow Ke^+e^-$   
under hypothesis of maximal  
Lepton Flavour Universal Violation

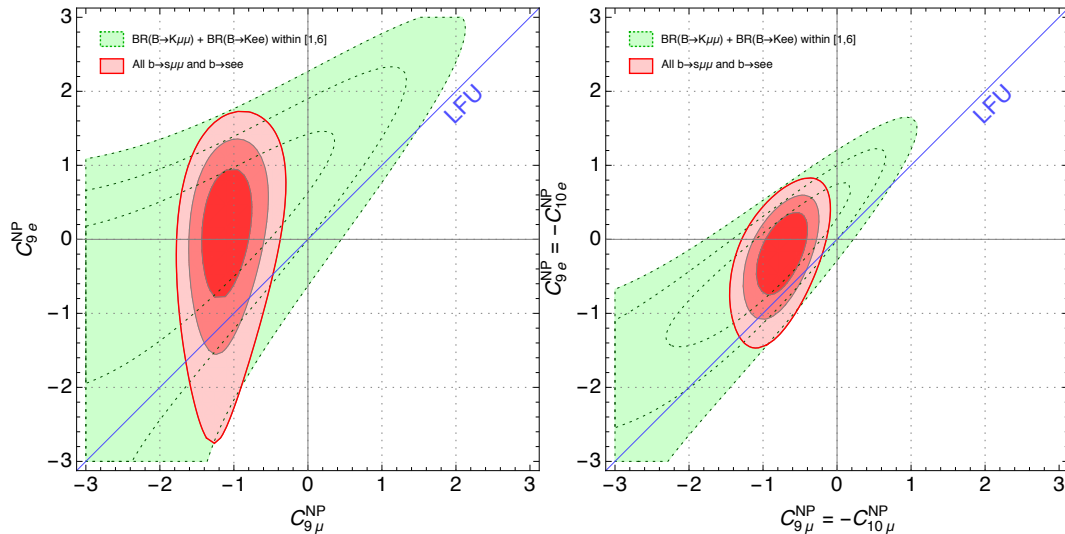
1D-Coefficient	Best fit	$1\sigma$	$3\sigma$	Pull <sub>SM</sub>
$C_9^{\text{NP}}$	-1.14	[-1.34, -0.93]	[-1.71, -0.47]	<b>4.5</b> → <b>4.9</b>
$C_9^{\text{NP}} = -C_{10}^{\text{NP}}$	-0.66	[-0.81, -0.50]	[-1.15, -0.21]	<b>4.1</b> → <b>4.6</b>
$C_9^{\text{NP}} = -C_{9'}^{\text{NP}}$	-1.09	[-1.28, -0.88]	[-1.62, -0.43]	<b>4.9</b>
$C_9^{\text{NP}} = -C_{10}^{\text{NP}} = -C_{9'}^{\text{NP}} = -C_{10'}^{\text{NP}}$	-0.65	[-0.83, -0.49]	[-0.19, -0.19]	4.4

2D-Coefficient	Best Fit Point	Pull <sub>SM</sub>
$(C_7^{\text{NP}}, C_9^{\text{NP}})$	(0.00, -1.13)	<b>4.1</b> → <b>4.6</b>
$(C_9^{\text{NP}}, C_{10}^{\text{NP}})$	(-1.11, 0.32)	<b>4.3</b> → <b>4.8</b>
$(C_9^{\text{NP}}, C_{7'}^{\text{NP}})$	(-1.20, 0.03)	<b>4.2</b> → <b>4.7</b>
$(C_9^{\text{NP}}, C_{9'}^{\text{NP}})$	(-1.23, 0.61)	<b>4.5</b> → <b>4.9</b>
$(C_9^{\text{NP}}, C_{10'}^{\text{NP}})$	(-1.32, -0.34)	<b>4.5</b> → <b>4.9</b>
$(C_9^{\text{NP}} = -C_{9'}^{\text{NP}}, C_{10}^{\text{NP}} = C_{10'}^{\text{NP}})$	(-1.23, 0.39)	<b>4.6</b> → <b>5.0</b>
$(C_9^{\text{NP}} = -C_{9'}^{\text{NP}}, C_{10}^{\text{NP}} = -C_{10'}^{\text{NP}})$	(-0.99, 0.03)	<b>4.5</b>
$(C_9^{\text{NP}} = C_{9'}^{\text{NP}}, C_{10}^{\text{NP}} = C_{10'}^{\text{NP}})$	(-0.70, -0.22)	<b>3.8</b> → <b>4.3</b>
$(C_9^{\text{NP}} = -C_{10}^{\text{NP}}, C_{9'}^{\text{NP}} = C_{10'}^{\text{NP}})$	(-0.69, 0.27)	<b>3.7</b> → <b>4.2</b>

- The strong correlations among form factors of  $B \rightarrow K\mu\mu$  and  $B \rightarrow Kee$  assuming no NP in  $B \rightarrow Kee$  enhances the NP evidence in muons.
- Notice that we use all bins in  $B \rightarrow K\mu\mu$  while  $R_K$  is only [1,6]. **All theory correlations included.**
- Only scenarios explaining  $R_K$  get an extra enhancement of +0.4-0.5  $\sigma$



# Fits considering Lepton Flavour (non-) Universality



- If NP-LFUV is assumed, NP may enter both  $b \rightarrow see$  and  $b \rightarrow s\mu\mu$  decays with different values.

⇒ For each scenario, we see that there is no clear indication of a NP contribution in the electron sector, whereas one has clearly a non-vanishing contribution for the muon sector, with a deviation from the Lepton Flavour Universality line.

# Prediction for LFU tests observables

	$R_K[1, 6]$	$R_{K^*}[1.1, 6]$	$R_\phi[1.1, 6]$
SM	$1.00 \pm 0.01$	$1.00 \pm 0.01 [1.00 \pm 0.01]$	$1.00 \pm 0.01$
$C_9^{\text{NP}} = -1.11$	$0.79 \pm 0.01$	$0.87 \pm 0.08 [0.84 \pm 0.02]$	$0.84 \pm 0.02$
$C_9^{\text{NP}} = -C_{9'}^{\text{NP}} = -1.09$	$1.00 \pm 0.01$	$0.79 \pm 0.14 [0.74 \pm 0.04]$	$0.74 \pm 0.03$
$C_9^{\text{NP}} = -C_{10}^{\text{NP}} = -0.69$	$0.67 \pm 0.01$	$0.71 \pm 0.03 [0.69 \pm 0.01]$	$0.69 \pm 0.01$
$C_9^{\text{NP}} = -1.15, C_{9'}^{\text{NP}} = 0.77$	$0.91 \pm 0.01$	$0.80 \pm 0.12 [0.76 \pm 0.03]$	$0.76 \pm 0.03$
$C_9^{\text{NP}} = -1.16, C_{10}^{\text{NP}} = 0.35$	$0.71 \pm 0.01$	$0.78 \pm 0.07 [0.75 \pm 0.02]$	$0.76 \pm 0.01$
$C_9^{\text{NP}} = -1.23, C_{10'}^{\text{NP}} = -0.38$	$0.87 \pm 0.01$	$0.79 \pm 0.11 [0.75 \pm 0.02]$	$0.76 \pm 0.02$
$C_9^{\text{NP}} = -C_{9'}^{\text{NP}} = -1.17, C_{10}^{\text{NP}} = C_{10'}^{\text{NP}} = 0.26$	$0.88 \pm 0.01$	$0.76 \pm 0.12 [0.71 \pm 0.04]$	$0.71 \pm 0.03$

*Table: Predictions for  $R_K$ ,  $R_{K^*}$ ,  $R_\phi$  at the best fit point of different scenarios of interest, assuming that NP enters only in the muon sector, and using the inputs of our reference fit, in particular the KMPW form factors for  $B \rightarrow K$  and  $B \rightarrow K^*$ , and BSZ for  $B_s \rightarrow \phi$ . In the case of  $B \rightarrow K^*$ , we also indicate in brackets the predictions using the form factors in BSZ.*

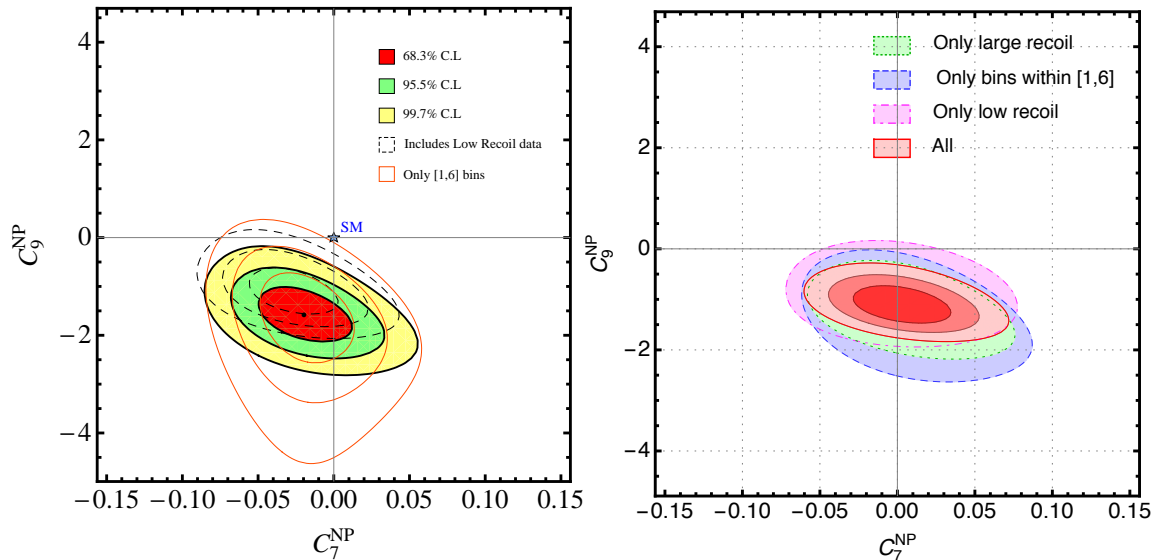


Figure: For the scenario where NP occurs in the two Wilson coefficients  $C_7$  and  $C_9$ , we compare the situation from the analysis in Fig. 1 of Ref. DMV'13 (on the left) and the current situation (on the right). On the right, we show the  $3\sigma$  regions allowed by large-recoil only (dashed green), by bins in the [1-6] range (long-dashed blue), by low recoil (dot-dashed purple) and by considering all data (red, with 1, 2, 3  $\sigma$  contours).

A CRUCIAL QUESTION:

How much the fit results  
depend on the details?

*Two first strong tests*

# TEST 1: Does the fit result depend on method IQCDF-KMPW or Full-FF-BSZ?

Two examples of Form Factor determinations (left KMPW, right BSZ):

form factor	$F_{BK^{(*)}}^i(0)$	$b_1^i$
$f_{BK}^+$	$0.34^{+0.05}_{-0.02}$	$-2.1^{+0.9}_{-1.6}$
$f_{BK}^0$	$0.34^{+0.05}_{-0.02}$	$-4.3^{+0.8}_{-0.9}$
$f_{BK}^T$	$0.39^{+0.05}_{-0.03}$	$-2.2^{+1.0}_{-2.00}$
$V^{BK^*}$	<b><math>0.36^{+0.23}_{-0.12}</math></b>	$-4.8^{+0.8}_{-0.4}$
$A_1^{BK^*}$	<b><math>0.25^{+0.16}_{-0.10}</math></b>	$0.34^{+0.86}_{-0.80}$
$A_2^{BK^*}$	$0.23^{+0.19}_{-0.10}$	$-0.85^{+2.88}_{-1.35}$
$A_0^{BK^*}$	$0.29^{+0.10}_{-0.07}$	$-18.2^{+1.3}_{-3.0}$
$T_1^{BK^*}$	$0.31^{+0.18}_{-0.10}$	$-4.6^{+0.81}_{-0.41}$
$T_2^{BK^*}$	$0.31^{+0.18}_{-0.10}$	$-3.2^{+2.1}_{-2.2}$
$T_3^{BK^*}$	$0.22^{+0.17}_{-0.10}$	$-10.3^{+2.5}_{-3.1}$

Table: The  $B \rightarrow K^{(*)}$  form factors from LCSR and their z-parameterization.

- Interestingly in this update from BZ to BSZ, relevant FF from BZ moved towards KMPW. For example:

$$V^{BZ}(0) = 0.41 \rightarrow 0.37 \quad T_1^{BZ}(0) = 0.33 \rightarrow 0.31$$

- The size of uncertainty in BSZ = size of error of p.c. we use.

	$B \rightarrow K^*$	$B_s \rightarrow \phi$	$B_s \rightarrow K^*$
$A_0(0)$	$0.391 \pm 0.035$	$0.433 \pm 0.035$	$0.336 \pm 0.032$
$A_1(0)$	<b><math>0.289 \pm 0.027</math></b>	$0.315 \pm 0.027$	$0.246 \pm 0.023$
$A_{12}(0)$	$0.281 \pm 0.025$	$0.274 \pm 0.022$	$0.246 \pm 0.023$
$V(0)$	<b><math>0.366 \pm 0.035</math></b>	$0.407 \pm 0.033$	$0.311 \pm 0.030$
$T_1(0)$	$0.308 \pm 0.031$	$0.331 \pm 0.030$	$0.254 \pm 0.027$
$T_2(0)$	$0.308 \pm 0.031$	$0.331 \pm 0.030$	$0.254 \pm 0.027$
$T_{23}(0)$	$0.793 \pm 0.064$	$0.763 \pm 0.061$	$0.643 \pm 0.058$

Table: Values of the form factors at  $q^2 = 0$  and their uncertainties.

# TEST 1: Does the fit result depend on method IQCDF-KMPW or Full-FF-BSZ?

## NO

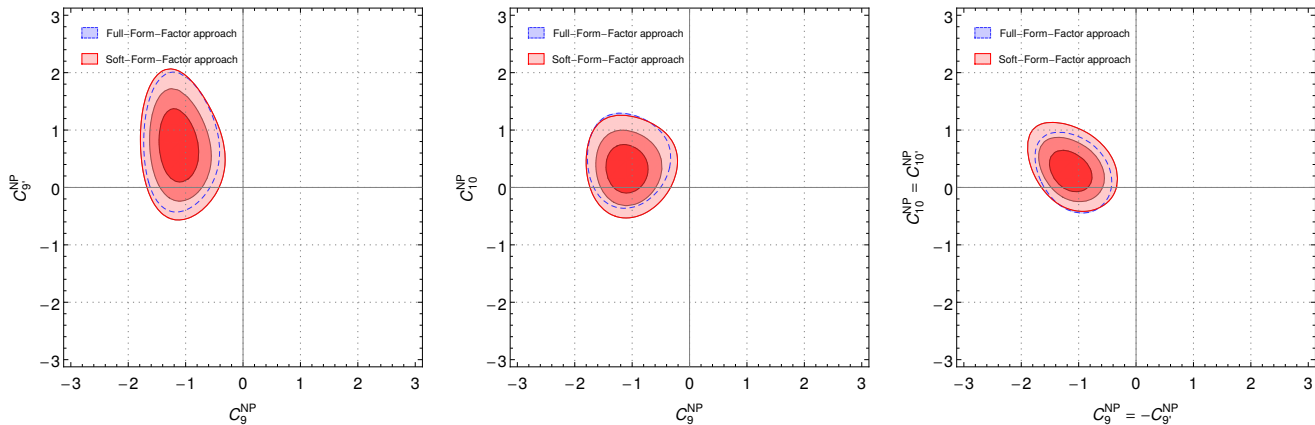
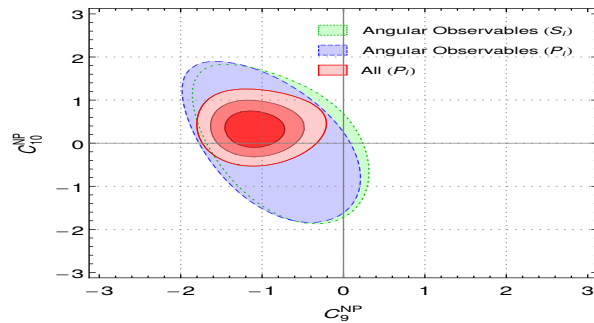
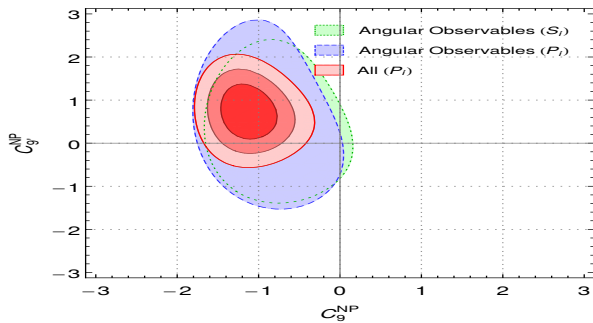


Figure: We show the  $3\sigma$  regions allowed using form factors in BSZ'15 in the full form factor approach (long-dashed blue) compared to our reference fit with the soft form factor approach (red, with 1,2,3  $\sigma$  contours).

- The results of the fit using (IQCDF-KMPW) or (Full-FF-BSZ) are perfectly consistent.
- The fact that our regions are slightly larger points that our estimate of uncertainties (power corrections, etc.) is conservative.

## TEST 2: Does the fit result depend on using $P_i$ or $S_i$ observables? **NO**



- The results of the fit using  $P_i$  observables or  $S_i$  observables are perfectly consistent.
- The highest sensitivity to NP of the optimized observables due to the shielding on FF details  
 $\Rightarrow$  induces a **small albeit systematic improvement in significance** for the  $P_i$ .
- Does the **error predictions on individual observables** depend significantly on **FF choice**?

anomaly [4,6] bin	$P'_5$ error SIZE [pull]	$S_5$ error SIZE [pull]
Full-FF-BSZ	10% [ $2.7\sigma$ ]	12% [ $2.0\sigma$ ]
IQCDF-KMPW	10% [ $2.9\sigma$ ]	40% [ $1.2\sigma$ ]

**Yes for  $S_i$ ,  
Not for  $P_i$ .**

Only in a **global fit** thanks to correlations it is basically the same to use:

- Optimized observables  $\mathbf{P}_i$ .
- FF dependent observables  $\mathbf{S}_i$ .

BUT when testing **individual observables** with data:

- Optimized observables  $\mathbf{P}_i$  are robust.
- FF dependent observables  $\mathbf{S}_i$  are largely choice dependent.



# Hadronic Uncertainties: Power corrections and charm loop

*Frequent naïve statement: Uncertainties are underestimated?  
It is important to understand what the uncertainties are  
and how they are treated before been able to **ask** the question.*

While in the past focusing in one single anomaly was logical...,

now it is not a good idea neither an acceptable approach to focus all the attention on one single observable.



# Hadronic uncertainties: power corrections and charm

While in the past focusing in one single anomaly was logical...,

now it is not a good idea neither an acceptable approach  
to focus all the attention on one single observable

BECAUSE now we have several deviations so **a global view** is compulsory. The correction question is:

**What is more natural a solution consistent with all anomalies and tensions or  
an ad-hoc (and unclear) solution different for each anomaly?**



# Hadronic Uncertainties I: Factorizable Power corrections

# Factorizable Power Corrections

**General idea:** : Parametrize power corrections to form factors:

$$F(q^2) = F^{\text{soft}}(\xi_{\perp,\parallel}(q^2)) + \Delta F^{\alpha_s}(q^2) + a_F + b_F \frac{q^2}{m_B^2} + c_F \frac{q^4}{m_B^4} \dots \quad (\text{JC}'12)$$

$\Rightarrow a_F, b_F, c_F \dots$  represent the deviation to the SFF+ known  $\alpha_s$  in the full form factor  $\mathbf{F}$  (taken e.g. from LCSR)

$$\mathbf{V}(q^2) = \frac{m_B + m_{K^*}}{m_B} \xi_{\perp}(q^2) + \Delta V^{\alpha_s}(q^2) + \Delta V^{\wedge}(q^2),$$

$$\mathbf{A}_1(q^2) = \frac{2E}{m_B + m_{K^*}} \xi_{\perp}(q^2) + \Delta A_1^{\alpha_s}(q^2) + \Delta A_1^{\wedge}(q^2),$$

$$\mathbf{A}_2(q^2) = \frac{m_B}{m_B - m_{K^*}} [\xi_{\perp}(q^2) - \xi_{\parallel}(q^2)] + \Delta A_2^{\alpha_s}(q^2) + \Delta A_2^{\wedge}(q^2),$$

$$\mathbf{A}_0(q^2) = \frac{E}{m_{K^*}} \xi_{\parallel}(q^2) + \Delta A_0^{\alpha_s}(q^2) + \Delta A_0^{\wedge}(q^2),$$

$$\mathbf{T}_1(q^2) = \xi_{\perp}(q^2) + \Delta T_1^{\alpha_s}(q^2) + \Delta T_1^{\wedge}(q^2) \dots$$

**STEP 1:** Define the SFF  $\xi_{\perp,\parallel}$  to all orders by means of a factorisation scheme CHOICE.

**STEP 2:** The **CHOICE of scheme** is fundamental

⇒ will yield accurate predictions for different observables depending on the scheme choice

⇒ if not all correlations among errors are known not all choices are appropriate.

In the scheme we use (Beneke-Feldmann'01) SFF are defined by:

$$\xi_{\perp}^{(1)}(q^2) \equiv \frac{m_B}{m_B + m_{K^*}} V(q^2) \quad \xi_{\parallel}^{(1)}(q^2) \equiv \frac{m_B + m_{K^*}}{2E} A_1(q^2) - \frac{m_B - m_{K^*}}{m_B} A_2(q^2),$$

⇒ Power corrections  $\Delta V^{\wedge}(q^2)$  and a combination of  $\Delta A_1^{\wedge}(q^2)$  and  $\Delta A_2^{\wedge}(q^2)$  are absorbed in  $\xi_{\perp, \parallel}$ .

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- **Size of power corrections:** The fit to the difference between SFF+ $\Delta F^{\alpha_s}$  and full-FF is  $\mathcal{O}(\Lambda/m_b)$  and is scheme dependent.

	$\hat{a}_F^{(1)}$	$\hat{b}_F^{(1)}$	$\hat{c}_F^{(1)}$	$r(0 \text{ GeV}^2)$	$r(4 \text{ GeV}^2)$	$r(8 \text{ GeV}^2)$
$A_1(\text{KMPW})$	$-0.013 \pm 0.025$	$-0.056 \pm 0.018$	$0.158 \pm 0.021$	5%	6%	5%
$A_1(\text{BZ})$	$-0.009 \pm 0.027$	$0.042 \pm 0.018$	$0.078 \pm 0.017$	3%	1%	3%
$A_2(\text{KMPW})$	$-0.018 \pm 0.023$	$-0.105 \pm 0.022$	$0.192 \pm 0.028$	8%	11%	10%
$A_2(\text{BZ})$	$-0.012 \pm 0.024$	$0.037 \pm 0.029$	$0.239 \pm 0.034$	5%	1%	5%
$T_1(\text{KMPW})$	$-0.006 \pm 0.031$	$-0.012 \pm 0.054$	$-0.034 \pm 0.095$	2%	2%	2%
$T_1(\text{BZ})$	$-0.024 \pm 0.032$	$-0.019 \pm 0.045$	$-0.014 \pm 0.092$	8%	7%	6%

where  $r = (a_F + b_F q^2/m_B^2 + c_F/m_B^4)/FF(q^2)$  represents the percentage of p.c.

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⇒ This confirms **power corrections are typically of order  $\lesssim 10\%$**  (or smaller in BZ) for **relevant FF** as expected from dimensional arguments.



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In the older scheme (first Beneke-Feldmann) SFF are defined by (also JC'14):

$$\xi_{\perp}^{(2)}(\mathbf{q}^2) \equiv \mathbf{T}_1(\mathbf{q}^2), \quad \xi_{\parallel}^{(2)}(\mathbf{q}^2) \equiv \frac{m_{K^*}}{E} \mathbf{A}_0(\mathbf{q}^2).$$

⇒ Power corrections associated to  $\Delta T_1^{\Lambda}(\mathbf{q}^2)$  and  $\Delta A_0^{\Lambda}(\mathbf{q}^2)$  are absorbed in  $\xi_{\perp, \parallel}$ .

### Problems of $T_1$ choice:

- Extracting  $T_1(0)$  from data on  $B \rightarrow K^* \gamma$  is plagued of assumptions (as done in JC'12):
  - 1) assumption of no NP in  $C_7^{(\prime)}$
  - 2) ignoring possible non-factorizable power corrections.
- Taking  $T_1$  from LCSR and use it to define  $\xi_{\perp}$  is also **non-optimal** (as done in JC'14).

$$A_{\perp}^{L,R} = \mathcal{N}_{\perp} \left[ C_{9\pm 10}^+ [\mathbf{V}^{\text{sff}+\alpha_s}(\mathbf{q}^2) + \Delta V^{\Lambda}] + C_7^+ [\mathbf{T}_1^{\text{sff}+\alpha_s}(\mathbf{q}^2) + \Delta T_1^{\Lambda}] \right] + \mathcal{O}(\alpha_s, \Lambda/m_b, \dots)$$

If one is interested in obtaining accurated predictions for observables dominated by  $C_9$  (like  $P'_5$ ) better to have a good control of p.c on  $V$  than in  $T_1$ .

⇒  $T_1$  may be a good choice for observables dominated by  $C_7$ .

### Problem of $A_0$ choice (minor):

$P_i$  observables do not depend on  $A_0(\mathbf{q}^2)$  FF. ⇒  $A_0$  choice would be a good choice for lepton-mass suppressed observables.

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**STEP 3:** A **correct treatment** of power corrections require to respect **the correlations** among them:

- a) *kinematic correlations* among QCD form factors at maximum recoil
- b) from the *renormalization scheme definition* of the soft form factors  $\xi_{\perp}$  and  $\xi_{\parallel}$ .

**STEP 4:** The error estimate in previous table

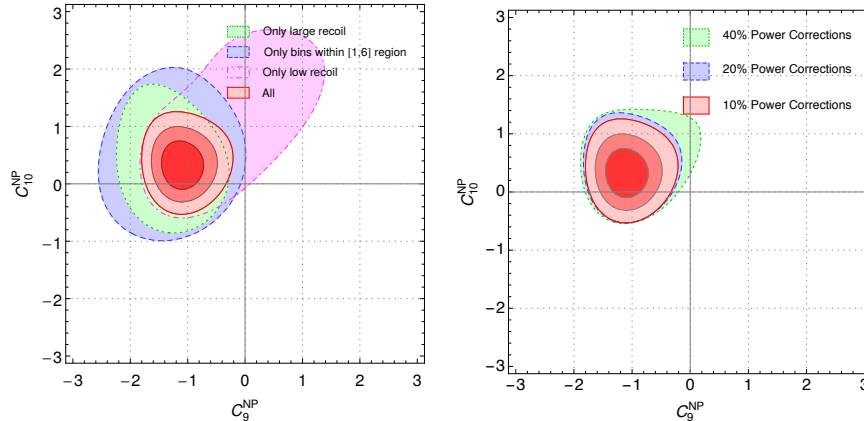
$$\begin{aligned}\hat{a}_F - \Delta\hat{a}_F &\leq a_F \leq \hat{a}_F + \Delta\hat{a}_F, \\ \hat{b}_F - \Delta\hat{b}_F &\leq b_F \leq \hat{b}_F + \Delta\hat{b}_F, \\ \hat{c}_F - \Delta\hat{c}_F &\leq c_F \leq \hat{c}_F + \Delta\hat{c}_F.\end{aligned}$$

comes from  $\Delta F^{\Lambda} \sim F \times \mathcal{O}(\Lambda/m_b) \sim 0.1F \Rightarrow$  error assignment larger than size of p.c. itself for  $\Delta\hat{a}$ .

## IN SUMMARY:

- *Each set of observables has an optimal scheme choice, a non-optimal choice may induce artificially large corrections.*
- *Interestingly an independent computation using full-FF (BSZ) that has embedded the correlations of a specific LCSR computation gives predictions in good agreement with us for the  $P_i$ .*

# What does the fit tells you about IMPACT of POWER CORRECTIONS:

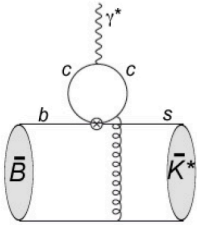


- We show the impact **in the fit** of increasing power corrections up to 40%
- At a certain point p.c.-sensitive observable become subdominant and low-recoil dominates.  
→ even if power corrections diverge we still get a pull from low-recoil.

# Hadronic Uncertainties II:

## Non-factorizable power corrections and long distance charm contributions

- **Non-factorizable power corrections (amplitudes)**: subleading new unknown non-perturbative. SCET/QCDF at leading power in  $1/m_b$ : Factorization of matrix elements into form factors, light-cone distribution amplitudes and hard-scattering kernels.
- $c\bar{c}$  loops (resonant and non-resonant contributions)



$\Rightarrow$  Single out in the amplitude  $\mathcal{T}_i$  in  $\langle K^* \gamma^* | H_{eff} | B \rangle$  the piece not associated to FF:  $\mathcal{T}_i^{\text{had}} = \mathcal{T}_i|_{C_7^{(\prime)} \rightarrow 0}$

Multiply each amplitude  $i = 0, \perp, \parallel$  with a complex  $q^2$ -dependent factor.

$$\mathcal{T}_i^{\text{had}} \rightarrow \left(1 + r_i(q^2)\right) \mathcal{T}_i^{\text{had}}$$

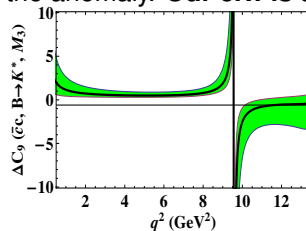
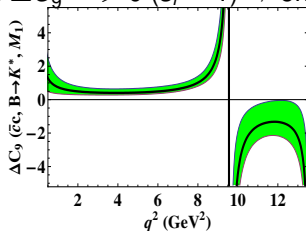
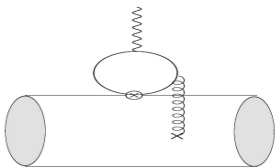
where  $r_i(s) = r_i^a e^{i\phi_i^a} + r_i^b e^{i\phi_i^b} (s/m_B^2) + r_i^c e^{i\phi_i^c} (s/m_B^2)^2$  and  $r_i^{a,b,c} \in [0, 0.1]$  and  $\phi_i^{a,b,c} \in [-\pi, \pi]$

## General considerations on resonances:

- Low- $q^2$ :  $q^2 \leq 7 - 8 \text{ GeV}^2$  to limit impact of  $J/\psi$  tail.  
 $\Rightarrow$  LHCb interesting test split  $[4.3, 8.68] \rightarrow [4, 6], [6, 8]$
- Large- $q^2$ : quark-hadron duality violations
  - Model estimate yield 2-5% for  $\text{BR}(B \rightarrow K\mu\mu)$  [BBF, GP]
  - Assumed similar size for BR and angular observables  $B \rightarrow K^*\mu\mu$ .  
 $\Rightarrow$  We enlarge it up to 10% as a correction to each amplitude.

**At Large-recoil two type of contributions:**  $\Delta C_9^{BK^*} = \delta C_{9,\text{pert}}^{BK^*} + s_i \delta C_{9,\text{non pert}}^{BK^*,i}$

- Short distance (hard-gluons)
  - LO included in  $C_9 \rightarrow C_9 + Y(q^2)$
  - higher-order corrections via QCDF/HQET.
- Long distance (soft-gluons)
  - Only existing computation KMPW'10 using LCSR.
  - Partial computation yields  $\Delta C_9^{BK^*} > 0$  ( $s_i = 1$ )  $\Rightarrow$  enlarges the anomaly. **Our c.v. is  $s_i = 0$  to be conservative**



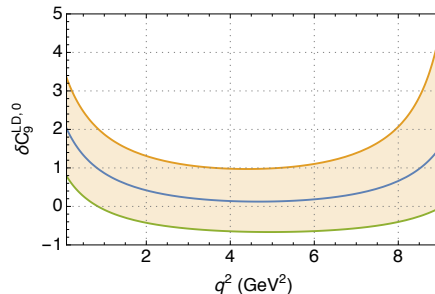
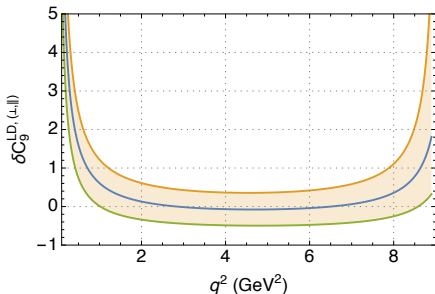


Inspired by Khodjamirian et al (KMPW):  $C_9 \rightarrow C_9 + s_i \delta C_9^{\text{LD}(i)}(q^2)$

Notice that KMPW implies  $s_i = 1$ , but we vary it independently  $s_i = 0 \pm 1$ ,  $i = 0, \perp, \parallel$  (Zwicky)

$$\delta C_9^{\text{LD},(\perp,\parallel)}(q^2) = \frac{a^{(\perp,\parallel)} + b^{(\perp,\parallel)} q^2 [c^{(\perp,\parallel)} - q^2]}{b^{(\perp,\parallel)} q^2 [c^{(\perp,\parallel)} - q^2]}$$

$$\delta C_9^{\text{LD},0}(q^2) = \frac{a^0 + b^0 [q^2 + s_0] [c^0 - q^2]}{b^0 [q^2 + s_0] [c^0 - q^2]}$$

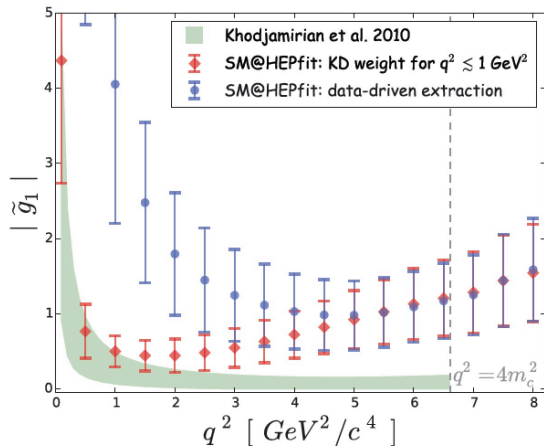


Obtaining from fitting the long-distance part to KMPW.

# Is reasonable to expect a huge-charm contribution??

Attempt 1 (Valli, Silvestrini et al.):

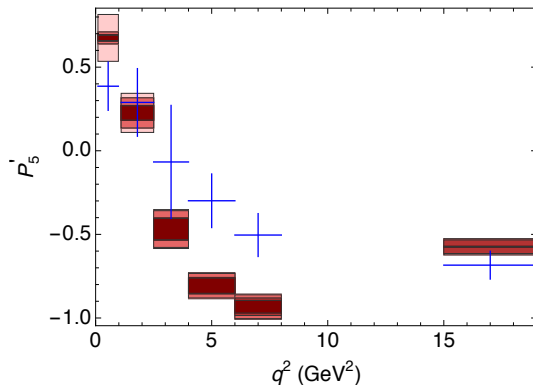
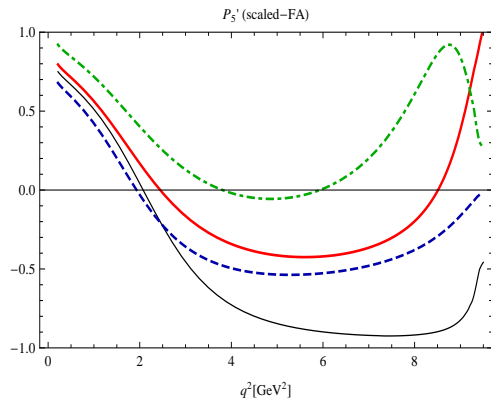
- Introduce an arbitrary parametrization of charm-loop  $h_\lambda = h_\lambda^{(0)} + h_\lambda^{(1)} q^2 + h_\lambda^{(2)} q^4$  with  $(\lambda = 0, \pm)$ 
  - So many free parameters allows to fit any shape  $\Rightarrow$  predictivity is rather low.
  - **This approach is in trouble if  $R_K$  and other sensitive LFV observables like  $R_{K^*}$  are confirmed.**



- $\tilde{g} = \Delta C_9^{non\,pert.} / (2C_1)$
- They force the fit (red points) to agree on the very low- $q^2$  with KMPW. This has two problems:
  - At very low- $q^2$  there are other problems they forgot (lepton mass effects).
  - By forcing the fit to agree at very low- $q^2$  can induce an artificial tilt of your fit.
- More interestingly the blue points where KMPW is not imposed is perfectly compatible with  $C_9 - C_9^{SM} \simeq \text{constant} + \text{KMPW}$  **similar to us!!**

Attempt 2 (Lyon, Zwicky'14 unpublished):

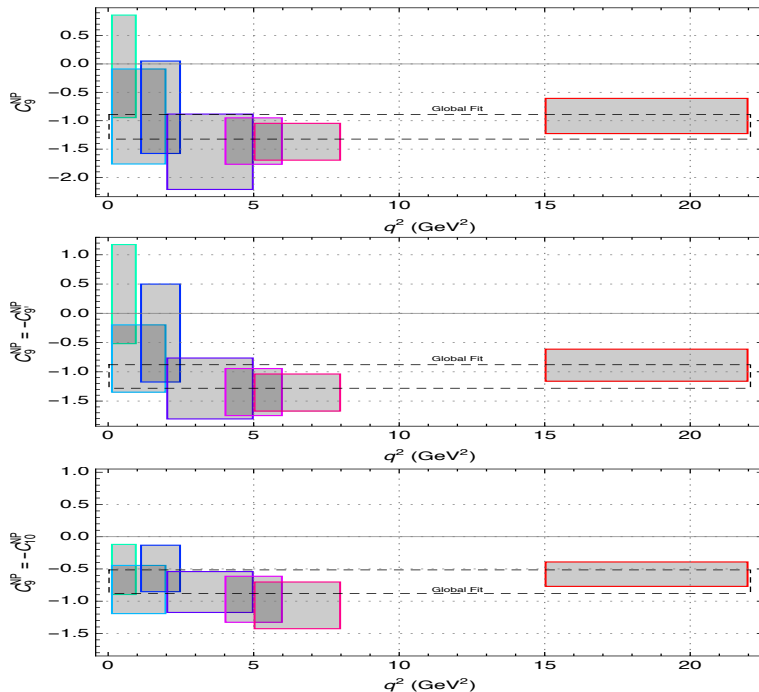
- An attempt more near to a prediction was presented by LZ'14 (left plot). Using  $e^+e^- \rightarrow$  hadrons to build a model of  $c\bar{c}$  resonances at low-recoil in  $B \rightarrow K\mu\mu$ , then extrapolating it at large-recoil via dispersion relations, and assuming that it holds in the same way for  $B \rightarrow K^*\mu\mu$   
⇒ However a large charm contribution ( $q^2$ ) should be seen in bin [6,8] being above [4,6] bin.



Different curves on left correspond to different hypothesis of the impact at low- $q^2$  from high- $q^2$ .

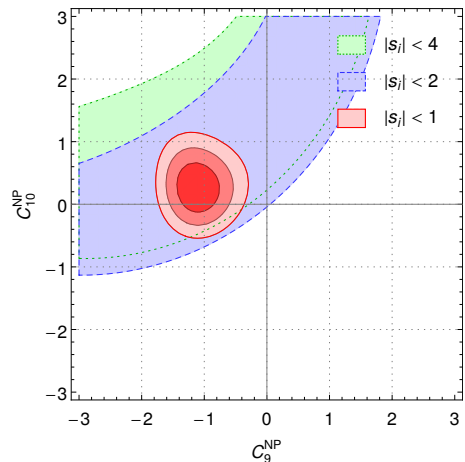
Smooth behaviour of data does not favour claims on large-long distance charm  $q^2$  effects in [6,8] bin.

# Cross check: Bin by Bin analysis of $C_9$ in three scenarios



Result of bin-by-bin analysis of  $C_9$  in 3 scenarios.

- **Notice the excellent agreement of bins [2,5], [4,6], [5,8].**  
*Strong argument in favour of including the [5,8] region-bin.*
- **First bin is afflicted by lepton-mass effects.** (see Back-up slides)
- **We do not find indication for a  $q^2$ -dependence in  $C_9$  neither in the plots nor in a 6D fit adding  $a^i + b^i s$  to  $C_9^{\text{eff}}$  for  $i = K^*, K, \phi$ .**  
→ disfavour again charm explanation.
- 2nd and 3rd plot test if you allow for NP in other WC the agreement of  $C_9$  bin by bin improves as compared to 1st plot.



- $\Delta C_9^{BK^*} = \delta C_{9,\text{pert}}^{BK^*} + s_i \delta C_{9,\text{non pert}}^{BK^*,i}$   
(same for  $B_s \rightarrow \phi$ )
- Increasing the range allowed for  $s_i$  makes low-recoil and  $B \rightarrow K\mu\mu$  dominate more and more

A comparison of our charm error estimate with other estimates (BSZ) shows a good agreement even if systematically we take a larger error size and we have an extra non-factorizable error. Example in the anomaly bin  $P'_{5[4,6]}$ :

- our estimate hep-ph/1503.03328 is  $^{+0.07}_{-0.08}$
- BSZ estimate in hep-ph/1503.05534 is  $\pm 0.05$

## Factorizable power corrections:

- The fit to factorizable power corrections show they are of order 10% as expected from dimensional arguments.
- The freedom to define  $\xi_{\perp,\parallel}$  allows you find an optimal scheme with minimal sensitivity to power corrections.
- Our results are in excellent agreement with a different approach/methodology/FF set.

In summary a careful computation of power corrections shows they are perfectly under control.

## Charm-loop contributions:

- $R_K$ , nor the future  $R_{K^*}$  or  $R_\phi$  cannot be explained with a charm contribution.
- The behaviour of bin [6,8] versus [4,6] in observables like  $P_5'$  precludes it.
- A 6-D fit or a bin-by-bin analysis does not find indication for a  $q^2$ -dependence in  $C_9$ .

In summary three arguments **against a large-charm explanation** of all the anomalies.

**Even if one can try to find alternative explanations for individual deviations (with not much success...), at the end of the day one has to rely on a different explanation for each deviation, **contrary to a shift in the Wilson Coefficients which explains all at the same time.****

Does the **alternative explanations** to NP  
raised in literature:  
factorizable power corrections and charm  
**stand a serious and accurate analysis?**

**NO**

# A glimpse into the future: Wilson coefficients versus Anomalies

	$R_K$	$\langle P'_5 \rangle_{[4,6],[6,8]}$	$\mathcal{B}_{B_s \rightarrow \phi \mu \mu}$	$\mathcal{B}_{B_s \rightarrow \mu \mu}$	best-fit-point of global fit	
$C_9^{NP}$	+					
	-	✓	✓ [100%]	✓	X	
$C_{10}^{NP}$	+	✓	[36%]	✓	✓	X
	-		✓ [32%]			
$C_{9'}$	+		[21%]	✓		X
	-	✓	✓ [36%]			
$C_{10'}$	+	✓	✓ [75%]			
	-		[75%]	✓	✓	X

Table: A checkmark (✓) indicates that a shift in the Wilson coefficient with this sign moves the prediction in the right direction to solve the corresponding anomaly.  $\mathcal{B}_{B_s \rightarrow \mu \mu}$  is not an anomaly but a very mild tension.

- $C_9^{NP} < 0$  is consistent with all anomalies. This is the reason why it gives a strong pull.
- $C_{10}^{NP}$ ,  $C'_{9,10}$  fail in some anomaly. BUT
  - ⇒  $C_{10}^{NP}$  is the most promising coefficient after  $C_9$ .
  - ⇒  $C'_9, C'_{10}$  seems quite inconsistent between the different anomalies and the global fit.

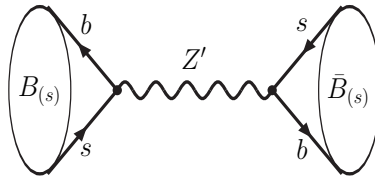
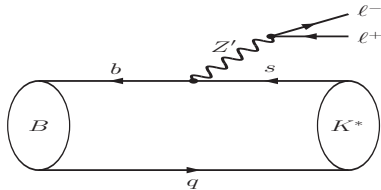


# Z' particle a possible explanation

In [DMV'13] we proposed to explain the anomaly in  $B \rightarrow K^* \mu \mu$  with a **Z' gauge boson** contributing to

$$\mathcal{O}_9 = e^2 / (16\pi^2) (\bar{s} \gamma_\mu P_L b) (\bar{\ell} \gamma^\mu \ell),$$

with specific couplings as a possible explanation of the anomaly in  $P'_5$ .



$$\mathcal{L}^q = \left( \bar{s} \gamma_\nu P_L b \Delta_L^{sb} + \bar{s} \gamma_\nu P_R b \Delta_R^{sb} + h.c. \right) Z'^{\nu} \quad \mathcal{L}^{lep} = \left( \bar{\mu} \gamma_\nu P_L \mu \Delta_L^{\mu\mu} + \bar{\mu} \gamma_\nu P_R \mu \Delta_R^{\mu\mu} + \dots \right) Z'^{\nu}$$

The Wilson coefficients of the semileptonic operators are:

$$C_{\{9,10\}}^{\text{NP}} = -\frac{1}{s_W^2 g_{SM}^2} \frac{1}{M_{Z'}^2} \frac{\Delta_L^{sb} \Delta_{\{V,A\}}^{\mu\mu}}{\lambda_{ts}}, \quad C_{\{9',10'\}}^{\text{NP}} = -\frac{1}{s_W^2 g_{SM}^2} \frac{1}{M_{Z'}^2} \frac{\Delta_R^{sb} \Delta_{\{V,A\}}^{\mu\mu}}{\lambda_{ts}},$$

with the vector and axial couplings to muons:  $\Delta_{V,A}^{\mu\mu} = \Delta_R^{\mu\mu} \pm \Delta_L^{\mu\mu}$ .

$\Delta_L^{sb}$  with same phase as  $\lambda_{ts} = V_{tb} V_{ts}^*$  (to avoid  $\phi_s$ ) like in MFV. Main constraint from  $\Delta M_{B_s}$  ( $\Delta_{L,R}^{sb}$ ).

A  $Z'$  model can belong to the following categories:

	no-coupling	non-zero couplings	Pull <sub>SM</sub>
$C_9$	<b>no-right-handed quark &amp; no-muon-axial coupling</b>	$\Delta_L^{sb} \neq 0, \Delta_V^{\mu\mu} \neq 0$	4.9 $\sigma$
$(C_9, C_{10})$	<b>no-right-handed quark coupling</b>	$\Delta_L^{sb} \neq 0, \Delta_V^{\mu\mu} \neq 0, \Delta_A^{\mu\mu} \neq 0$	4.8 $\sigma$
$(C_9, C'_9)$	<b>no-muon-axial coupling</b>	$\Delta_L^{sb} \neq 0, \Delta_R^{sb} \neq 0, \Delta_V^{\mu\mu} \neq 0$	4.9 $\sigma$
$(C_{10}, C'_{10})$	<b>no-muon-vector coupling</b>	$\Delta_L^{sb} \neq 0, \Delta_R^{sb} \neq 0, \Delta_A^{\mu\mu} \neq 0$	-
$(C'_9, C'_{10})$	<b>no-left-handed quark coupling</b>	$\Delta_R^{sb} \neq 0, \Delta_V^{\mu\mu} \neq 0, \Delta_A^{\mu\mu} \neq 0$	-

Example:  $C_9^{\text{NP}} = -1.1, \Delta_V^{\mu\mu}/M'_Z = -0.6 \text{ TeV}^{-1}$  and  $\Delta_L^{bs}/M'_Z = 0.003 \text{ TeV}^{-1}$

- If NP enters **all** four semileptonic coefficients, the following relationships hold:

$$\frac{C_9^{\text{NP}}}{C_{10}^{\text{NP}}} = \frac{C_{9'}^{\text{NP}}}{C_{10'}^{\text{NP}}} = \frac{\Delta_V^{\mu\mu}}{\Delta_A^{\mu\mu}}, \quad \frac{C_9^{\text{NP}}}{C_{9'}^{\text{NP}}} = \frac{C_{10}^{\text{NP}}}{C_{10'}^{\text{NP}}} = \frac{\Delta_L^{sb}}{\Delta_R^{sb}}.$$

Many ongoing attempts to embed this kind of  $Z'$  inside a model [U.Haisch, W.Altmannshofer, A.Buras, D. Straub,..]

- The global analysis of  $b \rightarrow sl^+\ell^-$  with  $3 \text{ fb}^{-1}$  dataset **shows that the solution** we proposed in 2013 to solve the anomaly with a contribution  $\mathbf{C}_9^{\text{NP}} \simeq -1$  **is confirmed** and reinforced.
  - We take full dataset and optimized basis of observables.
- The **fit result is very robust** and does not show a significant dependence nor on the method used to compute observables neither on the observables used once correlations are correctly included.
- We have shown that the **treatment of uncertainties** entering the observables in  $B \rightarrow K^*\mu\mu$  is indeed **under good control** and the **alternative explanations** to New Physics are indeed **not in very solid ground**.
- Near future? **Maybe  $C_{10}$  will become significant soon**. A heavy  $Z'$  (1-2 TeV) with  $bs$ -coupling is a **viable explanation** for many (not all) scenarios.

- Robustness of the FIT:
  - The results of the fit using  $P_i$  (optimized-FFI) or  $S_i$  (FFD) and IQCDF-KMPW or Full-FF-BSZ results are in very good agreement. **Low sensitivity to details of FF computation.**
  - Any scenario including  $C_9^{NP}$  gets a large-pull. 2D scenarios still indistinguishable  $\Rightarrow$  More data needed.
- Robustness of hadronic uncertainties of OBSERVABLES:
  - Factorizable power corrections:
    - The **Fit** to full FF in an **appropriate scheme** gives  $\leq 10\%$  in agreement with dimensional arguments.
    - A correct FF choice + correlations among p.c. is essential not to artificially inflate errors.
  - $c\bar{c}$  loops: We include LO and NLL perturbative contributions to  $C_9^{\text{eff}}$  also long distance (following KMPW) with both signs to be conservative.
    - Three reasons why a huge charm contribution cannot explain all deviations:  $R_K$  cannot be explained. The behaviour of bin [6,8] versus [4,6] in observables like  $P'_5$ . A 6-D fit or a bin-by-bin analysis does not find evidence for a new- $q^2$  dependence.
  - Any set of observables is equivalent in terms of accuracy for the predictions on individual observables?  
**NO,  $P_i$  observables are stable under FF changes,  $S_i$  depend largely on the choice.**

# Back-up slides

LHCb naturally given the limited statistics takes the massless lepton limit. They measure:

$$\frac{1}{d(\Gamma + \bar{\Gamma})/dq^2} \frac{d^3(\Gamma + \bar{\Gamma})}{d\Omega} = \frac{9}{32\pi} \left[ \frac{3}{4}(1 - F_L^{LHCb}) \sin^2 \theta_K + F_L^{LHCb} \cos^2 \theta_K \right. \\ \left. + \frac{1}{4}(1 - F_L^{LHCb}) \sin^2 \theta_K \cos 2\theta_l - F_L^{LHCb} \cos^2 \theta_K \cos 2\theta_l + \dots \right]$$

which is modified once lepton masses are considered

$$\frac{1}{d(\Gamma + \bar{\Gamma})/dq^2} \frac{d^3(\Gamma + \bar{\Gamma})}{d\Omega} = \frac{9}{32\pi} \left[ \frac{3}{4}\hat{F}_T \sin^2 \theta_K + \hat{F}_L \cos^2 \theta_K \right. \\ \left. + \frac{1}{4}F_T \sin^2 \theta_K \cos 2\theta_l - F_L \cos^2 \theta_K \cos 2\theta_l + \dots \right]$$

where  $\hat{F}_{T,L}$  and  $F_{L,T}$  are [JM'12]. All our observables are thus written and computed in terms of the longitudinal and transverse polarisation fractions  $F_{L,T}$

$$F_L = -\frac{J_{2c}}{d(\Gamma + \bar{\Gamma})/dq^2} \quad F_T = 4\frac{J_{2s}}{d(\Gamma + \bar{\Gamma})/dq^2} \quad \Rightarrow \quad \hat{F}_L = \frac{J_{1c}}{d(\Gamma + \bar{\Gamma})/dq^2}$$

WHEN measured value  $\hat{F}_L$  is used instead of  $F_L$  SM prediction is shifted towards the data in 1st bin

$$\langle F_L \rangle_{[0.1,0.98]} = 0.21 \rightarrow 0.26, \quad \langle P_2 \rangle_{[0.1,0.98]} = 0.12 \rightarrow 0.09, \\ \langle P'_4 \rangle_{[0.1,0.98]} = -0.49 \rightarrow -0.38, \quad \langle P'_5 \rangle_{[0.1,0.98]} = 0.68 \rightarrow 0.53.$$

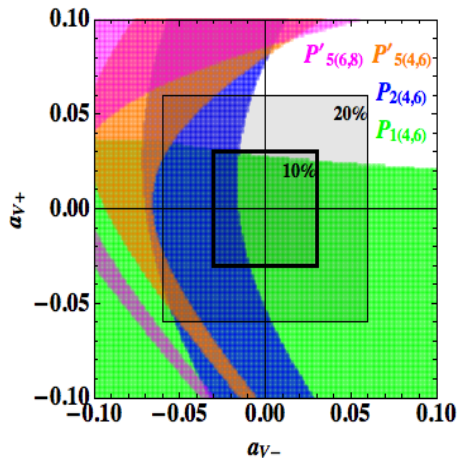
		$ \delta\mathcal{C}_7  = 0.1$	$ \delta\mathcal{C}_9  = 1$	$ \delta\mathcal{C}_{10}  = 1$	$ \delta\mathcal{C}_{7'}  = 0.1$	$ \delta\mathcal{C}_{9'}  = 1$	$ \delta\mathcal{C}_{10'}  = 1$
$\langle P_1 \rangle_{[0.1, .98]}$	$+\delta\mathcal{C}_i$	--	--	--	-0.53	-0.05	--
	$-\delta\mathcal{C}_i$	--	--	--	+0.52	+0.05	--
$\langle P_1 \rangle_{[6,8]}$	$+\delta\mathcal{C}_i$	--	--	--	+0.11	+0.16	- <b>0.37</b>
	$-\delta\mathcal{C}_i$	--	--	--	- <b>0.12</b>	- <b>0.17</b>	+0.37
$\langle P_1 \rangle_{[15,19]}$	$+\delta\mathcal{C}_i$	--	--	--	+ <b>0.03</b>	+ <b>0.15</b>	-0.14
	$-\delta\mathcal{C}_i$	--	--	--	-0.03	-0.11	+ <b>0.19</b>
$\langle P_2 \rangle_{[2.5,4]}$	$+\delta\mathcal{C}_i$	-0.31	-0.21	+ <b>0.05</b>	--	--	--
	$-\delta\mathcal{C}_i$	+ <b>0.19</b>	+ <b>0.15</b>	-0.04	-0.03	--	--
$\langle P_2 \rangle_{[6,8]}$	$+\delta\mathcal{C}_i$	-0.07	-0.09	-0.06	--	--	--
	$-\delta\mathcal{C}_i$	+ <b>0.11</b>	+ <b>0.17</b>	+ <b>0.05</b>	--	--	--
$\langle P_2 \rangle_{[15,19]}$	$+\delta\mathcal{C}_i$	--	--	--	--	-0.05	+0.06
	$-\delta\mathcal{C}_i$	--	+0.04	--	--	+0.05	-0.06
$\langle P'_4 \rangle_{[6,8]}$	$+\delta\mathcal{C}_i$	+ <b>0.04</b>	--	--	-0.11	-0.10	+ <b>0.17</b>
	$-\delta\mathcal{C}_i$	-0.05	--	--	+ <b>0.09</b>	+ <b>0.10</b>	-0.20
$\langle P'_4 \rangle_{[15,19]}$	$+\delta\mathcal{C}_i$	--	--	--	--	- <b>0.06</b>	+0.05
	$-\delta\mathcal{C}_i$	--	--	--	--	+0.04	- <b>0.08</b>
$\langle P'_5 \rangle_{[4,6]}$	$+\delta\mathcal{C}_i$	-0.11	-0.15	-0.10	-0.11	-0.06	+ <b>0.21</b>
	$-\delta\mathcal{C}_i$	+ <b>0.16</b>	+ <b>0.28</b>	+ <b>0.09</b>	+ <b>0.15</b>	+ <b>0.10</b>	-0.21
$\langle P'_5 \rangle_{[6,8]}$	$+\delta\mathcal{C}_i$	-0.04	-0.07	-0.07	-0.08	-0.08	+ <b>0.19</b>
	$-\delta\mathcal{C}_i$	+ <b>0.07</b>	+ <b>0.19</b>	+ <b>0.09</b>	+ <b>0.10</b>	+ <b>0.11</b>	-0.18

# Correlations play a central role

If one wants to solve the anomalies exhibited in  $b \rightarrow s\mu\mu$  processes through power corrections, it is important not to focus on one single observable, like  $P'_5$ , alone but on the full set.

Illustrative example. Let's do the following exercise: Assume you take the non-optimal scheme-2 as in (JC'14) and helicity basis

$$a_{V_{\pm}} = \frac{1}{2} \left[ \left( 1 + \frac{m_{K^*}}{m_B} \right) a_1 \mp \left( 1 - \frac{m_{K^*}}{m_B} \right) a_V \right].$$



- Notice that taking  $a_{V-}$  in a range  $\pm 0.1$  correspond to an absurd 33% power correction in KMPW.
  - because a 10% in KMPW corresponds to 0.03 in  $a_{V-}$ .
  - accepting values like  $(a_{V-} = -0.1, a_{V+} = 0)$  would imply that **BSZ computation of  $A_1(q^2)$  is wrong by several sigmas.**
- An explanation of  $\langle P'_5 \rangle_{[4,6]}$ ,  $\langle P_2 \rangle_{[4,6]}$  and  $\langle P_1 \rangle_{[4,6]}$  within SM requires a 20% correction. Adding  $\langle P'_5 \rangle_{[6,8]}$  no common solution found even beyond 20%.



### Another important aspect is the error associated to SFF.

- Our error comes from KMPW. Example:  $\xi_{\perp}(\mathbf{0}) = 0.31^{+0.20}_{-0.10}$
- On the contrary in (JC'14): Error from **spread of central values with different inputs** and **not considering errors**,  $\xi_{\perp}(\mathbf{0}) = 0.31 \pm 0.04$ . Factor of 5 to 3 smaller error than us.
  - This will give a **very small error** (due to this error definition) for FFD observables.

$$\begin{aligned}
J_{1s} &= \frac{(2 + \beta_\ell^2)}{4} \left[ |A_\perp^L|^2 + |A_\parallel^L|^2 + (L \rightarrow R) \right] + \frac{4m_\ell^2}{q^2} \operatorname{Re} \left( A_\perp^L A_\perp^{R*} + A_\parallel^L A_\parallel^{R*} \right), \\
J_{1c} &= |A_0^L|^2 + |A_0^R|^2 + \frac{4m_\ell^2}{q^2} \left[ |A_t|^2 + 2\operatorname{Re}(A_0^L A_0^{R*}) \right] + \beta_\ell^2 |A_S|^2, \\
J_{2s} &= \frac{\beta_\ell^2}{4} \left[ |A_\perp^L|^2 + |A_\parallel^L|^2 + (L \rightarrow R) \right], \quad J_{2c} = -\beta_\ell^2 \left[ |A_0^L|^2 + (L \rightarrow R) \right], \\
J_3 &= \frac{1}{2} \beta_\ell^2 \left[ |A_\perp^L|^2 - |A_\parallel^L|^2 + (L \rightarrow R) \right], \quad J_4 = \frac{1}{\sqrt{2}} \beta_\ell^2 \left[ \operatorname{Re}(A_0^L A_\parallel^{L*}) + (L \rightarrow R) \right], \\
J_5 &= \sqrt{2} \beta_\ell \left[ \operatorname{Re}(A_0^L A_\perp^{L*}) - (L \rightarrow R) - \frac{m_\ell}{\sqrt{q^2}} \operatorname{Re}(A_\parallel^L A_S^* + A_\parallel^R A_S^*) \right], \\
J_{6s} &= 2\beta_\ell \left[ \operatorname{Re}(A_\parallel^L A_\perp^{L*}) - (L \rightarrow R) \right], \quad J_{6c} = 4\beta_\ell \frac{m_\ell}{\sqrt{q^2}} \operatorname{Re} \left[ A_0^L A_S^* + (L \rightarrow R) \right], \\
J_7 &= \sqrt{2} \beta_\ell \left[ \operatorname{Im}(A_0^L A_\parallel^{L*}) - (L \rightarrow R) + \frac{m_\ell}{\sqrt{q^2}} \operatorname{Im}(A_\perp^L A_S^* + A_\perp^R A_S^*) \right], \\
J_8 &= \frac{1}{\sqrt{2}} \beta_\ell^2 \left[ \operatorname{Im}(A_0^L A_\perp^{L*}) + (L \rightarrow R) \right], \quad J_9 = \beta_\ell^2 \left[ \operatorname{Im}(A_\parallel^{L*} A_\perp^L) + (L \rightarrow R) \right]
\end{aligned}$$

In red lepton mass terms and  $\beta_\ell = \sqrt{1 - 4m_\ell^2/q^2}$

The corresponding spin amplitudes  $A_{\perp}, A_{\parallel}, A_0$  are function:

- Wilson Coefficients:  $C_7^{\text{eff}}, C_7^{\text{eff}'}, C_9^{\text{eff}}, C_{10}$
- Form factors  $A_{1,2}(s), V(s), T_{1,2,3}(s)$

$$\mathbf{A}_{\perp\text{L,R}} = N\sqrt{2}\lambda^{1/2} \left[ (C_9^{\text{eff}} \mp C_{10}) \frac{V(q^2)}{m_B + m_{K^*}} + \frac{2m_b}{q^2} (C_7^{\text{eff}} + C_7^{\text{eff}'}) T_1(q^2) \right]$$

$$\mathbf{A}_{\parallel\text{L,R}} = -N\sqrt{2}(m_B^2 - m_{K^*}^2) \left[ (C_9^{\text{eff}} \mp C_{10}) \frac{A_1(q^2)}{m_B - m_{K^*}} + \frac{2m_b}{q^2} (C_7^{\text{eff}} - C_7^{\text{eff}'}) T_2(q^2) \right],$$

$$\begin{aligned} \mathbf{A}_{0\text{L,R}} = & -\frac{N}{2m_{K^*}\sqrt{q^2}} \times \left[ (C_9^{\text{eff}} \mp C_{10}) \left\{ (m_B^2 - m_{K^*}^2 - q^2)(m_B + m_{K^*}) A_1(q^2) - \right. \right. \\ & \left. \left. - \lambda \frac{A_2(q^2)}{m_B + m_{K^*}} \right\} + 2m_b (C_7^{\text{eff}} - C_7^{\text{eff}'}) \left\{ (m_B^2 + 3m_{K^*}^2 - q^2) T_2(q^2) - \right. \right. \\ & \left. \left. - \frac{\lambda}{m_B^2 - m_{K^*}^2} T_3(q^2) \right\} \right], \end{aligned}$$

The **hadronic matrix elements** are in naive factorization:

$$\begin{aligned} \langle K^*(p_{K^*}) | \bar{s} \gamma_\mu P_{L,R} b | B(p) \rangle &= i \epsilon_{\mu\nu\alpha\beta} \epsilon^{\nu*} p^\alpha q^\beta \frac{V(q^2)}{m_B + m_{K^*}} \mp \\ &\mp \frac{1}{2} \left\{ \epsilon_\mu^* (m_B + m_{K^*}) A_1(q^2) - (\epsilon^* \cdot q) (2p - q)_\mu \frac{A_2(q^2)}{m_B + m_{K^*}} - \right. \\ &\quad \left. - \frac{2m_{K^*}}{q^2} (\epsilon^* \cdot q) [A_3(q^2) - A_0(q^2)] q_\mu \right\}, \end{aligned}$$

$$\begin{aligned} \langle K^*(p_{K^*}) | \bar{s} i \sigma_{\mu\nu} q^\nu P_{R,L} b | B(p) \rangle &= -i \epsilon_{\mu\nu\alpha\beta} \epsilon^{\nu*} p^\alpha q^\beta T_1(q^2) \pm \\ &\pm \frac{1}{2} \left\{ [\epsilon_\mu^* (m_B^2 - m_{K^*}^2) - (\epsilon^* \cdot q) (2p - q)_\mu] T_2(q^2) + \right. \\ &\quad \left. + (\epsilon^* \cdot q) \left[ q_\mu - \frac{q^2}{m_B^2 - m_{K^*}^2} (2p - q)_\mu \right] T_3(q^2) \right\}. \end{aligned}$$

where  $A_3(q^2) = \frac{m_B + m_{K^*}}{2m_{K^*}} A_1(q^2) - \frac{m_B - m_{K^*}}{2m_{K^*}} A_2(q^2)$

$K^*$  Spin Amplitudes ( $A_{0,\perp,\parallel}$ ) related Helicity Amplitudes ( $H_{0,\pm}$ ):

$$A_0 = H_0 \quad A_{\perp,\parallel} = \frac{H_+ \mp H_-}{\sqrt{2}}$$

They follow in naive factorisation a  $\Lambda/m_b$  hierarchy:

$$H_0 : H_- : H_+ = 1 : \frac{\Lambda}{m_b} : \left(\frac{\Lambda}{m_b}\right)^2$$

due to *spectator quark flip*, broken by electromagnetic effects.

At quark level in SM in the limit  $m_B \rightarrow \infty$  and  $E_K^* \rightarrow \infty$ :

$$H_+ = 0 \quad \Rightarrow \quad A_{\perp} = -A_{\parallel}$$

At hadron level  $A_{\perp} \approx -A_{\parallel}$ .

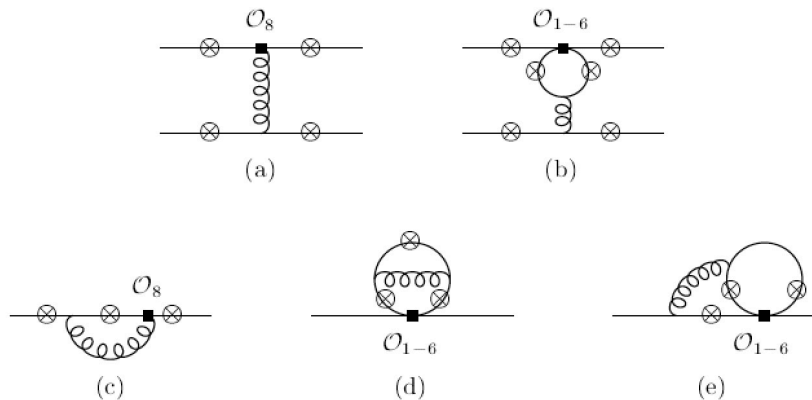


Figure 2: Non-factorizable contributions to  $\langle \gamma^* \bar{K}^* | H_{\text{eff}} | \bar{B} \rangle$ . The circled cross marks the possible insertions of the virtual photon line. Diagrams that follow from (c) and (e) by symmetry are not shown. Upper line: hard spectator scattering. Lower line: diagrams involving a  $B \rightarrow K^*$  form factor (the spectator quark line is not drawn for these diagrams).