Theoretical Predictions for the Weak Radiative B-Meson Decays

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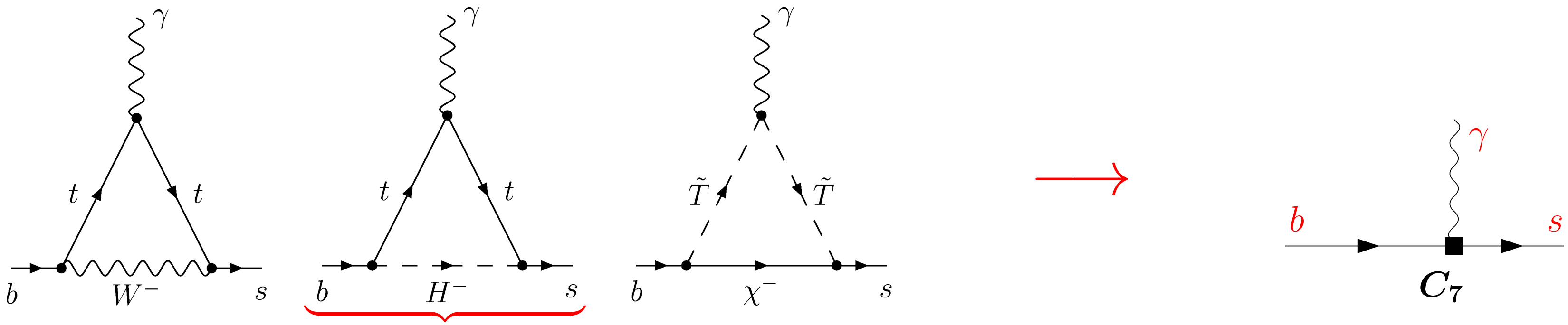
based on the articles:

[1] "Updated NNLO QCD Predictions for the Weak Radiative B-Meson Decays"
Phys. Rev. Lett. 114 (2015) 221801, arXiv:1503.01789
MM, H. M. Asatrian, R. Boughezal, M. Czakon, T. Ewerth, A. Ferroglia, P. Fiedler, P. Gambino, C. Greub, U. Haisch, T. Huber, M. Kamiński, G. Ossola, M. Poradziński, A. Rehman, T. Schutzmeier, M. Steinhauser and J. Virto.

[2] "The $(Q_7, Q_{1,2})$ contribution to $\bar{B} \to X_s\gamma$ at $\mathcal{O}\left(\alpha_s^2\right)$"
JHEP 1504 (2015) 168, arXiv:1503.01791
M. Czakon, P. Fiedler, T. Huber, MM, T. Schutzmeier and M. Steinhauser.

1. Introduction
2. $\bar{B} \to X_s \gamma$ at $\mathcal{O}(\alpha_s^2)$ in the SM
3. Sample bounds on beyond-SM physics: $M_{H^\pm}$ vs. $\tan\beta$ in 2HDM-II
4. Direct CP asymmetry in $\bar{B} \to X_s \gamma$
5. $\bar{B} \to X_d \gamma$
6. The photon energy spectrum and non-perturbative effects
7. Summary

Information on electroweak-scale physics in the $b \to s\gamma$ transition is encoded in an effective low-energy local interaction:



$\Rightarrow M_{H^\pm} > \ \sim 500\,\mathrm{GeV}$
in the 2HDM-II

$$b \in \bar{B} \equiv (\bar{B}^0 \text{ or } B^-)$$

The inclusive $\bar{B} \to X_s\gamma$ decay rate for $E_\gamma > E_0$ is well approximated by the corresponding perturbative decay rate of the b-quark:

$$\Gamma(\bar{B} \to X_s\gamma) \;=\; \Gamma(b \to X_s^p\gamma) \;+ \begin{pmatrix} \text{non-perturbative effects} \\ (3 \pm 5)\% \end{pmatrix}$$

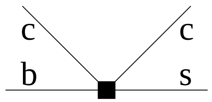
[G. Buchalla, G. Isidori and S.-J. Rey, Nucl. Phys. B511 (1998) 594]
[M. Benzke, S.J. Lee, M. Neubert and G. Paz, JHEP 1008 (2010) 099]

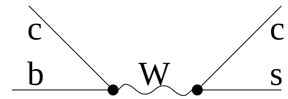
provided E_0 is large ($E_0 \sim m_b/2$)
but not too close to the endpoint $(m_b - 2E_0 \gg \Lambda_{\mathrm{QCD}})$.

Conventionally, $E_0 = 1.6\,\mathrm{GeV} \simeq m_b/3$ is chosen.

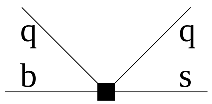
Resummation of $\left(\alpha_s \ln M_W^2/m_b^2\right)^n$ is most conveniently performed in the framework of an effective theory that arises from the SM after decoupling of the heavy electroweak bosons and the top quark. The Lagrangian of such a theory reads:

$$\mathcal{L}_{\rm eff} \;=\; \mathcal{L}_{\rm QCD\times QED}(u,d,s,c,b) \;+\; \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{8} C_i(\mu)\, Q_i \;+\; \left(\begin{array}{c}\textbf{EW-suppressed,}\\ \textbf{higher-dimensional,}\\ \textbf{on-shell vanishing,}\\ \textbf{evanescent}\end{array}\right).$$

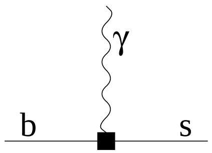
$Q_{1,2} =$ 

$= (\bar{s}\Gamma_i c)(\bar{c}\Gamma_i' b)$, **from** 

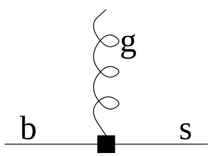
, $\quad |C_i(m_b)| \sim 1$

$Q_{3,4,5,6} =$ 

$= (\bar{s}\Gamma_i b)\Sigma_q(\bar{q}\Gamma_i' q), \quad |C_i(m_b)| < 0.07$

$Q_7 =$ 

$= \dfrac{e m_b}{16\pi^2}\, \bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu}, \quad C_7(m_b) \simeq -0.3$

$Q_8 =$ 

$= \dfrac{g m_b}{16\pi^2}\, \bar{s}_L \sigma^{\mu\nu} T^a b_R G^a_{\mu\nu}, \quad C_8(m_b) \simeq -0.15$

Three steps of the calculation:

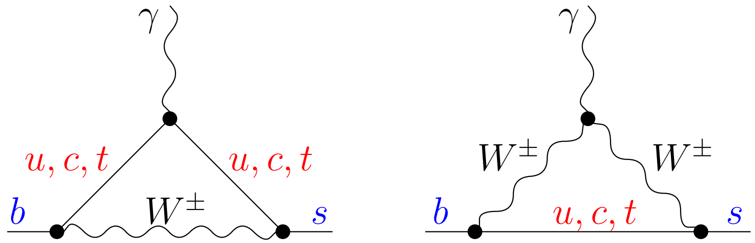
Matching: **Evaluating $C_i(\mu_0)$ at $\mu_0 \sim M_W$ by requiring equality of the SM and the effective theory Green functions.**

Mixing: **Deriving the effective theory Renormalization Group Equations** $\left(C_j^{\rm bare} = C_i Z_{ij}\right)$ **and evolving $C_i(\mu)$ from μ_0 to $\mu_b \sim m_b$.**

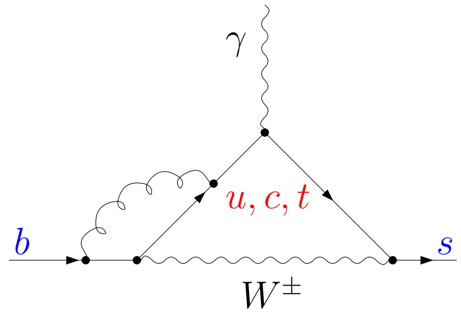
Matrix elements: **Evaluating the on-shell amplitudes at $\mu_b \sim m_b$.**

Examples of SM diagrams for the matching of $C_7(\mu_0)$

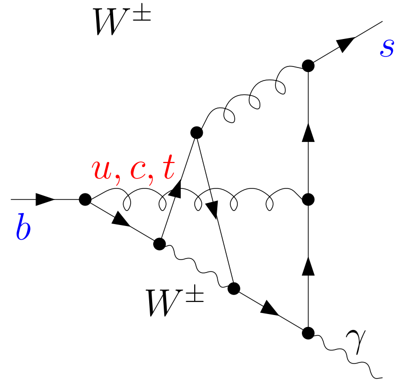
LO:
[Inami, Lim, 1981]



NLO:
[Adel, Yao, 1993]



NNLO:
[Steinhauser, MM, 2004]



NNLO method:

- Taylor expansion in the off-shell external momenta is applied before integration.
- The UV and spurious IR divergences are regulated dimensionally.
- $\Rightarrow$ In the effective theory, only tree-level diagrams survive (tree vertices and UV counterterms). The UV renormalization constants are known from former anomalous-dimension calculations.
- All the $1/\epsilon$ poles cancel in the matching equation, i.e. in the difference between the effective theory and the full SM Green functions.
- At the 3-loop level, the difference $m_t - M_W$ is taken into account with the help of expansions in y^n and $(1-y^2)^n$ up to $n = 8$, where $y = M_W/m_t$.

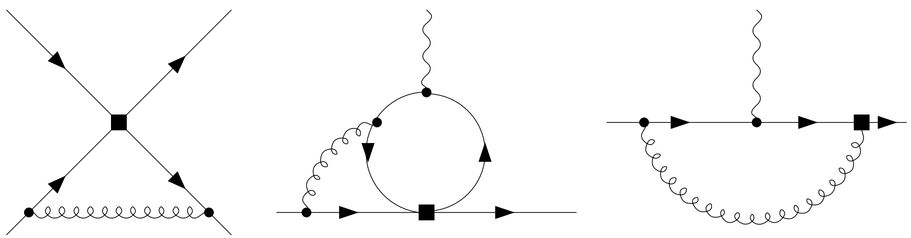
Resummation of large logarithms $\left(\alpha_s \ln \frac{M_W^2}{m_b^2}\right)^n$ in the $b \to s\gamma$ amplitude.

RGE for the Wilson coefficients: $\mu \frac{d}{d\mu} C_j(\mu) = C_i(\mu)\gamma_{ij}(\mu)$

The anomalous dimension matrix γ_{ij} is found from the effective theory renormalization constants, e.g.:

Z_{22} Z_{27} Z_{87}

LO

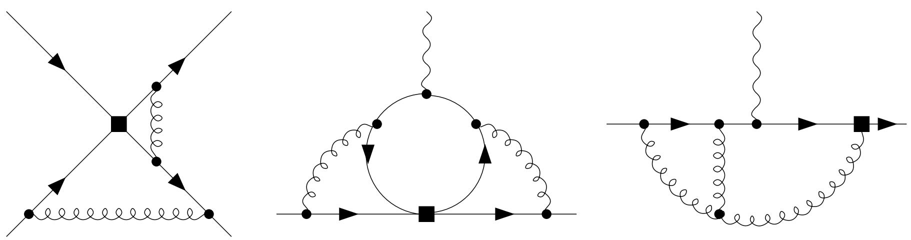


[Gaillard, Lee, 1974] [Altarelli, Maiani, 1974]

[Grinstein *et al.*, 1990]

[Shifman *et al.*, 1978] [Grigjanis *et al.*, 1988]

NLO

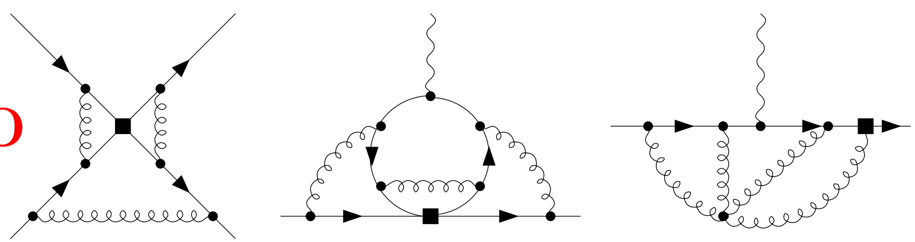


[Altarelli *et al.*, 1981] [Buras, Weisz, 1990]

[Chetyrkin, MM, Münz, 1997]

[MM, Münz, 1995]

NNLO



[Gorbahn, Haisch, 2004]

[Czakon, Haisch, MM, 2006]

$\sim 2 \times 10^4$ **diagrams,**
-4% **effect in the BR**

[Gorbahn, Haisch, MM, 2005]

All the Wilson coefficients $C_1(\mu_b), \ldots, C_8(\mu_b)$ are known at the NNLO in the SM.

NNLO QCD corrections to $\bar{B} \to X_s \gamma$

The relevant perturbative quantity $P(E_0)$:

$$\frac{\Gamma[b \to X_s \gamma]_{E_\gamma > E_0}}{\Gamma[b \to X_u e \bar{\nu}]} = \left| \frac{V_{ts}^* V_{tb}}{V_{ub}} \right|^2 \frac{6\alpha_{\rm em}}{\pi} \underbrace{\sum_{i,j} C_i(\mu_b)\, C_j(\mu_b)\, K_{ij}}_{P(E_0)}$$

Expansions of the Wilson coefficients and K_{ij} in $\widetilde{\alpha}_s \equiv \frac{\alpha_s(\mu_b)}{4\pi}$:

$$C_i(\mu_b) = C_i^{(0)} + \widetilde{\alpha}_s\, C_i^{(1)} + \widetilde{\alpha}_s^2\, C_i^{(2)} + \ldots$$

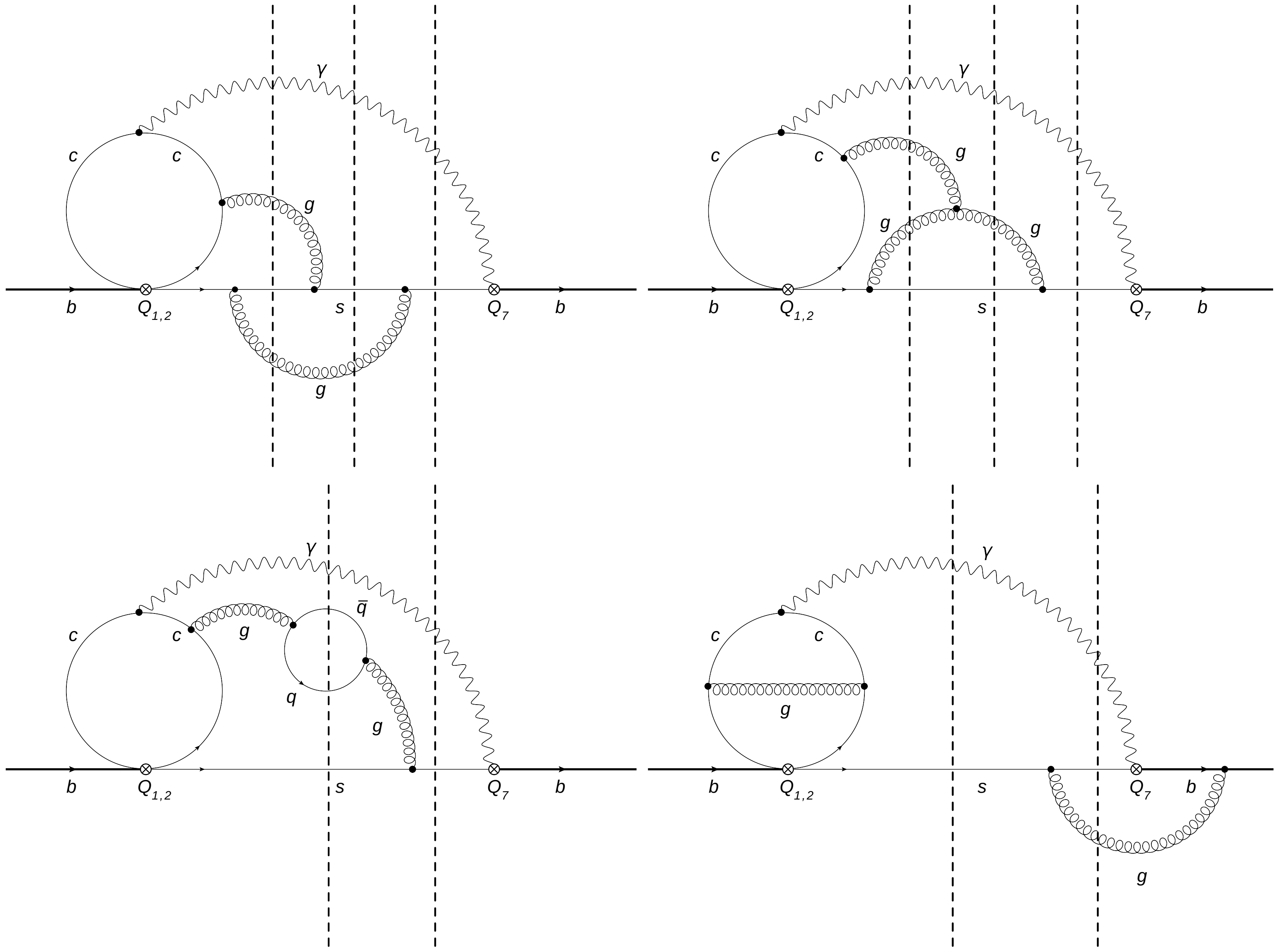
$$K_{ij} = K_{ij}^{(0)} + \widetilde{\alpha}_s\, K_{ij}^{(1)} + \widetilde{\alpha}_s^2\, K_{ij}^{(2)} + \ldots$$

Most important at the NNLO: $K_{77}^{(2)}$, $K_{27}^{(2)}$ and $K_{17}^{(2)}$.

They depend on $\frac{\mu_b}{m_b}$, $\delta = 1 - \frac{2E_0}{m_b}$ and $z = \frac{m_c^2}{m_b^2}$.

Evaluation of $K_{27}^{(2)}$ and $K_{17}^{(2)}$ for $m_c = 0$ and $\delta = 1$:

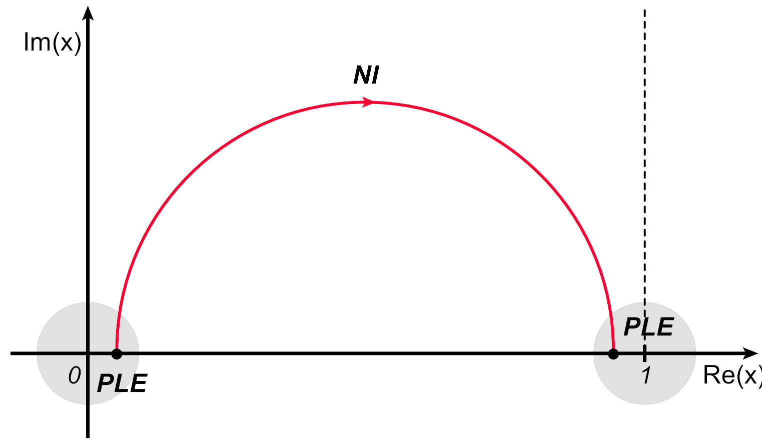
[M. Czakon, P. Fiedler, T. Huber, MM, T. Schutzmeier, M. Steinhauser, JHEP 1504 (2015) 168]



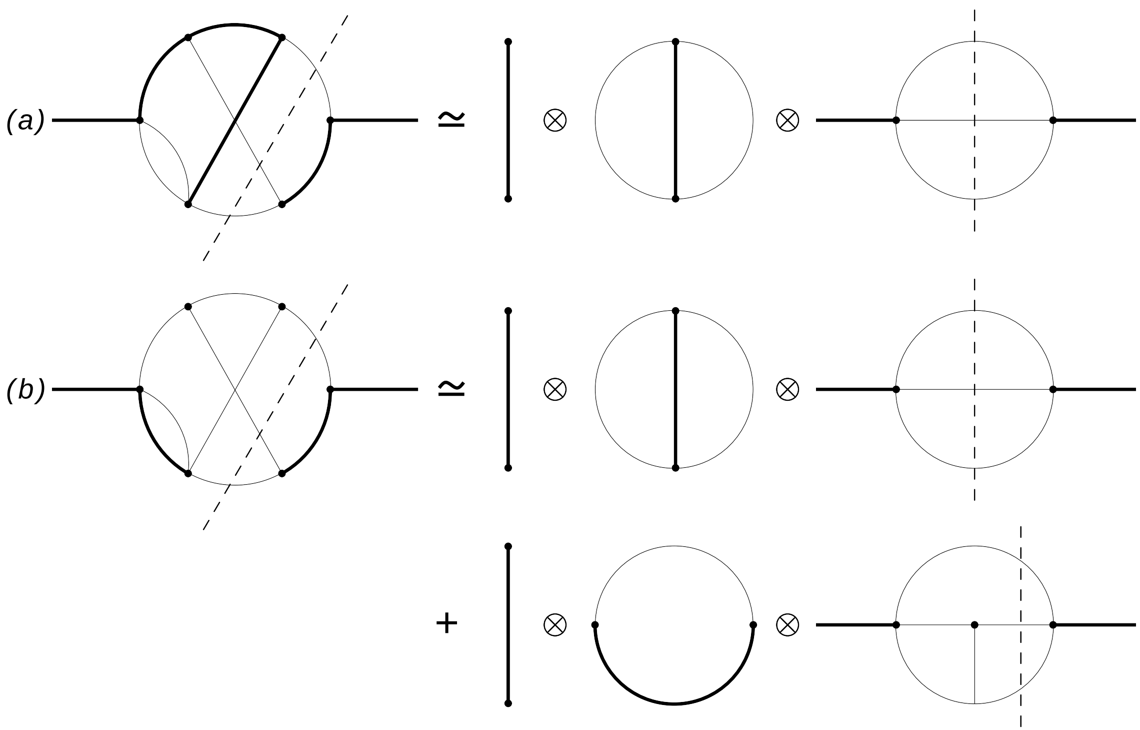
Master integrals and differential equations:

	n_D	n_{OS}	n_{eff}	$n_{massless}$
2-particle cuts	**292**	**92**	**143**	**9**
3-particle cuts	**267**	**54**	**110**	**11**
4-particle cuts	**292**	**17**	**37**	**7**
total	**851**	**163**	**290**	**27**

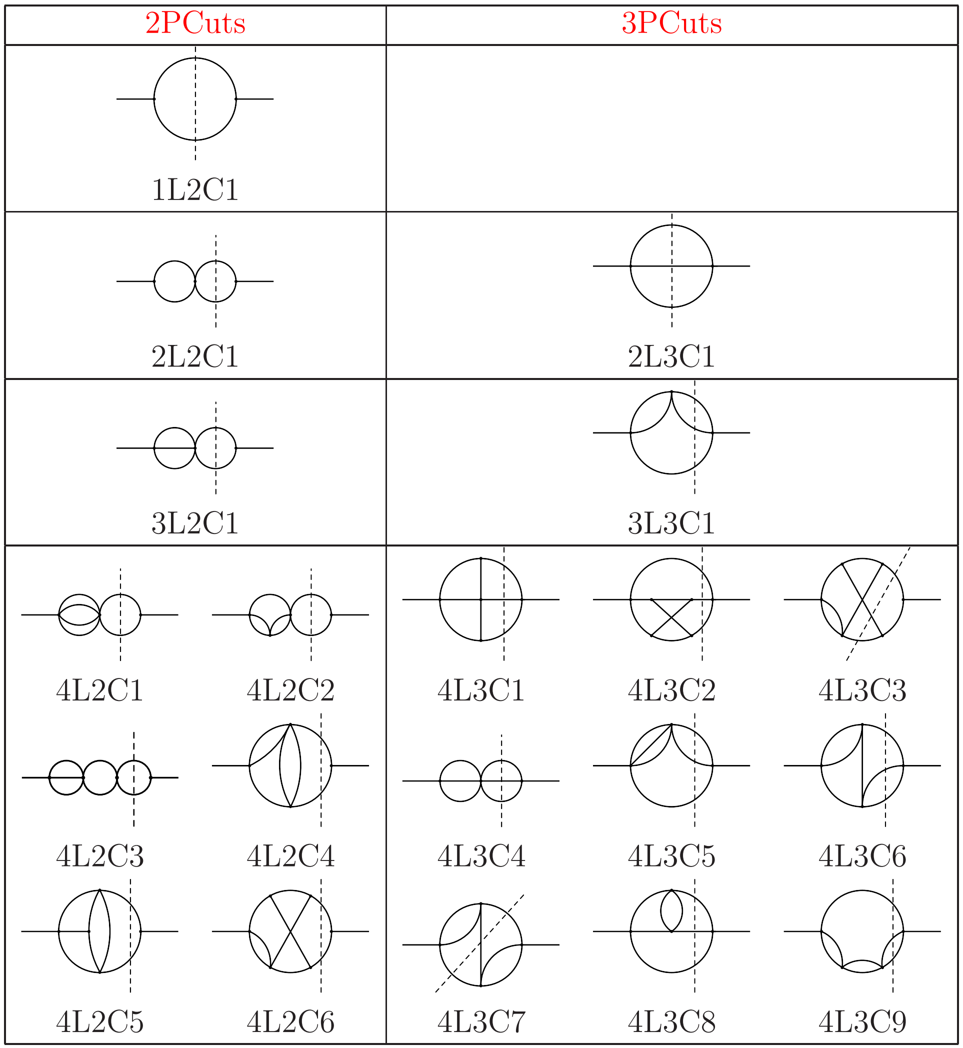
$$\frac{d}{dx} I_i(x) = \sum_j R_{ij}(x) I_j(x), \qquad x = \frac{p^2}{m_b^2}.$$

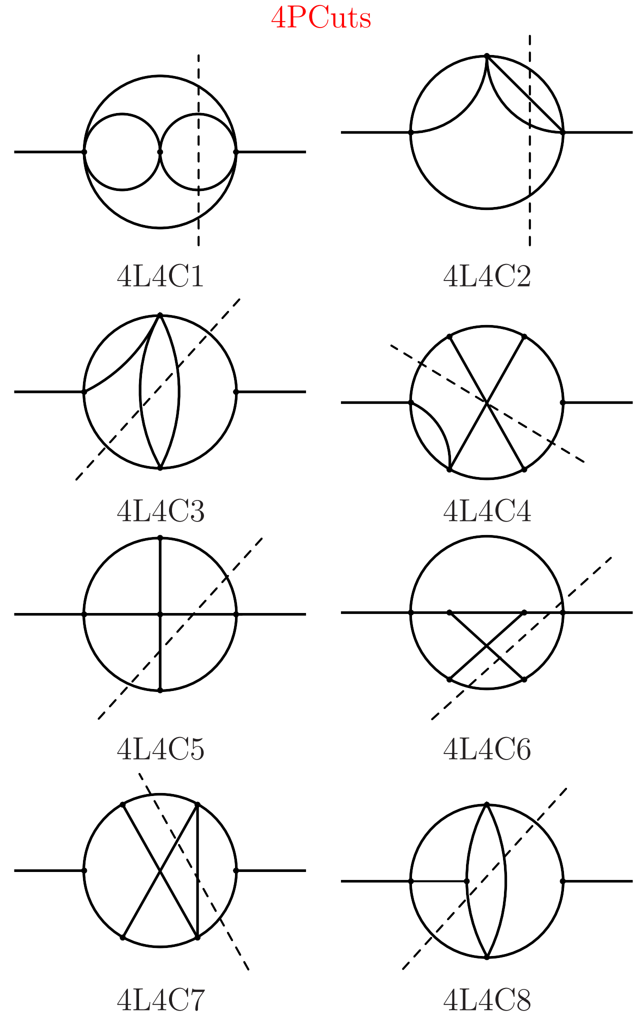


Boundary conditions in the vicinity of $x = 0$:



Massless integrals for the boundary conditions:





Results for the NNLO corrections:

$$K_{27}^{(2)}(z,\delta) \;=\; A_2 + F_2(z,\delta) \underbrace{-\tfrac{27}{2} f_q(z,\delta) + f_b(z) + f_c(z) + \tfrac{4}{3}\phi_{27}^{(1)}(z,\delta)\ln z}_{\text{quark loops on the gluon lines } \;\&\; \text{ BLM approximation}}$$

$$+\;\left[\text{terms} \;\sim\; \left(\ln\frac{\mu_b}{m_b},\;\; \ln^2\frac{\mu_b}{m_b},\;\; \ln\frac{\mu_c}{m_c}\right) \text{ or vanishing when } m_b \to m_b^{\text{pole}}\right],$$

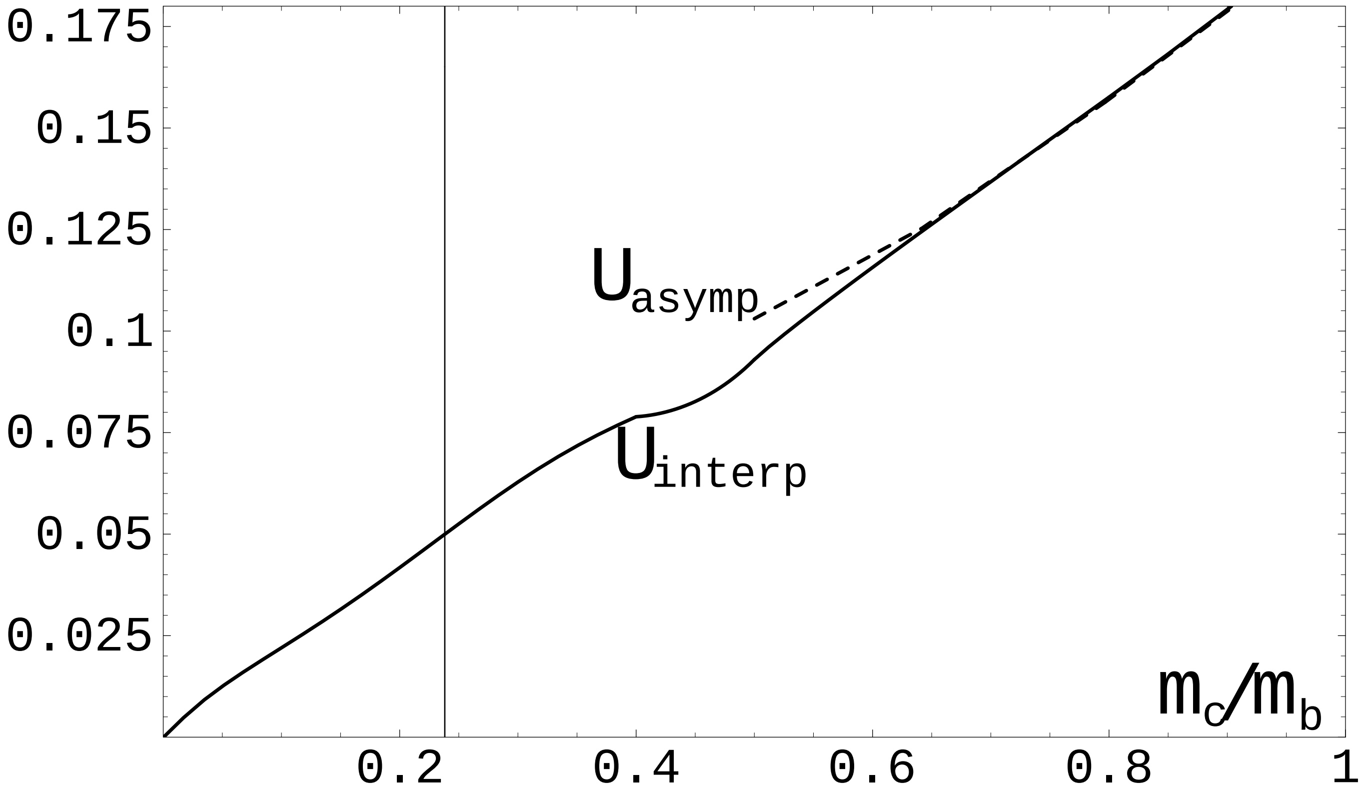
$$K_{17}^{(2)}(z,\delta) \;=\; -\tfrac{1}{6}K_{27}^{(2)}(z,\delta) + A_1 + F_1(z,\delta) + \left[\text{terms} \;\sim\; \left(\ln\frac{\mu_b}{m_b},\;\; \ln^2\frac{\mu_b}{m_b}\right)\right].$$

$F_i(0,1) \equiv 0$, $A_1 \simeq 22.605$, $A_2 \simeq 75.603$ from the present calculation.

Next, we interpolate in $z = m_c^2/m_b^2$ by assuming that $F_i(z,1)$ are linear combinations of $f_q(z,1)$, $K_{27}^{(1)}(z,1)$, $z\frac{d}{dz}K_{27}^{(1)}(z,1)$ and a constant term. The known large-z behaviour of F_i [hep-ph/0609241] and the condition $F_i(0,1) \equiv 0$ fix these linear combinations in a unique manner.

Effect of the interpolated contribution on the branching ratio

$$\frac{\Delta\mathcal{B}_{s\gamma}}{\mathcal{B}_{s\gamma}} \simeq U(z,\delta) \equiv \frac{\alpha_s^2(\mu_b)}{8\pi^2} \, \frac{C_1^{(0)}(\mu_b)F_1(z,\delta)+\left(C_2^{(0)}(\mu_b)-\frac{1}{6}C_1^{(0)}(\mu_b)\right)F_2(z,\delta)}{C_7^{(0)\mathrm{eff}}(\mu_b)}$$

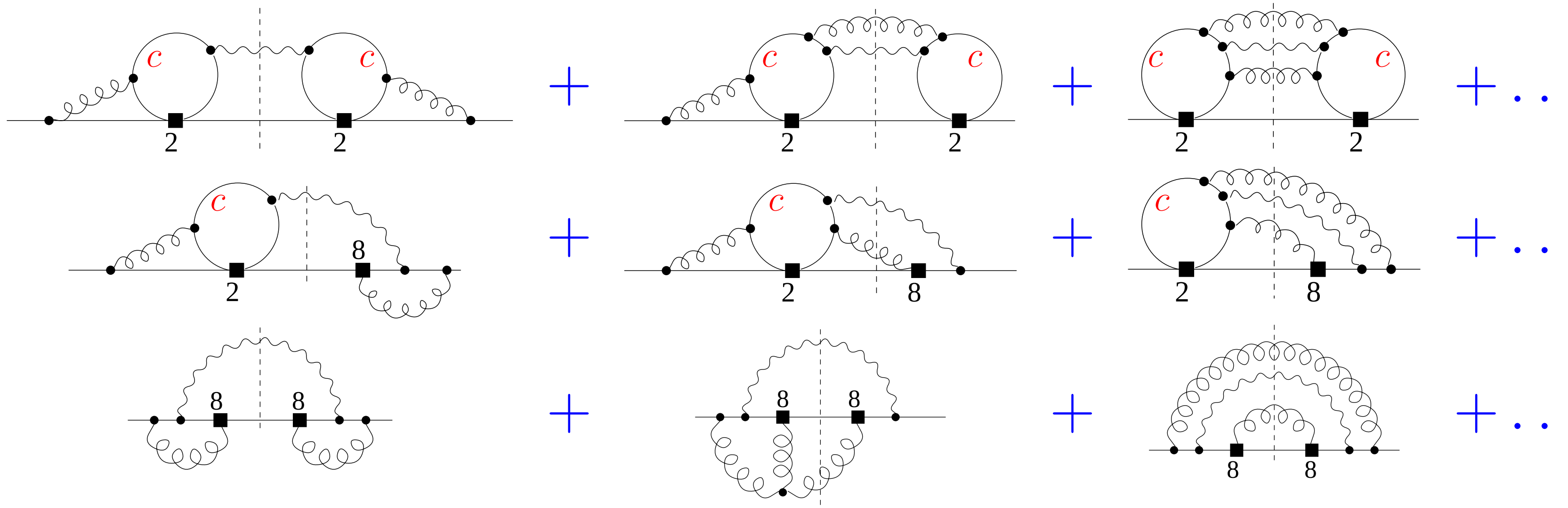


Interferences not involving the photonic dipole operator are treated as follows:

K_{22}**:**
(and analogous K_{11} & K_{12})

K_{28}**:**
(and analogous K_{18})

K_{88}**:**



Two-particle cuts are known (just $|\mathrm{NLO}|^2$**).**

Three- and four-particle cuts are known in the BLM approximation only. The NLO+(NNLO BLM) corrections are not big $(+3.8\%)$**.**

Incorporating other perturbative contributions evaluated after the previous phenomenological analysis in hep-ph/0609232:

1. Four-loop mixing (current-current) $\to$ (gluonic dipole)
   M. Czakon, U. Haisch, MM, JHEP 0703 (2007) 008 [hep-ph/0612329]
2. Diagrams with massive quark loops on the gluon lines
   R. Boughezal, M. Czakon and T. Schutzmeier, JHEP 0709 (2007) 072 [arXiv:0707.3090]
   H. M. Asatrian, T. Ewerth, H. Gabrielyan and C. Greub, Phys. Lett. B 647 (2007) 173 [hep-ph/0611123]
   T. Ewerth, Phys. Lett. B 669 (2008) 167 [arXiv:0805.3911]
3. Complete interference (photonic dipole)–(gluonic dipole)
   H. M. Asatrian, T. Ewerth, A. Ferroglia, C. Greub and G. Ossola,
   Phys. Rev. D 82 (2010) 074006 [arXiv:1005.5587]
4. New BLM corrections to contributions from 3-body and 4-body final states for interferences not involving the photonic dipole
   A. Ferroglia and U. Haisch, Phys. Rev. D 82 (2010) 094012 [arXiv:1009.2144]
   MM and M. Poradziński, Phys. Rev. D 83 (2011) 014024 [arXiv:1009.5685]
5. LO contributions from $b \to s\gamma q\bar{q}$, $(q = u, d, s)$ from 4-quark operators ("penguin" or CKM-suppressed)
   M. Kamiński, MM and M. Poradziński, Phys. Rev. D 86 (2012) 094004 [arXiv:1209.0965]
6. NLO contributions from $b \to s\gamma q\bar{q}$, $(q = u, d, s)$ from interferences of the above operators with $Q_{1,2,7,8}$
   T. Huber, M. Poradziński, J. Virto, JHEP 1501 (2015) 115 [arXiv:1411.7677]

Taking into account new non-perturbative analyses:

M. Benzke, S. J. Lee, M. Neubert and G. Paz, JHEP 1008 (2010) 099 [arXiv:1003.5012]
T. Ewerth, P. Gambino and S. Nandi, Nucl. Phys. B 830 (2010) 278 [arXiv:0911.2175]

Updating the parameters (Parametric uncertainties go down to 2.0%)

P. Gambino, C. Schwanda, Phys. Rev. D 89 (2014) 014022
A. Alberti, P. Gambino, K. J. Healey, S. Nandi, Phys. Rev. Lett. 114 (2015) 061802

Updated SM estimate for the CP- and isospin-averaged branching ratio of $\bar{B} \to X_s\gamma$ [arXiv:1503.01789, arXiv:1503.01791]:

$$\mathcal{B}_{s\gamma}^{\mathrm{SM}} = (3.36 \underbrace{\pm 0.23}_{\pm 6.9\%}) \times 10^{-4}$$

Contributions to the total TH uncertainty (summed in quadrature):

5% non-perturbative, **3%** from the interpolation in m_c

3% higher order $\mathcal{O}(\alpha_s^3)$, **2%** parametric

It is very close the the experimental world average(s):

(a) $\mathcal{B}_{s\gamma}^{\mathrm{exp}} = (3.43 \underbrace{\pm 0.21 \pm 0.07}_{\pm 6.5\%}) \times 10^{-4}$ [HFAG, arXiv:1412.7515]

(b) $\mathcal{B}_{s\gamma}^{\mathrm{exp}} = (3.41 \underbrace{\pm 0.15 \pm 0.04}_{\pm 4.6\%}) \times 10^{-4}$ [Karim Trabelsi, talk at EPS 2015]

Experiment agrees with the SM to much better than $\sim 1\sigma$ level.

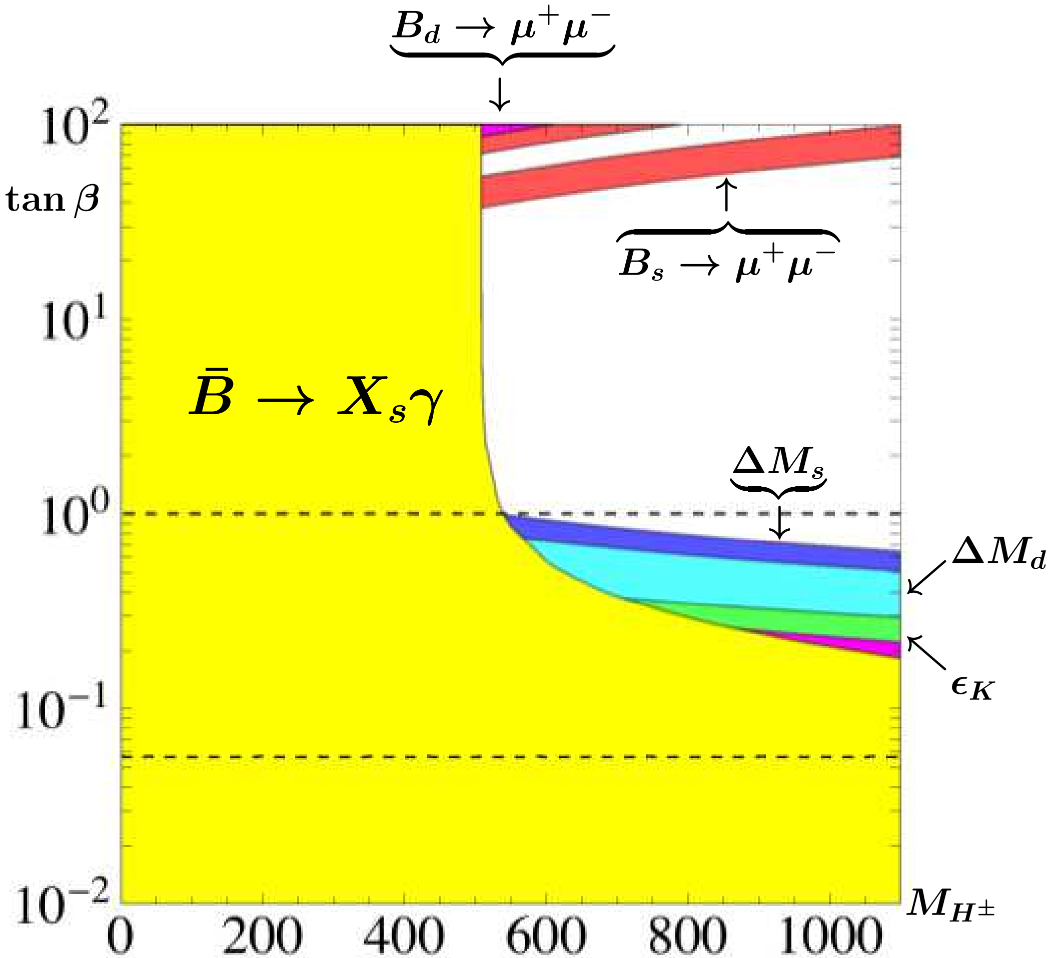
$\Rightarrow$ Strong bounds on the $H^\pm$ mass in the Two-Higgs-Doublet-Model II:

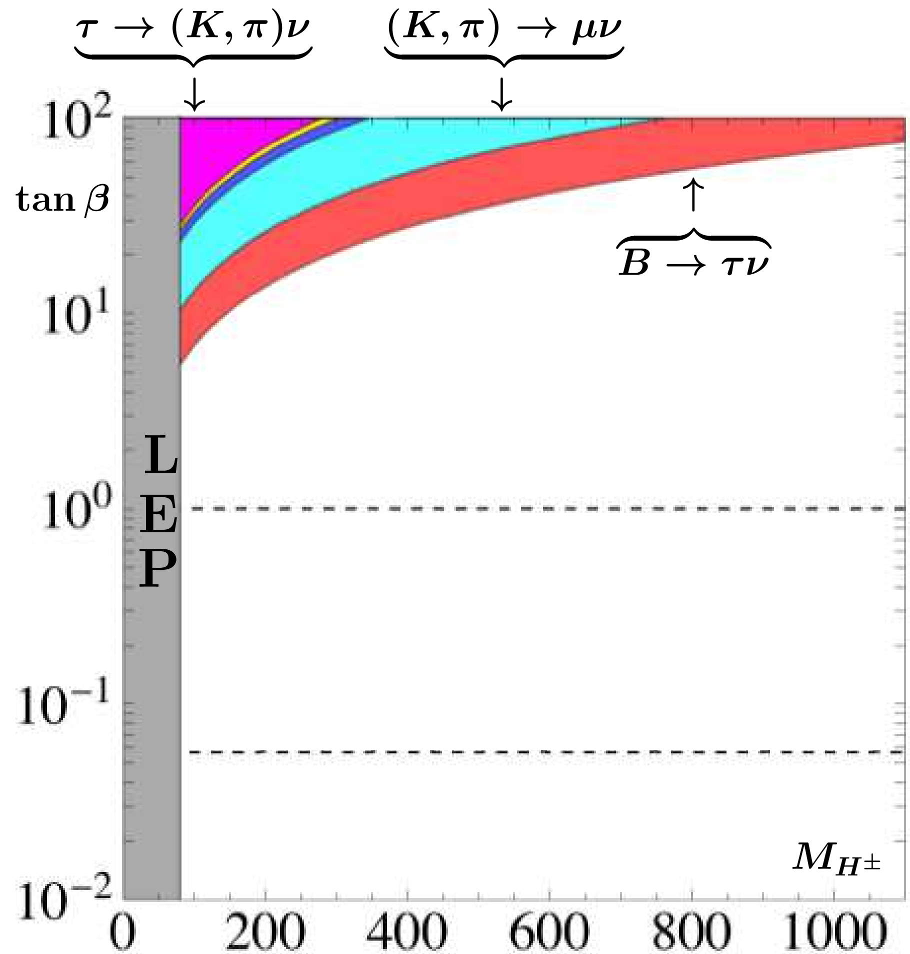
(a) $M_{H^\pm} > 480\,\mathrm{GeV}$ at 95%C.L.

(b) $M_{H^\pm} > 540\,\mathrm{GeV}$ at 95%C.L.

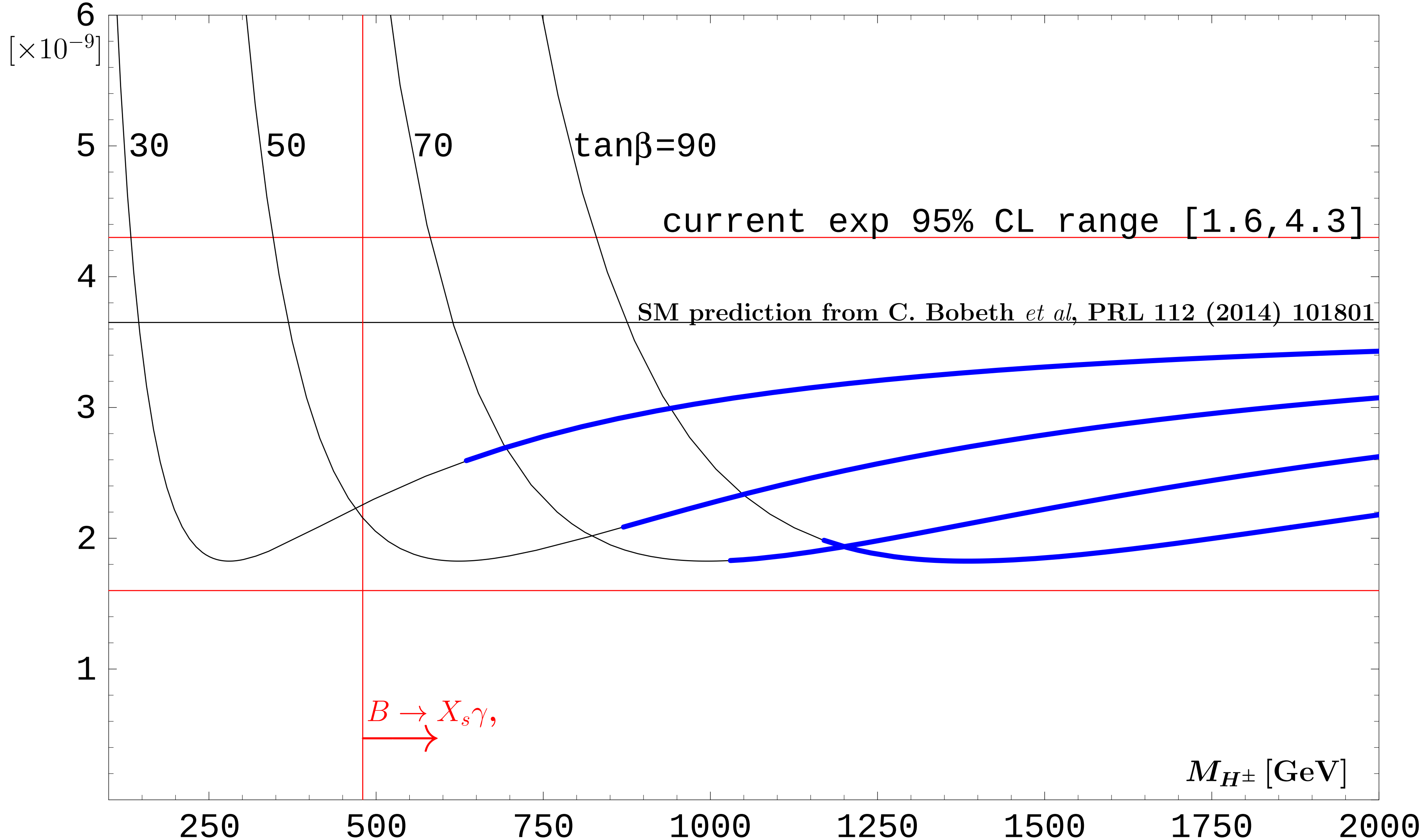
Current flavour-physics bounds in the $M_{H^\pm}-\tan\beta$ plane of the 2HDM-II

[from T. Enomoto and R. Watanabe, corrected w.r.t. arXiv:1511.05066v1]

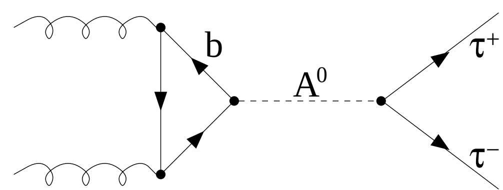




$\mathcal{B}(B_s \to \mu^+\mu^-)$ in the Two-Higgs-Doublet Model II



Blue lines — still allowed for $M_{H^\pm} = \sqrt{M_A^2 + M_W^2}$ after taking into account the LHC searches for $pp \to A^0 \to \tau^+\tau^-$
[CMS arXiv:1408.3316, ATLAS arXiv:1409.6064].



The direct CP asymmetry in $\bar{B} \to X_s\gamma$

$$A_{X_s\gamma} = \frac{\Gamma(\bar{B}\to X_s\gamma) - \Gamma(B\to X_{\bar{s}}\gamma)}{\Gamma(\bar{B}\to X_s\gamma) + \Gamma(B\to X_{\bar{s}}\gamma)}$$

Semi inclusive measurements $\Rightarrow$ $A^{\rm exp}_{X_s\gamma} = +(1.5 \pm 2.0)\%$ (HFAG 2014 average)

SM estimate [Benzke, Lee, Neubert, Paz, arXiv:1012.3167]:

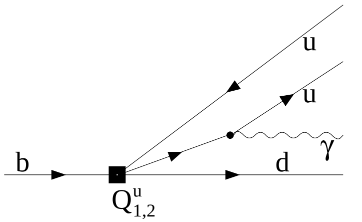
$$A^{\rm SM}_{X_s\gamma} \simeq {\rm Im}\left(\frac{V^*_{us}V_{ub}}{V^*_{ts}V_{tb}}\right)\pi\left|\frac{C_1^{\rm their}}{C_7}\right|\left[\frac{\tilde{\Lambda}^u_{17}-\tilde{\Lambda}^c_{17}}{m_b} + \frac{40\alpha_s}{9\pi}\frac{m_c^2}{m_b^2}\left(1-\frac{2}{5}\ln\frac{m_b}{m_c}+\frac{4}{5}\ln^2\frac{m_b}{m_c}-\frac{\pi^2}{15}\right)\right]$$

$$\simeq \left(1.15\,\frac{\tilde{\Lambda}^u_{17}-\tilde{\Lambda}^c_{17}}{300\ {\rm MeV}} + 0.71\right)\% \in [-0.6\%, +2.8\%] \quad \text{using} \begin{cases} -330\ {\rm MeV} < \tilde{\Lambda}^u_{17} < +525\ {\rm MeV} \\ -9\ {\rm MeV} < \tilde{\Lambda}^u_{17} < +11\ {\rm MeV} \end{cases}$$

Despite the uncertainties, $A_{X_s\gamma}$ provides constraints on models with non-minimal flavour violation. Such models are also constrained by:

$$A_{X_{(s+d)}\gamma} = \frac{\Gamma(\bar{B}\to X_{(s+d)}\gamma) - \Gamma(B\to X_{(\bar{s}+\bar{d})}\gamma)}{\Gamma(\bar{B}\to X_{(s+d)}\gamma) + \Gamma(B\to X_{(\bar{s}+\bar{d})}\gamma)} \qquad (A^{\rm SM}_{X_{(s+d)}\gamma} \simeq 0)$$

$\bar{B} \to X_d \gamma$



$$\mathcal{L}_{\rm eff} \sim V_{td}^* V_{tb} \left[\sum_{i=1}^{8} C_i Q_i + \kappa_d \sum_{i=1}^{2} C_i (Q_i - Q_i^u) \right]$$

$$\kappa_d = (V_{ud}^* V_{ub})/(V_{td}^* V_{tb}) = \left(0.007^{+0.015}_{-0.011}\right) + i \left(-0.404^{+0.012}_{-0.014}\right)$$

$$\left. \begin{array}{l} \mathcal{B}_{d\gamma}^{\rm SM} = \left(1.73^{+0.12}_{-0.22}\right) \times 10^{-5} \\ \mathcal{B}_{d\gamma}^{\rm exp} = (1.41 \pm 0.57) \times 10^{-5} \end{array} \right\} \text{for } E_0 = 1.6\,\text{GeV}$$

- $\mathcal{B}_{d\gamma}^{\rm SM}$ is rough: m_b/m_q varied between $10 \sim m_B/m_K$ and $50 \sim m_B/m_\pi$ $\Longrightarrow$ 2% to 11% of $\mathcal{B}_{d\gamma}$.
- Fragmentation functions give a similar range [H. M. Asatrian and C. Greub, arXiv:1305.6464].
- Collinear logarithms and isolated photons

The ratio R_γ

$$R_\gamma^{\rm SM} \equiv \left(\mathcal{B}_{s\gamma}^{\rm SM} + \mathcal{B}_{d\gamma}^{\rm SM} \right) / \mathcal{B}_{c\ell\nu} = (3.31 \pm 0.22) \times 10^{-3}$$

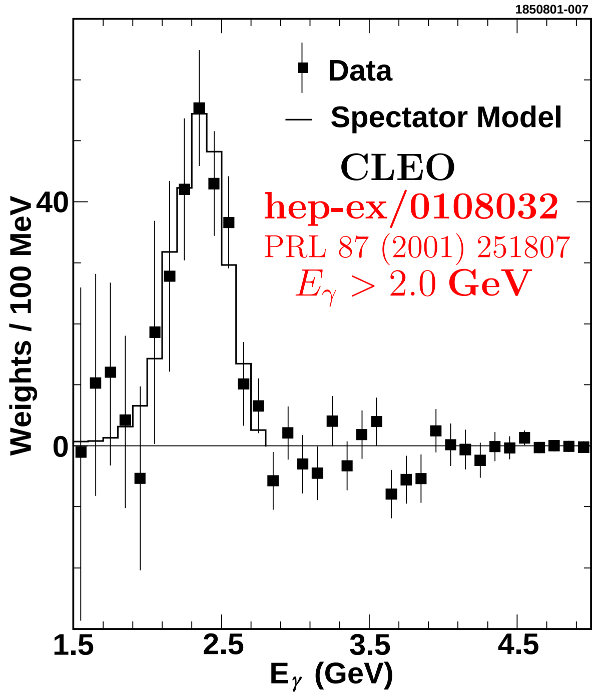
Generic (but CP-conserving) beyond-SM effects:

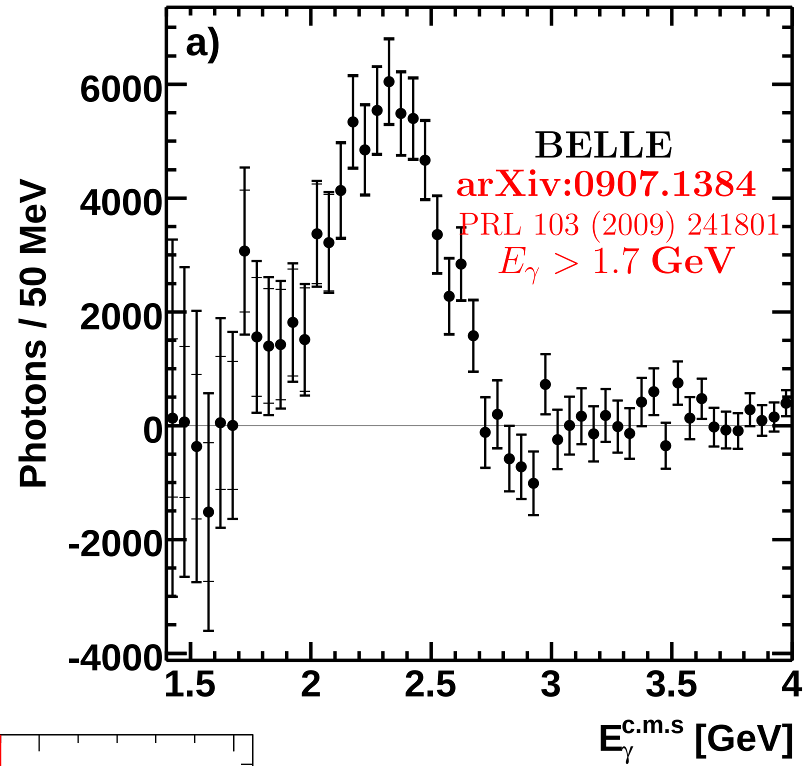
$$\mathcal{B}_{s\gamma} \times 10^4 = (3.36 \pm 0.23) - 8.22\, \Delta C_7 - 1.99\, \Delta C_8,$$

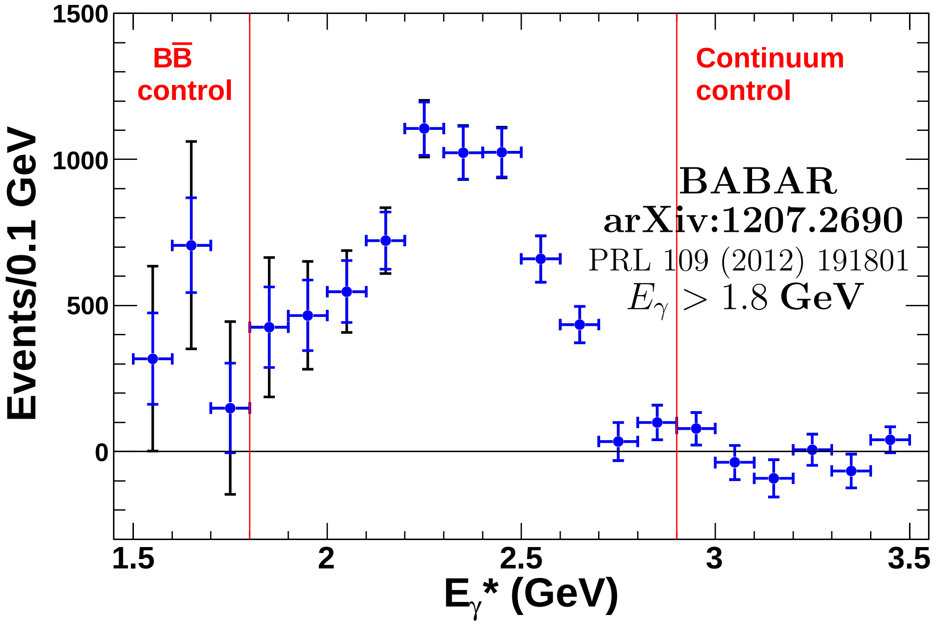
$$R_\gamma \times 10^3 = (3.31 \pm 0.22) - 8.05\, \Delta C_7 - 1.94\, \Delta C_8.$$

The "raw" photon energy spectra in the inclusive measurements of $\mathcal{B}_{s\gamma}$

(The CP- and isospin-averaged branching ratio of $\bar{B} \to X_s\gamma$)







The peaks are centred around

$$\tfrac{1}{2}m_b \simeq 2.35 \text{ GeV}$$

which corresponds to a two-body $b \to s\gamma$ decay.

Broadening is due to (mainly):

- perturbative gluon bremsstrahlung,
- motion of the b quark inside the $\bar{B}$ meson,
- motion of the $\bar{B}$ meson in the $\Upsilon(4S)$ frame.

Experimental world averages for $\mathcal{B}_{s\gamma}$:

$$\mathcal{B}_{s\gamma}^{\rm exp} = (3.43 \underbrace{\pm 0.21 \pm 0.07}_{\pm 6.5\%}) \times 10^{-4}$$

[HFAG, arXiv:1412.7515]
(lowest E_0 only from each exp)

$$\mathcal{B}_{s\gamma}^{\rm exp} = (3.41 \underbrace{\pm 0.15 \pm 0.04}_{\pm 4.6\%}) \times 10^{-4}$$

[Karim Trabelsi, talk at EPS 2015]
($E_0 = 1.9\,\text{GeV}$ only from each exp)

for the photon energy $E_\gamma > E_0 = 1.6\,\text{GeV}$. The averaging involves an extrapolation from the measurements performed at $E_0 \in [1.7, 2.0]\,\text{GeV}$.

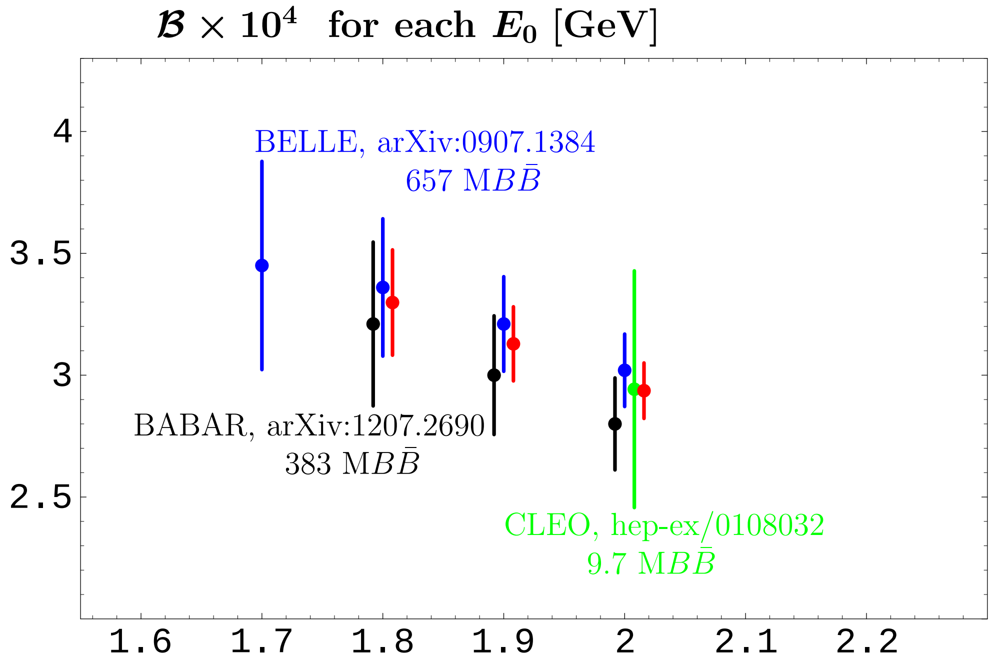
Applying the HFAG extrapolation method to the available $\bar{B} \to X_d\,\gamma$ measurement [BABAR, arXiv:1005.4087]**, one finds** [A. Crivellin, L. Mercolli, arXiv:1106.5499]**:**

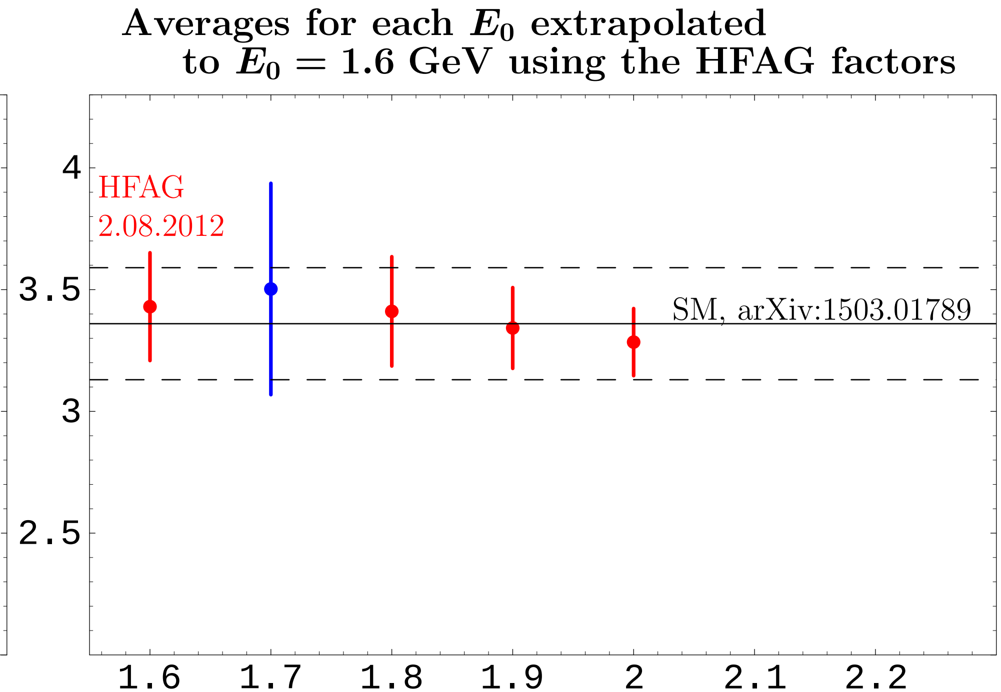
$$\mathcal{B}_{d\gamma}^{\rm exp} = (1.41 \underbrace{\pm 0.57}_{\pm 40\%}) \times 10^{-5}.$$

The HFAG average includes the following measurements:

Reference	Method	# of $B\bar{B}$	E_0 [GeV]	$\mathcal{B}_{s\gamma} \times 10^4$ at E_0
CLEO [PRL 87 (2001) 251807]	inclusive	9.70×10^6	2.0	$3.06 \pm 0.41 \pm 0.26$
BABAR [PRL 109 (2012) 191801]	inclusive	3.83×10^8	1.8 1.9 2.0	$3.21 \pm 0.15 \pm 0.29 \pm 0.08$ $3.00 \pm 0.14 \pm 0.19 \pm 0.06$ $2.80 \pm 0.12 \pm 0.14 \pm 0.04$
BELLE [PRL 103 (2009) 241801]	inclusive	6.57×10^8	1.7 1.8 1.9 2.0	$3.45 \pm 0.15 \pm 0.40$ $3.36 \pm 0.13 \pm 0.25$ $3.21 \pm 0.11 \pm 0.16$ $3.02 \pm 0.10 \pm 0.11$
BABAR [PRD 77 (2008) 051103]	inclusive with a hadronic tag (hadronic decay of the recoiling B ($\bar{B}$))	2.32×10^8, which gives 6.8×10^5 tagged events	1.9 2.0 2.1 2.2 2.3	$3.66 \pm 0.85 \pm 0.60$ $3.39 \pm 0.64 \pm 0.47$ $2.78 \pm 0.48 \pm 0.35$ $2.48 \pm 0.38 \pm 0.27$ $2.07 \pm 0.30 \pm 0.20$
BABAR [PRD 86 (2012) 052012]	semi-inclusive	4.71×10^8	1.9	$3.29 \pm 0.19 \pm 0.48$
BELLE [PLB 511 (2001) 151] BELLE [PRD 91 (2015) 052004]	semi-inclusive	6.07×10^6 7.72×10^8	2.24→1.6 1.9	$3.69 \pm 0.58 \pm 0.46 \pm 0.60$ $3.51 \pm 0.17 \pm 0.33$

Comparison of the inclusive measurements of $\mathcal{B}(\bar{B} \to X_s\gamma)$ by CLEO, BELLE and BABAR for each E_0 separately





The HFAG factors

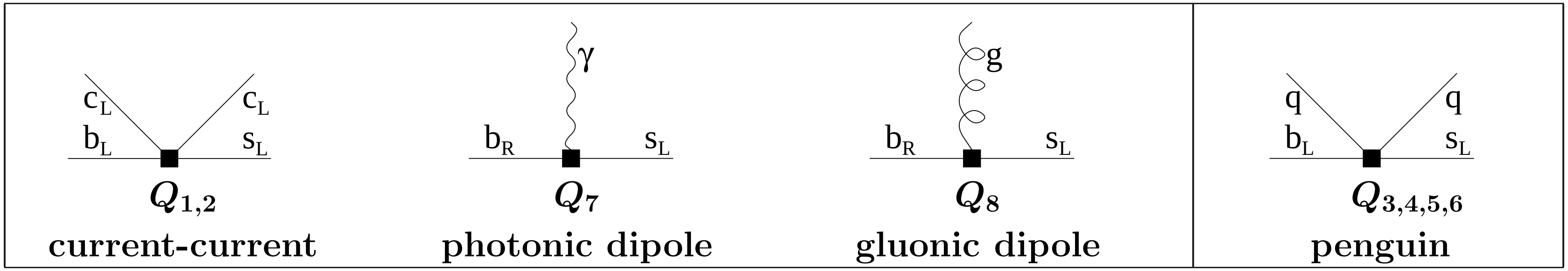
Scheme	$E_\gamma < 1.7$	$E_\gamma < 1.8$	$E_\gamma < 1.9$	$E_\gamma < 2.0$	$E_\gamma < 2.242$
Kinetic	0.986 ± 0.001	0.968 ± 0.002	0.939 ± 0.005	0.903 ± 0.009	0.656 ± 0.031
Neubert SF	0.982 ± 0.002	0.962 ± 0.004	0.930 ± 0.008	0.888 ± 0.014	0.665 ± 0.035
Kagan-Neubert	0.988 ± 0.002	0.970 ± 0.005	0.940 ± 0.009	0.892 ± 0.014	0.643 ± 0.033
Average	0.985 ± 0.004	0.967 ± 0.006	0.936 ± 0.010	0.894 ± 0.016	0.655 ± 0.037

- **Are the HFAG factors trustworthy?**

Decoupling of W, Z, t, H^0 $\Rightarrow$ effective weak interaction Lagrangian:

$$L_{\rm weak} \sim \sum_i C_i Q_i$$

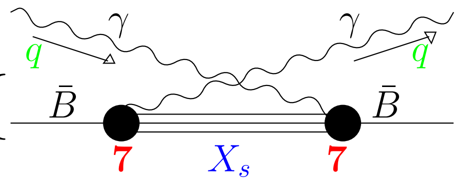
Eight operators Q_i matter for $\mathcal{B}_{s\gamma}^{\rm SM}$ when the NLO EW and/or CKM-suppressed effects are neglected:



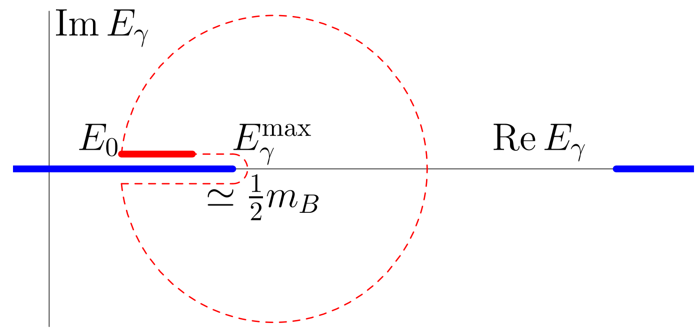
$$\Gamma(\bar{B} \to X_s\gamma)_{E_\gamma > E_0} = |C_7(\mu_b)|^2\, \Gamma_{77}(E_0) + (\text{other}) \qquad (\mu_b \sim m_b/2)$$

Optical theorem:

$$\frac{d\Gamma_{77}}{dE_\gamma} \sim \mathrm{Im}\{\ \cdots\ \} \equiv \mathrm{Im}\, A$$



Integrating the amplitude A over E_γ:



OPE on the ring $\Rightarrow$ **Non-perturbative corrections to $\Gamma_{77}(E_0)$ form a series in $\frac{\Lambda_{\rm QCD}}{m_b}$ and α_s that begins with**

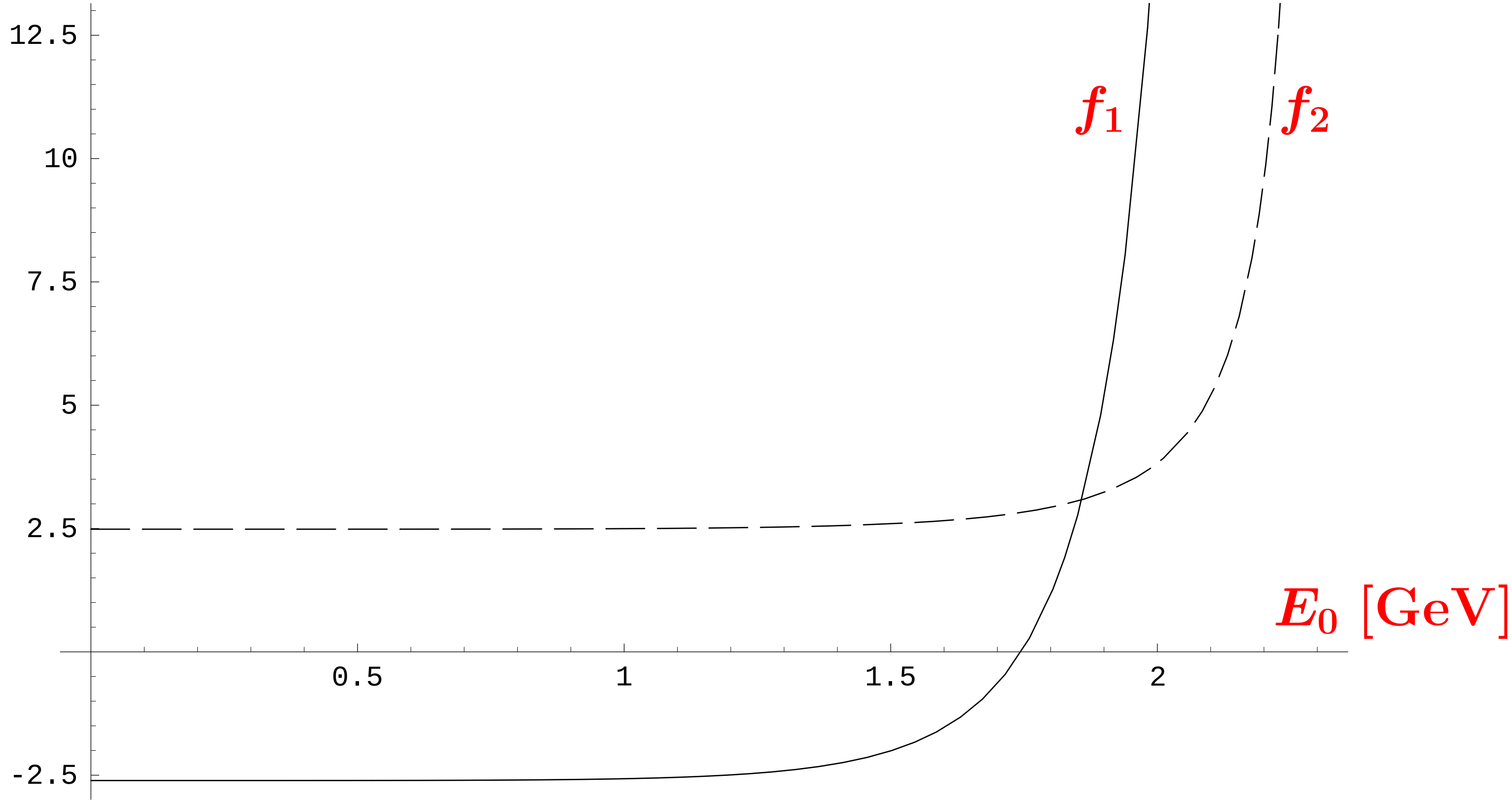
$$\frac{\mu_\pi^2}{m_b^2},\ \frac{\mu_G^2}{m_b^2},\ \frac{\rho_D^3}{m_b^3},\ \frac{\rho_{LS}^3}{m_b^3},\ldots;\ \frac{\alpha_s\mu_\pi^2}{(m_b-2E_0)^2},\ \frac{\alpha_s\mu_G^2}{m_b(m_b-2E_0)};\ldots,$$

where $\mu_\pi, \mu_G, \rho_D, \rho_{LS} = \mathcal{O}(\Lambda_{\rm QCD})$ are extracted from the semileptonic $\bar{B} \to X_c e\bar{\nu}$ spectra and the B–$B^\star$ mass difference.

The $\mathcal{O}\left(\frac{\alpha_s \mu_\pi^2}{(m_b-2E_0)^2}\right)$ and $\mathcal{O}\left(\frac{\alpha_s \mu_G^2}{m_b(m_b-2E_0)}\right)$ corrections

[T. Ewerth, P. Gambino and S. Nandi, arXiv:0911.2175]

$$\Gamma_{77}(E_0) = \Gamma_{77}^{\rm tree}\left\{1 + (\text{pert. corrections}) - \frac{\mu_\pi^2}{2m_b^2}\left[1 + \frac{\alpha_s}{\pi}\left(f_1(E_0) - \frac{4}{3}\ln\frac{\mu}{m_b}\right)\right] - \frac{3\mu_G^2(\mu)}{2m_b^2}\left[1 + \frac{\alpha_s}{\pi}\left(f_2(E_0) + \frac{1}{6}\ln\frac{\mu}{m_b}\right)\right]\right\}$$



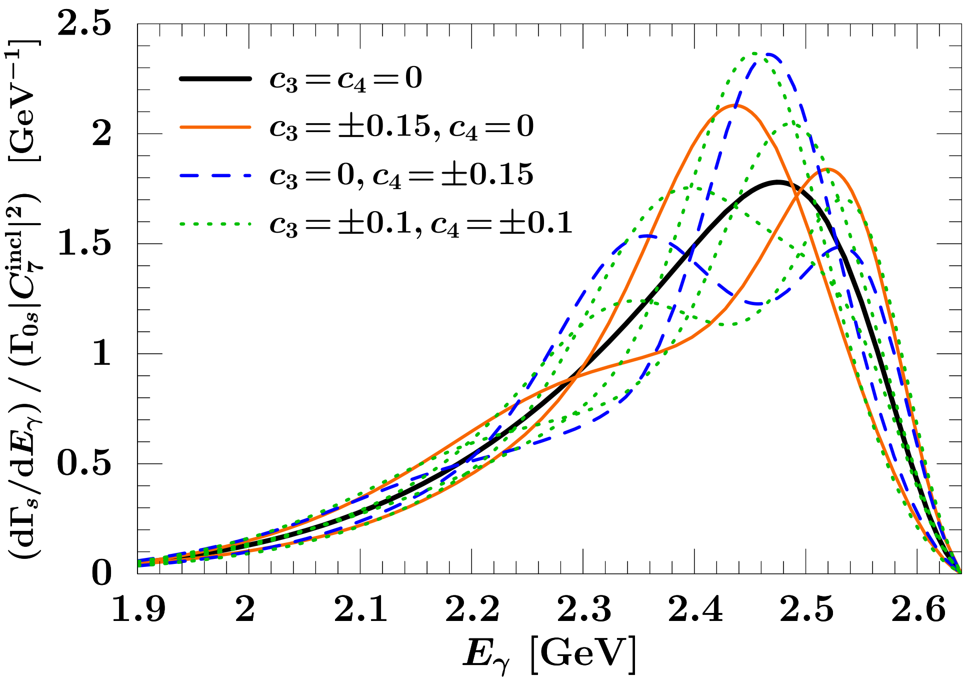
When $(m_b - 2E_0) \sim \Lambda \equiv \Lambda_{\rm QCD}$, no OPE can be applied.

Local operators $\longrightarrow$ Non-local operators

Non-perturbative parameters $\longrightarrow$ Non-perturbative functions

$$\frac{d}{dE_\gamma}\Gamma_{77} \;=\; N \underbrace{H(E_\gamma)}_{\text{pert.}} \int_0^{M_B-2E_\gamma} dk\;\; \underbrace{P(M_B - 2E_\gamma - k)}_{\text{pert.}}\; \underbrace{F(k)}_{\text{non-pert.}} + \mathcal{O}\left(\frac{\Lambda}{m_b}\right)$$

Photon spectra from models of $F(k)$ [Ligeti, Stewart, Tackmann, arXiv:0807.1926]



The function $F(k)$ is:

- perturbatively related to the standard shape function $S(\omega)$,
- exponentially suppressed for $k \gg \Lambda$,
- positive definite,
- constrained by measured moments of the $\bar{B} \to X_c e \bar{\nu}$ spectrum (local OPE),
- constrained by measured properties of the $\bar{B} \to X_u e \bar{\nu}$ and $\bar{B} \to X_s \gamma$ spectra (not imposed in the plot).

Upgrading the HFAG factors by fitting $F(k)$ to data:

- The SIMBA Collaboration [arXiv:1101.3310, arXiv:1303.0958] (work in progress)

  $F(k) = \frac{1}{\lambda}\left[\sum_{n=0}^{\infty} c_n f_n\left(\frac{k}{\lambda}\right)\right]^2$, $\quad f_n$ – basis functions. Truncate and fit.

- Another way: $F(k) = A(k)B(k)$ and use the SIMBA approach for $B(k)$.

  perfect fit

Why do we need to upgrade the HFAG factors?

- The old models (Kagan-Neubert 1998, ...) are not generic enough (too few parameters).
- Inclusion of $\mathcal{O}\left(\frac{\Lambda}{m_b}\right)$ effects and and taking other operators $(Q_i \neq Q_7)$ into account is necessary [Benzke, Lee, Neubert, Paz, arXiv:1003.5012].

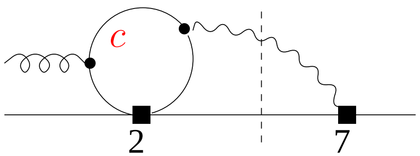
What about just fitting C_7 without extrapolation to any particular E_0?

- Fine, but measurements at low E_0 (even less precise) are still going to be crucial for constraining the parameter space.
- The fits are going to give the extrapolation factors anyway. Publishing them is necessary for cross-checks/upgrades by other groups.

Non-perturbative contributions from the photonic dipole operator alone ("77" term) are well controlled for $E_0 = 1.6$ GeV:

$\mathcal{O}\left(\frac{\alpha_s^n \Lambda}{m_b}\right)_{n=0,1,2,\ldots}$ vanish, $\mathcal{O}\left(\frac{\Lambda^2}{m_b^2}\right)$ [Bigi, Blok, Shifman, Uraltsev, Vainshtein, 1992], [Falk, Luke, Savage, 1993], $\mathcal{O}\left(\frac{\Lambda^3}{m_b^3}\right)$ [Bauer, 1997], $\mathcal{O}\left(\frac{\alpha_s \Lambda^2}{m_b^2}\right)$ [Ewerth, Gambino, Nandi, 2009].

The dominant non-perturbative uncertainty originates from the "27" interference term:



$$\frac{\Delta \mathcal{B}}{\mathcal{B}} = -\frac{6C_2 - C_1}{54 C_7}\left[\frac{\lambda_2}{m_c^2} + \sum_n b_n \mathcal{O}\left(\frac{\Lambda^2}{m_c^2}\left(\frac{m_b \Lambda}{m_c^2}\right)^n\right)\right]$$

$\lambda_2 \simeq 0.12\,\mathrm{GeV}^2$

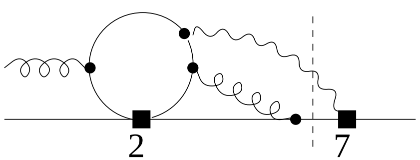
from B–B^* mass splitting

The coefficients b_n decrease fast with n.

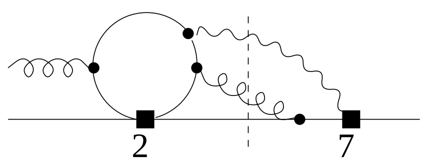
[Voloshin, 1996], [Khodjamirian, Rückl, Stoll, Wyler, 1997]
[Grant, Morgan, Nussinov, Peccei, 1997]
[Ligeti, Randall, Wise, 1997], [Buchalla, Isidori, Rey, 1997]

Claims by Benzke, Lee, Neubert and Paz in arXiv:1003.5012:

One cannot really expand in $m_b\Lambda/m_c^2$. All such corrections should be treated as Λ/m_b ones and estimated using models of subleading shape functions. Dominant contributions to the estimated $\pm 5\%$ non-perturbative uncertainty in $\mathcal{B}$ are found this way, with the help of alternating-sign subleading shape functions that undergo weaker suppression at large gluon momenta.



correction to the above



phase-space suppressed

$\mathcal{O}\left(\frac{\alpha_s \Lambda}{m_b}\right)$ **Main worry in hep-ph/0609232, and reason for the $\pm 5\%$ non-perturbative uncertainty.**

Summary

- **The dominant NNLO corrections to $\mathcal{B}_{s\gamma}$ are now known not only in the large m_c limit, but also at $m_c = 0$. However, no reduction of uncertainties with respect to the 2006 estimate is possible, except for the parametric one.**

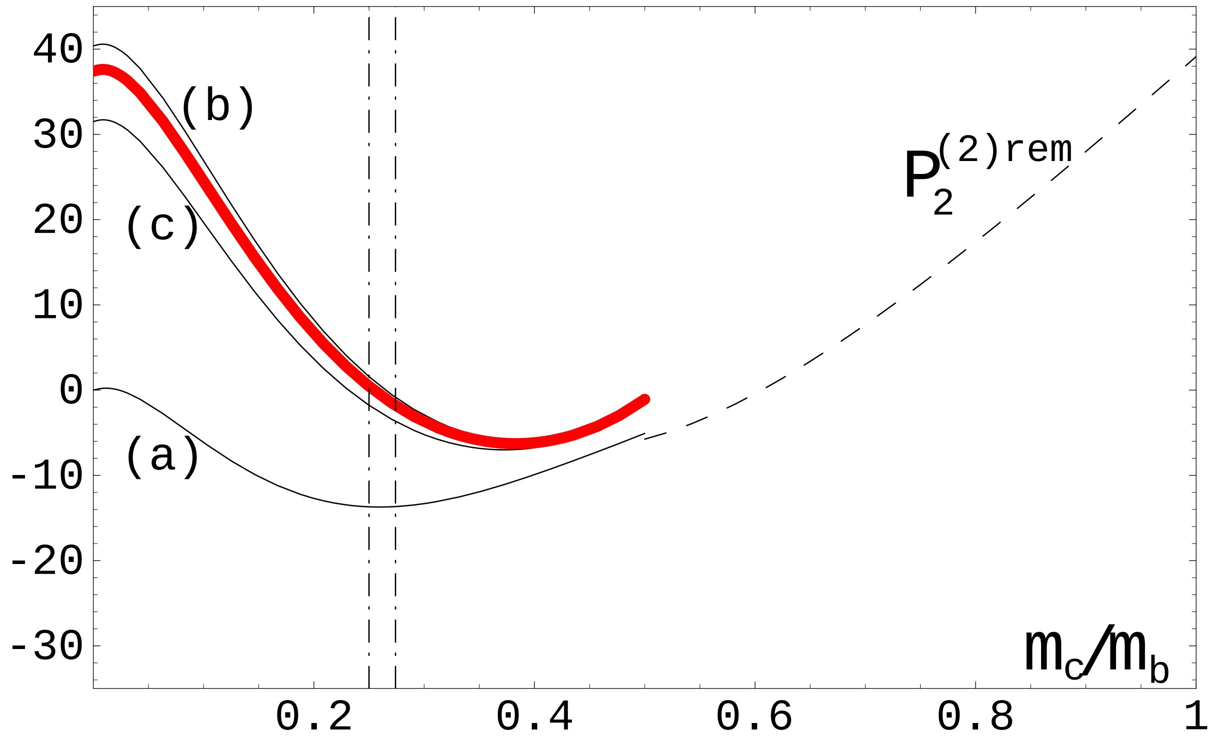
- **Updated predictions:**
  $\mathcal{B}_{s\gamma}^{\rm SM} = (3.36 \pm 0.23) \times 10^{-4}$
  $\mathcal{B}_{d\gamma}^{\rm SM} = \left(1.73^{+0.12}_{-0.22}\right) \times 10^{-5}$
  $R_{\gamma}^{\rm SM} = (3.31 \pm 0.22) \times 10^{-3}$

- **Completing the calculation of $K_{17}^{(2)}$ and $K_{27}^{(2)}$ for arbitrary $z = m_c^2/m_b^2$ is necessary to further reduce the perturbative uncertainties in $\mathcal{B}_{s\gamma}$.**

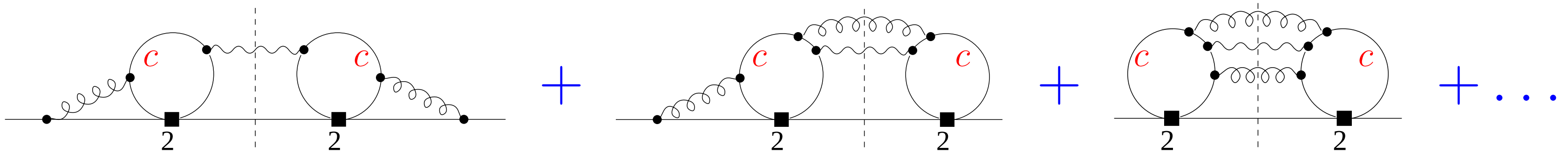
- **New experimental averages of $\mathcal{B}_{s\gamma}$ and R_γ should be based on an improved extrapolation in E_0. It will be necessary to take full advantage of the awaited precise measurements at BELLE-II.**

BACKUP SLIDES

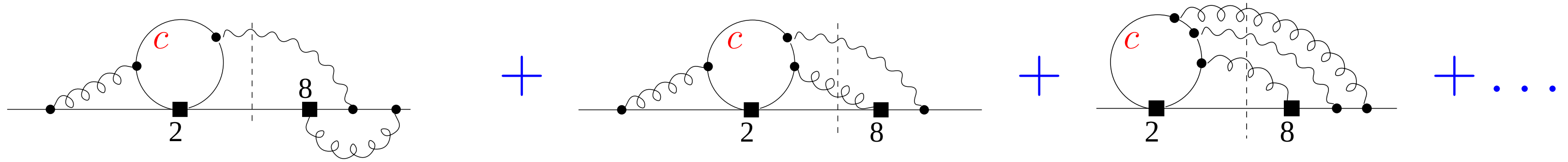
Comparison to the interpolation in hep-ph/0609241



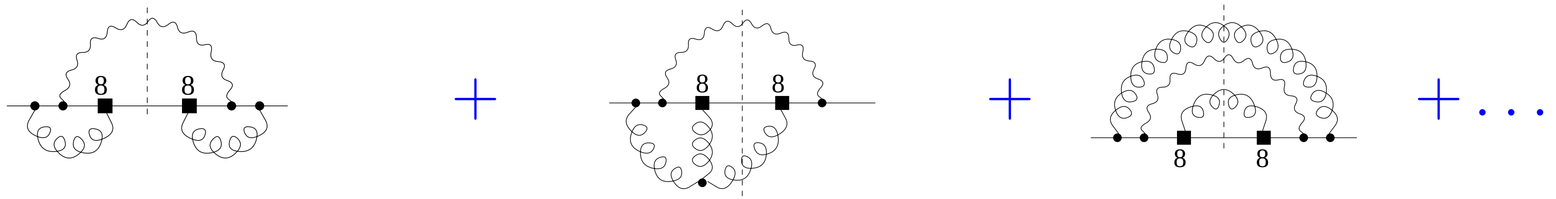
G_{22}**:**
(and analogous
G_{11} & G_{12})



G_{28}**:**
(and analogous G_{18})



G_{88}**:**



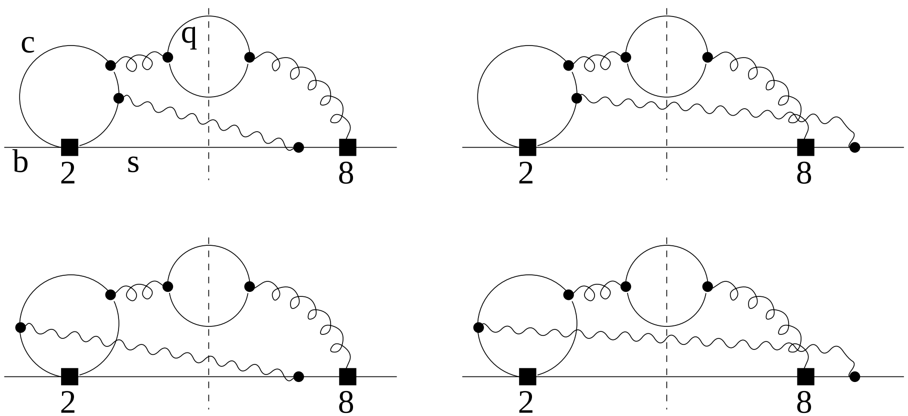
Two-particle cuts are known (just $|\mathrm{NLO}|^2$).

Three- and four-particle cuts are known in the BLM approximation only: [Ligeti, Luke, Manohar, Wise, 1999], [Ferroglia, Haisch, arXiv:1009.2144], [Poradziński, MM, arXiv:1009.5685].

NLO+(NNLO BLM) corrections are not big (+3.8%).

Example:

Evaluation of the $(n > 2)$-particle cut contributions to G_{28} in the Brodsky-Lepage-Mackienzie (BLM) approximation ("naive nonabelianization", large-β_0 approximation) [Poradziński, MM, arXiv:1009.5685]:



q – **massless quark,**

N_q – **number of massless flavours (equals to 3 in practice because masses of u, d, s are neglected).**

Replacement in the final result:

$$-\frac{2}{3}N_q \quad \longrightarrow \quad \beta_0 = 11 - \frac{2}{3}(N_q + 2).$$

The diagrams have been evaluated using the method of Smith and Voloshin [hep-ph/9405204].

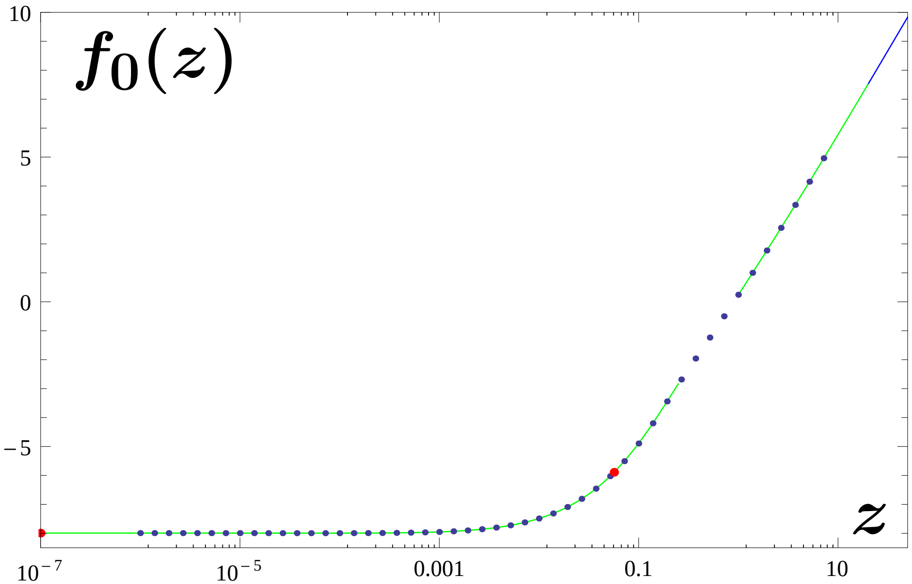
Non-BLM contributions to G_{ij} from quark loops on the gluon lines are quasi-completely known.
[Boughezal, Czakon, Schutzmeier, 2007], [Asatrian, Ewerth, Gabrielyan, Greub, 2007], [Ewerth, 2008].

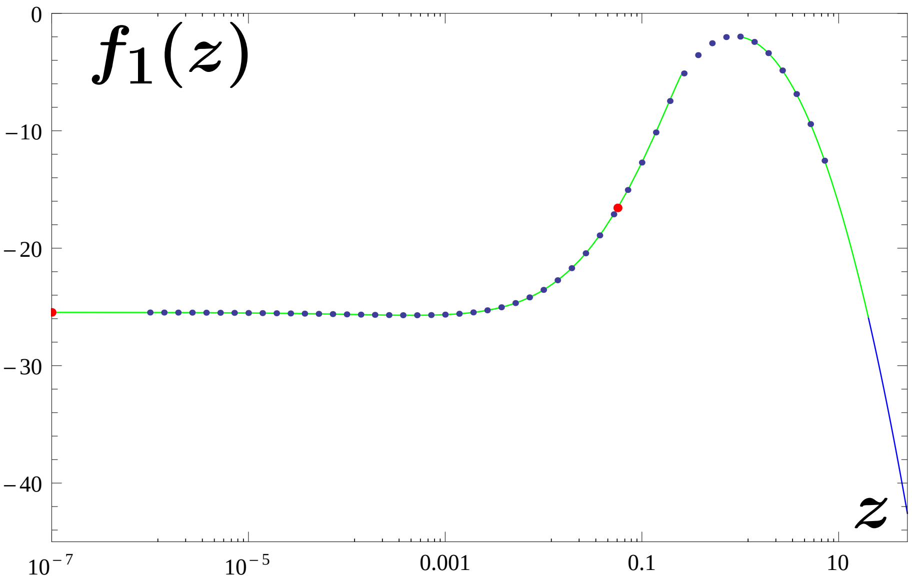
Outlook: generalizing the K_{27} NNLO calculation to arbitrary $z = m_c^2/m_b^2$.

Method: differential equations in z for the master integrals.

Results for the bare NLO contributions up to $\mathcal{O}(\epsilon)$:

$$\tilde{G}_{27}^{(1)2P} = -\frac{92}{81\epsilon} + f_0(z) + \epsilon f_1(z) \xrightarrow{z\to 0} -\frac{92}{81\epsilon} - \frac{1942}{243} + \epsilon\left(-\frac{26231}{729} + \frac{259}{243}\pi^2\right)$$

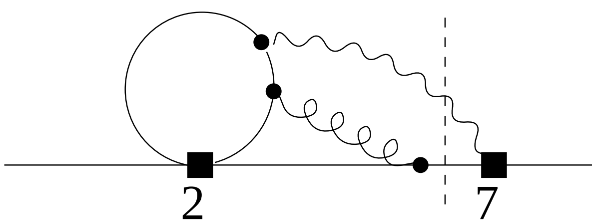




Dots: solutions to the differential equations and/or the exact $z \to 0$ limit.

Lines: large- and small-z asymptotic expansions

Large-z expansions of the 11 master integrals are from M. Steinhauser.



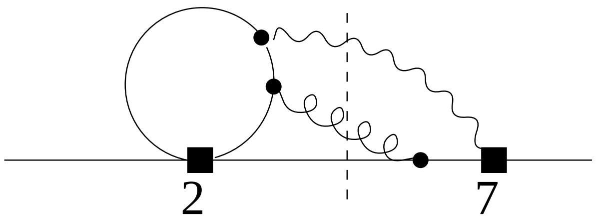
Small-z expansions of $\tilde{G}_{27}^{(1)2P}$:

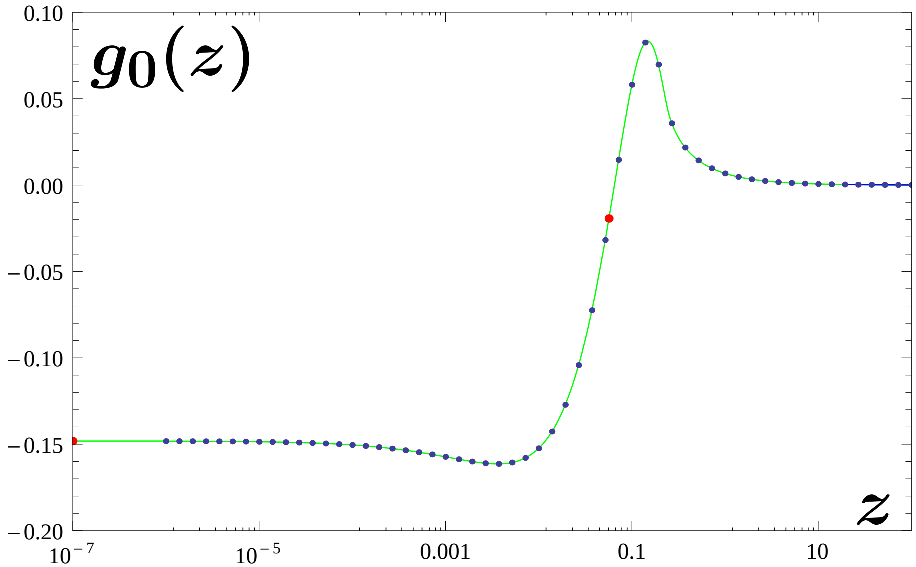
f_0 from C. Greub, T. Hurth, D. Wyler, hep-ph/9602281, hep-ph/9603404,
A. J. Buras, A. Czarnecki, MM, J. Urban, hep-ph/0105160,

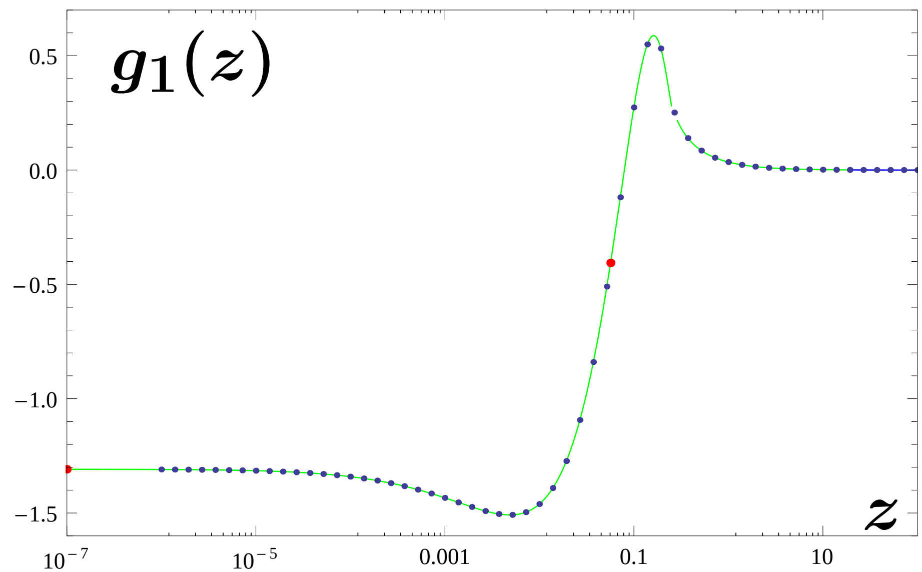
f_1 from H.M. Asatrian, C. Greub, A. Hovhannisyan, T. Hurth and V. Poghosyan, hep-ph/0505068.

Analogous results for the 3-body final state contributions ($\delta = 1$):

$$\tilde{G}_{27}^{(1)3P} = g_0(z) + \epsilon g_1(z) \xrightarrow{z\to 0} -\frac{4}{27} - \frac{106}{81}\epsilon$$







Dots: solutions to the differential equations and/or the exact $z \to 0$ limit.
Lines: exact result for g_0, as well as large- and small-z asymptotic expansions for g_1 from A. Rehman.

$$g_0(z) = \begin{cases} -\frac{4}{27} - \frac{14}{9}z + \frac{8}{3}z^2 + \frac{8}{3}z(1-2z)\, s\, L + \frac{16}{9}z(6z^2-4z+1)\left(\frac{\pi^2}{4} - L^2\right), & \text{for } z \le \frac{1}{4}, \\ -\frac{4}{27} - \frac{14}{9}z + \frac{8}{3}z^2 + \frac{8}{3}z(1-2z)\, t\, A + \frac{16}{9}z(6z^2-4z+1)\, A^2, & \text{for } z > \frac{1}{4}, \end{cases}$$

where $s = \sqrt{1-4z}$, $L = \ln(1+s) - \frac{1}{2}\ln 4z$, $t = \sqrt{4z-1}$, **and** $A = \arctan(1/t)$.

CP-averaged decay rates

$$\Gamma_0 = \frac{\Gamma(\bar{B}^0 \to X_s\gamma) + \Gamma(B^0 \to X_{\bar{s}}\gamma)}{2}, \qquad \Gamma_\pm = \frac{\Gamma(B^- \to X_s\gamma) + \Gamma(B^+ \to X_{\bar{s}}\gamma)}{2}.$$

CP- and isospin-averaged branching ratio in an untagged measurement at $\Upsilon(4S)$

$$\mathcal{B}_{s\gamma} = \tau_{B^0}\,\Gamma\,\left(\frac{1+r_f r_\tau}{1+r_f} + \Delta_{0\pm}\frac{1-r_f r_\tau}{1+r_f}\right).$$

where

$\Gamma = (\Gamma_0 + \Gamma_\pm)/2$ (isospin average)

$\Delta_{0\pm} = (\Gamma_0 - \Gamma_\pm)/(\Gamma_0 + \Gamma_\pm)$ (isospin asymmetry)

$r_\tau = \tau_{B^+}/\tau_{B^0} = 1.076 \pm 0.004$ (measured lifetime rate)

$r_f = f^{+-}/f^{00} = 1.059 \pm 0.027$ (measured production rate at $\Upsilon(4S)$)

The term proportional to $\Delta_{0\pm}$ contributes only at a permille level, which follows from the measured value of $\Delta_{0\pm} = -0.01 \pm 0.06$ (for $E_\gamma > 1.9\,\mathrm{GeV}$).

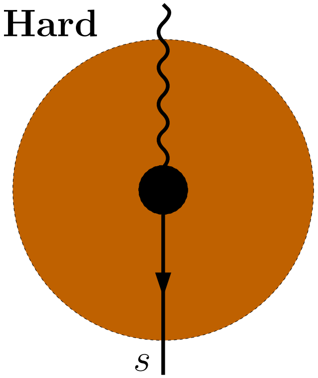
The final state strangeness (-1 for X_s and $+1$ for $X_{\bar{s}}$) and neutral B-meson flavours have been specified upon ignoring effects of the $B^0\bar{B}^0$ and $K^0\bar{K}^0$ mixing. Taking the $K^0\bar{K}^0$ mixing into account amounts to replacing X_s and $X_{\bar{s}}$ by $X_{|s|}$ with an unspecified strangeness sign, which leaves Γ_0 and $\Gamma_\pm$ invariant. Next, taking the $B^0\bar{B}^0$ mixing into account amounts to using in Γ_0 the time-integrated decay rates of mesons whose flavour is fixed at the production time. Such a change leaves Γ_0 practically unaffected because mass eigenstates in the $B^0\bar{B}^0$ system are very close to being orthogonal ($|p/q| = 1$) and having the same decay width.

Energetic photon production in charmless decays of the $\bar{B}$-meson

$(E_\gamma \gtrsim \frac{m_b}{3} \simeq 1.6\,\mathrm{GeV})$ [see MM, arXiv:0911.1651]

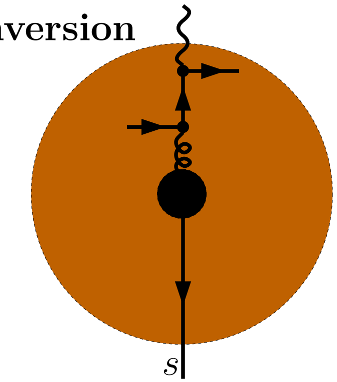
A. Without long-distance charm loops:

1. **Hard**



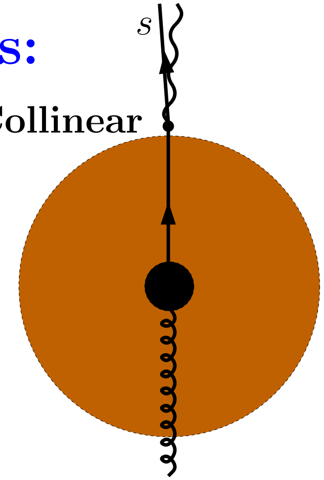
Dominant, well-controlled.

2. **Conversion**



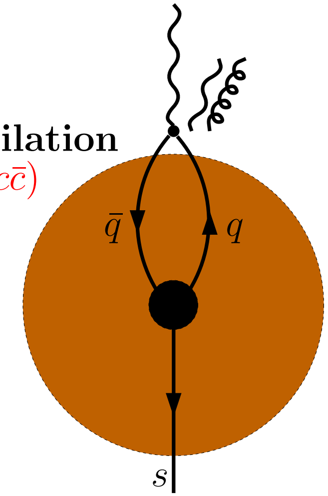
$\mathcal{O}(\alpha_s \Lambda / m_b)$, $(-1.6 \pm 1.2)\%$.
[Benzke, Lee, Neubert, Paz, 2010]

3. **Collinear**



$\sim -0.2\%$ or $(+0.8 \pm 1.1)\%$.
[Kapustin,Ligeti,Politzer, 1995]
[Benzke, Lee, Neubert, Paz, 2010]

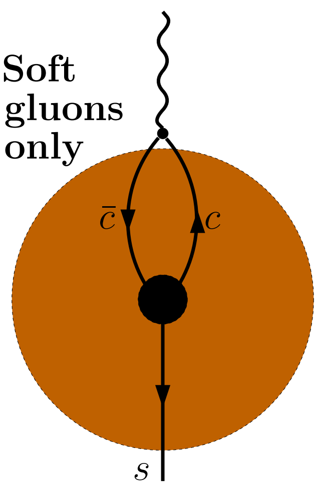
4. **Annihilation** $(q\bar{q} \neq c\bar{c})$



Exp. π^0, η, η', ω subtracted.
Perturbatively $\sim 0.1\%$.

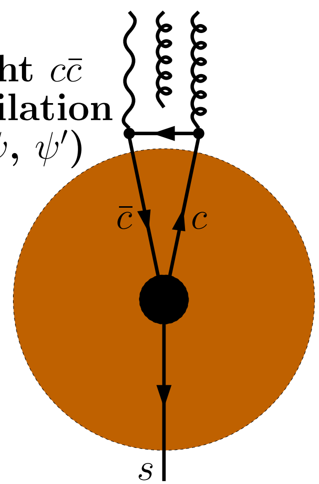
B. With long-distance charm loops:

5. **Soft gluons only**



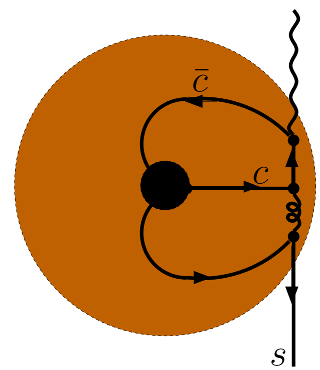
$\mathcal{O}(\Lambda^2/m_c^2)$, $\sim +3.1\%$.
[Voloshin, 1996], [...],
[Buchalla, Isidori, Rey, 1997]
[Benzke, Lee, Neubert, Paz, 2010]: add $(+1.1 \pm 2.9)\%$

6. **Boosted light $c\bar{c}$ state annihilation (e.g. η_c, J/ψ, ψ')**

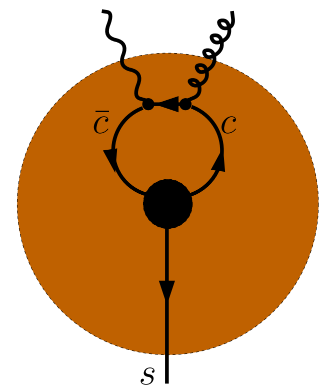


Exp. J/ψ subtracted ($< 1\%$).
Perturbatively (including hard): $\sim +3.6\%$.

7. **Annihilation of $c\bar{c}$ in a heavy $(\bar{c}s)(\bar{q}c)$ state**



$\mathcal{O}(\alpha_s(\Lambda/M)^2)$



$\mathcal{O}(\alpha_s \Lambda/M)$

$M \sim 2m_c, 2E_\gamma, m_b$.

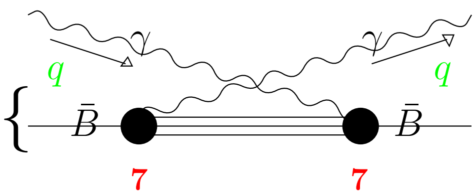
e.g. $\mathcal{B}[B^- \to D_{sJ}(2457)^- \, D^*(2007)^0 \;] \simeq 1.2\%$,
$\mathcal{B}[B^0 \to D^*(2010)^+ \bar{D}^*(2007)^0 K^-] \simeq 1.2\%$.

The “hard” contribution to $\bar{B} \to X_s \gamma$

J. Chay, H. Georgi, B. Grinstein PLB 247 (1990) 399.
A.F. Falk, M. Luke, M. Savage, PRD 49 (1994) 3367.

Goal: calculate the inclusive sum $\Sigma_{X_s} \left| C_7(\mu_b)\langle X_s\gamma|O_7|\bar{B}\rangle + C_2(\mu_b)\langle X_s\gamma|O_2|\bar{B}\rangle + \ldots \right|^2$

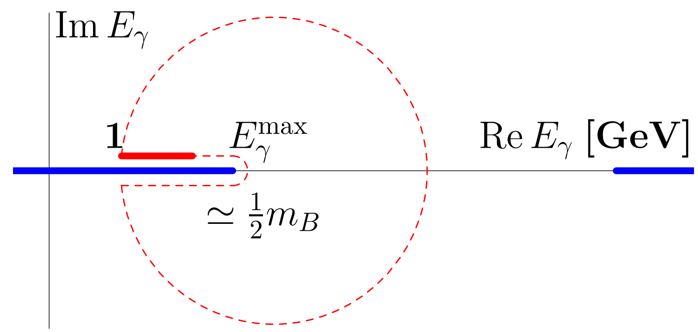
The “77” term in this sum is “hard”. It is related via the optical theorem to the imaginary part of the elastic forward scattering amplitude $\bar{B}(\vec{p}=0)\gamma(\vec{q}) \to \bar{B}(\vec{p}=0)\gamma(\vec{q})$:

$\mathrm{Im}\{$  $\} \equiv \mathrm{Im}\, A$

When the photons are soft enough, $m_{X_s}^2 = |m_B(m_B - 2E_\gamma)| \gg \Lambda^2 \Rightarrow$ **Short-distance dominance** $\Rightarrow$ **OPE.**
However, the $\bar{B} \to X_s\gamma$ **photon spectrum is dominated by hard photons** $E_\gamma \sim m_b/2$.

Once $A(E_\gamma)$ **is considered as a function of arbitrary complex** E_γ**,** $\mathrm{Im}A$ **turns out to be proportional to the discontinuity of** A **at the physical cut. Consequently,**

$$\int_{1\ \mathrm{GeV}}^{E_\gamma^{\max}} dE_\gamma\ \mathrm{Im}\, A(E_\gamma) \sim \oint_{\mathrm{circle}} dE_\gamma\ A(E_\gamma).$$



Since the condition $|m_B(m_B - 2E_\gamma)| \gg \Lambda^2$ **is fulfilled along the circle, the OPE coefficients can be calculated perturbatively, which gives**

$$A(E_\gamma)\big|_{\mathrm{circle}} \simeq \sum_j \left[\frac{F^{(j)}_{\mathrm{polynomial}}(2E_\gamma/m_b)}{m_b^{n_j}(1 - 2E_\gamma/m_b)^{k_j}} + \mathcal{O}\left(\alpha_s(\mu_{\mathrm{hard}})\right) \right] \langle \bar{B}(\vec{p}=0)|Q^{(j)}_{\mathrm{local\ operator}}|\bar{B}(\vec{p}=0)\rangle.$$

Thus, contributions from higher-dimensional operators are suppressed by powers of Λ/m_b.

At $(\Lambda/m_b)^0$: $\langle \bar{B}(\vec{p})|\bar{b}\gamma^\mu b|\bar{B}(\vec{p})\rangle = 2p^\mu \quad \Rightarrow \quad \Gamma(\bar{B} \to X_s\gamma) = \Gamma(b \to X_s^{\mathrm{parton}}\gamma) + \mathcal{O}(\Lambda/m_b)$.

At $(\Lambda/m_b)^1$: **Nothing! All the possible operators vanish by the equations of motion.**

At $(\Lambda/m_b)^2$:

$$\langle \bar{B}(\vec{p})|\bar{b}_v D^\mu D_\mu b_v|\bar{B}(\vec{p})\rangle \sim m_B\, \mu_\pi^2,$$
$$\langle \bar{B}(\vec{p})|\bar{b}_v g_s G_{\mu\nu}\sigma^{\mu\nu} b_v|\bar{B}(\vec{p})\rangle \sim m_B\, \mu_G^2,$$

The HQET heavy-quark field $b_v(x)$ **is defined by** $b_v(x) = \frac{1}{2}(1 + \not{v})b(x)\exp(im_b\, v\cdot x)$ **with** $v = p/m_B$.

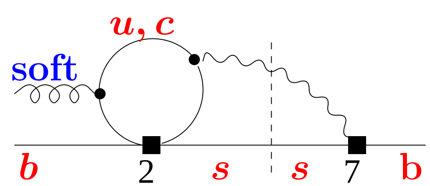
Non-perturbative effects in the presence of other operators $(Q_i \neq Q_7)$

[Benzke, Lee, Neubert, Paz, arXiv:1003.5012].

$$\frac{d}{dE_\gamma}\Gamma(\bar{B} \to X_s\gamma) = (\Gamma_{77}\text{-like term}) + \tilde{N} E_\gamma^3 \sum_{i \leq j} \mathrm{Re}\left(C_i^* C_j\right) F_{ij}(E_\gamma).$$

Remarks:

- The SCET approach is valid for large E_γ only. It is fine for $E_\gamma > E_0 \sim \frac{1}{3} m_b \simeq 1.6$ GeV. Lower cutoffs are academic anyway.
- For such E_0, non-perturbative effects in the integrated decay rate are estimated to remain within 5%. They scale like:
    - $\frac{\Lambda^2}{m_b^2}$, $\frac{\Lambda^2}{m_c^2}$ (known),
    - $\frac{\Lambda}{m_b} \frac{V_{us}^* V_{ub}}{V_{ts}^* V_{tb}}$ (negligible),
    - $\frac{\Lambda}{m_b}$, $\frac{\Lambda^2}{m_b^2}$, $\alpha_s \frac{\Lambda}{m_b}$ but suppressed by tails of subleading shape functions ("27"),
    - $\alpha_s \frac{\Lambda}{m_b}$ to be constrained by future measurements of the isospin asymmetry ("78"),
    - $\alpha_s \frac{\Lambda}{m_b}$ but suppressed by $Q_d^2 = \frac{1}{9}$ ("88").



- **Extrapolation factors?** Tails of subleading functions are less important for them.