Theoretical uncertainties in

b to s leptonic decays

Aoife Bharucha

LIO international conference on flavour physics, composite models and dark matter
23$^{\text{rd}}$ April 2015

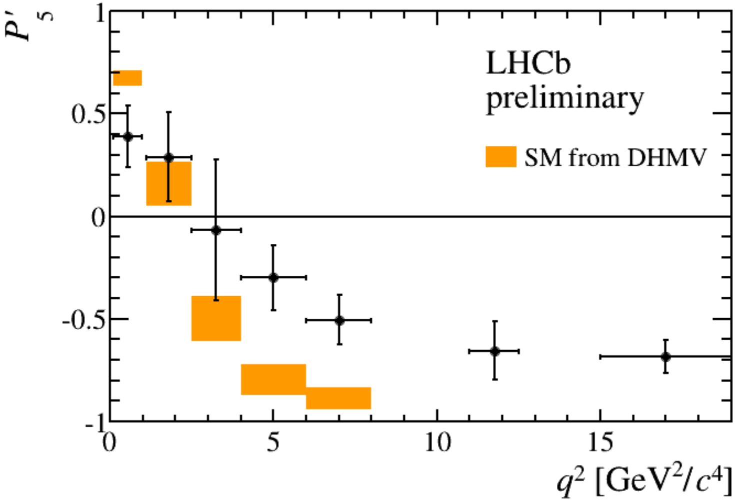
Based on work with David M. Straub, Roman Zwicky
(arXiv:1503.05534 [hep-ph])

Outline

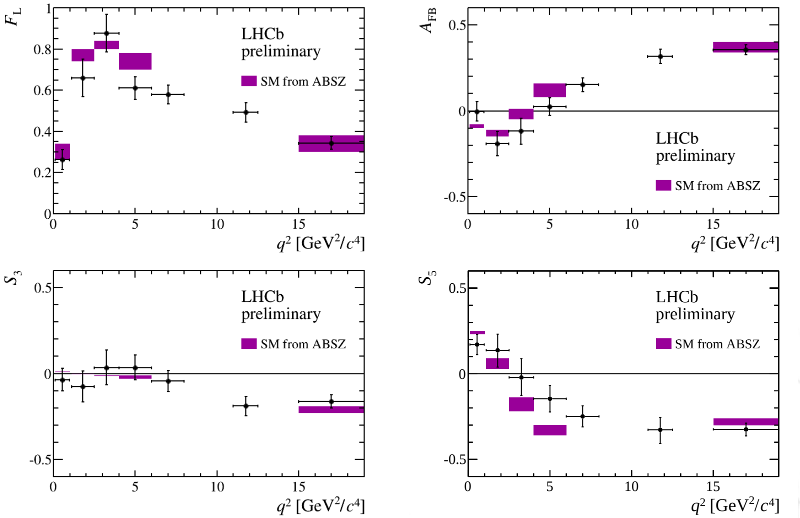
- Status of $b \to s$ decays at LHCb
- Overview of uncertainties
- Form factors with light cone sum rules

How the discrepancy evolved

Results from Moriond 2015



More results from Moriond 2015



Taken from LHCb-CONF-2015-002

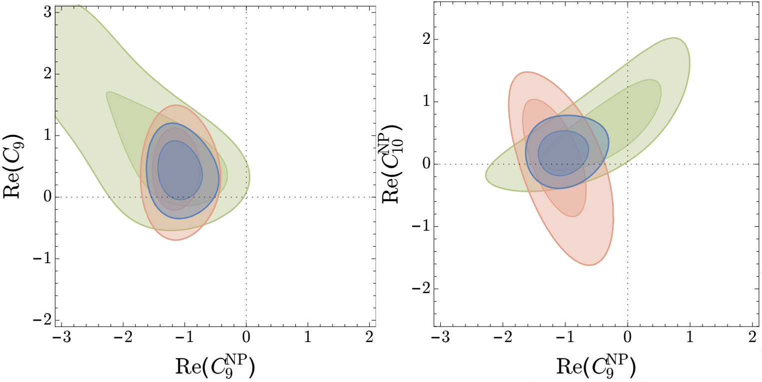
Summary of devations $> 1.9\sigma$

W. Altmannshofer and D. Straub, arXiv:1411.3161, arXiv:1503.06199v2

Decay	obs.	q^2 bin	SM pred.	measurement		pull
$\bar{B}^0 \to \bar{K}^{*0}\mu^+\mu^-$	F_L	$[2, 4.3]$	0.81 ± 0.02	0.26 ± 0.19	ATLAS	$+2.9$
$\bar{B}^0 \to \bar{K}^{*0}\mu^+\mu^-$	F_L	$[4, 6]$	0.74 ± 0.04	0.61 ± 0.06	LHCb	$+1.9$
$\bar{B}^0 \to \bar{K}^{*0}\mu^+\mu^-$	S_5	$[4, 6]$	-0.33 ± 0.03	-0.15 ± 0.08	LHCb	-2.2
$\bar{B}^0 \to \bar{K}^{*0}\mu^+\mu^-$	P_5'	$[1.1, 6]$	-0.44 ± 0.08	-0.05 ± 0.11	LHCb	-2.9
$\bar{B}^0 \to \bar{K}^{*0}\mu^+\mu^-$	P_5'	$[4, 6]$	-0.77 ± 0.06	-0.30 ± 0.16	LHCb	-2.8
$B^- \to K^{*-}\mu^+\mu^-$	$10^7 \frac{dBR}{dq^2}$	$[4, 6]$	0.54 ± 0.08	0.26 ± 0.10	LHCb	$+2.1$
$\bar{B}^0 \to \bar{K}^0\mu^+\mu^-$	$10^8 \frac{dBR}{dq^2}$	$[0.1, 2]$	2.71 ± 0.50	1.26 ± 0.56	LHCb	$+1.9$
$\bar{B}^0 \to \bar{K}^0\mu^+\mu^-$	$10^8 \frac{dBR}{dq^2}$	$[16, 23]$	0.93 ± 0.12	0.37 ± 0.22	CDF	$+2.2$
$B_s \to \phi\mu^+\mu^-$	$10^7 \frac{dBR}{dq^2}$	$[1, 6]$	0.48 ± 0.06	0.23 ± 0.05	LHCb	$+3.1$

Interpretation in term of NP

W. Altmannshofer and D. Straub, arXiv:1411.3161 [hep-ph]

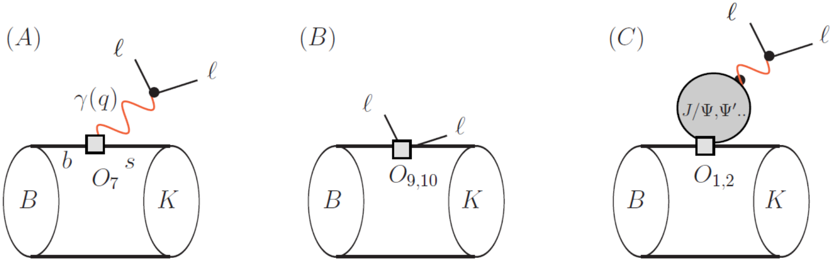


Dependence of Observables on Wilson coefficients

Observable	mostly affected by
S_1^s, S_1^c, S_2^s, S_2^c	C_7, C_7', C_9, C_9', C_{10}, C_{10}'
S_3	C_7', C_9', C_{10}'
S_4	C_7, C_7', C_{10}, C_{10}'
S_5	C_7, C_7', C_9, C_{10}'
S_6^s	C_7, C_9
A_7	C_7, C_7', C_{10}, C_{10}'
A_8	C_7, C_7', C_9, C_9', C_{10}'
A_9	C_7', C_9', C_{10}'
S_6^c	$C_S - C_S'$

Resonances gone topsy turvy

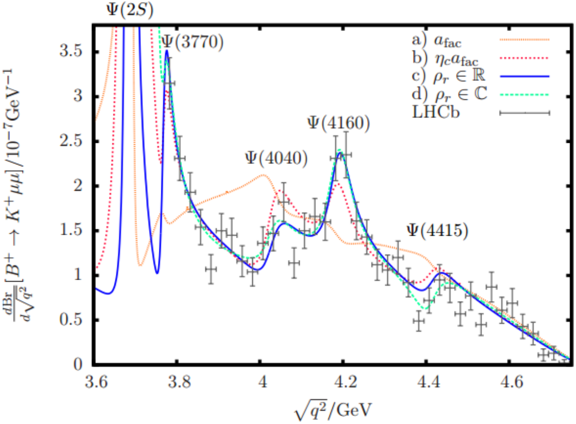
J. Lyon and R. Zwicky, arXiv:1406.0566 [hep-ph]



- Fit (C) using BES II data on e^+e^- to hadrons
- Defining $R(s) = \dfrac{\sigma(e^+e^- \to \mathrm{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$
- Can relate $R_c(s) = \frac{3}{2\pi i}\mathrm{Disc}(\mathrm{h_c(s)})$ where $h_c(s)$ parametrizes the charm loop contribution to $B \to K\ell^+\ell^-$

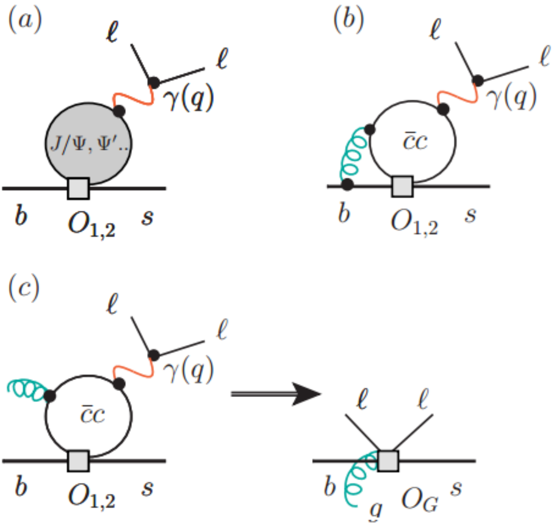
Resonances gone topsy turvy

J. Lyon and R. Zwicky, arXiv:1406.0566 [hep-ph]



Fit	η_B	η_c	$\rho_{\Psi(2S)}$	$\rho_{\Psi(3370)}$	$\rho_{\Psi(4040)}$	$\rho_{\Psi(4160)}$	$\rho_{\Psi(4415)}$	χ^2/d.o.f.	d.o.f.	pts	p-value
a)	0.98	$\equiv 1$	$\equiv 1$	$\equiv 1$	$\equiv 1$	$\equiv 1$	$\equiv 1$	3.59	99	117	$\simeq 10^{-30}$
b)	1.08	-2.55	$\equiv 1$	$\equiv 1$	$\equiv 1$	$\equiv 1$	$\equiv 1$	1.334	98	117	1.5%
c)	0.81	$\equiv 1$	-1.3	-7.2	-1.9	-4.6	-3.0	1.169	94	117	12%
d)	1.06	$\equiv 1$	$3.8-5.1i$ $6.4e^{-i53.3°}$	$-0.1-2.3i$ $2.0e^{-i92°}$	$-0.5-1.2i$ $1.3e^{-i111°}$	$-3.0-3.1i$ $4.3e^{-i135°}$	$-4.5+2.3i$ $5.1e^{i153°}$	1.124	89	117	20%

Additional QCD corrections



Resonances gone topsy turvy

J. Lyon and R. Zwicky, arXiv:1406.0566 [hep-ph]

Observable	q^2	LHCb	SM	η_c =-1.25(1, 1)	-2.5(0, 1)	-2.5(1, 0)
$\langle P_2 \rangle$	$[1.00, 6.00]$	$0.33^{+0.11}_{-0.12}$	0.0085	0.16	-0.013	0.33
$\langle P_2 \rangle$	$[2.00, 4.30]$	$0.50^{+0.00}_{-0.07}$	0.15	0.25	0.067	0.39
$\langle P_2 \rangle$	$[4.30, 8.68]$	$-0.25^{+0.07}_{-0.08}$	-0.44	-0.05	-0.23	0.29
$\langle P_2 \rangle$	$[14.18, 16.00]$	$-0.50^{+0.03}_{-0.00}$	-0.42	-0.39	-0.36	-0.36
$\langle P_2 \rangle$	$[16.00, 19.00]$	$-0.32^{+0.08}_{-0.08}$	-0.34	-0.31	-0.25	-0.25
$\langle P'_4 \rangle$	$[1.00, 6.00]$	$0.58^{+0.32}_{-0.36}$	0.57	0.66	0.8	0.64
$\langle P'_4 \rangle$	$[2.00, 4.30]$	$0.74^{+0.54}_{-0.60}$	0.61	0.69	0.82	0.67
$\langle P'_4 \rangle$	$[4.30, 8.68]$	$1.18^{+0.26}_{-0.32}$	1.0	1.0	1.2	0.98
$\langle P'_4 \rangle$	$[14.18, 16.00]$	$-0.18^{+0.54}_{-0.70}$	1.2	1.2	1.2	1.2
$\langle P'_4 \rangle$	$[16.00, 19.00]$	$0.70^{+0.44}_{-0.52}$	1.3	1.3	1.3	1.3
$\langle P'_5 \rangle$	$[1.00, 6.00]$	$0.21^{+0.20}_{-0.21}$	-0.44	-0.15	-0.33	0.17
$\langle P'_5 \rangle$	$[2.00, 4.30]$	$0.29^{+0.40}_{-0.39}$	-0.47	-0.17	-0.36	0.13
$\langle P'_5 \rangle$	$[4.30, 8.68]$	$-0.19^{+0.16}_{-0.16}$	-0.88	-0.31	-0.44	0.26
$\langle P'_5 \rangle$	$[14.18, 16.00]$	$-0.79^{+0.27}_{-0.22}$	-0.7	-0.66	-0.59	-0.61
$\langle P'_5 \rangle$	$[16.00, 19.00]$	$-0.60^{+0.21}_{-0.18}$	-0.53	-0.49	-0.39	-0.38
$\langle A_{FB} \rangle$	$[1.00, 6.00]$	$0.17^{+0.06}_{-0.06}$	0.0026	0.054	-0.0033	0.14
$\langle A_{FB} \rangle$	$[2.00, 4.30]$	$0.20^{+0.08}_{-0.08}$	0.034	0.069	0.014	0.15
$\langle A_{FB} \rangle$	$[4.30, 8.68]$	$-0.16^{+0.05}_{-0.06}$	-0.21	-0.025	-0.098	0.19
$\langle A_{FB} \rangle$	$[14.18, 16.00]$	$-0.51^{+0.05}_{-0.07}$	-0.43	-0.40	-0.36	-0.37
$\langle A_{FB} \rangle$	$[16.00, 19.00]$	$-0.30^{+0.08}_{-0.08}$	-0.35	-0.33	-0.26	-0.26

Form factors for exclusive $B \to V$

- Largest uncertainty in calculation is from form factors: non-perturbative quantities
- LCSR[1] at low q^2, Lattice[2] at high q^2
- Best coverage in q^2: fit to LCSR and Lattice using e.g. series expansion, coefficients satisfy dispersive bounds.[3]
- Many people resorting to using soft form factors with corrections in order to include correlations[4]
- **Our Aim:** resolve this by making correlations available!

---

[1] see e.g. P. Ball and R. Zwicky, Phys. Rev. D **71** (2005) 014015 [arXiv:hep-ph/0406232] and Phys. Rev. D **71** (2005) 014029 [arXiv:hep-ph/0412079]

[2] see e.g. A. Al-Haydari *et al.* [QCDSF Collaboration], Eur. Phys. J. A **43**, 107 (2010) [arXiv:0903.1664 [hep-lat]]

[3] AB, T. Feldmann, M. Wick, JHEP **1009** (2010) 090 [arXiv:1004.3249 [hep-ph]]

[4] e.g. S. Descotes-Genon, T. Hurth, J. Matias and J. Virto, JHEP **1305** (2013) 137 [arXiv:1303.5794 [hep-ph]], S. Jaeger and J. Martin Camalich, JHEP **1305** (2013) 043 [arXiv:1212.2263 [hep-ph]].

Form Factor Definitions

Express hadronic matrix elements via:

$$\langle K^*(p)|\bar{s}\gamma^\mu(1\mp\gamma_5)b|\bar{B}(p_B)\rangle = P_1^\mu\,\mathcal{V}_1(q^2) \pm P_{2,3}^\mu\,\mathcal{V}_{2,3}(q^2) \pm P_P^\mu \mathcal{V}_P(q^2)$$

$$\langle K^*(p)|\bar{s}iq_\nu\sigma^{\mu\nu}(1\pm\gamma_5)b|\bar{B}(p_B)\rangle = P_1^\mu\,T_1(q^2) \pm P_{2,3}^\mu\,T_{2,3}(q^2)$$

where the Lorentz structures P_i^μ are

$$P_P^\mu = i(\eta^*\cdot q)q^\mu\,,$$
$$P_1^\mu = 2\epsilon^\mu{}_{\alpha\beta\gamma}\eta^{*\alpha}p^\beta q^\gamma\,,$$
$$P_2^\mu = i\{(m_B^2 - m_{K^*}^2)\eta^{*\mu} - (\eta^*\cdot q)(p+p_B)^\mu\}\,,$$
$$P_3^\mu = i(\eta^*\cdot q)\{q^\mu - \frac{q^2}{m_B^2 - m_{K^*}^2}(p+p_B)^\mu\}$$

- Bjorken & Drell convention for the Levi-Civita tensor $\epsilon_{0123} = +1$
- η is the polarization of K^*
- Only 7 independent FFs

The equation of motion

Starting from

$$i\partial^\nu(\bar{s} i\sigma_{\mu\nu}(\gamma_5)b) \quad = -(m_s \pm m_b)\bar{s}\gamma_\mu(\gamma_5)b + i\partial_\mu(\bar{s}(\gamma_5)b) - 2\bar{s} i \overleftarrow{D}_\mu (\gamma_5)b\,,$$

We obtain the four equation of motion relations:

$$\begin{aligned} T_1(q^2) + (m_b + m_s)\mathcal{V}_1(q^2) + \mathcal{D}_1(q^2) &= 0\,, \\ T_2(q^2) + (m_b - m_s)\mathcal{V}_2(q^2) + \mathcal{D}_2(q^2) &= 0\,, \\ T_3(q^2) + (m_b - m_s)\mathcal{V}_3(q^2) + \mathcal{D}_3(q^2) &= 0\,, \\ (m_b - m_s)\mathcal{V}_P(q^2) + \left(\mathcal{D}_P(q^2) - \frac{q^2}{m_b + m_s}\mathcal{V}_P(q^2)\right) &= 0\,. \end{aligned}$$

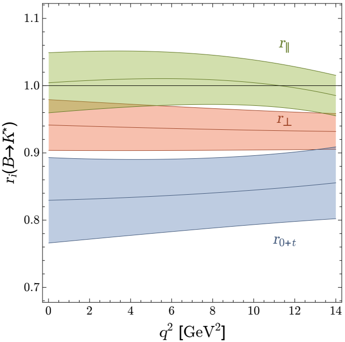
where the $\mathcal{D}_\iota$s are defined via

$$\langle K^*(p,\eta)|\bar{s}(2i\overleftarrow{D})^\mu(1\pm\gamma_5)b|\bar{B}(p_B)\rangle = P_1^\mu\,\mathcal{D}_1(q^2) \pm P_{2,3}^\mu\,\mathcal{D}_{2,3}(q^2) \pm P_P^\mu\mathcal{D}_P(q^2)$$

- Isgur-Wise relations at low recoil follow from $\mathcal{D}_\iota/(\mathcal{V}_\iota \text{ or } T_\iota) \sim \mathcal{O}(\Lambda_{\rm QCD}/m_b)$
- Are certain combinations of $\mathcal{D}_\iota$'s small at large recoil?
- $\iota = 1, 2$ are direct candidates, but $\iota = 3, P$ more tricky

Quantifying the EOM

Combine $\iota = 3, P$ to obtain potentially small ratio of $\mathcal{D}/T$



$$, r_{\perp}(q^2) = -\frac{(m_b + m_s)\mathcal{V}_1(q^2)}{T_1(q^2)},$$

$$r_{\parallel}(q^2) = -\frac{(m_b - m_s)\mathcal{V}_2(q^2)}{T_2(q^2)},$$

$$r_{0+t}(q^2) = -\frac{(m_b - m_s)(\mathcal{V}_2(q^2) - c_{23}(q^2)(\mathcal{V}_3(q^2) + \mathcal{V}_P(q^2)))}{T_2(q^2) - c_{23}(q^2)T_3(q^2)}$$

$$= -\frac{(m_b - m_s)(\mathcal{V}_0(q^2) - c_{23}\mathcal{V}_P(q^2))}{T_0(q^2)},$$

The deviation from unity (shown for $B \to K^*$) is a measure of the relative size of the derivative form factor with respect to the tensor and vector form factors.

Parameters and uncertainties

Choosing s_0 and $M2$

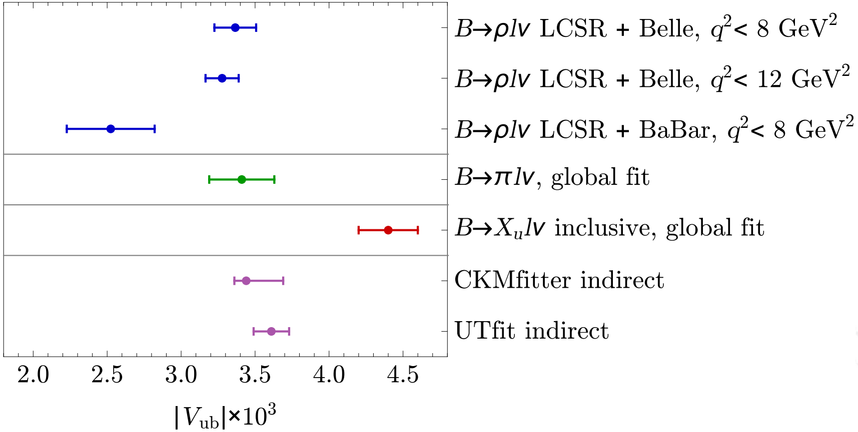
We carefully choose the sum rules parameters using the following:

- SR depends little on, but is clear extremum as fn of s_0, M^2, SR for m_B fulfilled, $(m_B^2 = \int_{m_b^2}^{s_0} ds\, s\, \rho^{\rm tot}(s) / \int_{m_b^2}^{s_0} ds\, \rho^{\rm tot}(s))$;
- the continuum and higher twist contributions should be under control$\lesssim 30\%, 10\%$ respectively;
- Correlate s_0 for EOM related FFs, and M^2 for $FF \times f_B$ and f_B 50%.

Dominant uncertainties arise due to varying the following:

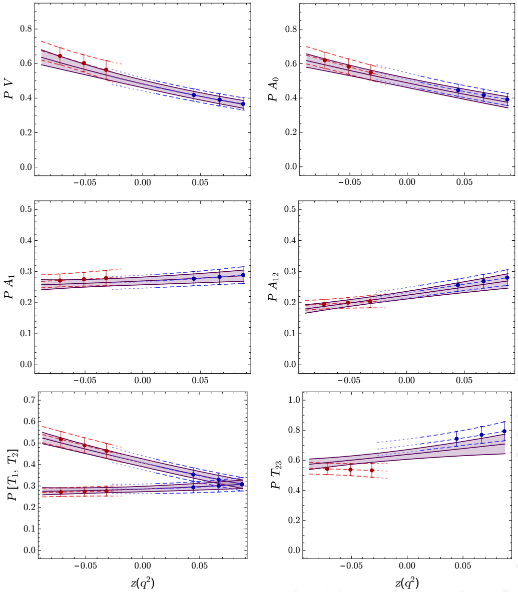
- the continuum threshold s_0 by $\pm 2\,\mathrm{GeV}^2$ and the Borel parameter M_2 by $\pm 1\,\mathrm{GeV}^2$;
- the condensates $\langle \bar{q}q \rangle = (-0.24 \pm 0.01)^3 \mathrm{GeV}^3$, $\frac{\langle \bar{q}\sigma g G q\rangle}{\langle \bar{q}q\rangle} = (0.8 \pm 0.2)$
- the twist-3 parameter η_3 by $\pm 50\%$;
- the factorisation scale in the range $\mu/2$ to 2μ.

The V_{ub} test



$B\to\rho l\nu$ LCSR + Belle, $q^2 < 8$ GeV^2
$B\to\rho l\nu$ LCSR + Belle, $q^2 < 12$ GeV^2
$B\to\rho l\nu$ LCSR + BaBar, $q^2 < 8$ GeV^2
$B\to\pi l\nu$, global fit
$B\to X_u l\nu$ inclusive, global fit
CKMfitter indirect
UTfit indirect
2.0
2.5
3.0
3.5
4.0
4.5
$|V_{ub}|\times 10^3$

Results for the form factors



Resulting Observables

$B^0 \to K^{*0}\mu^+\mu^-$		
Observable	q^2 bin	SM prediction
$10^7\ \frac{dBR}{dq^2}$	[0.1, 1]	$1.083 \pm 0.064 \pm 0.147 \pm 0.058$
	[1, 2]	$0.511 \pm 0.025 \pm 0.069 \pm 0.020$
	[2, 3]	$0.459 \pm 0.022 \pm 0.064 \pm 0.018$
	[3, 4]	$0.467 \pm 0.023 \pm 0.062 \pm 0.021$
	[4, 5]	$0.494 \pm 0.026 \pm 0.062 \pm 0.026$
	[5, 6]	$0.530 \pm 0.031 \pm 0.062 \pm 0.032$
A_{FB}	[0.1, 1]	$-0.088 \pm 0.001 \pm 0.009 \pm 0.001$
	[1, 2]	$-0.140 \pm 0.004 \pm 0.028 \pm 0.010$
	[2, 3]	$-0.078 \pm 0.005 \pm 0.018 \pm 0.019$
	[3, 4]	$0.002 \pm 0.005 \pm 0.008 \pm 0.025$
	[4, 5]	$0.077 \pm 0.004 \pm 0.016 \pm 0.029$
	[5, 6]	$0.144 \pm 0.004 \pm 0.025 \pm 0.030$
F_L	[0.1, 1]	$0.308 \pm 0.012 \pm 0.052 \pm 0.017$
	[1, 2]	$0.738 \pm 0.009 \pm 0.044 \pm 0.021$
	[2, 3]	$0.831 \pm 0.002 \pm 0.031 \pm 0.012$
	[3, 4]	$0.820 \pm 0.002 \pm 0.033 \pm 0.007$
	[4, 5]	$0.776 \pm 0.004 \pm 0.039 \pm 0.013$
	[5, 6]	$0.723 \pm 0.004 \pm 0.045 \pm 0.019$

Resulting Observables

$B^0 \to K^{*0}\mu^+\mu^-$		
Observable	q^2 bin	SM prediction
S_4	$[0.1, 1]$	$0.097 \pm 0.001 \pm 0.004 \pm 0.003$
	$[1, 2]$	$0.023 \pm 0.005 \pm 0.007 \pm 0.010$
	$[2, 3]$	$-0.081 \pm 0.005 \pm 0.012 \pm 0.013$
	$[3, 4]$	$-0.151 \pm 0.003 \pm 0.016 \pm 0.014$
	$[4, 5]$	$-0.198 \pm 0.002 \pm 0.017 \pm 0.013$
	$[5, 6]$	$-0.228 \pm 0.001 \pm 0.016 \pm 0.011$
S_5	$[0.1, 1]$	$0.247 \pm 0.002 \pm 0.008 \pm 0.005$
	$[1, 2]$	$0.119 \pm 0.008 \pm 0.016 \pm 0.021$
	$[2, 3]$	$-0.077 \pm 0.007 \pm 0.015 \pm 0.028$
	$[3, 4]$	$-0.212 \pm 0.005 \pm 0.019 \pm 0.028$
	$[4, 5]$	$-0.300 \pm 0.005 \pm 0.021 \pm 0.026$
	$[5, 6]$	$-0.356 \pm 0.004 \pm 0.019 \pm 0.022$

Alternative Observables

$B^0 \to K^{*0}\mu^+\mu^-$		
Observable	q^2 bin	SM prediction
P_4'	$[0.1, 1]$	$0.252 \pm 0.004 \pm 0.005 \pm 0.008$
	$[1, 2]$	$0.058 \pm 0.013 \pm 0.018 \pm 0.023$
	$[2, 3]$	$-0.232 \pm 0.015 \pm 0.026 \pm 0.043$
	$[3, 4]$	$-0.413 \pm 0.007 \pm 0.020 \pm 0.036$
	$[4, 5]$	$-0.487 \pm 0.003 \pm 0.016 \pm 0.023$
	$[5, 6]$	$-0.518 \pm 0.002 \pm 0.013 \pm 0.016$
P_5'	$[0.1, 1]$	$0.643 \pm 0.002 \pm 0.009 \pm 0.016$
	$[1, 2]$	$0.297 \pm 0.017 \pm 0.027 \pm 0.042$
	$[2, 3]$	$-0.223 \pm 0.023 \pm 0.044 \pm 0.086$
	$[3, 4]$	$-0.579 \pm 0.013 \pm 0.039 \pm 0.078$
	$[4, 5]$	$-0.738 \pm 0.014 \pm 0.033 \pm 0.056$
	$[5, 6]$	$-0.809 \pm 0.011 \pm 0.031 \pm 0.040$

Interpretation in term of NP

W. Altmannshofer and D. Straub, arXiv:1411.3161 [hep-ph]

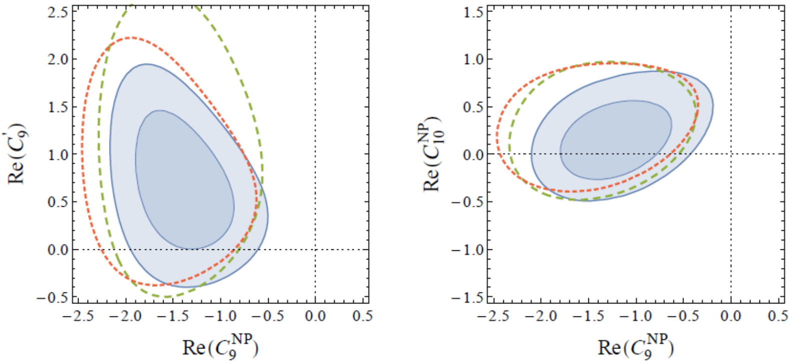


Figure 4: Allowed regions in the Re(C_9^{NP})-Re(C_9') plane (left) and the Re(C_9^{NP})-Re(C_{10}^{NP}) plane (right). The blue contours correspond to the 1 and 2σ best fit regions. The green and red short-dashed contours correspond to the 2σ regions in scenarios with doubled form factor uncertainties and doubled uncertainties from sub-leading non-factorizable corrections, respectively.

Summary

and Outlook

Anomalies in $b \to s$ transitions

- Anomalies in $B \to K^* \ell\ell$ angular observables
- NP in $C_{9/10}^{(\prime)}$ or large charm contribution?

[5] and to David Straub and Roman Zwicky; and Flip Tanedo for letting me use his beamer theme

Summary

and Outlook

Anomalies in $b \to s$ transitions

- Anomalies in $B \to K^*\ell\ell$ angular observables
- NP in $C^{(\prime)}_{9/10}$ or large charm contribution?

Updated LCSR calculation:

- Prevent community from resorting to soft form factors
- Full correlated errors and fit with Lattice using various parameterizations
- Latest input parameters and use of equation of mation

[5] and to David Straub and Roman Zwicky; and Flip Tanedo for letting me use his beamer theme

Summary

and Outlook

Anomalies in $b \to s$ transitions

- Anomalies in $B \to K^* \ell\ell$ angular observables
- NP in $C_{9/10}^{(\prime)}$ or large charm contribution?

Updated LCSR calculation:

- Prevent community from resorting to soft form factors
- Full correlated errors and fit with Lattice using various parameterizations
- Latest input parameters and use of equation of mation

Things for the future:

- Wait and see how experimental results unfold
- Improve understanding of non-perturbative charm contribution

Thanks for listening![5]

[5] and to David Straub and Roman Zwicky; and Flip Tanedo for letting me use his beamer theme

Summary

and Outlook

Anomalies in $b \to s$ transitions

- Anomalies in $B \to K^* \ell\ell$ angular observables
- NP in $C^{(\prime)}_{9/10}$ or large charm contribution?

Updated LCSR calculation:

- Prevent community from resorting to soft form factors
- Full correlated errors and fit with Lattice using various parameterizations
- Latest input parameters and use of equation of mation

Things for the future:

- Wait and see how experimental results unfold
- Improve understanding of non-perturbative charm contribution

Thanks for listening![5]

[5] and to David Straub and Roman Zwicky; and Flip Tanedo for letting me use his beamer theme