Institute for Research in Fundamental Sciences

Constraints on Wilson Coefficients from $b \to s$ transitions

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$b \to s$ transitions

loop suppressed in the SM

⇨ Very sensitive to New Physics

$b \to s$ transitions are multi-scale processes ($M_W, m_b, \Lambda_{\text{QCD}}$)

Low and high energies separated with **O**perator **P**roduct **E**xpansion

$$H_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{1,10,S,P} (C_i\, O_i + C_i'\, O_i')$$

- Long distance: represented by local operators - O_i
- Short distance: Wilson coefficients - C_i

  contain all the high energy physics effects

New physics in the effective framework:

- Modified Wilson coefficients: $C_i = C_i^{\text{SM}} + C_i^{\text{NP}}$
- Additional New Physics operators: $\Sigma_i C_i^{\text{NP}}\, O_i^{\text{NP}}$

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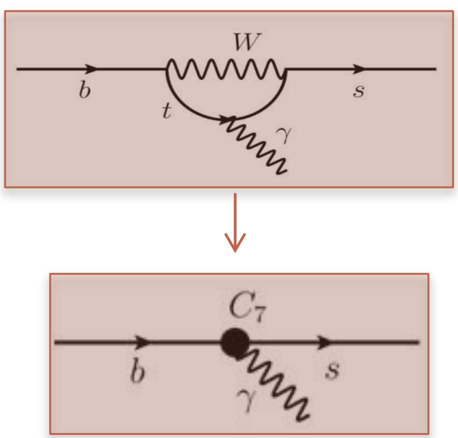
$b \to s$ transitions are multi-scale processes $(M_W, m_b, \Lambda_{\mathrm{QCD}})$

Low and high energies separated with **O**perator **P**roduct **E**xpansion

$$H_{\mathrm{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{1,10,S,P} (C_i \, O_i + C_i' \, O_i')$$

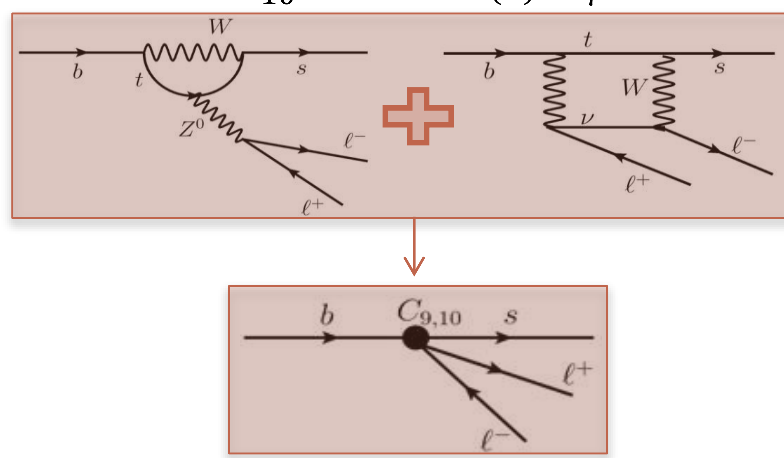
- Electromagnetic dipole

$$O_7^{(\prime)} \propto m_b \, \bar{s} \sigma_{\mu\nu} P_{R(L)} b \, F^{\mu\nu}$$



- Semileptonic

$$O_9^{(\prime)} \propto \bar{s} \, \gamma^{\mu} P_{L(R)} \, \bar{\ell} \gamma_{\mu} \ell$$
$$O_{10}^{(\prime)} \propto \bar{s} \, \gamma^{\mu} P_{L(R)} \, \bar{\ell} \gamma_{\mu} \gamma_5 \ell$$



Many experimental data available, each sensitive to one or more Wilson coefficient

Inclusive decays

- $B \to X_s \gamma\ (BR)$ → $C_7^{(\prime)}$
- $B \to X_s \ell^+\ell^-\ (BR)$ → fixed combination of $C_7^{(\prime)}, C_9^{(\prime)}, C_{10}^{(\prime)}$

Large experimental uncertainties, improvement expected from Belle II

Exclusive decays

- $B \to K^* \gamma\ (BR)$ → $C_7^{(\prime)}$
- $B_s \to \mu^+\mu^-\ (BR)$ → fixed combination of $C_{10}^{(\prime)}, C_S^{(\prime)}, C_P^{(\prime)}$
- $B \to K\ \ell^+\ell^-$ (BR, angular obs.)
- $B \to K^*\ \ell^+\ell^-$ (BR, angular obs.)
- $B_s \to \phi\ \ell^+\ell^-$ (BR, angular obs.)

} various combinations of $C_7^{(\prime)}, C_9^{(\prime)}, C_{10}^{(\prime)}$

Observed in experiment: $B \to K^* (\to K^+\pi^-)\ell^+\ell^-$

Angular behavior of K^+ and π^- $\longrightarrow$ additional information on the helicity of K^*

Diff. decay distribution described by dilepton invariant mass q^2 and three angles $\theta_{K^*}, \theta_\ell, \phi$

$$\frac{d^4\Gamma}{dq^2 dcos\theta_{K^*} dcos\theta_\ell dcos\phi} = \frac{9}{32\pi} J(q^2, \theta_{K^*}, \theta_\ell, \phi)$$

$J(q^2, \theta_{K^*}, \theta_\ell, \phi) = \Sigma_i J_i(q^2) f_i(\theta_{K^*}, \theta_\ell, \phi)$:

Angular coefficients J_{1-9} $\longrightarrow$ transversity amplitudes $A_\perp^{L,R}, A_\parallel^{L,R}, A_0^{L,R}, A_t^{L,R}, A_S^{L,R}$

- Wilson coefficients $C^{(\prime)}_{1-6,\,8}, C^{(\prime)}_7, C^{(\prime)}_9, C^{(\prime)}_{10}$
- 7 independent form factors : $V, A_{0,1,2}, T_{1,2,3}$

Standard observables:

Differential decay distribution: $\frac{d\Gamma}{dq^2} = \frac{3}{4}(J_1 - \frac{J_2}{3})$

Forward backward asymmetry: $A_{FB}(q^2) \equiv [\int_0^1 - \int_0^1]\, dcos\theta_\ell \frac{d^2\Gamma}{dq^2 dcos\theta_\ell} / \frac{d\Gamma}{dq^2} = \frac{3}{8} J_6 / \frac{d\Gamma}{dq^2}$

Forward backward asymmetry zero crossing : $q_0^2 \simeq -2 m_b m_B \frac{C_9^{eff}(q_0^2)}{C_7} + O(\alpha_s, \Lambda/m_b)$

Longitudinal polarization fraction: $F_L = \frac{|A_0^2|}{|A_\perp|^2 + |A_\parallel|^2 + |A_0|^2} = -2 J_2^c / \frac{d\Gamma}{dq^2}$

Besides zero crossing, at leading order dependent on Form Factors

Many other angular observables…

- minimize form factor uncertainties
- sensitive to specific Wilson coefficients

Optimized obervables:

$$\langle P_1 \rangle_{\rm bin} = \frac{1}{2} \frac{\int_{\rm bin} dq^2 [J_3 + \bar{J}_3]}{\int_{\rm bin} dq^2 [J_{2s} + \bar{J}_{2s}]} \qquad \langle P_2 \rangle_{\rm bin} = \frac{1}{8} \frac{\int_{\rm bin} dq^2 [J_{6s} + \bar{J}_{6s}]}{\int_{\rm bin} dq^2 [J_{2s} + \bar{J}_{2s}]}$$

$$\langle P_4' \rangle_{\rm bin} = \frac{1}{\mathcal{N}'_{\rm bin}} \int_{\rm bin} dq^2 [J_4 + \bar{J}_4] \qquad \langle P_5' \rangle_{\rm bin} = \frac{1}{2\mathcal{N}'_{\rm bin}} \int_{\rm bin} dq^2 [J_5 + \bar{J}_5]$$

$$\langle P_6' \rangle_{\rm bin} = \frac{-1}{2\mathcal{N}'_{\rm bin}} \int_{\rm bin} dq^2 [J_7 + \bar{J}_7] \qquad \langle P_8' \rangle_{\rm bin} = \frac{-1}{\mathcal{N}'_{\rm bin}} \int_{\rm bin} dq^2 [J_8 + \bar{J}_8]$$

with $\mathcal{N}'_{\rm bin} = \sqrt{- \int_{\rm bin} dq^2 [J_{2s} + \bar{J}_{2s}] \int_{\rm bin} dq^2 [J_{2c} + \bar{J}_{2c}]}$

+ CP violating observables and other combinations

U. Egede et al., JHEP 0811 (2008) 032
U. Egede et al.,JHEP 1010 (2010) 056
J. Matias et al., JHEP 1204 (2012) 104
S. Descotes-Genon et al., JHEP 1305 (2013) 137

Or alternatively :

$$S_i = \frac{J_i^{(s,c)} + \bar{J}_i^{(s,c)}}{\frac{d\Gamma}{dq^2} + \frac{d\bar{\Gamma}}{dq^2}}$$

W. Altmannshofer et al., JHEP 0901 (2009) 019

At low q^2, two theoretical approaches:

"Full FF" = "Soft FF" + $(\frac{1}{m_b}, \alpha_s)$

"Full FF"

all 7 indep. FF ($V, A_{0,1,2}, T_{1,2,3}$)

Con: correlations among FF
uncertainties need to be provided

Pro: includes factorizable corrections by default

"Soft FF"

Exploring symmetries → 2 indep. FF ($\xi_\perp, \xi_\parallel$)

factorisable corrections
α_s – known analytically
$\frac{1}{m_b}$ – unknown → *fact. power corrections*
↘ dimensional arguments
↘ fit with full FF

Con: analytically unknown $1/m_b$ corrections

Pro: correlations among the Full FF considered, only by construction (two indep. FF)

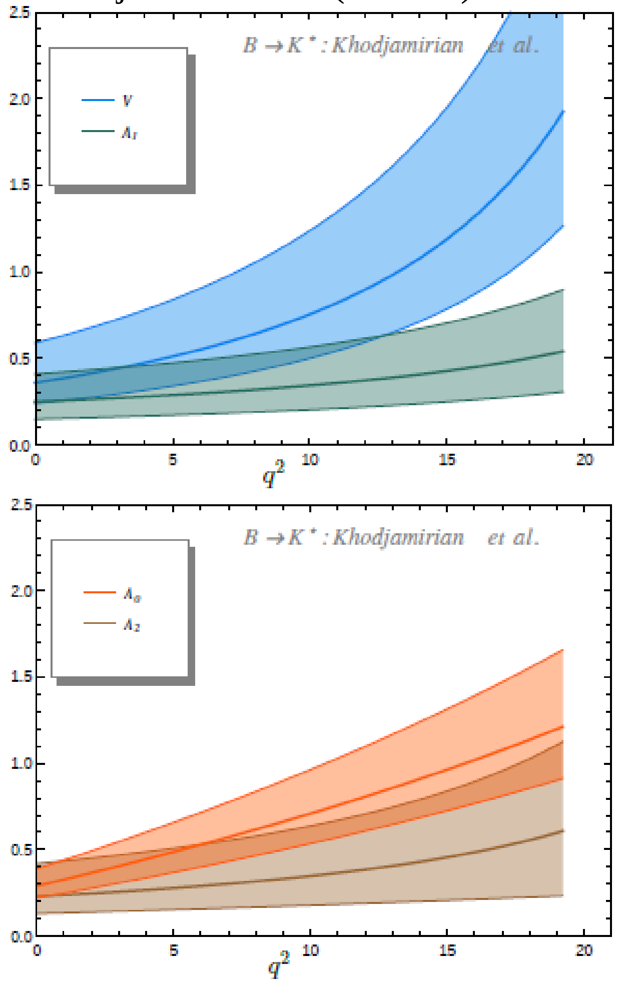
Both methods receive contributions from non-local 4-quark operators O_{1-6} & O_8

↘ non-fact. corrections → calculated in QCD factorization at LO in $\left(\frac{\Lambda}{m_b}, \frac{\Lambda}{E_{K^*}}\right)$

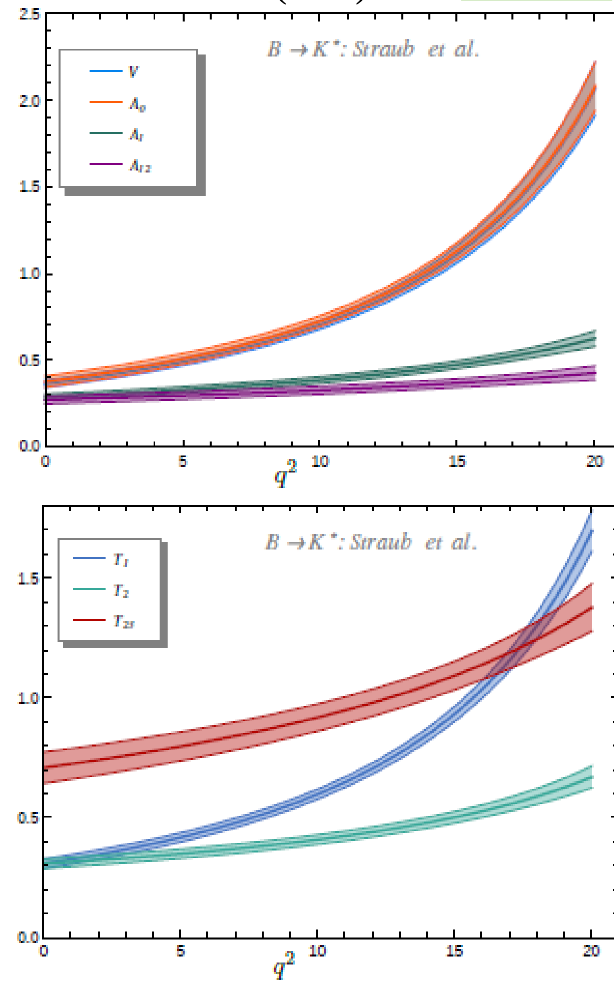
higher powers of $\left(\frac{\Lambda}{m_b}, \frac{\Lambda}{E_{K^*}}\right)$: unknown → *non-fact. power corrections*

Two main LCSR results for form factors at low q^2

Khodjamirian et al. (KMPW) 1006.4945



Bharucha et al. (BSZ) 1503.05534



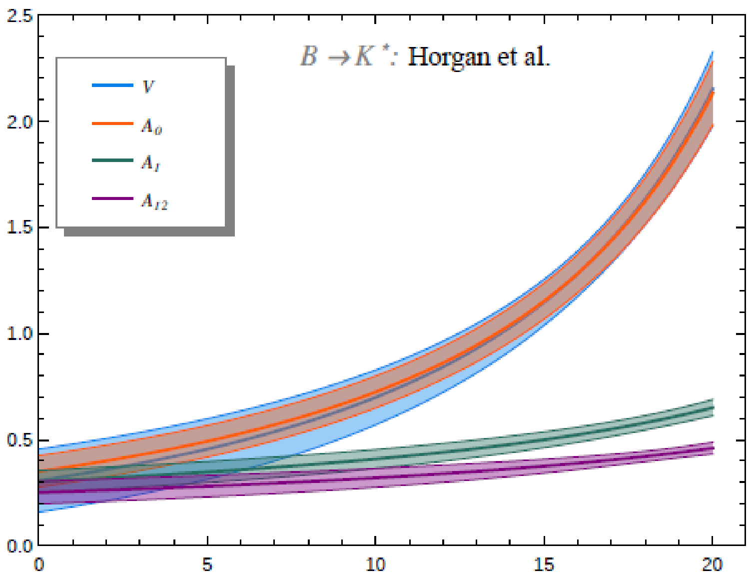
BSZ form factors, smaller th. uncertainty compared to KMPW:

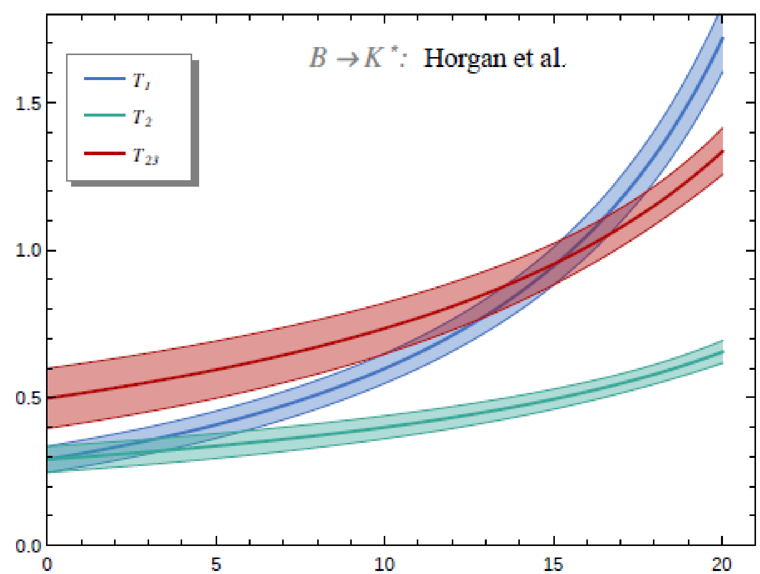
- Different choice of wave function
- Interpolation with lattice results

correlations of the uncertainties included

Form factors from lattice results

At high q^2 lattice results from Horgan et al. 1501.00367





correlations of the uncertainties included

"Lattice + LCSR" fit of BSZ applicable for whole q^2 region

SM predictions in soft FF and full FF using KMPW and BSZ form factor results

$B^0 \to K^{*0}\mu^+\mu^-$

Observable	Soft FF (KMPW)	Full FF (KMPW)	Soft FF (BSZ)	Full FF (BSZ)
		$\langle 10^7 \times BR \rangle$		
$q^2 \in [0.1\,,\,2.0]\,\mathrm{GeV}^2$	1.379	1.379	1.577	1.573
$q^2 \in [2.0\,,\,4.3]\,\mathrm{GeV}^2$	0.801	0.793	0.956	0.994
$q^2 \in [4.3\,,\,8.68]\,\mathrm{GeV}^2$	2.082	1.969	2.110	2.281
		$\langle F_L \rangle$		
$q^2 \in [0.1\,,\,0.98]\,\mathrm{GeV}^2$	0.176	0.175	0.241	0.249
$q^2 \in [1.1\,,\,2.5]\,\mathrm{GeV}^2$	0.645	0.634	0.716	0.725
$q^2 \in [2.5\,,\,4.0]\,\mathrm{GeV}^2$	0.768	0.769	0.814	0.809
$q^2 \in [4.0\,,\,6.0]\,\mathrm{GeV}^2$	0.714	0.731	0.758	0.741
$q^2 \in [6.0\,,\,8.0]\,\mathrm{GeV}^2$	0.612	0.635	0.646	0.626
		$\langle P_5' \rangle$		
$q^2 \in [0.1\,,\,0.98]\,\mathrm{GeV}^2$	0.658	0.658	0.655	0.661
$q^2 \in [1.1\,,\,2.5]\,\mathrm{GeV}^2$	0.252	0.246	0.252	0.252
$q^2 \in [2.5\,,\,4.0]\,\mathrm{GeV}^2$	−0.401	−0.413	−0.399	−0.387
$q^2 \in [4.0\,,\,6.0]\,\mathrm{GeV}^2$	−0.769	−0.821	−0.767	−0.718
$q^2 \in [6.0\,,\,8.0]\,\mathrm{GeV}^2$	−0.888	−0.948	−0.873	−0.816

- Soft FF and full FF approaches give very similar SM predictions (difference < 10%)
- SM predictions more sensitive to choice of form factor (KMPW or BSZ)

Experimental Results

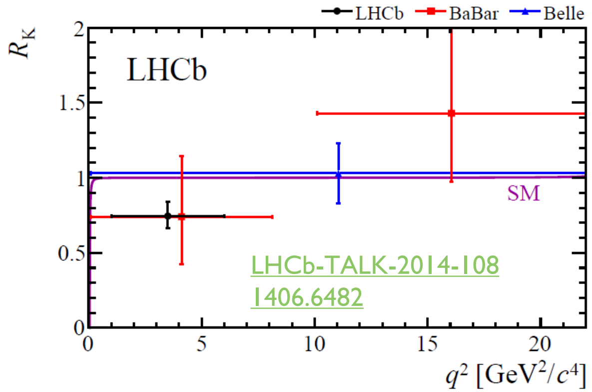
Most but not all, good agreement between SM prediction and measurement

Three main anomalies from LHCb:

- $R_K = \frac{BR(B^+ \to K^+ \mu^+ \mu^-)}{BR(B^+ \to K^+ e^+ e^-)}$

2.6σ tension in

[1-6] GeV^2 bin

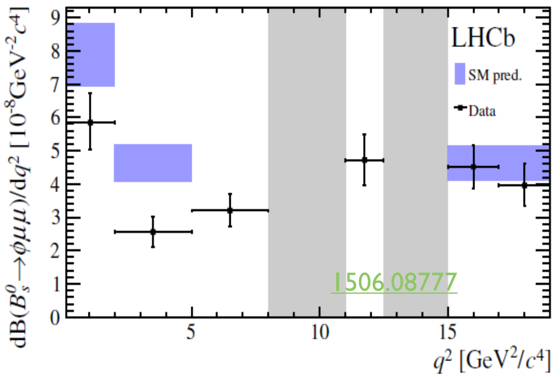


- Theoretically very clean

- $BR(B_s \to \phi\, \mu^+\, \mu^-)$

3.2σ tension in

[1-6] GeV^2 bin

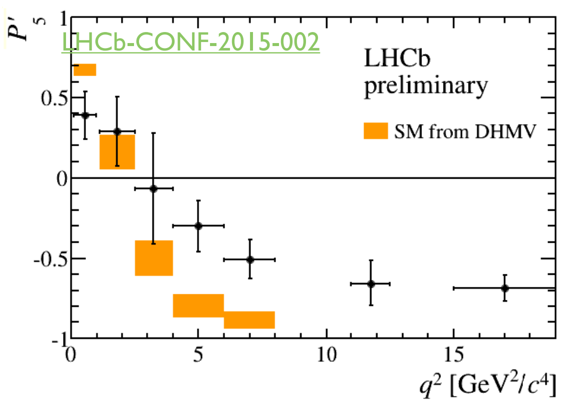


- Large theoretical uncertainty

- $P_5' \ (B \to K^* \mu^+\, \mu^-)$

3.7σ tension combining

[4-6] and [6-8] GeV^2 bins



- Theoretically clean?

→ possible issues from $c\bar{c}$ resonances

→ power corrections

Tensions depend on SM predictions, not the same for different groups

- Different SM Wilson coefficient
- Hadronic input parameters: decay constants, inverse moments, …
- Different choices for form factors
- Soft FF or Full FF approach
- …

Possible explanations for the LHCb anomalies

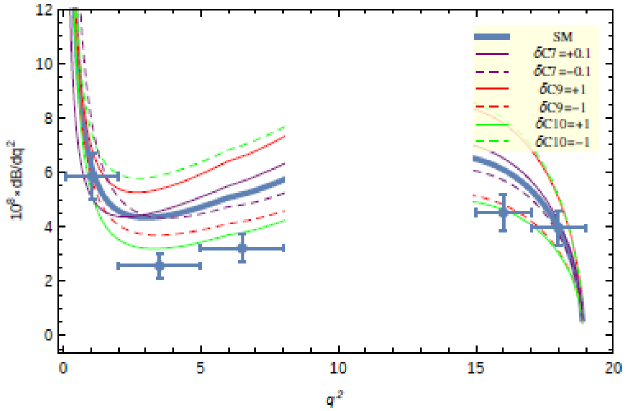
- Statistical fluctuations
- Theoretical issues
- New Physics!

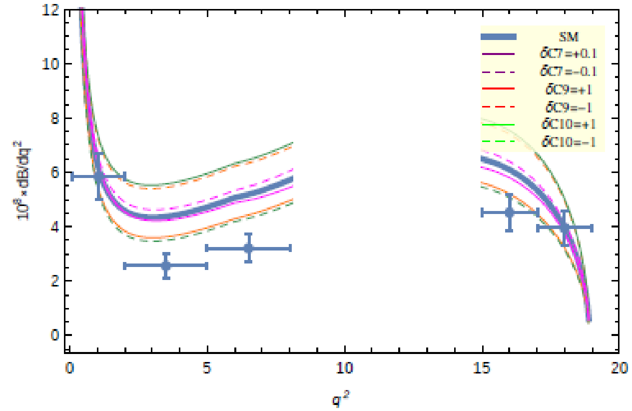
NP manifest itself in term of modified Wilson coefficients: $C_i = C_i^{SM} + \delta C_i$

- R_K ⟶ lepton non-universality $C_i^{\mu} \neq C_i^{e}$

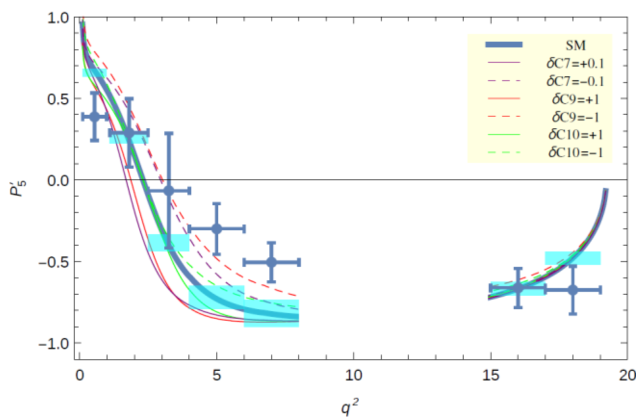
Effect of benchmark contributions to Wilson coefficients

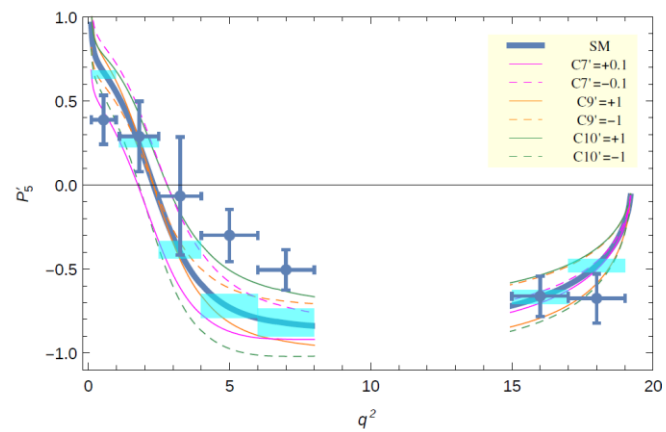
- $BR(B_s \to \phi\, \mu^+ \mu^-)$





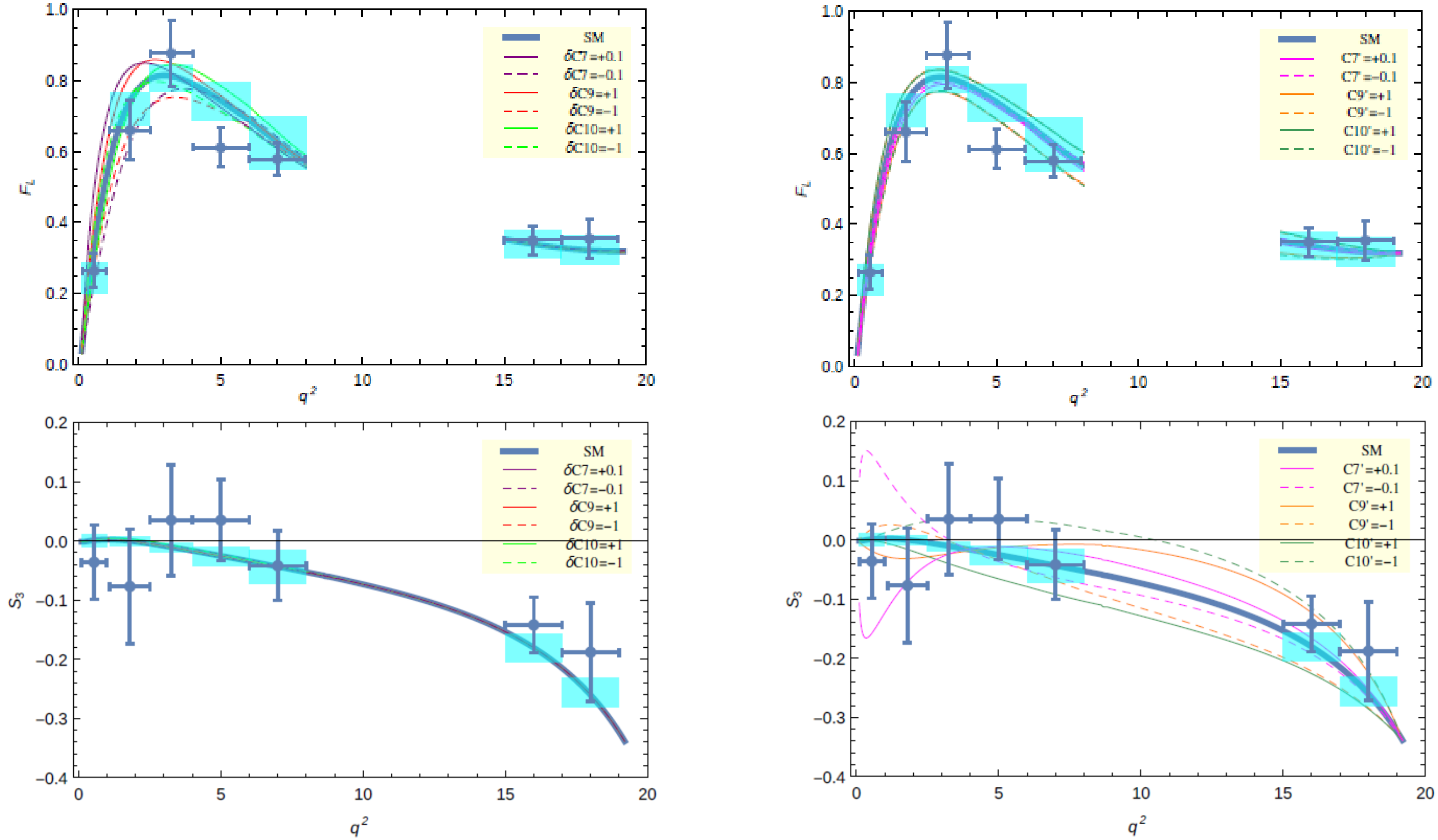
- $P_5'(B \to K^* \mu^+ \mu^-)$





Various observables are interdependent through Wilson coefficients

δC_i effect on some of the other observables (F_L, S_3)



- Sensitivity to C_i different for various obs. and bins
- a specific δC_i while reducing tension for one observable can increase tension in other observables

⮑ *global analysis required*

Global analysis of the latest LHCb data

Relevant Wilson coefficients:

$$C_7^{(\prime)}, C_9^{(\prime)}, C_{10}^{(\prime)}$$

With SuperIso

- Scan over the values of δC_i
- Calculation of flavour observables

- Comparison with experimental results
- Constraints on the Wilson coefficients C_i

Evaluations of uncertainties and correlations:

- Experimental errors and correlations

  3 fb^{-1} LHCb data for $B \to K^* \mu^+\mu^-$: provided in LHCb-CONF-2015-002

- Theoretical uncertainties and correlations
  - study of more than 100 observables

    (at a later stage, selection of the relevant operators for each fit)
  - Monte Carlo analysis
  - variation of the "standard" input parameters: masses, scales, CKM, ...
  - for $B_s \to \phi\, \mu^+\mu^-$ mixing effects taken into account from 1502.05509
  - decay constants taken from the latest lattice results
  - using the $B_{(s)} \to V$ form factors of the lattice+LCSR combinations from 1503.05534 (BSZ) including correlations
  - using the $B \to K$ form factors of the lattice+LCSR combinations from 1411.3161, (AS) including correlations
  - for the exclusive decays, two approaches: soft form factors, full form factors
  - two sets of hypotheses for the uncertainties associated to the factorisable and non-factorisable power corrections

$\Longrightarrow$ Computation of a (theory + exp) correlation matrix

For the exclusive semi-leptonic decays, two approaches and two evaluations of the uncertainties for each decay

At **low $\boldsymbol{q^2}$**:

- Soft form factor approach

  uncertainties of the **factorisable** and **non-factorisable power** corrections parametrised as

$$A_k \to A_k \left(1 + a_k \exp(i\phi_k) + \frac{q^2}{6\ \mathrm{GeV}^2} b_k \exp(i\theta_k)\right)$$

  where A_k are the transversity amplitudes $A_\perp, A_\parallel, A_0, A_t, A_S$

  $a_k \in [-10\%, +10\%]$ or $[-20\%, +20\%]$ $\qquad \phi_k, \theta_k \in [-\pi, +\pi]$

  $b_k \in [-25\%, +25\%]$ or $[-50\%, +50\%]$

- Full form factor approach

  uncertainties of the **non-factorisable power** corrections parametrised in a similar way

  $a_k \in [-5\%, +5\%]$ or $[-10\%, +10\%]$ $\qquad \phi_k, \theta_k \in [-\pi, +\pi]$

  $b_k \in [-10\%, +10\%]$ or $[-25\%, +25\%]$

At **high $\boldsymbol{q^2}$**, uncertainties parametrised as

$$A_k \to A_k\big(1 + a_k \exp(i\phi_k)\big)$$

$a_k \in [-10\%, +10\%]$ or $[-20\%, +20\%]$ $\qquad \phi_k \in [-\pi, +\pi]$

Global fits of observables by minimization of

$$\chi^2 = \left(\vec{O}^{\mathrm{th}} - \vec{O}^{\mathrm{exp}}\right) \cdot \left(\Sigma_{\mathrm{th}} + \Sigma_{\mathrm{exp}}\right)^{-1} \cdot \left(\vec{O}^{\mathrm{th}} - \vec{O}^{\mathrm{exp}}\right)$$

$\left(\Sigma_{\mathrm{th}} + \Sigma_{\mathrm{exp}}\right)^{-1}$ is the inverse covariance matrix

58 observables considered for leptonic and semileptonic decays:

- $\mathrm{BR}(B \to X_s\,\gamma)$
- $\mathrm{BR}(B \to X_d\,\gamma)$
- $\Delta_0(B \to K^*\,\gamma)$
- $\mathrm{BR}^{\mathrm{low}}(B \to X_s\,\mu^+\mu^-)$
- $\mathrm{BR}^{\mathrm{high}}(B \to X_s\,\mu^+\mu^-)$
- $\mathrm{BR}^{\mathrm{low}}(B \to X_s\,e^+e^-)$
- $\mathrm{BR}^{\mathrm{high}}(B \to X_s\,e^+e^-)$
- $\mathrm{BR}(B_s \to \mu^+\mu^-)$
- $\mathrm{BR}(B_d \to \mu^+\mu^-)$
- $\mathrm{BR}(B \to K^{*+}\mu^+\mu^-)$
- $\mathrm{BR}(B \to K^0\mu^+\mu^-)$
- $\mathrm{BR}(B^+ \to K^+\mu^+\mu^-)$
- $\mathrm{BR}(B \to K^*e^+e^-)$
- R_K
- $B \to K^{*0}\mu^+\mu^-$: $F_L, A_{FB}, S_3, S_4, S_5$ in five low q^2 and two high q^2 bins
- $B_s \to \phi\,\mu^+\mu^-$: BR, F_L in three low q^2 and two high q^2 bins

Statistical approach:

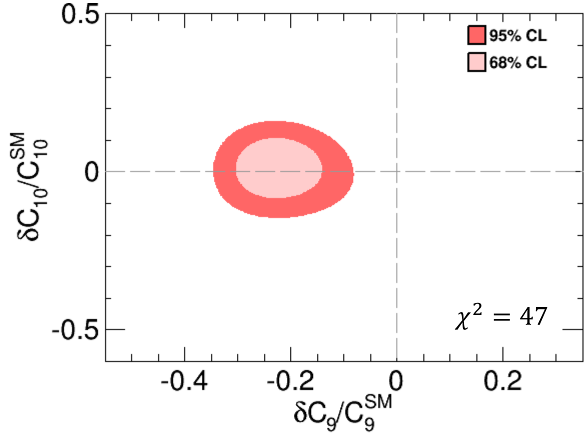
1. Determination of the minimum of χ^2 $\longrightarrow$ best fit point
2. Computation for each point of the scan the difference between χ^2 of that point with the χ^2 of best fit point
3. Find the $1-2\sigma$ regions corresponding to the number of d.o.f.

Interpretation: considering the best fit point gives the “real” description, which variations of the parameters are allowed

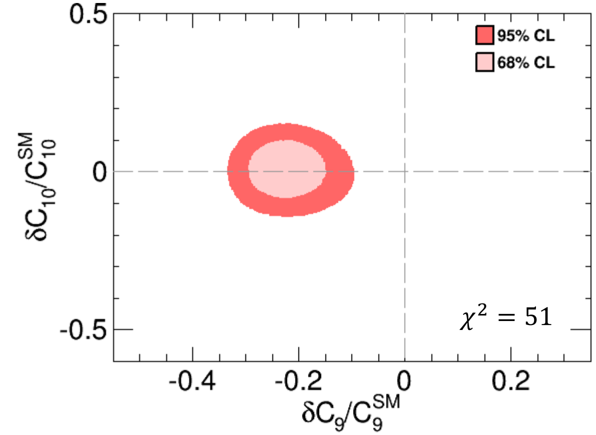
Fit results for two operators $\{C_9, C_{10}\}$

Soft form factor approach | **Full form factor approach**

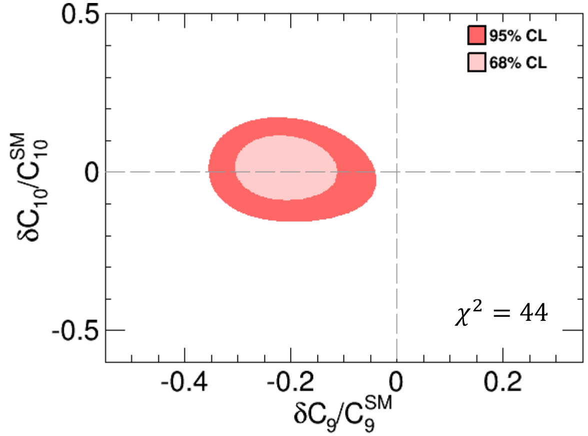
with 10% power corrections



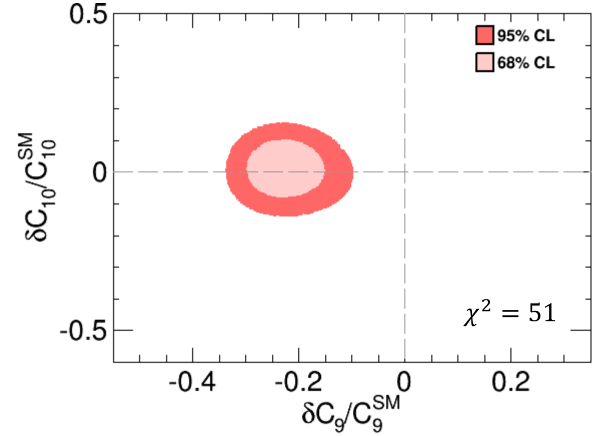
with 5% power corrections



with 20% power corrections



with 10% power corrections

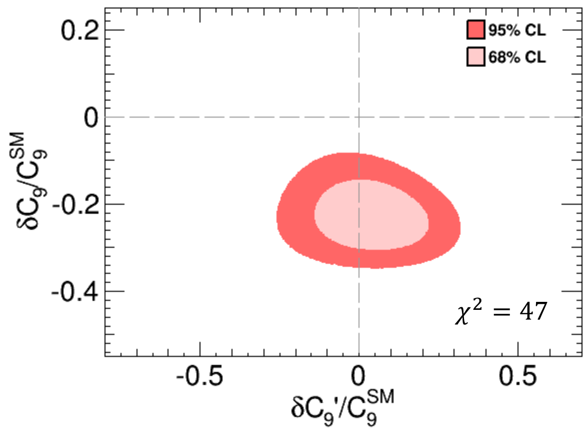


- C_9 in more than 2σ tension with SM value, no tension in C_{10}
- Going from 10% to 20% power correction in the soft FF approach slightly decrease tension
- Going from 5% to 10% power correction in the full FF approach has no significant effect

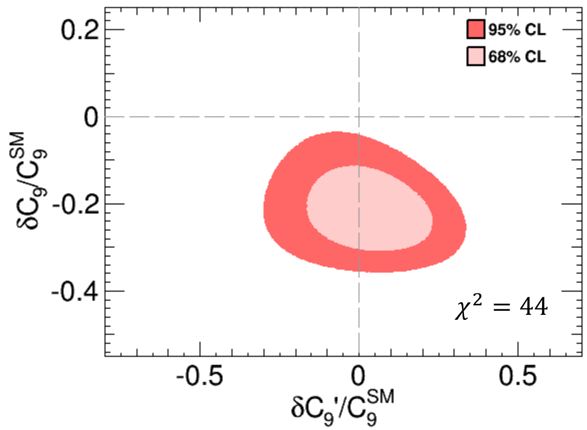
Fit results for two operators $\{C_9', C_9\}$

Soft form factor approach

with 10% power corrections

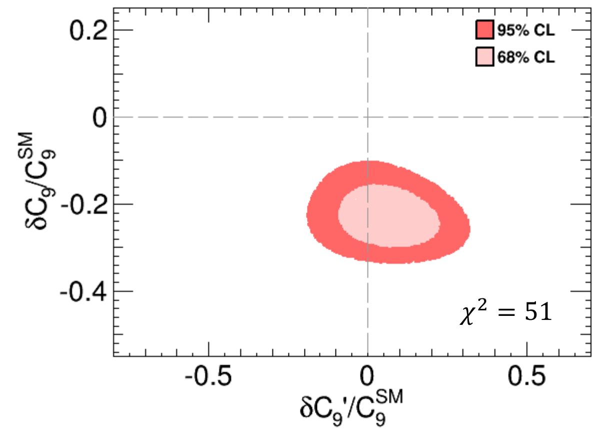


with 20% power corrections

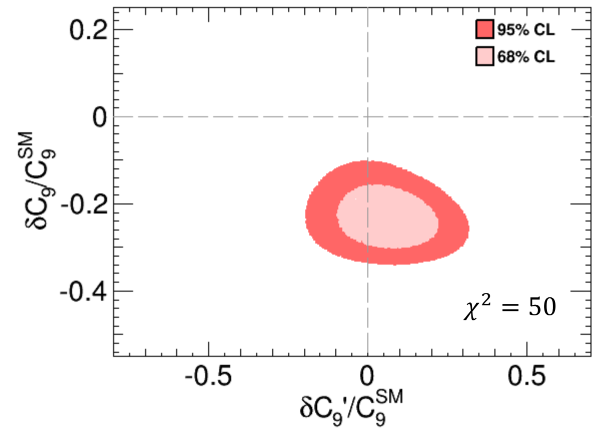


Full form factor approach

with 5% power corrections



with 10% power corrections

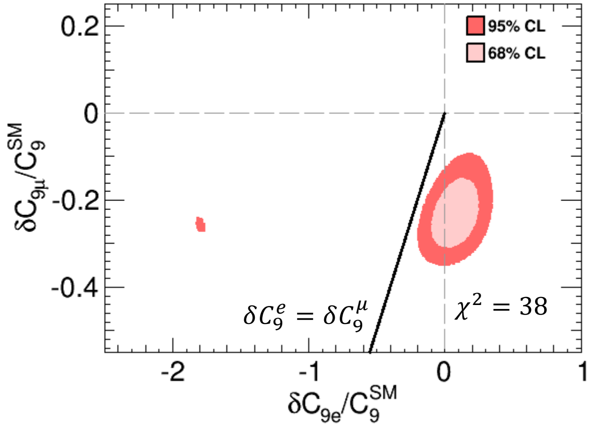


- Having C_9' in the fit still C_9 is in more than 2σ tension with SM value, no tension for C_9'
- Fit does not improve ⟶ no preference for a modified C_9' or C_{10}

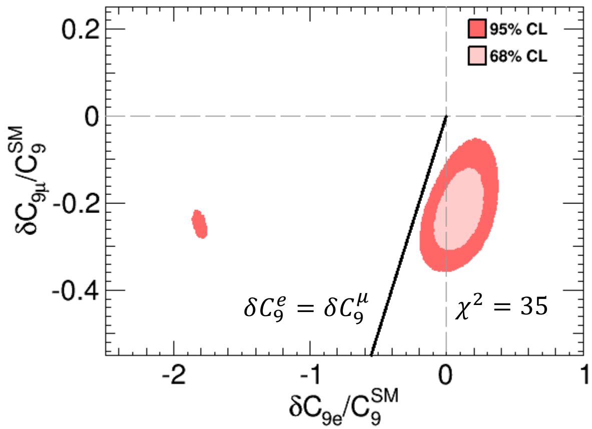
Fit results for two operators $\{C_9^e, C_9^\mu\}$

Soft form factor approach

with 10% power corrections

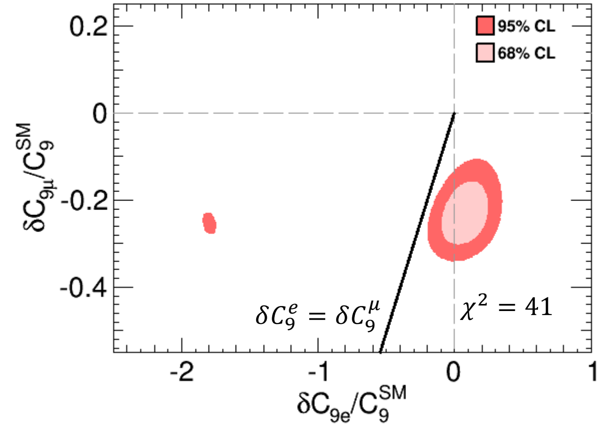


with 20% power corrections

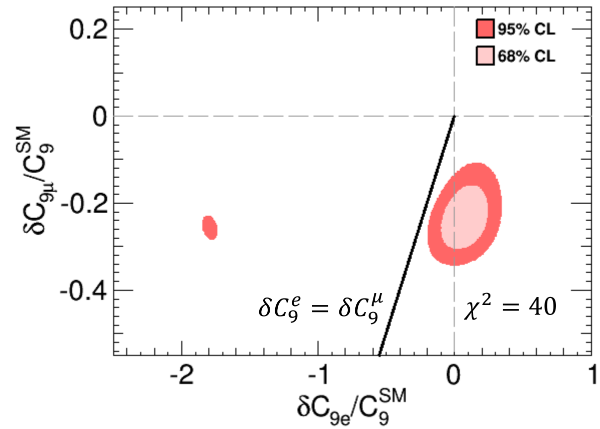


Full form factor approach

with 5% power corrections



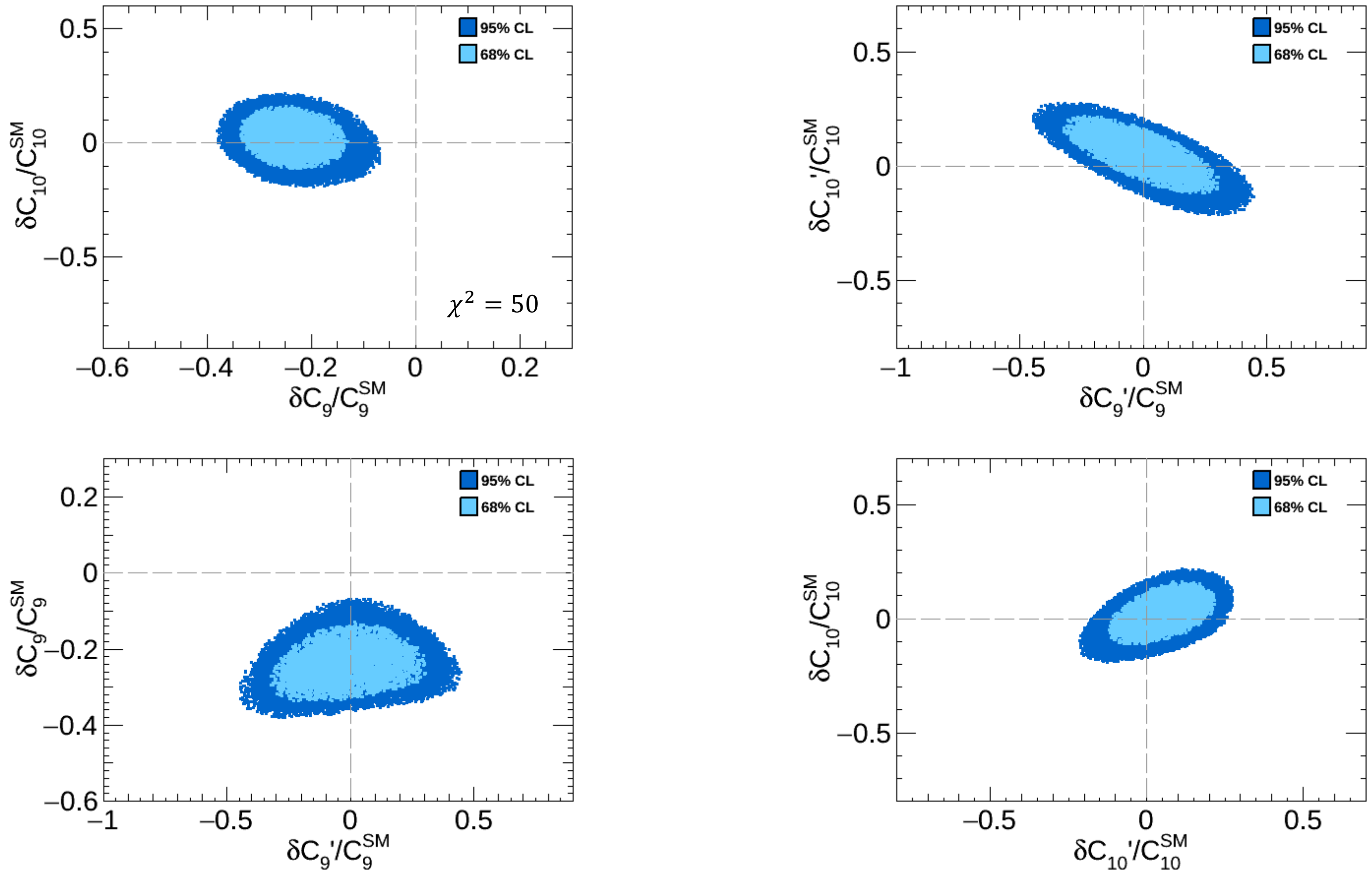
with 10% power corrections



- More than 2σ tension for C_9^μ, non-universality improves the fit
- Universality condition $\left(\delta C_9^e = \delta C_9^\mu\right)$ is barely allowed at 2σ level

Fit results for four operators $\{C_9, C_9', C_{10}, C_{10}'\}$

Full form factor approach with 5% power corrections

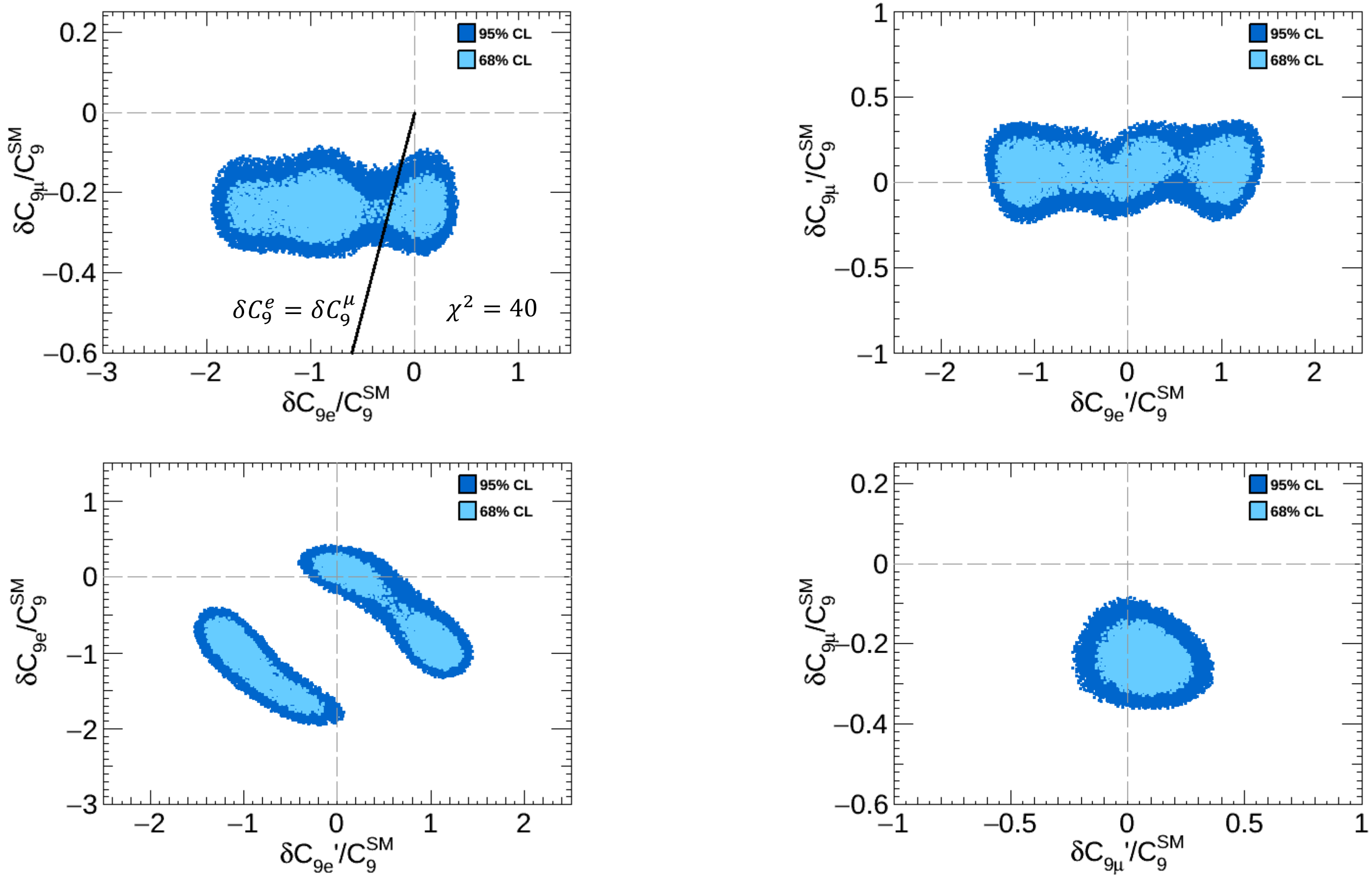


- More than 2σ tension for C_9, even in the four operator fit
- $\{C_9, C_{10}\} \rightarrow \chi^2 = 51$, $\{C_9, C_9', C_{10}, C_{10}'\} \rightarrow \chi^2 = 50$

  ⮡ adding primed WC doesn't improve the fit

Fit results for four operators $\{C_9^{\mu}, C_9^{e}, C_9^{\prime\mu}, C_9^{\prime e}\}$

Full form factor approach with 5% power corrections

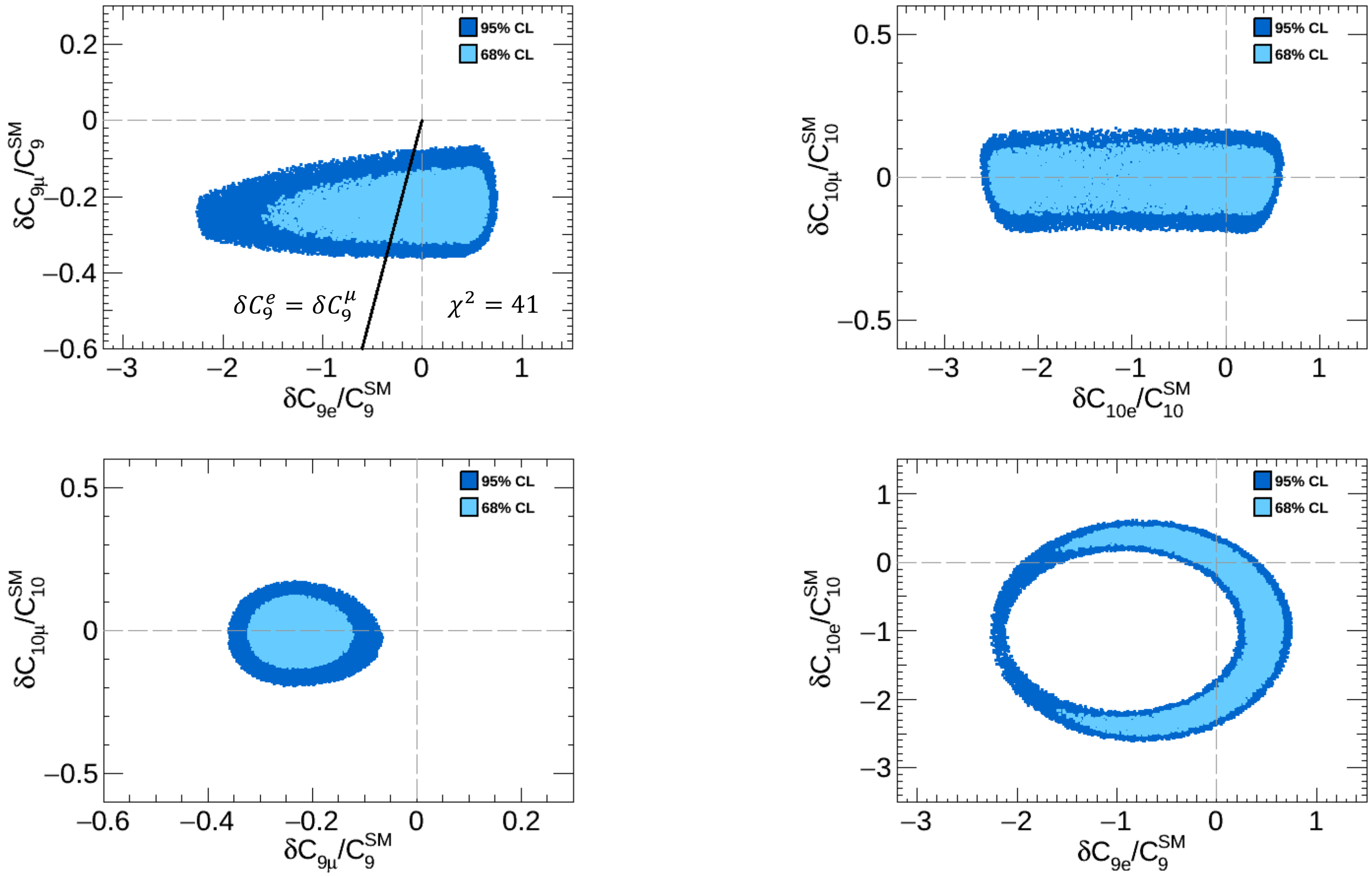


- More than 2σ tension for C_9
- In the four operator fit, it is possible to have $\delta C_9^e = \delta C_9^\mu$ → $\delta C_9^{\prime\mu} \neq \delta C_9^{\prime e}$
- $\{C_9, C_9'\} \rightarrow \chi^2 = 51$, $\{C_9^\mu, C_9^e, C_9^{\prime\mu}, C_9^{\prime e}\} \rightarrow \chi^2 = 40$

⟹ considering lepton flavour violation improves the fit

Fit results for four operators $\{C_9^\mu, C_9^e, C_{10}^\mu, C_{10}^e\}$

Full form factor approach with 5% power corrections



- More than 2σ tension for C_9
- In the four operator fit, it is possible to have $\delta C_9^e = \delta C_9^\mu$ ⟶ $\delta C_{10}^\mu \neq \delta C_{10}^e$
- $\{C_9, C_{10}\} \rightarrow \chi^2 = 51$, $\{C_9^\mu, C_9^e, C_{10}^\mu, C_{10}^e\} \rightarrow \chi^2 = 41$

  ⟹ considering lepton flavour violation improves the fit

Conclusions:

- Factorisable power corrections have small effect at observable level
- The fit results do not depend very much on wheather one uses soft or full form factor approach
- In the two operator fit going from 10% to 20% power correction in the soft FF approach slightly decrease tension in C_9, this is not case when going from 5% to10% in the full form factor approach
- In two operator fit there is a 2σ tension for $\delta C_9^e = \delta C_9^\mu$
- In four operator fit, possible to have $\delta C_9^e = \delta C_9^\mu$ but flavour violation takes place in C_9' or $C_{10}^{(\prime)}$
- Considering lepton flavour violation the fit is significantly improved

Thank you

Transversity amplitudes

$$A_\perp^{L,R} = N\sqrt{2}\sqrt{\lambda}\left[\left[(C_9^{\text{eff}} + C_9') \mp (C_{10} + C_{10}')\right]\frac{V}{M_B + m_{K^*}} + \frac{2m_b}{q^2}(C_7^{\text{eff}} + C_7')T_1\right]$$

$$A_\parallel^{L,R} = -N\sqrt{2}(M_B^2 - m_{K^*}^2)\left[\left[(C_9^{\text{eff}} - C_9') \mp (C_{10} - C_{10}')\right]\frac{A_1}{M_B - m_{K^*}} + \frac{4m_b}{M_B}(C_7^{\text{eff}} - C_7')\frac{E_{K^*}}{q^2}T_2\right]$$

$$A_0^{L,R} = -\frac{N}{2m_{K^*}\sqrt{q^2}}\left\{\left[(C_9^{\text{eff}} - C_9') \mp (C_{10} - C_{10}')\right]\right.$$

$$\times\left[(M_B^2 - m_{K^*}^2 - q^2)(M_B + m_{K^*})A_1 - \lambda\frac{A_2}{M_B + m_{K^*}}\right]$$

$$\left. + 2m_b(C_7^{\text{eff}} - C_7')\left[(M_B^2 + 3m_{K^*}^2 - q^2)T_2 - \frac{\lambda}{M_B^2 - m_{K^*}^2}T_3\right]\right\}$$

$$A_t = \frac{N}{\sqrt{q^2}}\sqrt{\lambda}\left[2(C_{10} - C_{10}') + \frac{q^2}{m_\ell m_b}(C_{Q_2} - C_{Q_2}')\right]A_0$$

$$A_S = -\frac{2N}{m_b}\sqrt{\lambda}(C_{Q_1} - C_{Q_1}')A_0$$

To compute the transversity amplitudes we need to have control over all the form factors

Transversity amplitudes (at LO for large recoil)

$$A_\perp^{L,R} = \frac{\sqrt{2}N(M_B^2-q^2)}{M_B}\left[\left[(C_9^{\rm eff}+C_9')\mp(C_{10}+C_{10}')\right]+\frac{2m_bM_B}{q^2}(C_7^{\rm eff}+C_7')\right]\xi_\perp(q^2)$$

$$A_\parallel^{L,R} = -\frac{\sqrt{2}N(M_B^2-q^2)}{M_B}\left[\left[(C_9^{\rm eff}+C_9')\mp(C_{10}+C_{10}')\right]+\frac{2m_bM_B}{q^2}(C_7^{\rm eff}-C_7')\right]\xi_\perp(q^2)$$

$$A_0^{L,R} = -\frac{NM_B(M_B^2-q^2)}{2m_{K^*}\sqrt{q^2}}\left[\left[(C_9^{\rm eff}+C_9')\mp(C_{10}+C_{10}')\right]+\frac{2m_b}{M_B}(C_7^{\rm eff}-C_7')\right]\xi_\parallel(q^2)$$

$$A_t = \frac{N(M_B^2-q^2)}{\sqrt{q^2}}\left[2(C_{10}-C_{10}')+\frac{q^2}{m_\ell m_b}(C_{Q_2}-C_{Q_2}')\right]\frac{E_{K^*}}{m_{K^*}}\xi_\parallel(q^2)$$

$$A_S = -\frac{2N(M_B^2-q^2)}{m_b}(C_{Q_1}-C_{Q_1}')\frac{E_{K^*}}{m_{K^*}}\xi_\parallel(q^2)$$

Backup

$$\sum_{\substack{\text{final state}\\ \text{spins}}} |M|^2 \longrightarrow \frac{d^4\Gamma}{dq^2\; d\cos\theta_l\; d\cos\theta_K\; d\phi} = \frac{9}{32\pi} J(q^2, \theta_l, \theta_K, \phi)$$

$$J(q^2, \theta_\ell, \theta_K, \phi) = \sum J_i(q^2)\, f_i(\theta_\ell, \theta_K, \phi)$$

$$\begin{aligned} J(q^2, \theta_\ell, \theta_{K^*}, \phi) = {} & J_1^s \sin^2\theta_{K^*} + J_1^c \cos^2\theta_{K^*} + (J_2^s \sin^2\theta_{K^*} + J_2^c \cos^2\theta_{K^*}) \cos 2\theta_\ell \\ & + J_3 \sin^2\theta_{K^*} \sin^2\theta_\ell \cos 2\phi + J_4 \sin 2\theta_{K^*} \sin 2\theta_\ell \cos\phi + J_5 \sin 2\theta_{K^*} \sin\theta_\ell \cos\phi \\ & + (J_6^s \sin^2\theta_{K^*} + J_6^c \cos^2\theta_{K^*}) \cos\theta_\ell + J_7 \sin 2\theta_{K^*} \sin\theta_\ell \sin\phi \\ & + J_8 \sin 2\theta_{K^*} \sin 2\theta_\ell \sin\phi + J_9 \sin^2\theta_{K^*} \sin^2\theta_\ell \sin 2\phi \end{aligned}$$

Backup

$$J_1^s = \frac{(2+\beta_\ell^2)}{4}\left[|A_\perp^L|^2 + |A_\parallel^L|^2 + (L\to R)\right] + \frac{4m_\ell^2}{q^2}\mathrm{Re}\left(A_\perp^L A_\perp^{R^*} + A_\parallel^L A_\parallel^{R^*}\right)$$

$$J_1^c = |A_0^L|^2 + |A_0^R|^2 + \frac{4m_\ell^2}{q^2}\left[|A_t|^2 + 2\mathrm{Re}(A_0^L A_0^{R^*})\right] + \beta_\ell^2|A_S|^2\,,$$

$$J_1 = 2J_1^s + J_1^c$$

$$J_2^s = \frac{\beta_\ell^2}{4}\left[|A_\perp^L|^2 + |A_\parallel^L|^2 + (L\to R)\right]\,,$$

$$J_2^c = -\beta_\ell^2\left[|A_0^L|^2 + (L\to R)\right]\,,$$

$$J_2 = 2J_2^s + J_2^c$$

$$J_3 = \frac{1}{2}\beta_\ell^2\left[|A_\perp^L|^2 - |A_\parallel^L|^2 + (L\to R)\right]\,,$$

$$J_4 = \frac{1}{\sqrt{2}}\beta_\ell^2\left[\mathrm{Re}(A_0^L A_\parallel^{L^*}) + (L\to R)\right]\,,$$

$$J_5 = \sqrt{2}\beta_\ell\left[\mathrm{Re}(A_0^L A_\perp^{L^*}) - (L\to R) - \frac{m_\ell}{\sqrt{q^2}}\mathrm{Re}(A_\parallel^L A_S^* + A_\parallel^R A_S^*)\right]\,,$$

$$J_6^s = 2\beta_\ell\left[\mathrm{Re}(A_\parallel^L A_\perp^{L^*}) - (L\to R)\right]\,,$$

$$J_6^c = 4\beta_\ell\frac{m_\ell}{\sqrt{q^2}}\mathrm{Re}\left[A_0^L A_S^* + (L\to R)\right]\,,$$

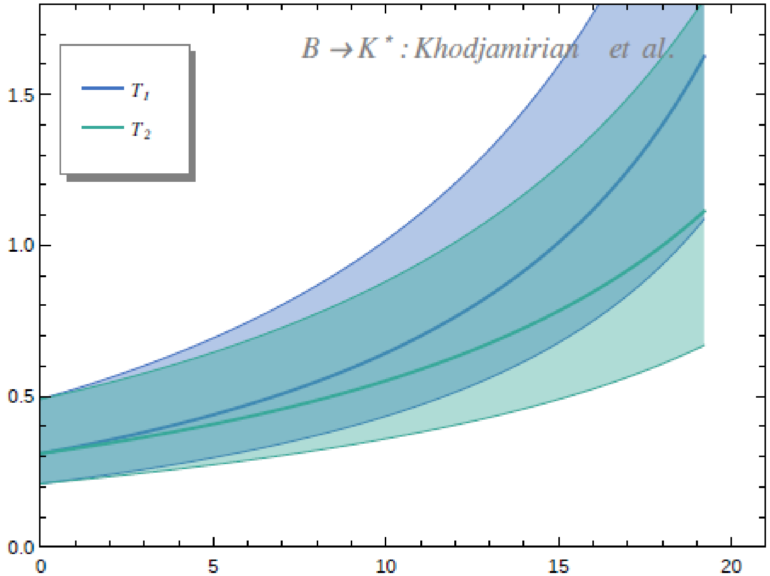
$$J_6 = 2J_6^s + J_6^c$$

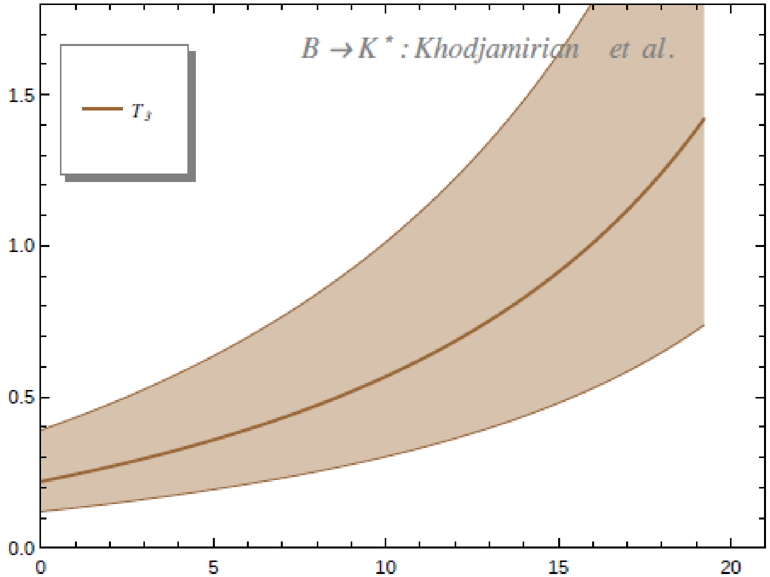
$$J_7 = \sqrt{2}\beta_\ell\left[\mathrm{Im}(A_0^L A_\parallel^{L^*}) - (L\to R) + \frac{m_\ell}{\sqrt{q^2}}\mathrm{Im}(A_\perp^L A_S^* + A_\perp^R A_S^*)\right]\,,$$

$$J_8 = \frac{1}{\sqrt{2}}\beta_\ell^2\left[\mathrm{Im}(A_0^L A_\perp^{L^*}) + (L\to R)\right]\,,$$

$$J_9 = \beta_\ell^2\left[\mathrm{Im}(A_\parallel^{L^*} A_\perp^L) + (L\to R)\right]\,.$$

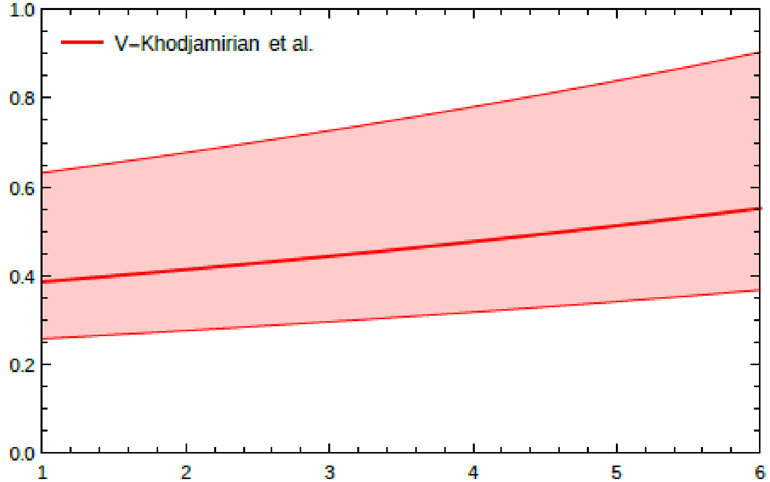
Khodjamirian et al. form factors $(T_1, T_2,\ T_3)$

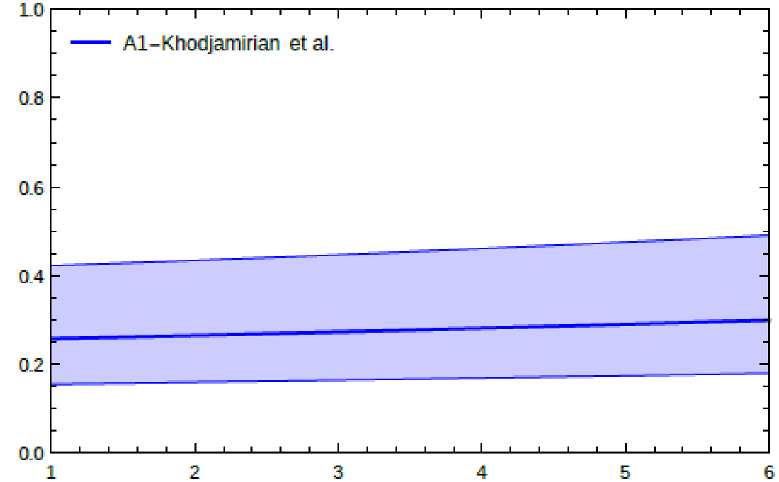




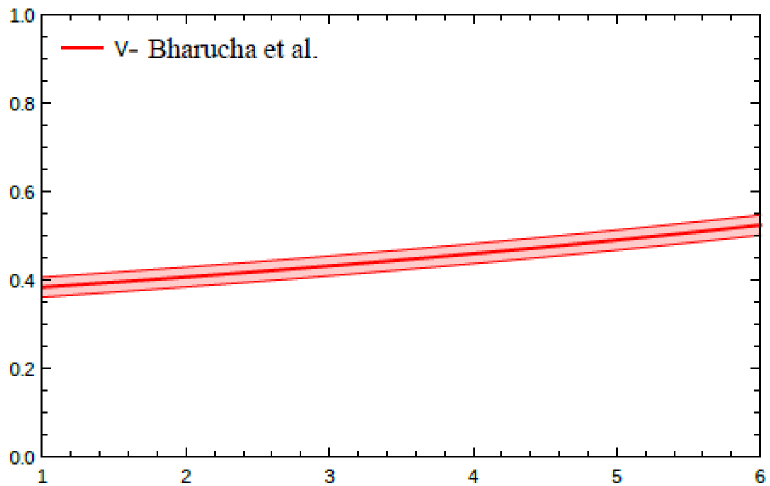
V, A_1 form factors at low q^2

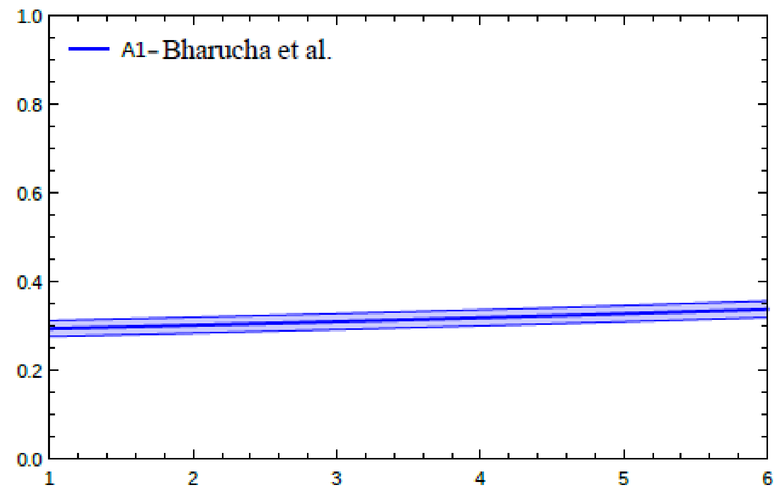
Khodjamirian et al





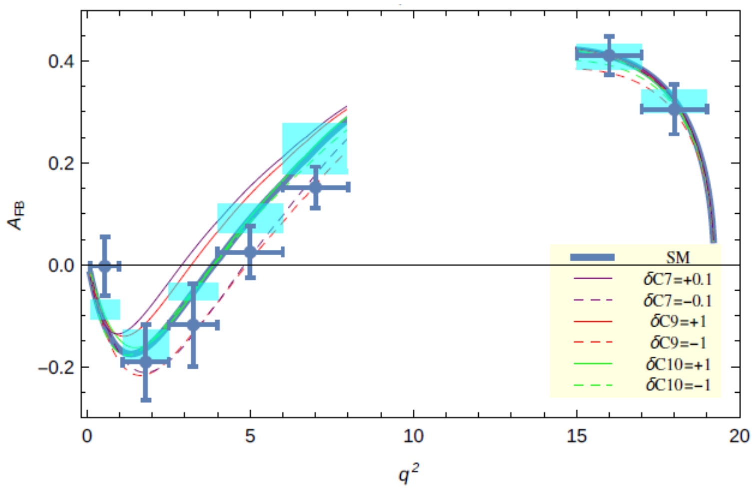
Bharucha et al.

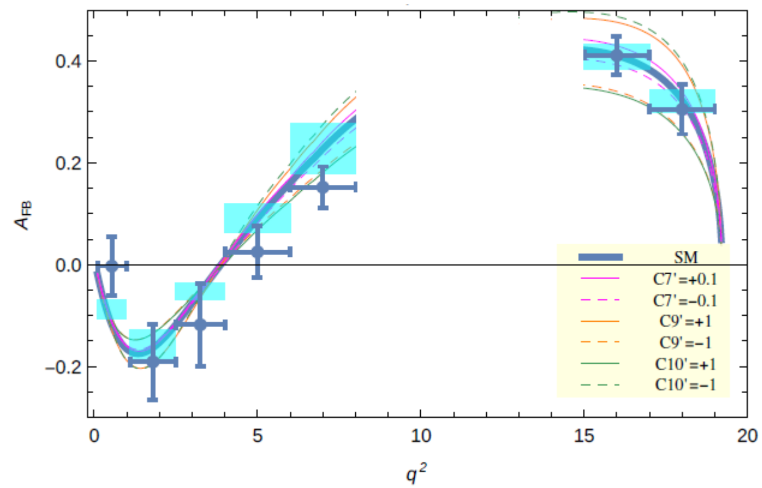




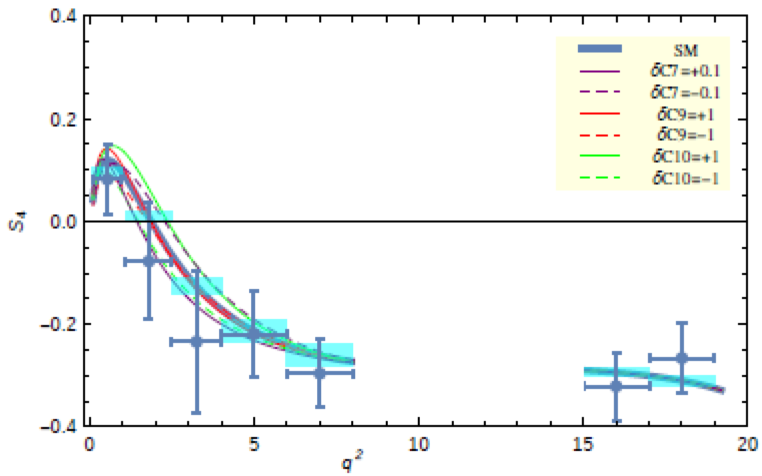
Modified Wilson coefficient effects

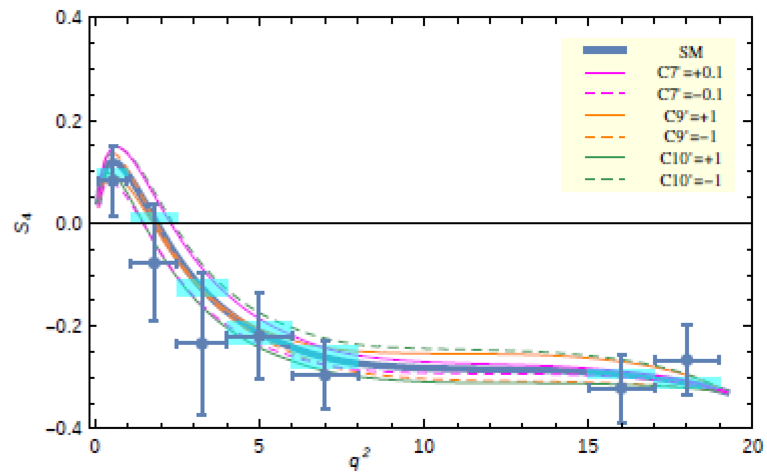
- A_{FB}





- S_4

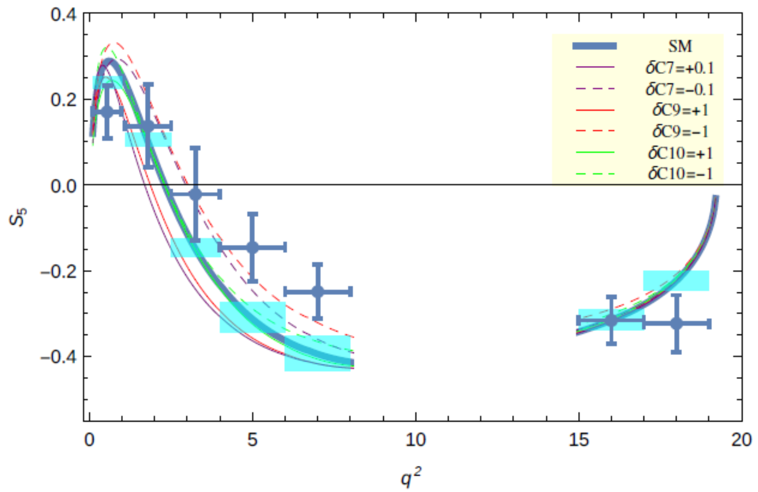


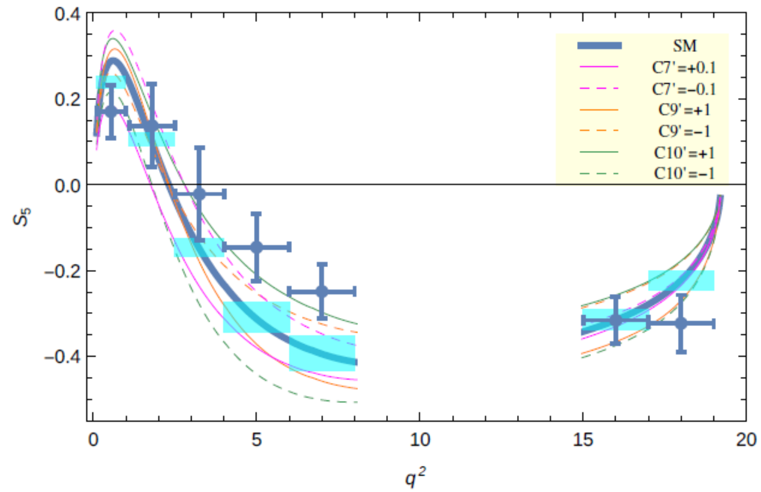


Backup

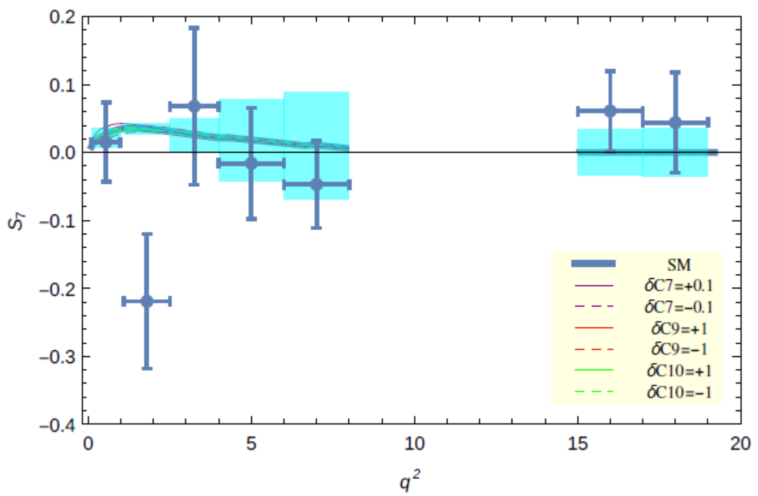
Modified Wilson coefficient effects

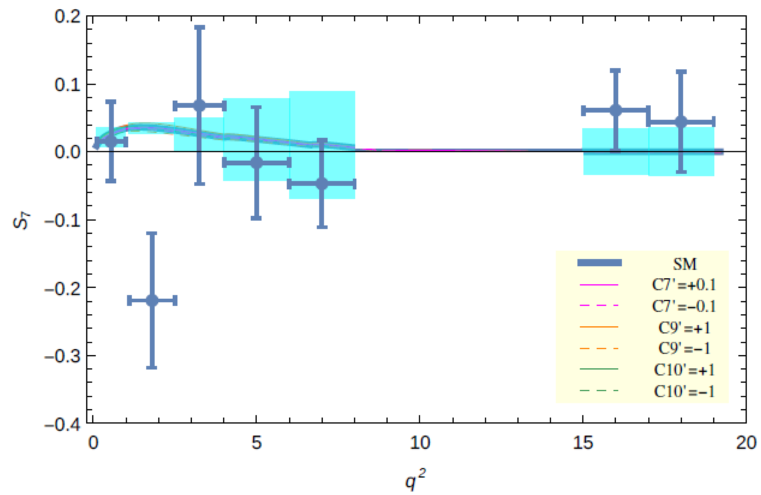
- S_5





- S_7

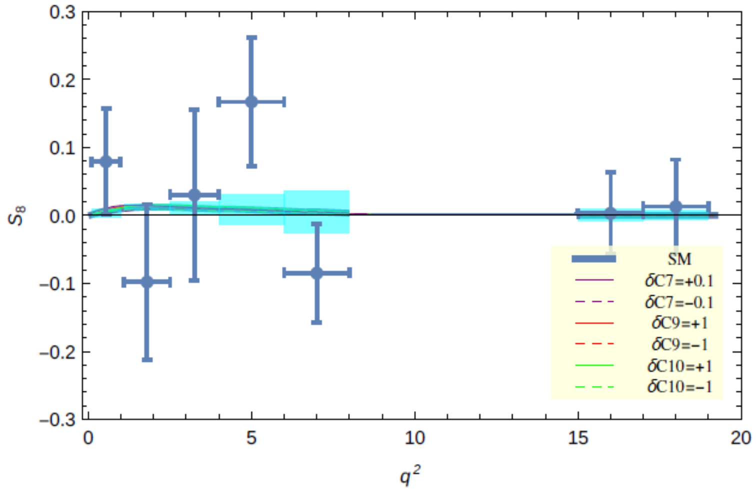


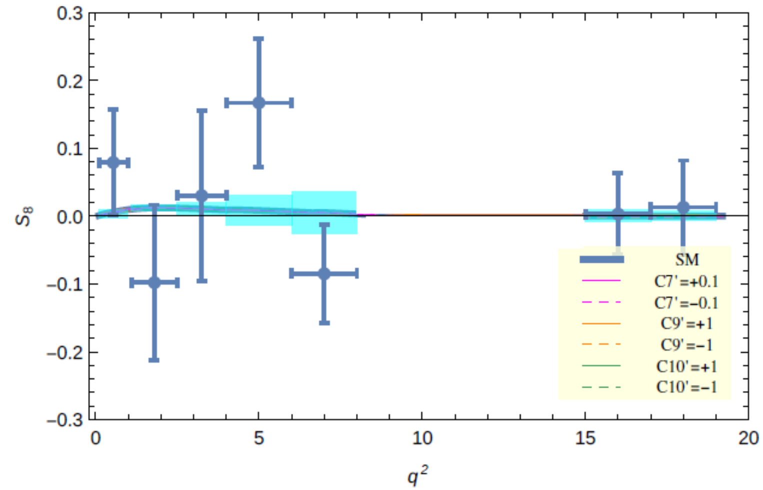


Backup

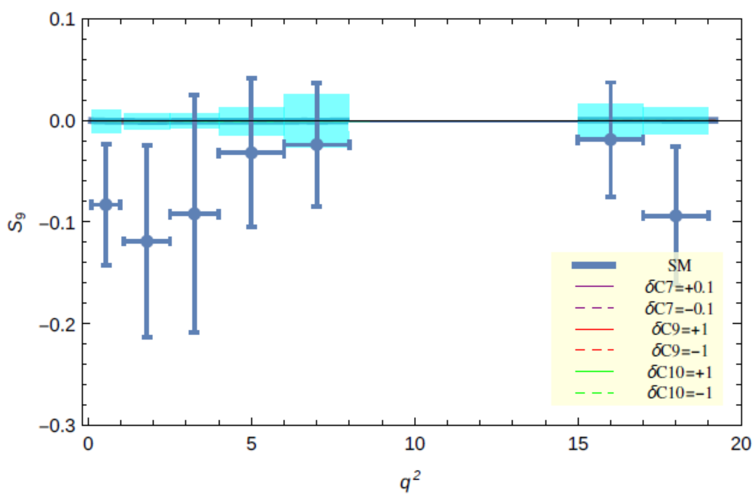
Modified Wilson coefficient effects

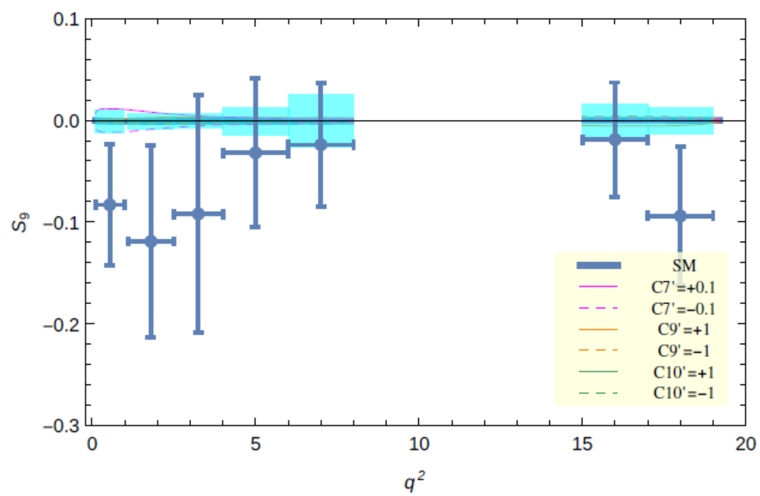
- S_8



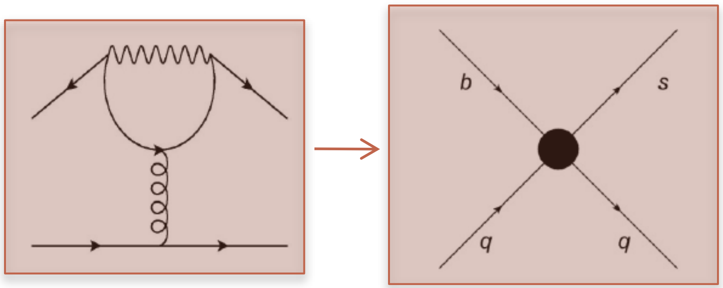


- S_9



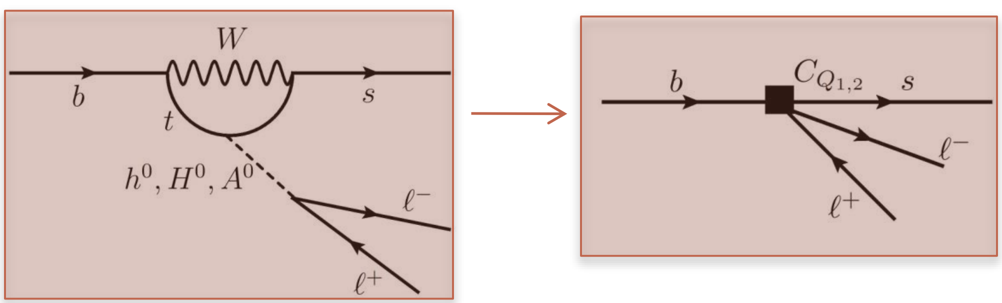


QCD penguins



$$
\begin{aligned}
\mathcal{O}_1 &= (\bar{s}\gamma_\mu T^a P_L c)\,(\bar{c}\gamma^\mu T_a P_L b)\\
\mathcal{O}_2 &= (\bar{s}\gamma_\mu P_L c)\,(\bar{c}\gamma^\mu P_L b)\\
\mathcal{O}_3 &= (\bar{s}\gamma_\mu P_L b)\,\textstyle\sum_q (\bar{q}\gamma^\mu q)\\
\mathcal{O}_4 &= (\bar{s}\gamma_\mu T^a P_L b)\,\textstyle\sum_q (\bar{q}\gamma^\mu T_a q)\\
\mathcal{O}_5 &= (\bar{s}\gamma_\mu \gamma_\nu \gamma_\rho P_L b)\,\textstyle\sum_q (\bar{q}\gamma^\mu \gamma^\nu \gamma^\rho q)\\
\mathcal{O}_6 &= (\bar{s}\gamma_\mu \gamma_\nu \gamma_\rho T^a P_L b)\,\textstyle\sum_q (\bar{q}\gamma^\mu \gamma^\nu \gamma^\rho T_a q)
\end{aligned}
$$

Scalar and pseudoscalar



$$Q_1 = \frac{e^2}{(4\pi)^2}(\bar{s}P_R b)(\bar{\ell}\,\ell)$$

$$Q_2 = \frac{e^2}{(4\pi)^2}(\bar{s}P_R b)(\bar{\ell}\gamma_5 \ell)$$