Lattice Four-Fermion Interactions in Beyond Standard Model Physics

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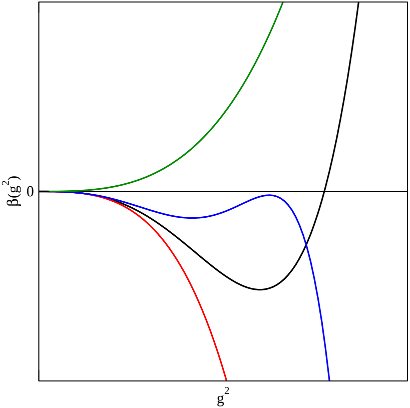
CP3 Origins

Walking and the conformal window



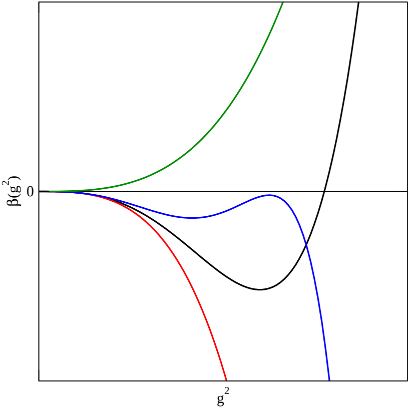
Depending on the number of flavors

No asymptotic freedom

Infrared fixed point

Chirally broken
(Walking, Running)

Walking and the conformal window



$$L_\gamma = \gamma^2\, \bar{\Psi}\Psi\bar{\Psi}\Psi$$

$\gamma < \gamma_c$:

$\langle \bar{\Psi}\Psi \rangle = 0$

$g^*(\gamma)$

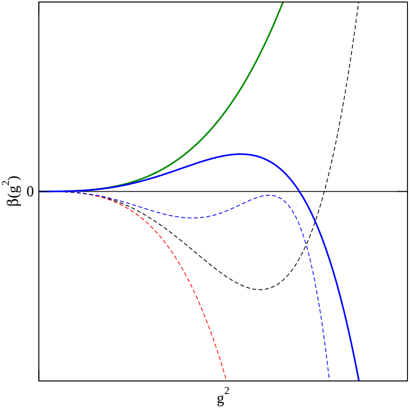
varying anomalous dimensions

$\gamma > \gamma_c$:

$\langle \bar{\Psi}\Psi \rangle \neq 0$

(Walking, Running)

Gauge Yukawa Models



UV limit of a
gauge Yukawa model
(Compositeness conditions)

Asymptotic safety without freedom

The Nambu Jona-Lasinio Model

$$L = \bar{\Psi}\partial\!\!\!/\Psi + \gamma^2 \left(\bar{\Psi}\Psi\bar{\Psi}\Psi + \bar{\Psi}i\gamma_5\tau_a\Psi\bar{\Psi}i\gamma_5\tau_a\Psi\right)$$

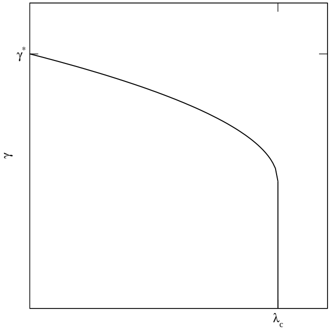
- Preserves a $SU(2) \times SU(2)$ chiral symmetry ($N_F = 2$)
- Spontaneous symmetry breaking at [1]

$$\gamma > \gamma^* \sim \sqrt{\frac{2\pi^2}{N\Lambda^2}}$$

[1] Nambu and G. Jona-Lasinio, Phys. Rev. **122** (1961) 345

Gauged NJL

$$L = F_{\mu\nu}F^{\mu\nu} + \bar{\Psi}\not{D}\Psi + \gamma^2\left(\bar{\Psi}\Psi\bar{\Psi}\Psi + \bar{\Psi}i\gamma_5\tau_a\Psi\bar{\Psi}i\gamma_5\tau_a\Psi\right)$$



Gauge coupling τ

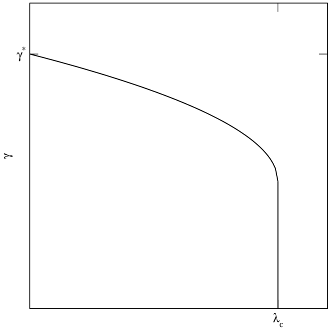
Chiral symmetry broken if $\tau(\mu) > \tau_c$

$$\gamma_c = \frac{1}{2}\left(1 + \sqrt{1 - \frac{\lambda}{\lambda_c}}\right)\gamma^{*2}$$

[2] K. Yamawaki, hep-ph/9603293
H. S. Fukano and F. Sannino, Phys. Rev. D **82** (2010) 035021

Gauged NJL

$$L = F_{\mu\nu}F^{\mu\nu} + \bar{\Psi}\not{D}\Psi + \gamma^2\left(\bar{\Psi}\Psi\bar{\Psi}\Psi + \bar{\Psi}i\gamma_5\tau_a\Psi\bar{\Psi}i\gamma_5\tau_a\Psi\right)$$



- Check the critical line
- Find the order of the transition
- Measure γ_m on the symmetric side

Now let's forget about the gauge

The Lattice Model

Regularization on the lattice:

- Non-renormalizable $\rightarrow$ no continuum limit
- Each regularization defines a separate model
- Divergent contributions to every operator

- Two scales, Λ and $1/a$, related by the coupling γ/a
- Wilson fermions: Regularization breaks chiral symmetry

The Lattice Model

Make quadratic

- auxiliary fields σ, π

Real fermion action

- limit $SU(2) \times SU(2)$ to $U(1) \times U(1)$

$$L = \bar{\Psi} \partial\!\!\!/ \Psi + \sigma \bar{\Psi} \Psi + \pi \bar{\Psi} i \gamma_5 \tau_3 \Psi + \frac{\sigma^2 + \pi^2}{4\gamma^2}$$

The Lattice Model

Wilson fermions:

$$L = \bar{\Psi} \partial\!\!\!/_W \Psi + (m_0 + \sigma)\bar{\Psi}\Psi + \pi \bar{\Psi} i\gamma_5\tau_3\Psi + \frac{\sigma^2 + \pi^2}{4\gamma^2}$$

$$\partial\!\!\!/_W = \partial\!\!\!/ - a\Delta_\mu\Delta_\mu$$

- Chiral symmetry broken
  $\rightarrow$ Chiral symmetry breaking corrections
- Restored when $\partial_\mu \left\langle A^3_\mu(x) O \right\rangle = 0$

The Lattice Model

Ward identities:

$$\partial_\mu \left\langle A^3_\mu(x) O \right\rangle = 2m_0 \left\langle P^3(x) O \right\rangle + 4\delta_\gamma \left\langle S^0(x) P^3(x) O \right\rangle + \left\langle a X^3(x) O \right\rangle$$

Variation of Wilson term X^3 renormalises as

$$\begin{aligned} aX^3(x) = {} & a\bar{X}^3(x) + \frac{c_m(\gamma/a)}{a} P^3(x) + (Z_A(\gamma/a) - 1)\,\partial_\mu A^3_\mu(x) \\ & + a c_A(\gamma/a)\,\partial_\mu \partial_\mu P^3(x) + a^2 c_m^{(\mu\nu\nu)}(\gamma/a)\,\partial_{(\mu}\partial_\nu\partial_{\nu)} A^3_\mu(x) \\ & + a^2 c_\gamma(\gamma/a)\, S^0(x) P^3(x) + \cdots \end{aligned}$$

Phase Structure

- Large N, meanfield [3]

$$
\begin{aligned}
&\sigma = \langle\sigma\rangle = \sigma_s, \quad \pi_3 = \langle\pi_3\rangle = \pi_s, \\
&V_{\text{eff}}(\sigma_s, \pi_s) = -4N \int \frac{d^4 p}{(2\pi)^4} \log[g(p)] + \frac{1}{8\gamma^2}\left(\sigma_s^2 + \pi_s^2\right), \\
&g(p) = \sum_\mu \sin^2 p_\mu + \pi_s^2 + [w(p) + \sigma_s + m]^2, \\
&w(p) = 4 - \sum_\mu \cos p_\mu
\end{aligned}
$$

[3] K. M. Bitar and P. M. Vranas, Phys. Rev. D **50** (1994) 3406
S. Aoki, S. Boettcher and A. Gocksch, Phys. Lett. B **331** (1994) 157

Phase Structure

- Gap Equations

$$\frac{\partial V_{\text{eff}}(\sigma_s, \pi_s))}{\partial \sigma_s} = 0 = \frac{\sigma_s}{32N\gamma^2} - \int \frac{d^4p}{(2\pi)^4} \frac{\sigma_s + m + w(p)}{g(p)},$$

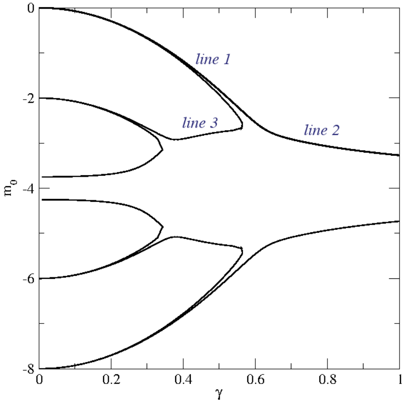
$$\frac{\partial V_{\text{eff}}(\sigma_s, \pi_s))}{\partial \pi_s} = 0 = \frac{\pi_s}{32N\gamma^2} - \int \frac{d^4p}{(2\pi)^4} \frac{\pi_s}{g(p)}$$

- Parity-flavor broken phase with $\pi_s \neq 0$, when

$$32N\gamma^2 \int \frac{d^4p}{(2\pi)^4} \frac{1}{g(p)} = 1$$

Phase Structure

Boundaries of the broken phase

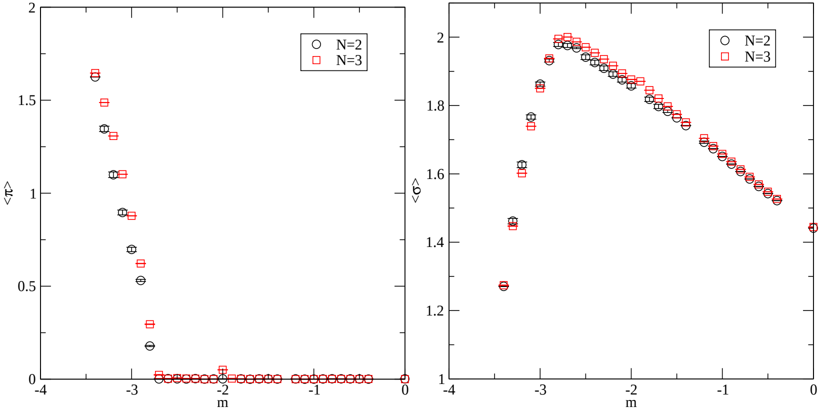


line 1 Small coupling zero mass line

line 2 Large coupling, expected chiral symmetry breaking

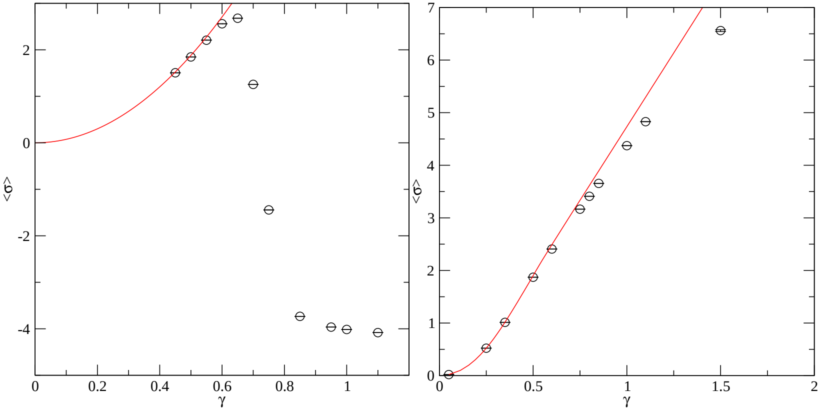
line 3 An unphysical zero mass line

Phase Structure



The pure NJL model and $N\gamma^2 = 0.25$. $\langle \pi \rangle$ serves as the order parameter for the parity-flavour broken phase.

Phase Structure



Values of $\langle\sigma\rangle$ from pure NJL simulations with N=2. On the left, $m_q = 0$, and on the right, $m = -2$. The mean field value given by the red line.

Phase Structure

To establish chiral symmetry breaking, measure m_π and m_ρ

$$C(t,t_0) = \left\langle \sum_{\mathbf{x},\mathbf{y}} O(\mathbf{x},t)O(\mathbf{y},t_0) \right\rangle = e^{-m_1(t-t_0)+\cdots}$$

m_ρ: $O(x) = \bar{\Psi}(x)\gamma_k \tau^3 \Psi(x)$

m_π: $O(x) = \bar{\Psi}(x)\gamma_5 \tau^3 \Psi(x)$, $O(x) = \bar{\Psi}(x)\gamma_5\gamma_0 \tau^3 \Psi(x)$

m_{π_2}: $O(x) = \pi(x)$

Disconnected Diagrams

The auxiliary field $\pi(x)$ induces disconnected diagrams

$$\begin{aligned} C(\Gamma)(x-y) &= \left\langle \bar{\Psi}(x)\Gamma\tau^3\Psi(x)\bar{\Psi}(y)\Gamma\tau^3\Psi(y)\right\rangle \\ &= \left\langle \mathrm{Tr}\,\left[S(y,x)\Gamma\tau^3 S(x,y)\Gamma\tau^3\right]\right\rangle \\ &+ \left\langle \mathrm{Tr}\,\left[S(x,x)\Gamma\tau^3\right]\mathrm{Tr}\,\left[S(y,y)\Gamma\tau^3\right]\right\rangle \\ S(x,y) &= \frac{1}{M}(x,y) \end{aligned}$$

In QCD, this is usually zero:

$$\mathrm{Tr}\,S(x,x)\Gamma\tau^3 = \mathrm{Tr}\,\left(S_u - S_d\right)\Gamma$$

Disconnected Diagrams

The auxiliary field $\pi(x)$ induces disconnected diagrams

$$M_{u,d} = \sum_{\mu} \partial_{\mu,y,x}\gamma_{\mu} + \delta_{x,y}\left(\sigma(x) \pm i\pi(x)\gamma_5\right)$$

The disconnected loop is

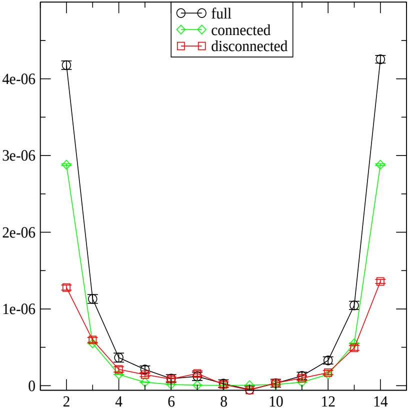
$$\mathrm{Tr}\,\left(S_u - S_d\right)\Gamma = -\mathrm{Tr}\,\frac{\delta_{x,y}2i\pi(x)\gamma_5}{M_u M_d}\Gamma$$

Noisy, need many configurations of π, σ

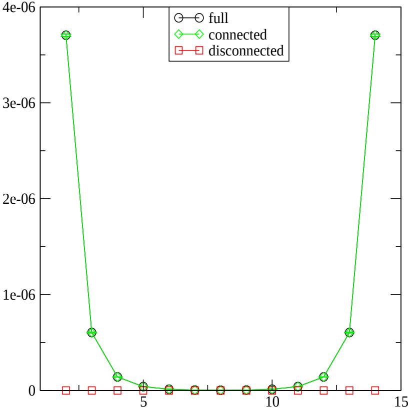
Disconnected Diagrams

$$\gamma = 0.65a, \quad \delta_\gamma = 0, \quad L = 8^3 \times 16, \quad m_0 = 2.9$$

Pseudoscalar



Vector



Phase Structure

To establish chiral symmetry breaking, measure m_π and m_ρ

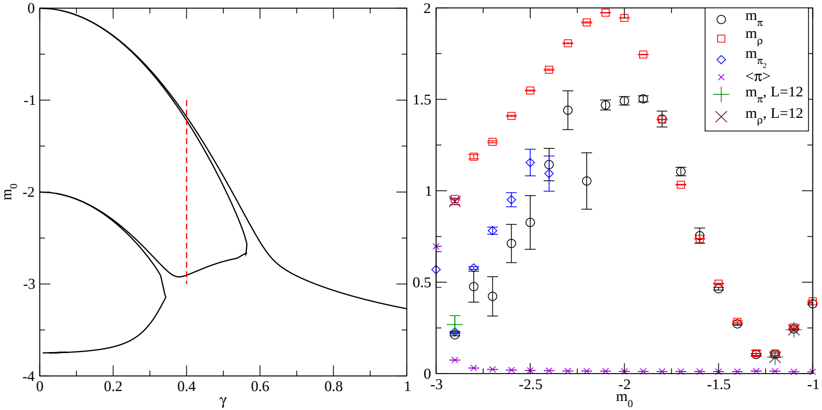
$$m_\rho: \; O(x) = \bar{\Psi}(x)\gamma_k\tau^3\Psi(x)$$

$$m_\pi: \; O(x) = \bar{\Psi}(x)\gamma_5\gamma_0\tau^3\Psi(x)$$

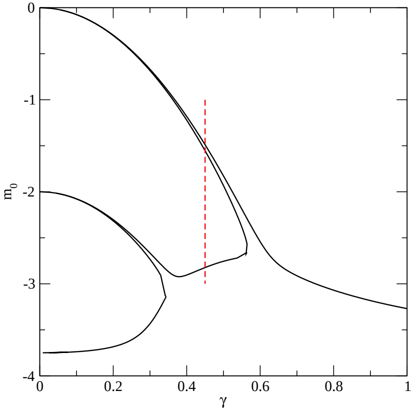
$$m_{\pi_2}: \; O(x) = \pi(x)$$

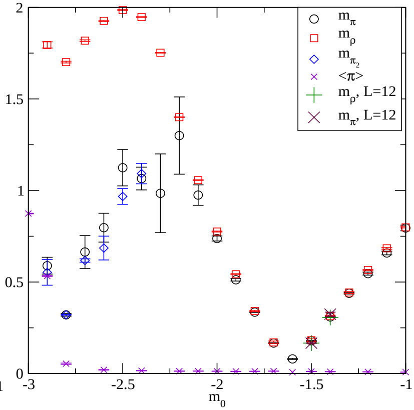
- Chiral symmetry restored when $m_\pi = 0$
- If spontaneously broken, $m_\rho \neq 0$

$$\gamma = 0.4a, \quad \delta_\gamma = 0, \quad L = 8^3 \times 16,$$

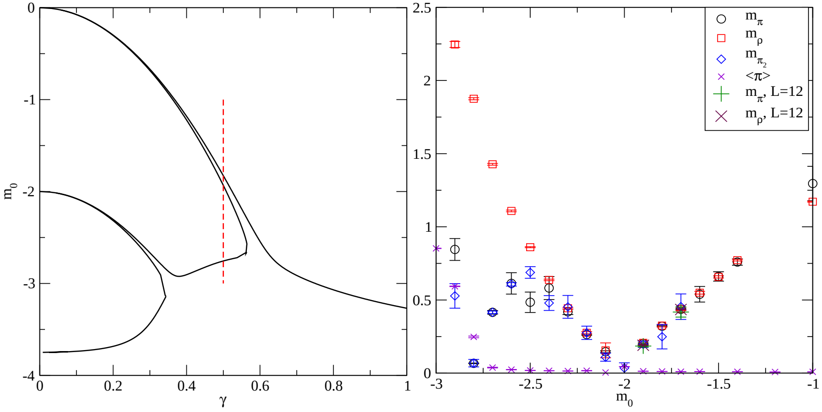


$$\gamma = 0.45a, \quad \delta_\gamma = 0, \quad L = 8^3 \times 16,$$

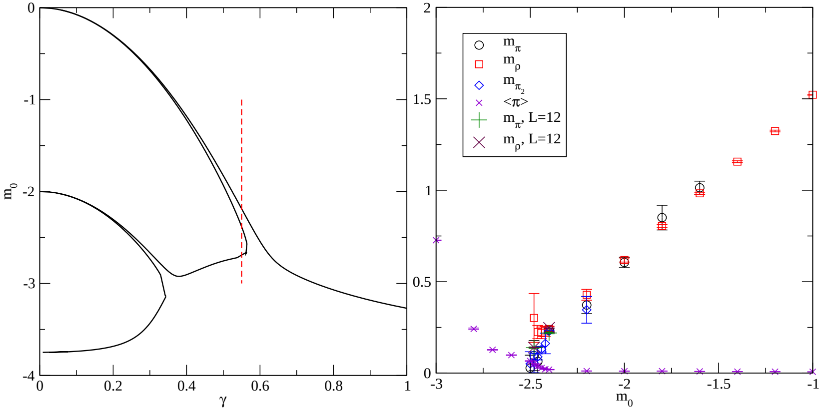




$$\gamma = 0.5a, \quad \delta_\gamma = 0, \quad L = 8^3 \times 16,$$



$$\gamma = 0.55a, \quad \delta_\gamma = 0, \quad L = 8^3 \times 16,$$



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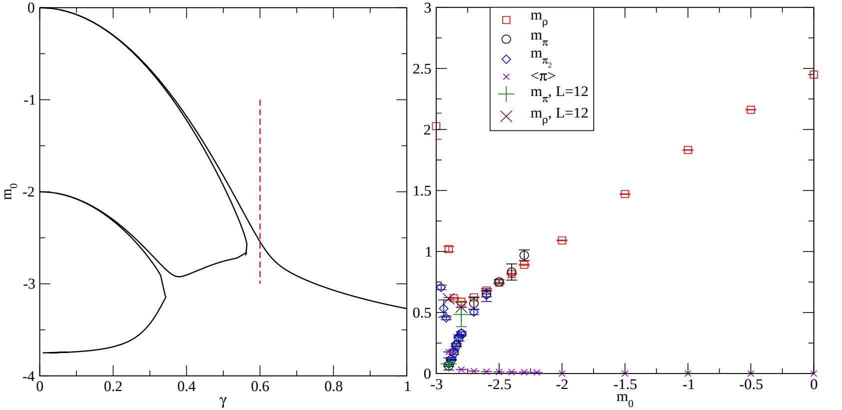
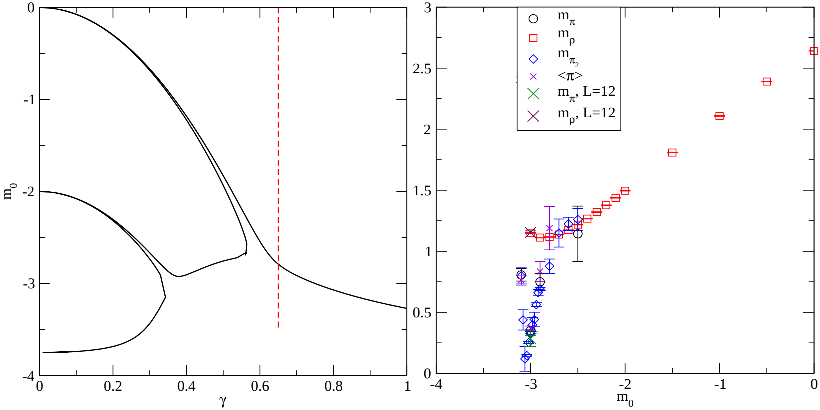
$$\gamma = 0.6a, \quad \delta_\gamma = 0, \quad L = 8^3 \times 16,$$



$$\gamma = 0.65a, \quad \delta_\gamma = 0, \quad L = 8^3 \times 16,$$



Four fermion interactions

- Present in many models (fermion masses)
- Conformal to chirally broken transition
- Critical line with varying anomalous dimensions

Ungauged NJL with Wilson fermions

- Spontaneous chiral symmetry breaking
- Large statistics needed for disconnected diagrams

Plans

- Full gauged model (SU(2) adjoint)
- Order of the transition
- Mass anomalous dimension