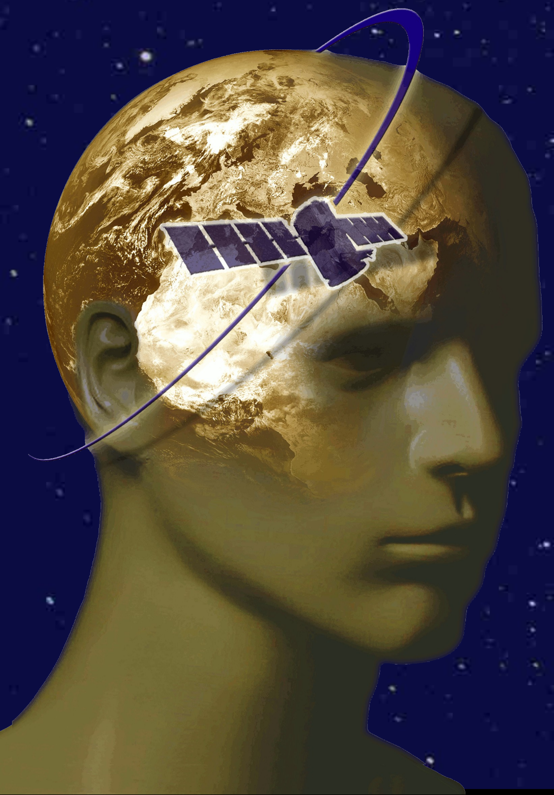
Chronometric Geodesy and new tests of the Gravitational Redshift

Systèmes de Référence Temps-Espace



Séminaire du LAPP

2015, 22nd May

Annecy

Pacôme DELVA

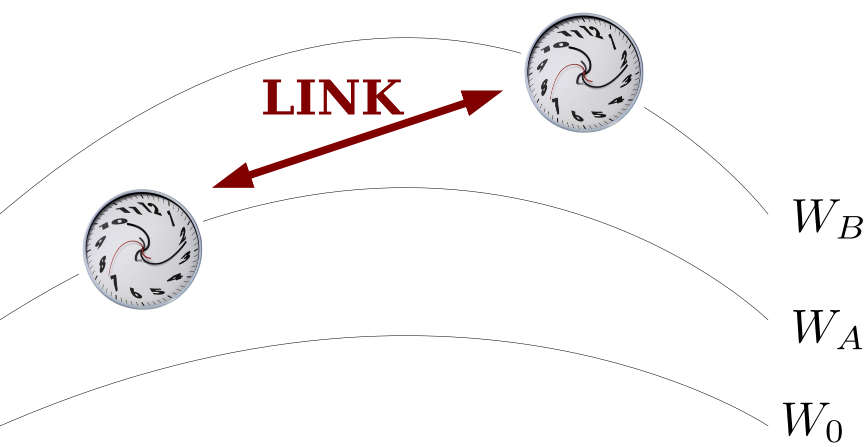
SYRTE

Observatoire de Paris

Université Pierre et Marie Curie

The flow of time, or the rate of a clock when compared to coordinate time, depends on the **velocity** of the clock and on the **space-time metric** (which depends on the mass/energy distribution). In the **weak-field approximation**:

$$\frac{\Delta\tau}{\tau} = \frac{\Delta f}{f} = \frac{U_B - U_A}{c^2} + \frac{v_B^2 - v_A^2}{2c^2} + O(c^{-4})$$

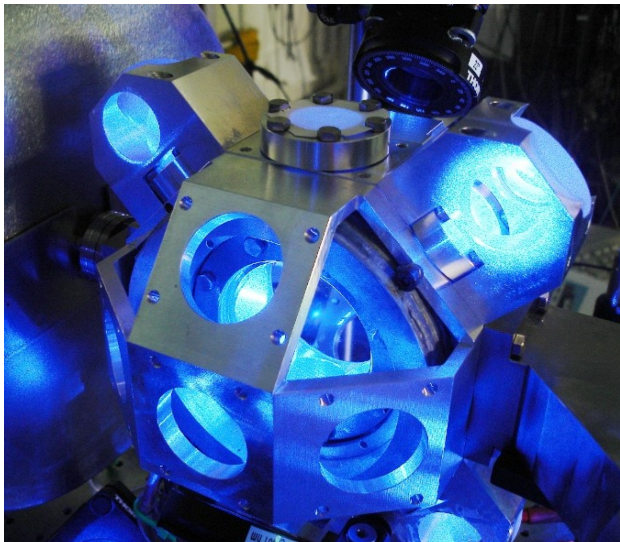


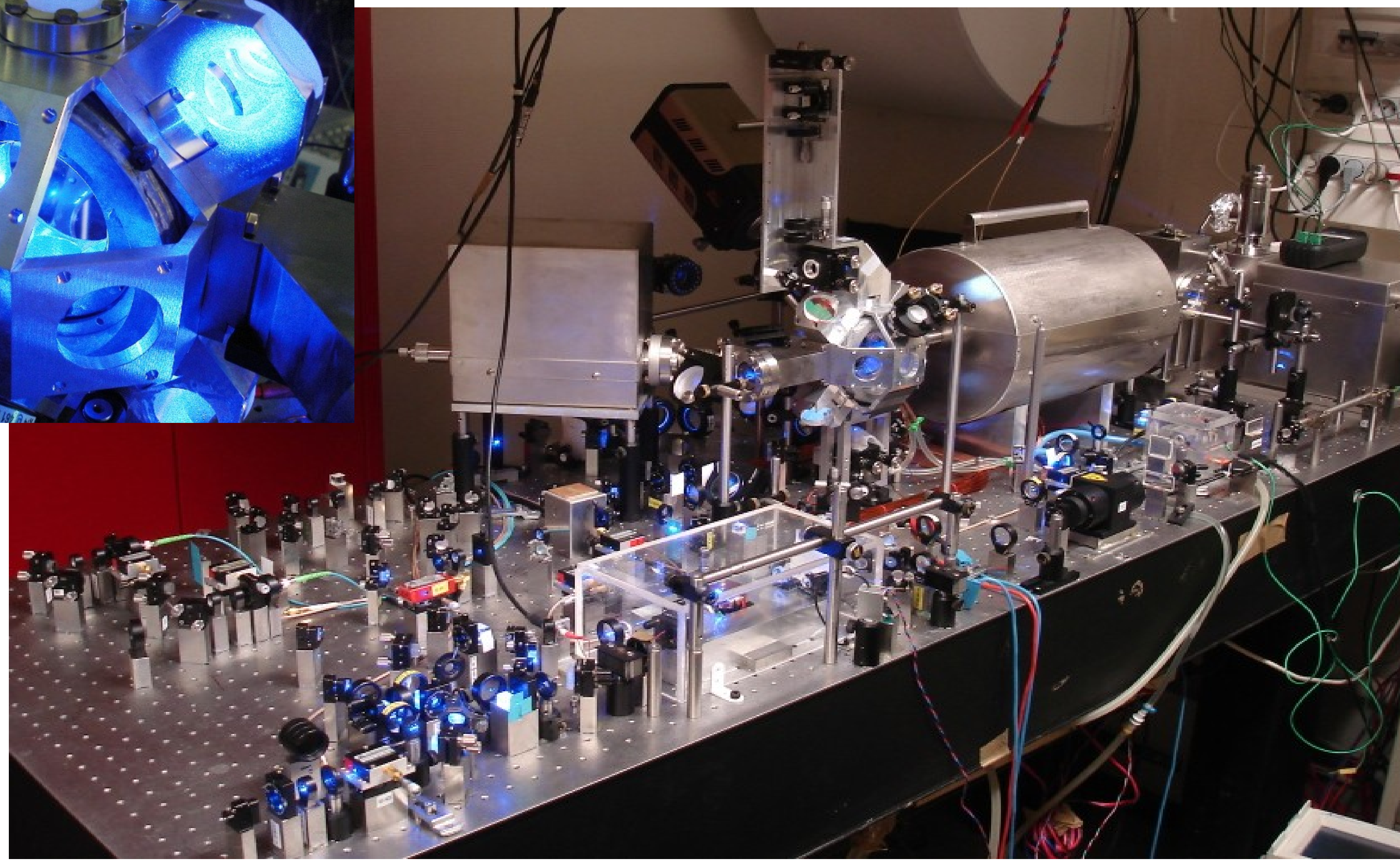
- U and v known → Δf prediction
  = **Clock syntonization**
- U, v and Δf known
  = **Gravitational redshift test**
- Δf known → ΔW prediction ($W=U+v^2/2$)
  = **Chronometric geodesy**

Contents

- Progress in time & frequency metrology
- A brief introduction to chronometric geodesy
- Proposal for a (new) test of gravitational redshift
- Some ideas for the future

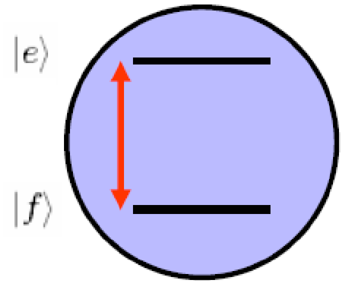
Progress in time and frequency metrology





→ Deliver a signal with stable and universal frequency

$$\hbar\omega_{ef} = E_e - E_f$$



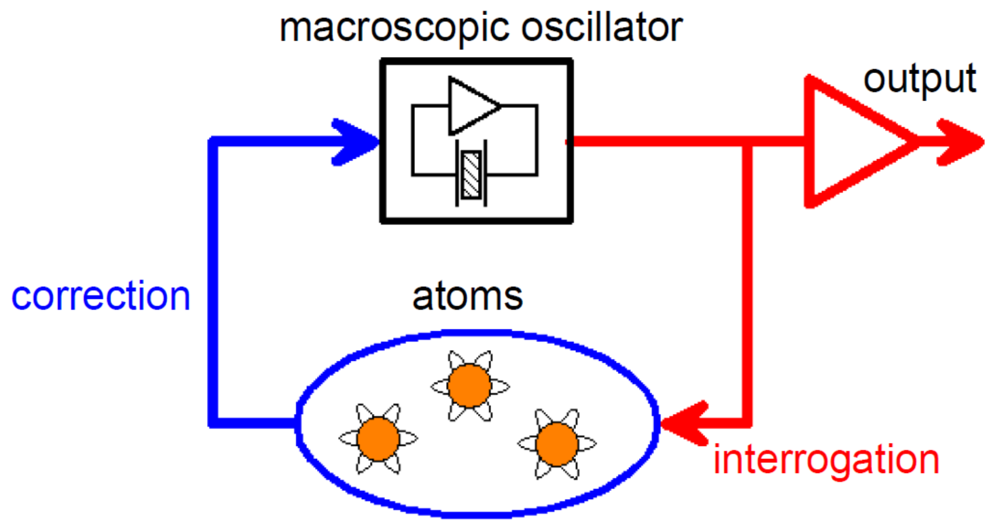
$$\omega(t) = \omega_{ef} \times (1 + \epsilon + y(t))$$

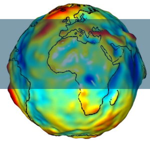
ε : fractional frequency offset

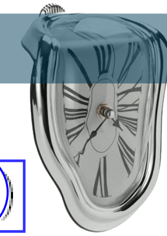
Accuracy: overall uncertainty on ε

y(t) : fractional frequency fluctuations

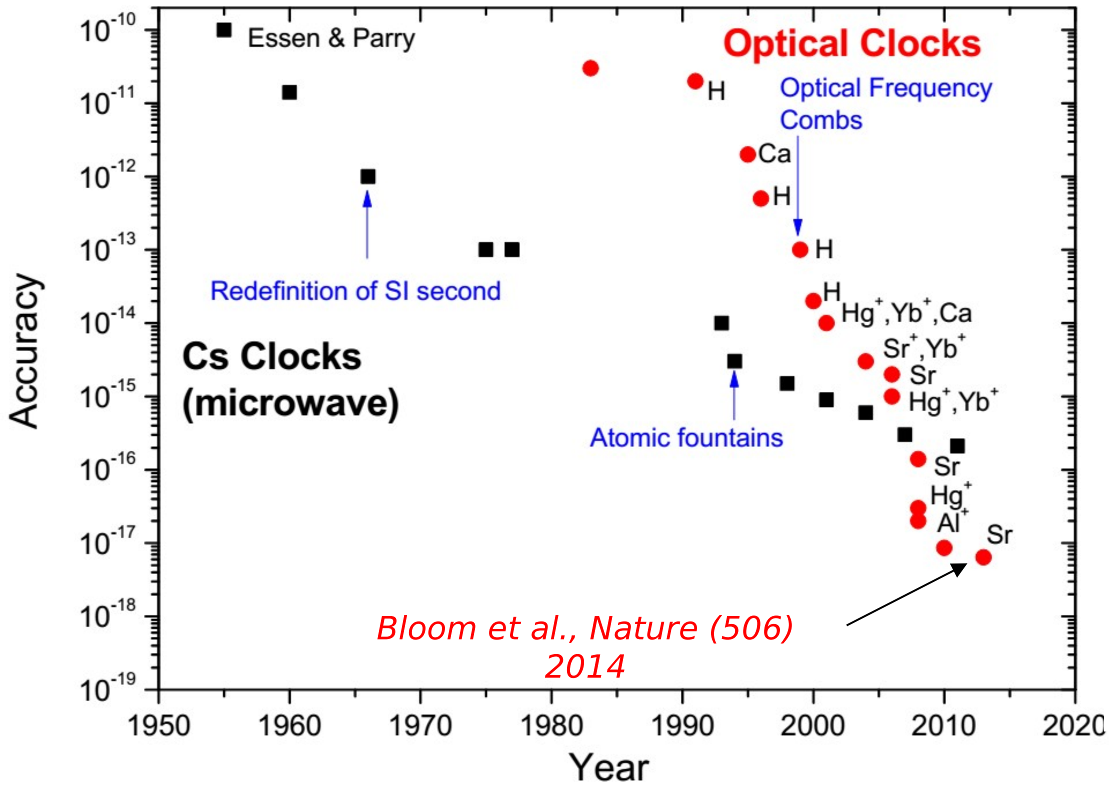
Stability: statistical properties of y(t), characterized by the Allan variance



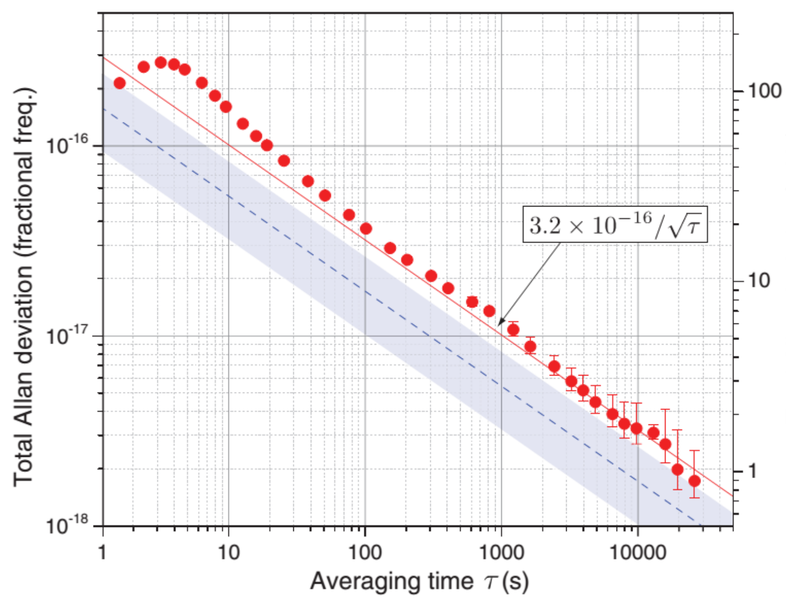




$$\omega(t) = \omega_{ef} \times (1 + \varepsilon + y(t))$$



Bloom et al., Nature (506) 2014

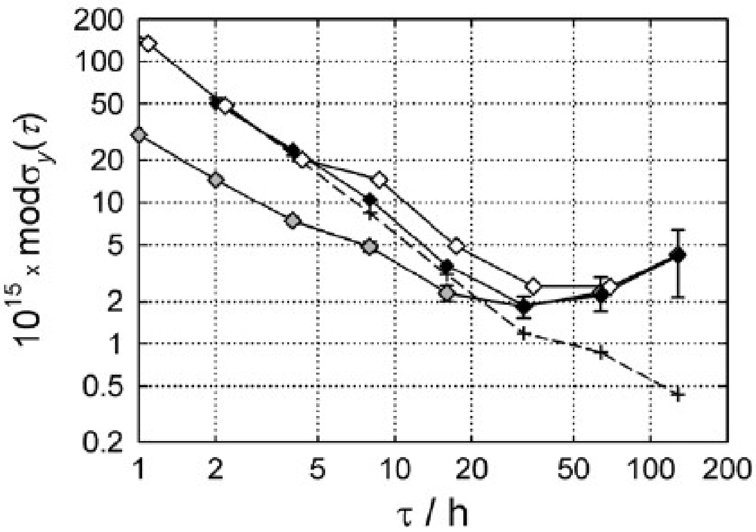


Hinkley et al., Science 341, 1215 (2013)

- Microwave clocks: 10^{-16} accuracy (Fountains)
- In space: microwave clocks with at best 10^{-14} stability at present (GNSS)
- Best performance of optical clocks to date:
  - Accuracy: Sr, 6.4×10^{-18} (JILA); Stability : Yb, 1.6×10^{-18} after 7 h averaging (NIST)
- Research in highly accurate clocks is an active, innovative and competitive field

- Best present satellite radio techniques (GNSS, TWSTFT) reach about $1x10^{-15}$ frequency stability after 1 day averaging ⇒ 3 years averaging required to reach $1x10^{-18}$!!! – and that is being very optimistic.
- Best present optical satellite link (T2L2) reaches about $3x10^{-13}$ after 10 s averaging ⇒ 25 days averaging required to reach $1x10^{-18}$!! – optimistic.
- ACES Microwave link is expected to reach $2x10^{-15}$ after 300 s averaging ⇒ 5 days to reach $1x10^{-18}$ – optimistic.

! 2-3 order of magnitudes improvement needed !

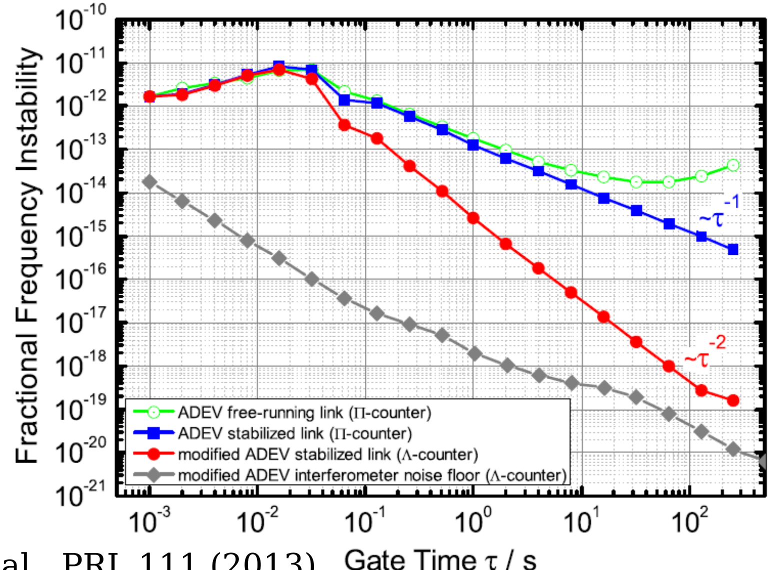


IEN-OP comparison with 3 techniques (GPS code, GPS phase, TWSTFT) (Bauch et al., Metrologia 2006)

- 100-2000 km phase coherent fibre links demonstrated
- Braunschweig-Munich: 1840 km → 4x10^{-19} (MDEV) in just 100s !!!
- Continental scales only
- Intensive development going on : (Western) Europe-wide network project Refimeve+
- Fibre costs : using existing fibres dedicated to research

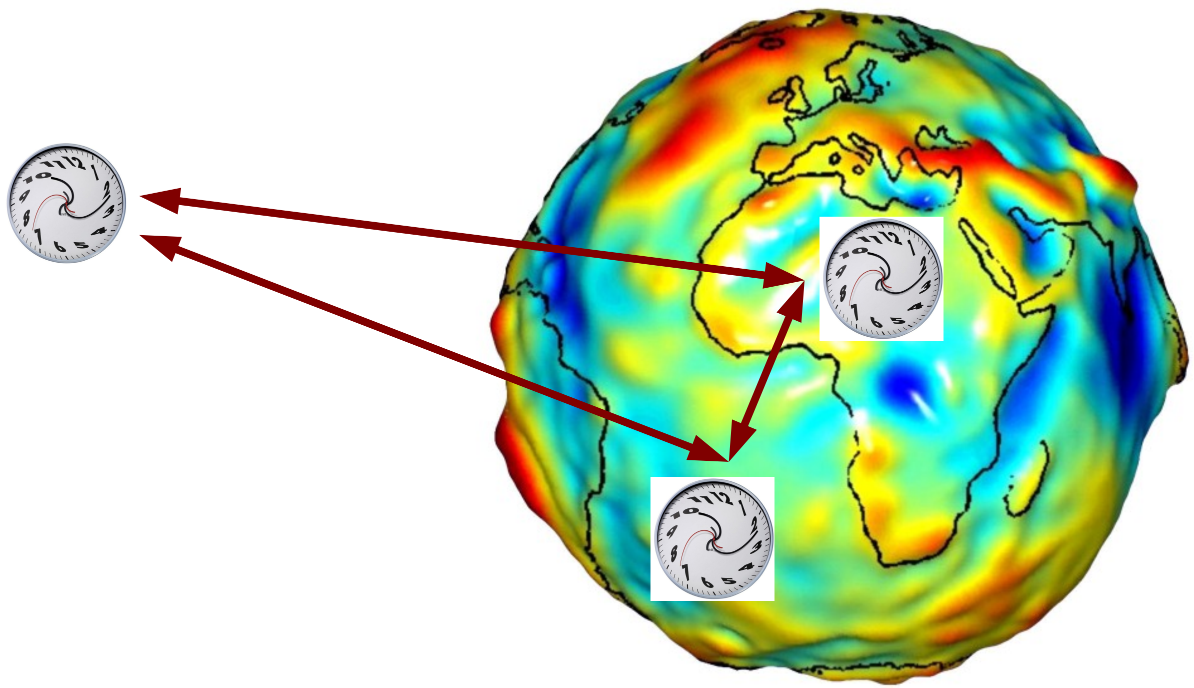
refimeve.fr

- Free space coherent optical links through turbulent atmosphere are in their infancy, but show potential for similar performance as fibre links (SYRTE-OCA, NIST)
- Transportable optical clocks are being developed (back to the future ?)



Droste et al., PRL 111 (2013)

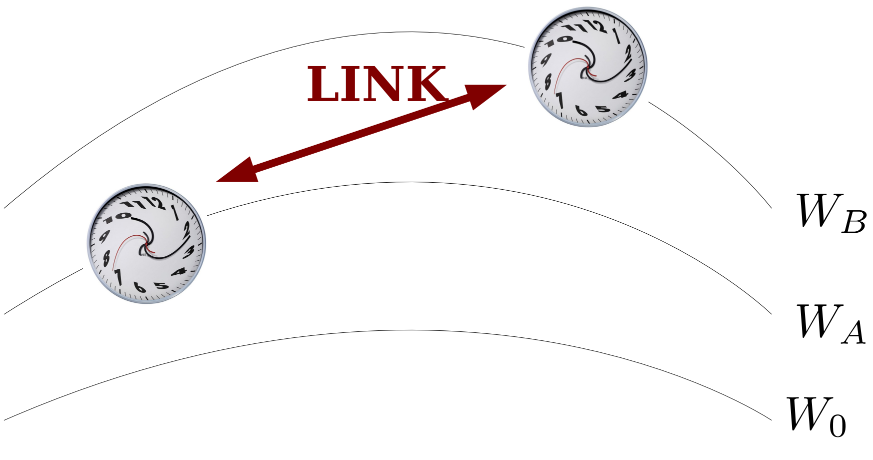
A brief introduction to chronometric geodesy



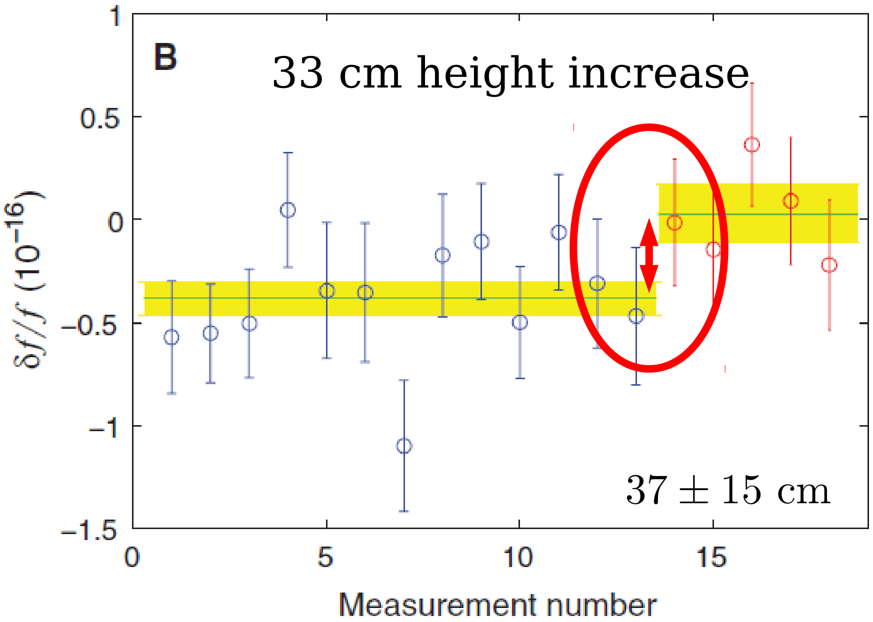
- Clock frequency comparison → measure directly gravity potential differences

$$\frac{\Delta f}{f} = \frac{W_B - W_A}{c^2} + O(c^{-4}) \, , \; W = U + \frac{v^2}{2}$$

$$10^{-18} \leftrightarrow 0.1 \text{ m}^2.\text{s}^{-2} \leftrightarrow 1 \text{ cm}$$



Chou et al, Science, 329 (2010)



Isochronometric surface and equipotential

- An **isochronometric surface** is a surface where all clocks beat at the same rate.
- They are almost equivalent to **equipotential surfaces of the gravity field** (differences of the order of 2 mm)

$$\left.\frac{\mathrm{d}\tau}{\mathrm{d}t}\right|_S = \mathrm{cst}$$

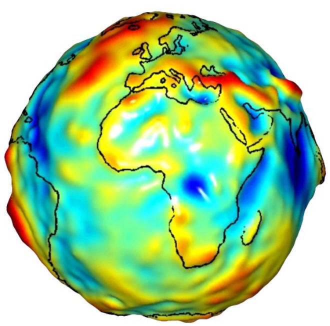
- Let t be the time given by a clock at infinity and at rest in the GCRS. Then the reference isochronometric surface (TT) defined by IAU is:

$$\frac{\mathrm{d}\tau}{\mathrm{d}t} = \mathrm{cst} = 1 - L_G$$

where $L_G = 6.969290134 \times 10^{-10}$ is a defining constant (IAU resolution B1.9, 2000)

- From this definition we get a reference equipotential

$$W_0 \equiv U + \frac{v^2}{2} = c^2 L_G + O(c^{-2})$$



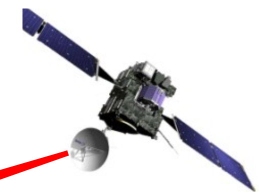
→ variation of the geoid ~ 2 mm/y → 2×10^{-18} in 10 years
→ use of clocks to **unify height systems**

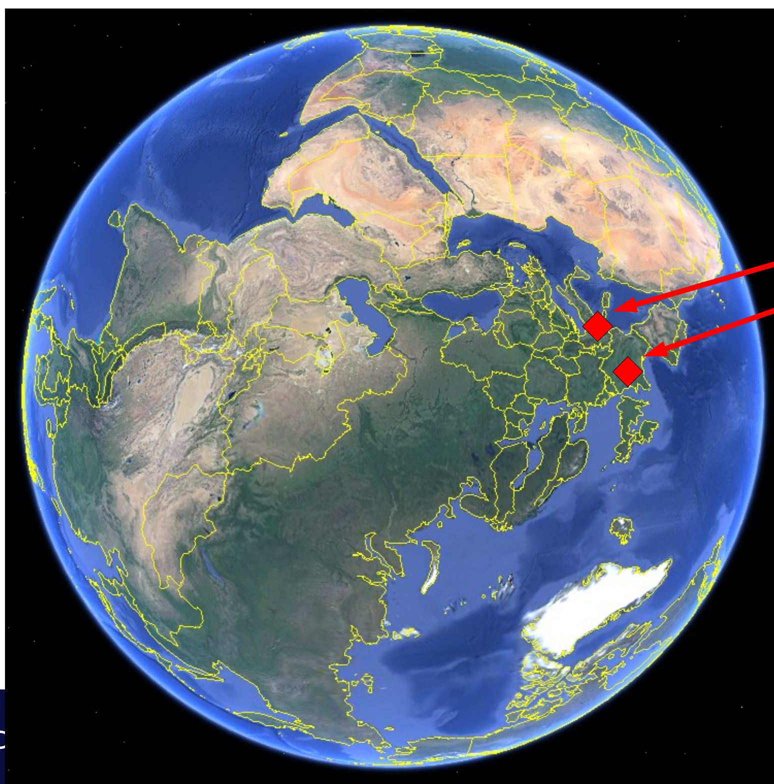
- As a proof-of-principle, one can determine (roughly) J_2 with two clocks:

$$\frac{\Delta f}{f} = \frac{W_B - W_A}{c^2} + O(c^{-4}) \ , \ W = U + \frac{v^2}{2}$$

$$U = \frac{GM_E}{r}\left[1 + \frac{J_2 R_E^2}{2r^2}\left(1 - 3\sin(\phi)^2\right)\right]$$

A: INRIM CsF1 (Turin, Italy)
B: SYRTE FO2 (Paris, France)





$$J_2 = (1.097 \pm 0.016) \times 10^{-3}$$

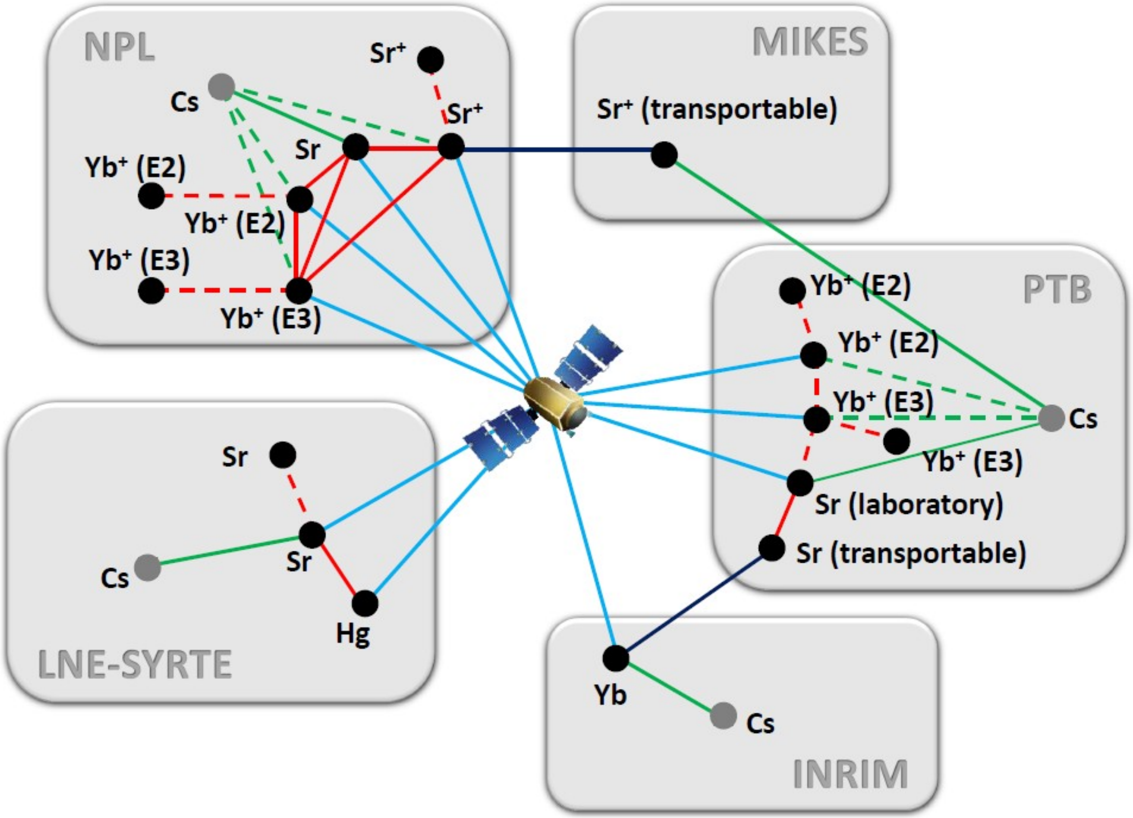
- Error of ~1.4% compare to best known value
- However, ground clocks are sensitive to higher order multipoles

- Measure "absolute" altitude of clocks (referenced to the space clock)
- Measure **ground-to-ground gravitational potential differences** up to 1 $m^2.s^{-2}$ accuracy (10 cm, 10^{-17} relative frequency shift)

A coordinated programme of optical clock comparisons

EMRP
European Metrology Research Programme
Programme of EURAMET

The EMRP is jointly funded by the EMRP participating countries within EURAMET and the European Union



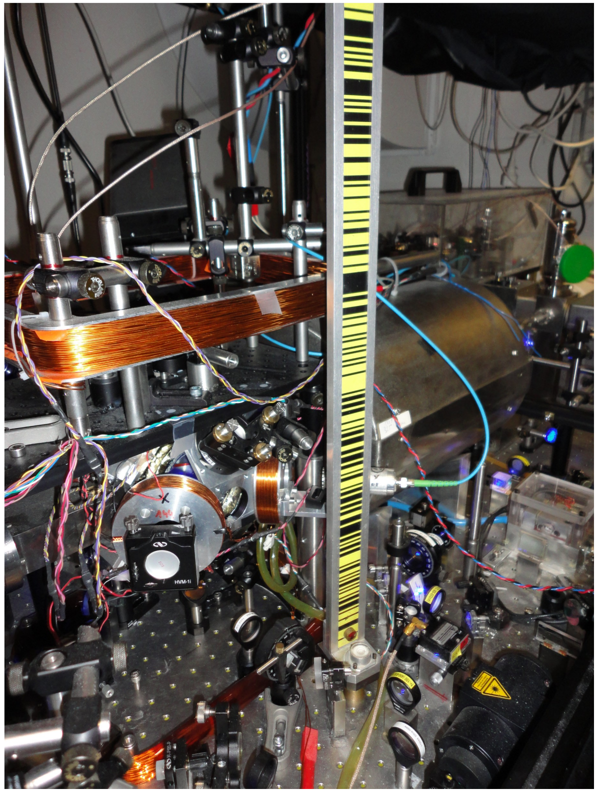
Local optical frequency comparisons using femtosecond combs

Frequency comparisons using transportable optical clocks

Optical frequency comparisons using broad bandwidth TWSTFT

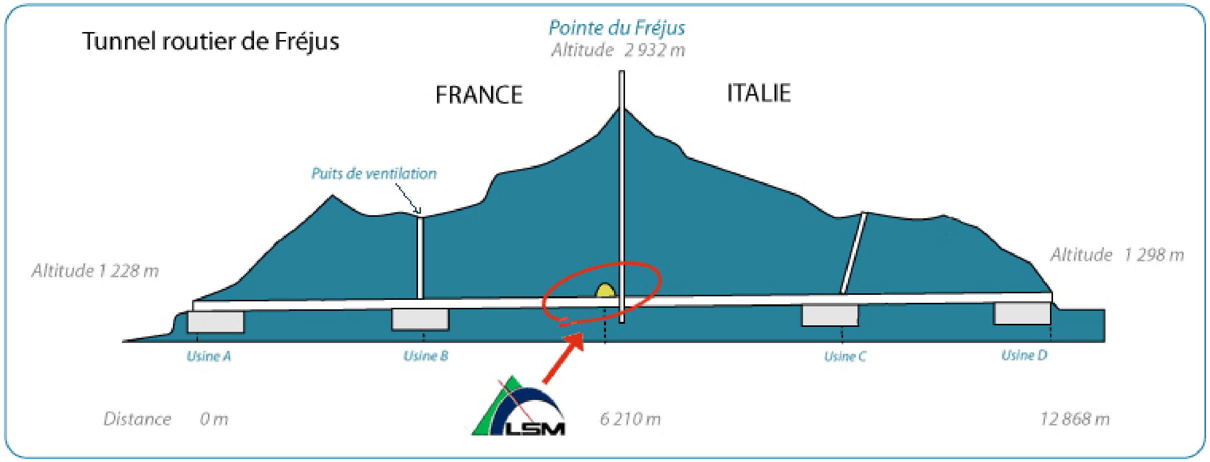
Absolute frequency measurements

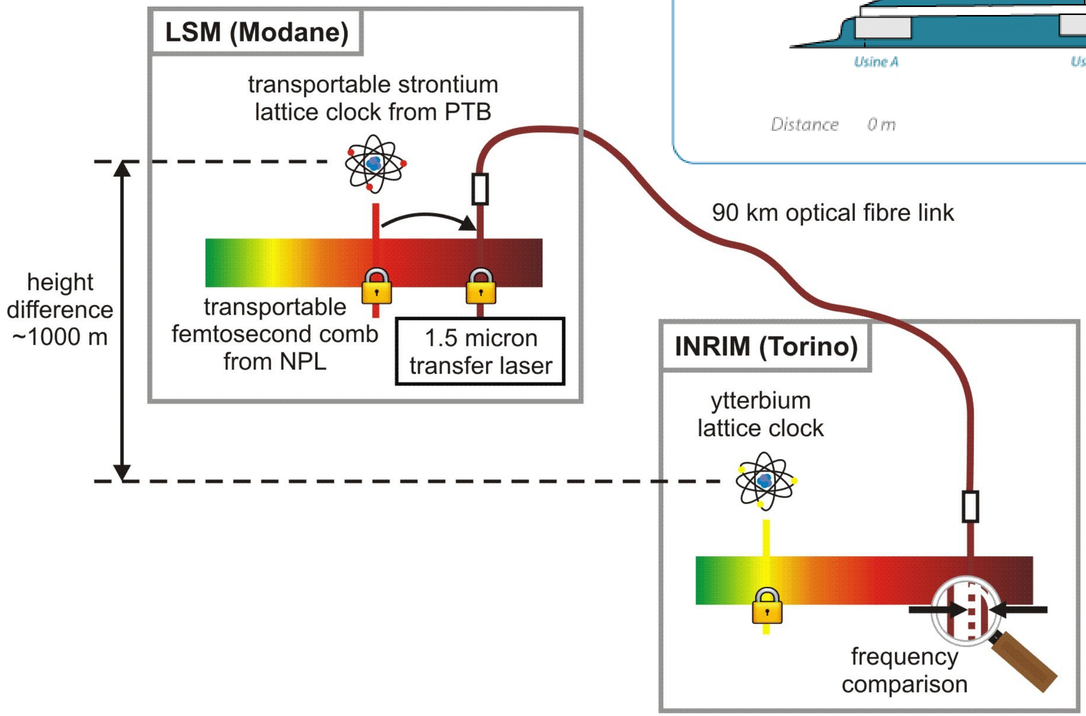
- **Determination of the static gravity potential at all clock locations**
- Development of a **refined European geoid model** including new gravity observations around all relevant clock sites (IFE)
- Investigation of time-variable components of the gravity potential, e.g. due to tides.



SYRTE clocks leveling campaign (IGN SGN Travaux Spéciaux)

- **Aim**: to demonstrate that optical clocks can be used to **measure gravity potential differences** over medium-long baselines with **high temporal resolution**.





- Height difference ~ 1000 m → Gravitational redshift ~ 10^{-13}
- Target → resolution of tens of cm in a few hours

Towards a high resolution geopotential model using chronometric geodesy

G. Lion[1,2], C. Guerlin[3], S. Bize[1], M. Holschneider[4], G. Métivier[2], P. Delva[1] and I. Panet[2]

[1]LNE–SYRTE, Observatoire de Paris, CNRS (UMR8630), UPMC

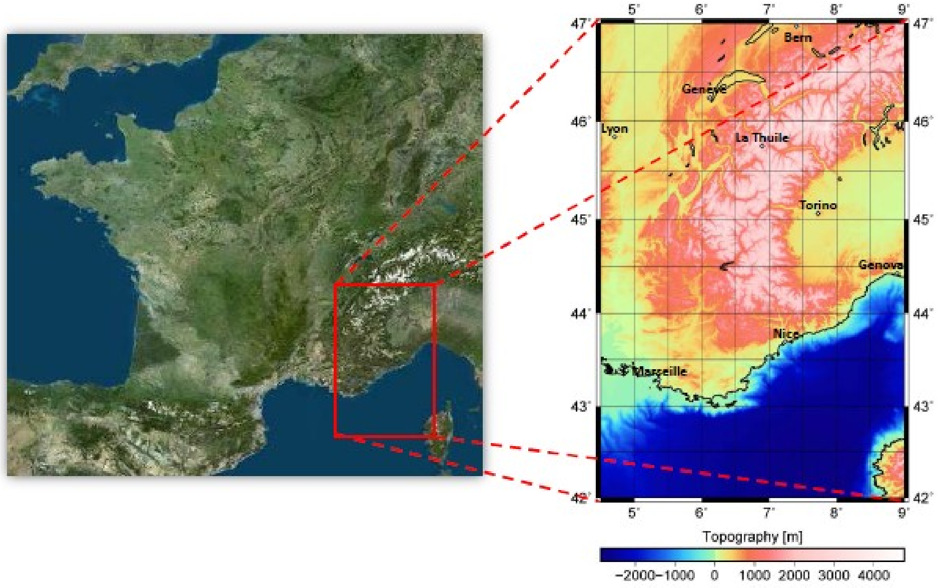
[2]Institut National de l'Information Géographique et Forestière, LAREG, Université Paris Diderot

[3]Laboratoire Kastler-Brossel, ENS, CNRS, Université Pierre et Marie Curie-Paris 6

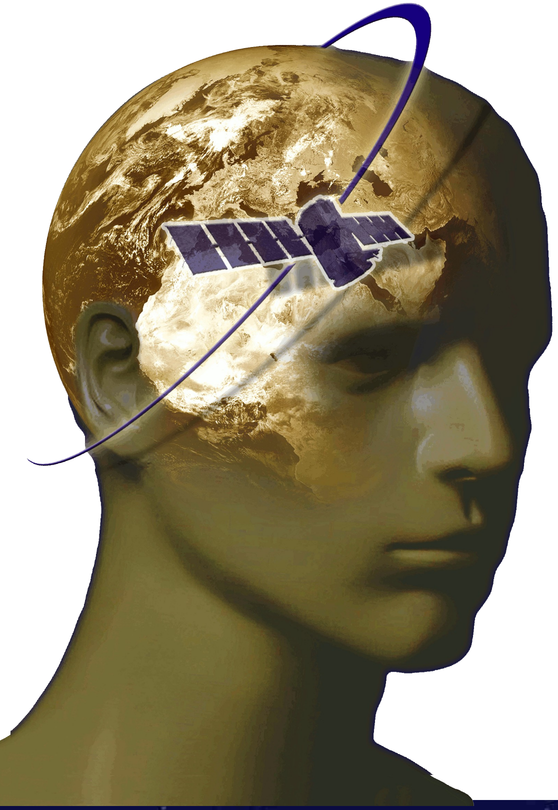
[4]Institutes of Applied and Industrial Mathematics, Universtität Potsdam

Region of interest

- French Alps to the Mediterranean Sea
- Region from 42° to 47° latitude and 4.5° to 9° longitude



Proposal for a test of the gravitational redshift with Galileo satellites 5 and 6



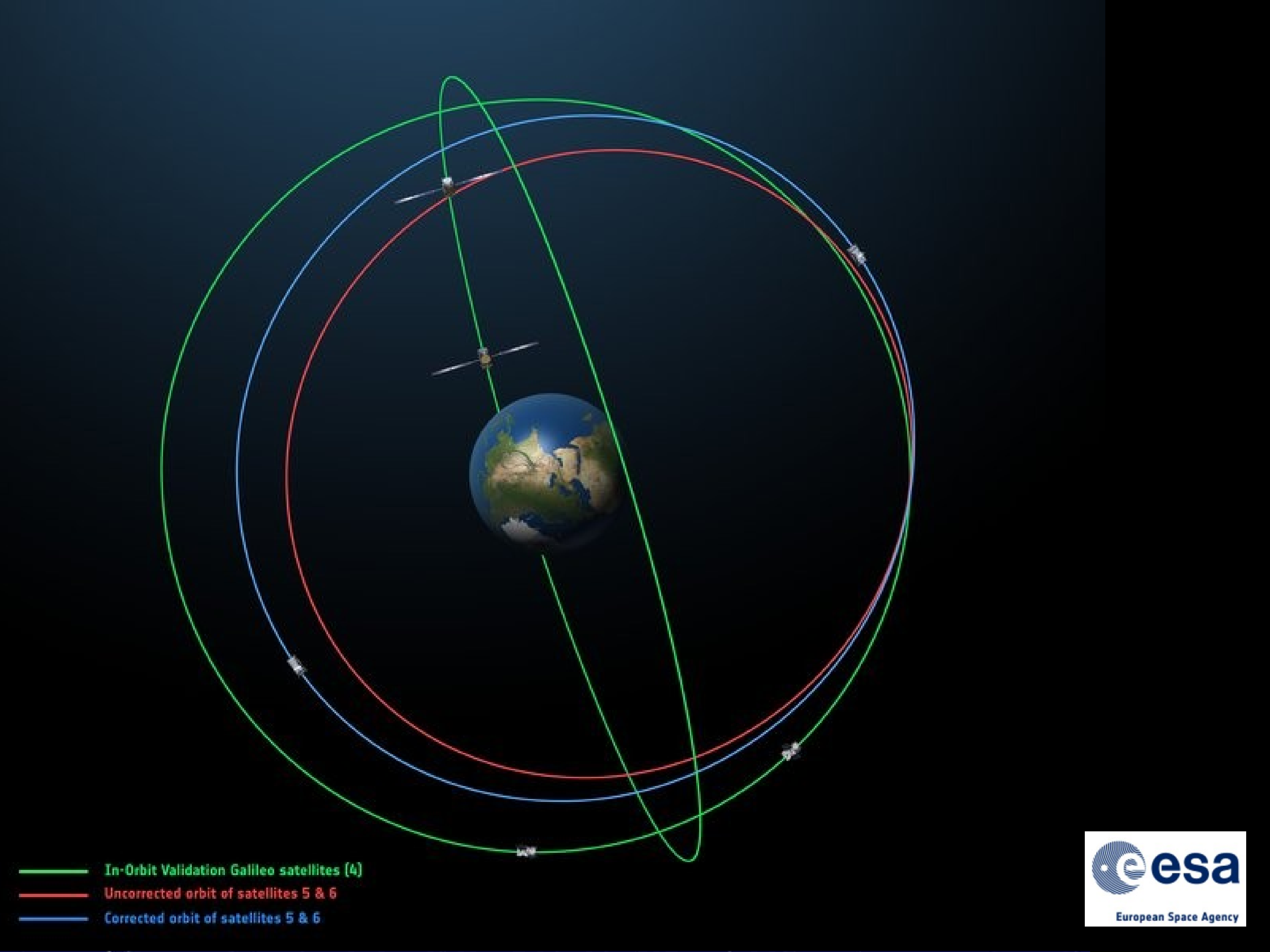
- **Galileo satellites 5 and 6** were launched with a Soyuz rocket on 22 august 2014 on the wrong orbit due to a technical problem
- Launch failure was due to a **temporary interruption of the** joint hydrazine **propellant supply** to the thrusters, caused by freezing of the hydrazine, which resulted from the proximity of hydrazine and cold helium feed lines.
- **Last launch** of Galileo satellites 7 and 8 occured on **friday 27 march**

European Space Agency

GALILEO

EGNOS

Navigation solutions powered by Europe


In-Orbit Validation Galileo satellites (4)
Uncorrected orbit of satellites 5 & 6
Corrected orbit of satellites 5 & 6
esa
European Space Agency

The eccentricity correction

- For a Keplerian orbit one shows that :

$$\tau(t) = \left(1 - \frac{3Gm}{2ac^2}\right) t - \frac{2\sqrt{Gma}}{c^2} e \sin E(t) + \mathrm{Cste}$$

constant frequency bias eccentricity correction

- One need an **accurate clock** to measure the constant frequency bias
- The eccentricity correction is a periodic term → **use the stability of the clock to “average” the random noise**
- Limitations are due to mismodeled **systematics effects**

- RMS deviation between theory and experiment is **~ 2.2 %**
- Evidence of **systematic bias** during some particular passes

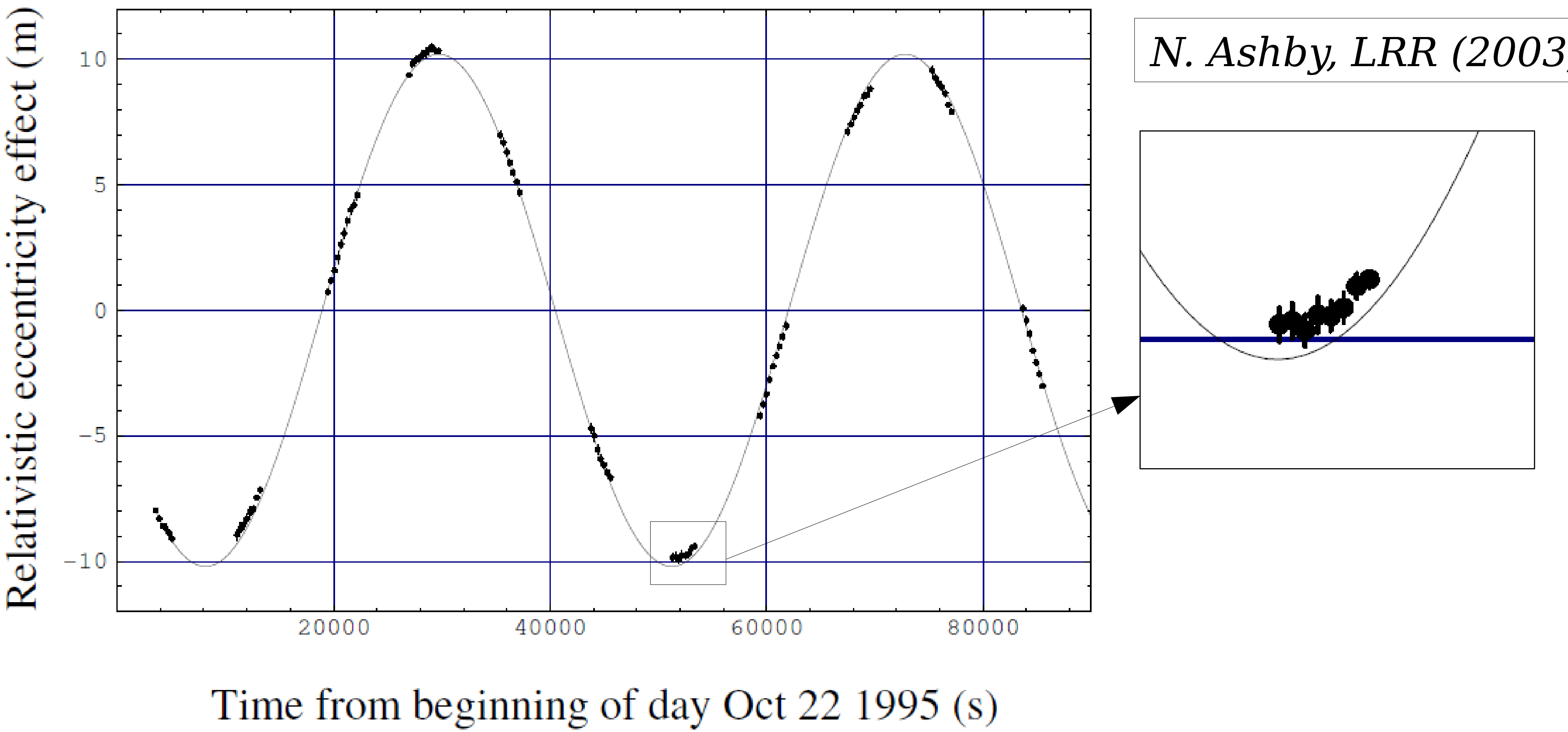
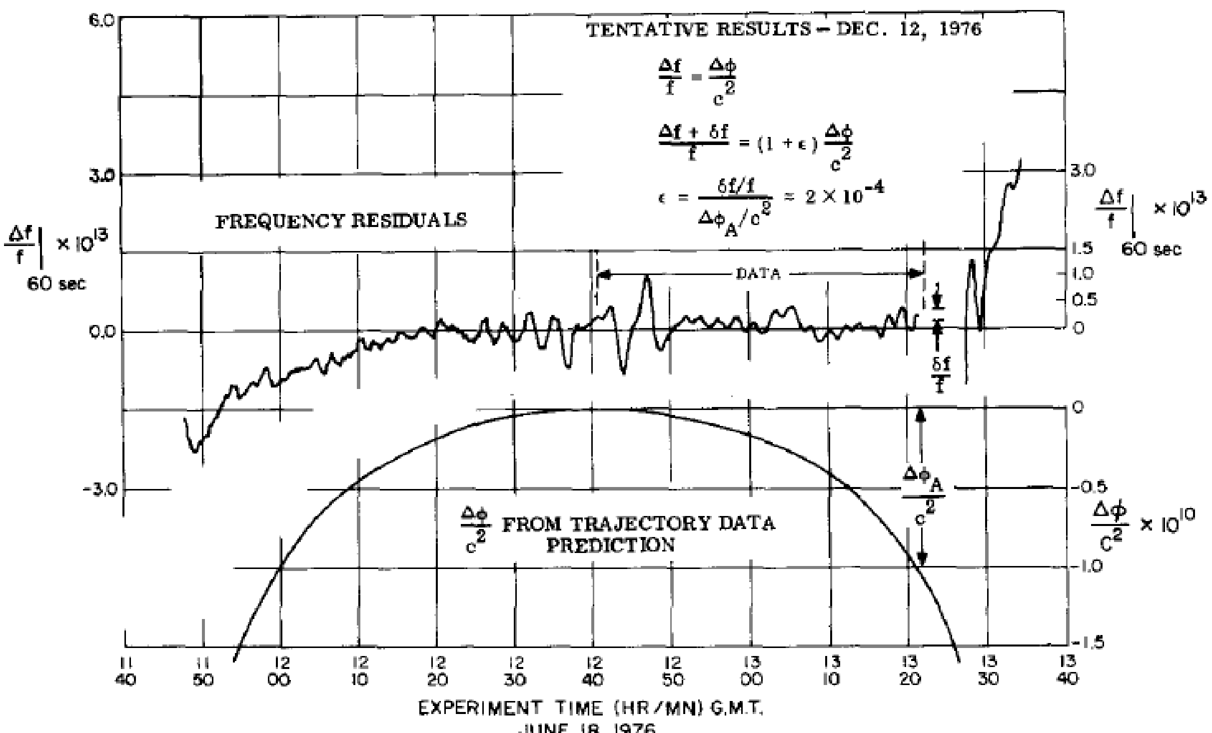


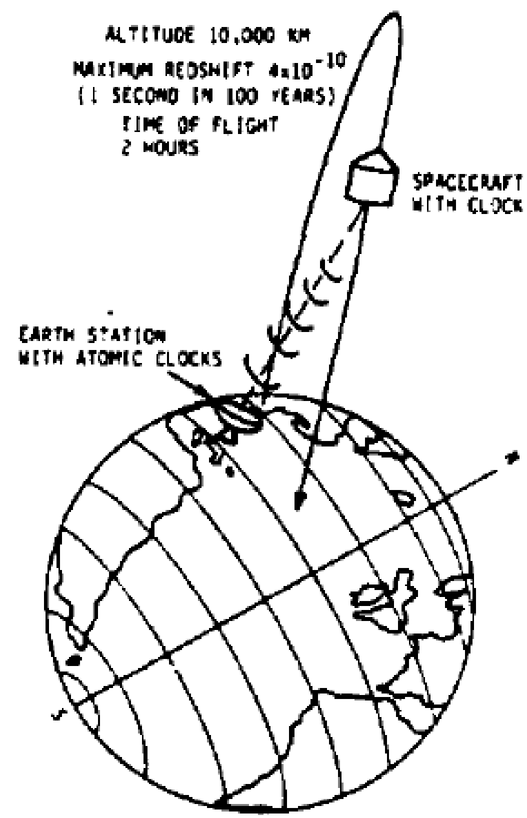
Figure 5: *Comparison of predicted and measured eccentricity effect for SV nr. 13.*

Gravity Probe A (1976)

R. Vessot et al., GRG 1979, PRL 1980, AdSR 1989

- Test of the redshift on a **single parabola**
- Continuous **two-way microwave link** between a spaceborne hydrogen maser clock and ground hydrogen masers
- Frequency shift verified to $7*10^{-5}$
- **Gravitational redshift verified to $1.4*10^{-4}$**





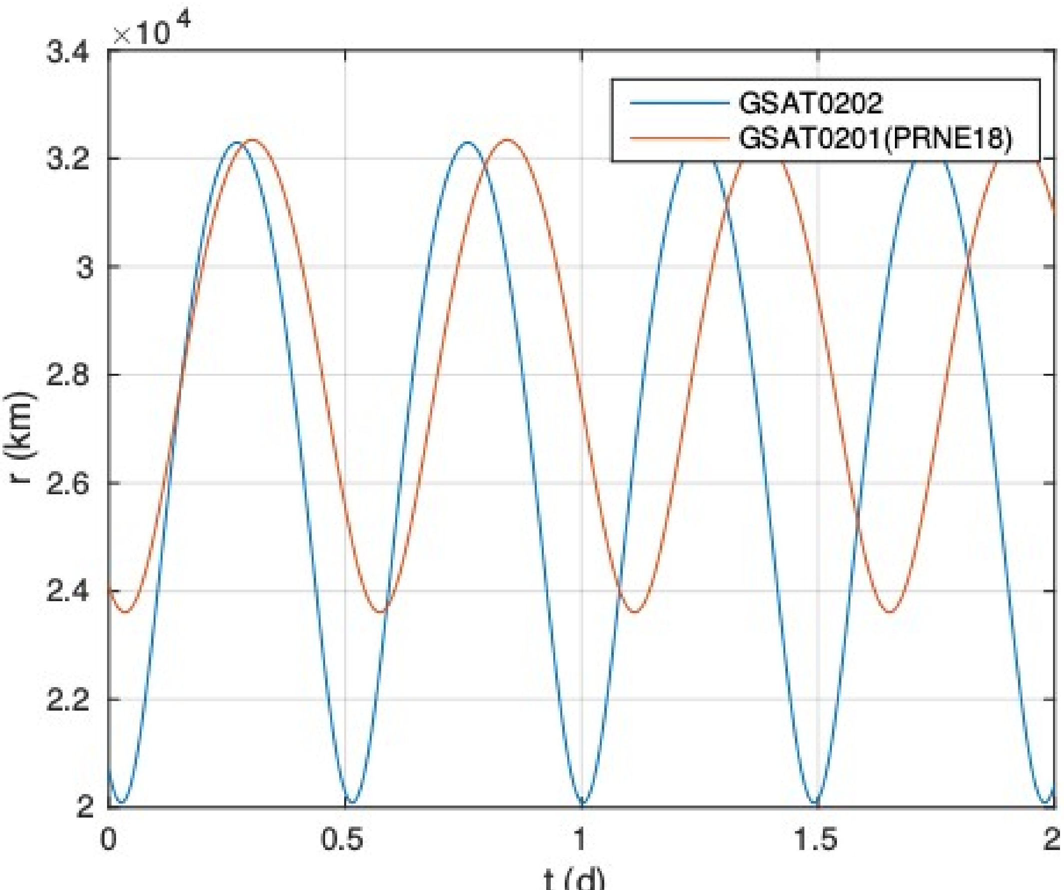
Simulation of :

1. Galileo 5 and 6 **orbits**
2. Realistic **onboard clock noise**
3. **Gravitational Redshit Signal** (including a Local Position Invariance violation, random noise and systematic effects)

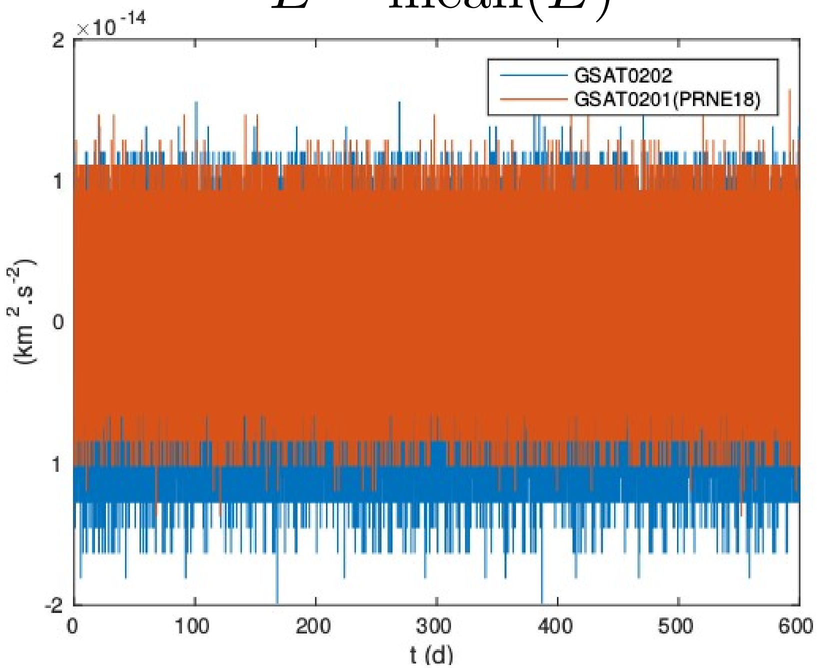
Analysis of the simulated signal with two different methods :

1. **Matched Filtering** in the frequency domain
2. **Linear Least-Square + Monte-Carlo** in the time domain

- Wrong orbit due to a technical problem : **eccentric orbit** (~0.16 today, ~0.23 initially)
- Two-Lines Elements (TLE) + Kepler equation for a duration of 2 years



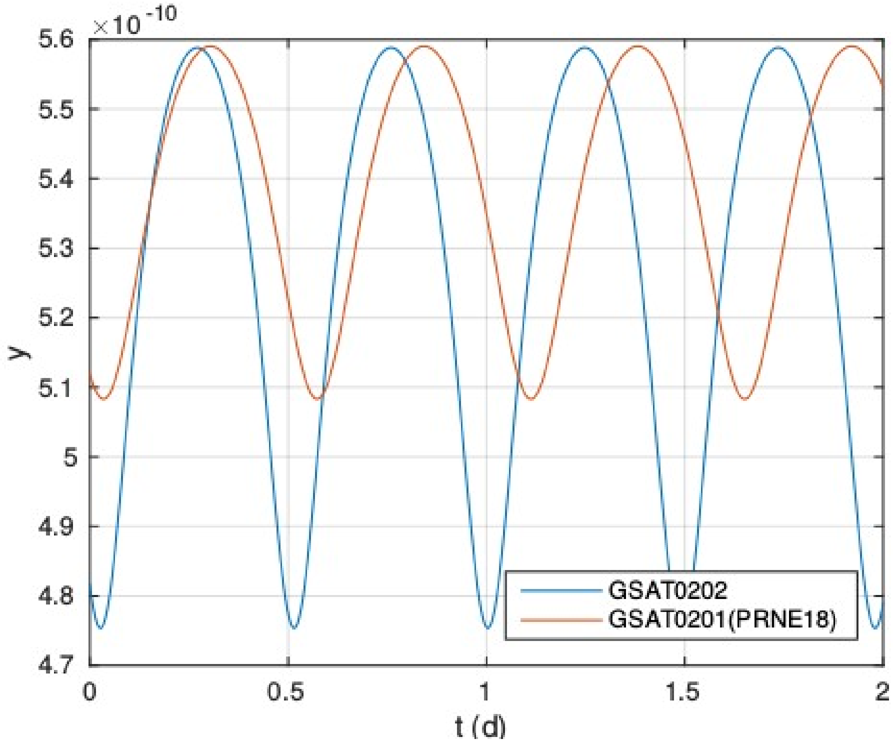
$$\tilde{E} - \mathrm{mean}(\tilde{E})$$

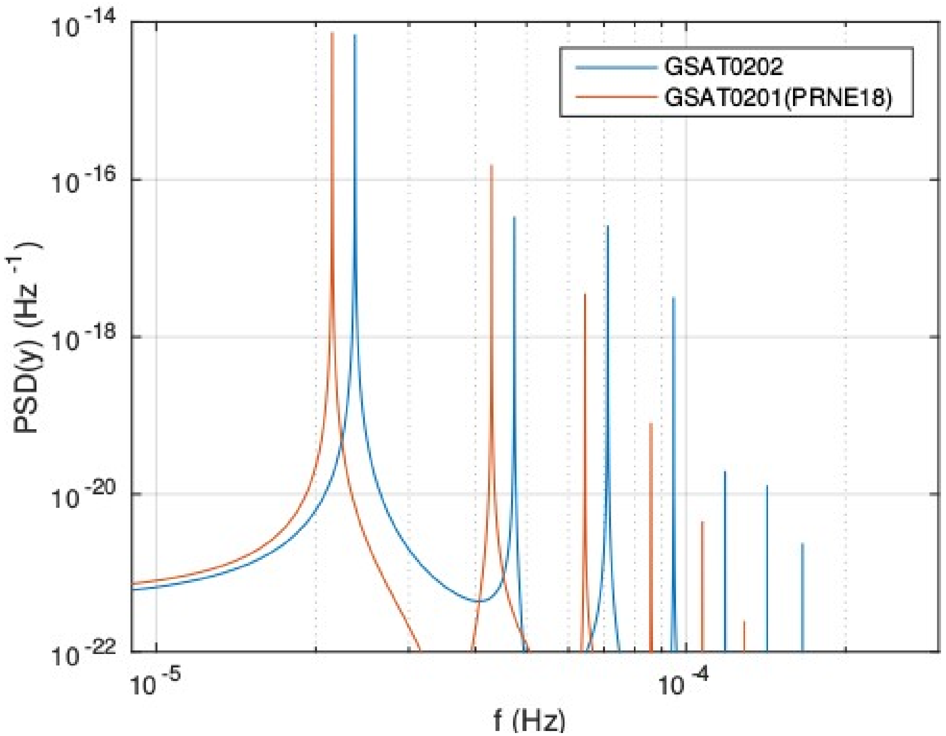


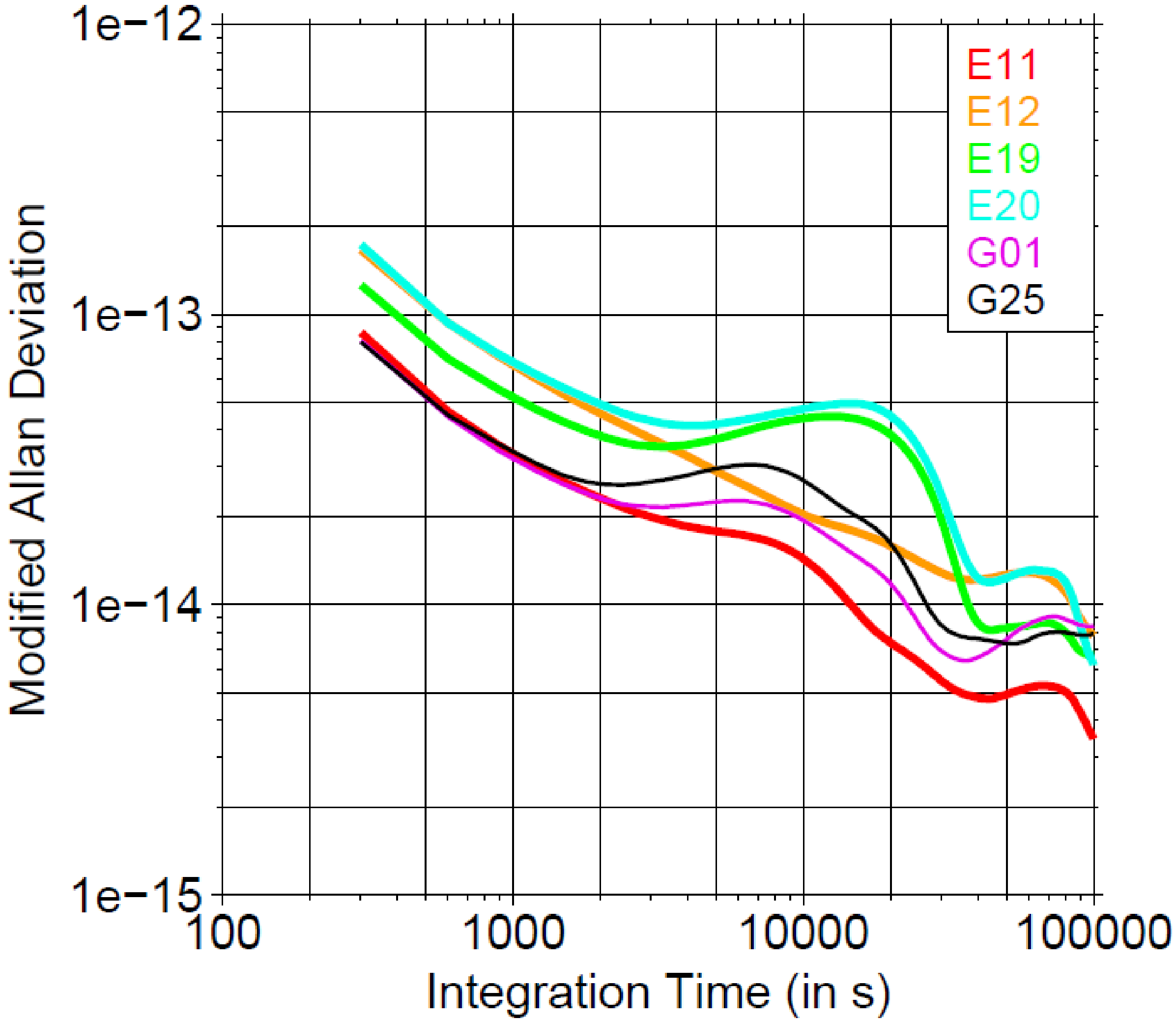
$$\tilde{E} = v^2/2 - GM/r$$

- Simple phenomenological model for LPI violation (C. Will, LRR 2014)
- Alpha is = 0 in GR
- GP-A limit : alpha < 1.4×10^{-4}

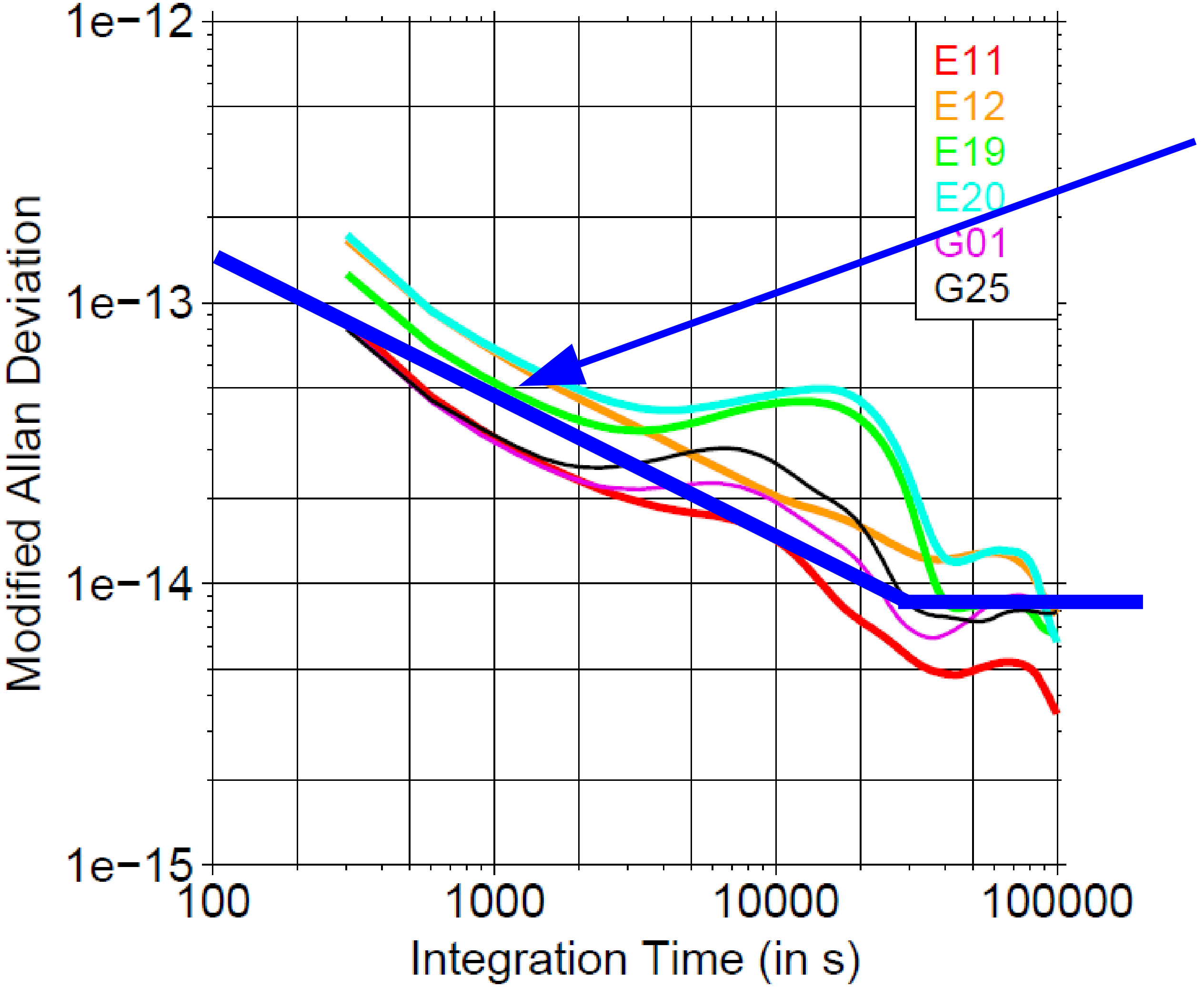
$$\tilde{y}(\alpha) = -(1+\alpha)\frac{GM}{c^2 r_s}$$







L. Prange et al., IAG Potsdam Proceedings, 2014, accepted for publication

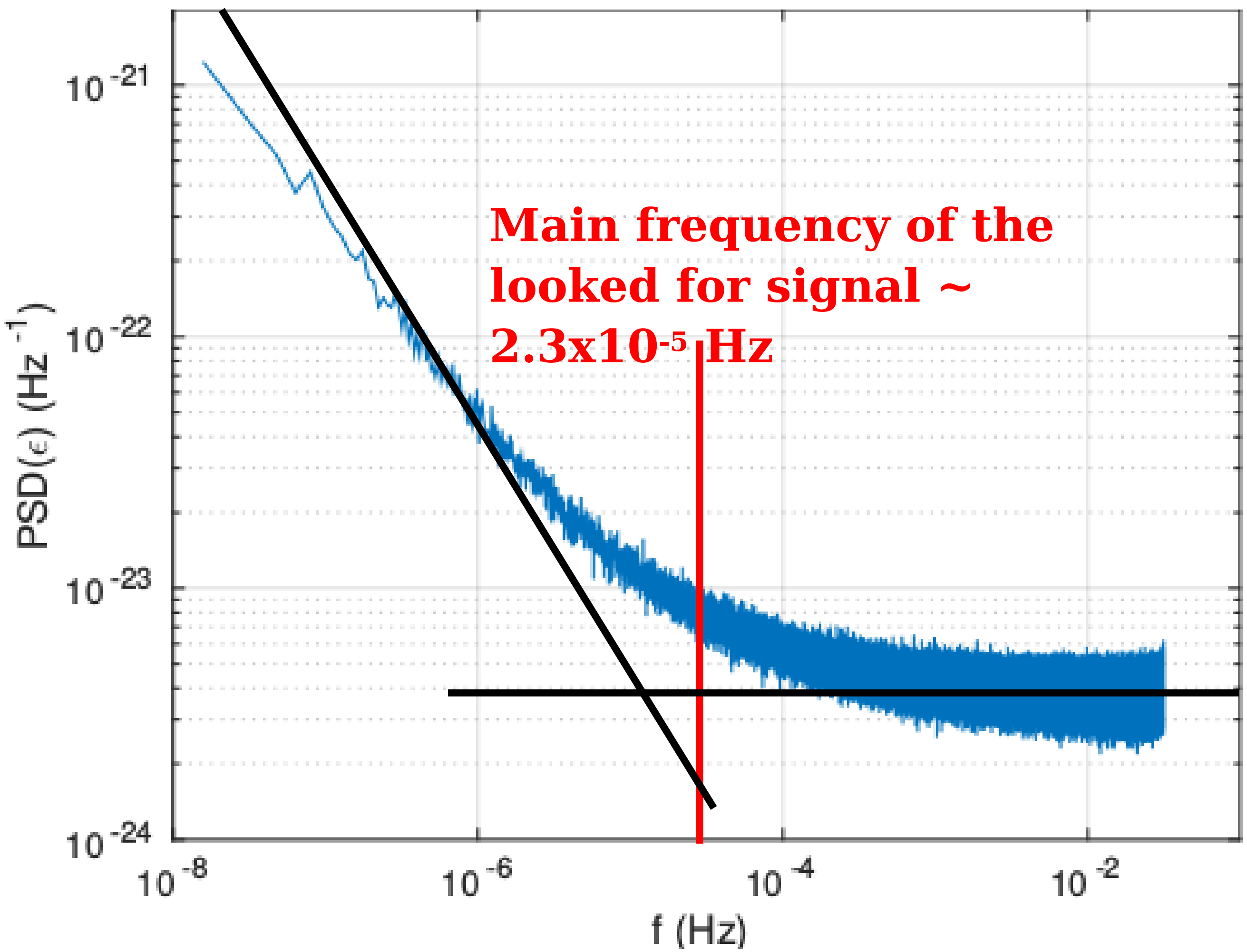


MDEV of the simulated clock noise

- White noise ~5x10^{-14} @ 1000s
- Flicker noise ~8x10^{-15}

L. Prange et al., IAG Potsdam Proceedings, 2014, accepted for publication

Simulation of onboard clock noise



PSD of the simulated clock noise

- White noise ~4x10^{-24}
- Flicker noise ~4x10^{-29} @ 1 Hz ~2x10^{-24} @ signal frequency

Matched filtering method

Sensitivity is the inverse of the signal-to-noise (SNR) ratio ρ, which is maximized with **matched filtering**

$$\rho^2 = \int_{-\infty}^{+\infty} \frac{|\tilde{X}(f)|^2}{S_N(f)} \mathrm{d}f$$

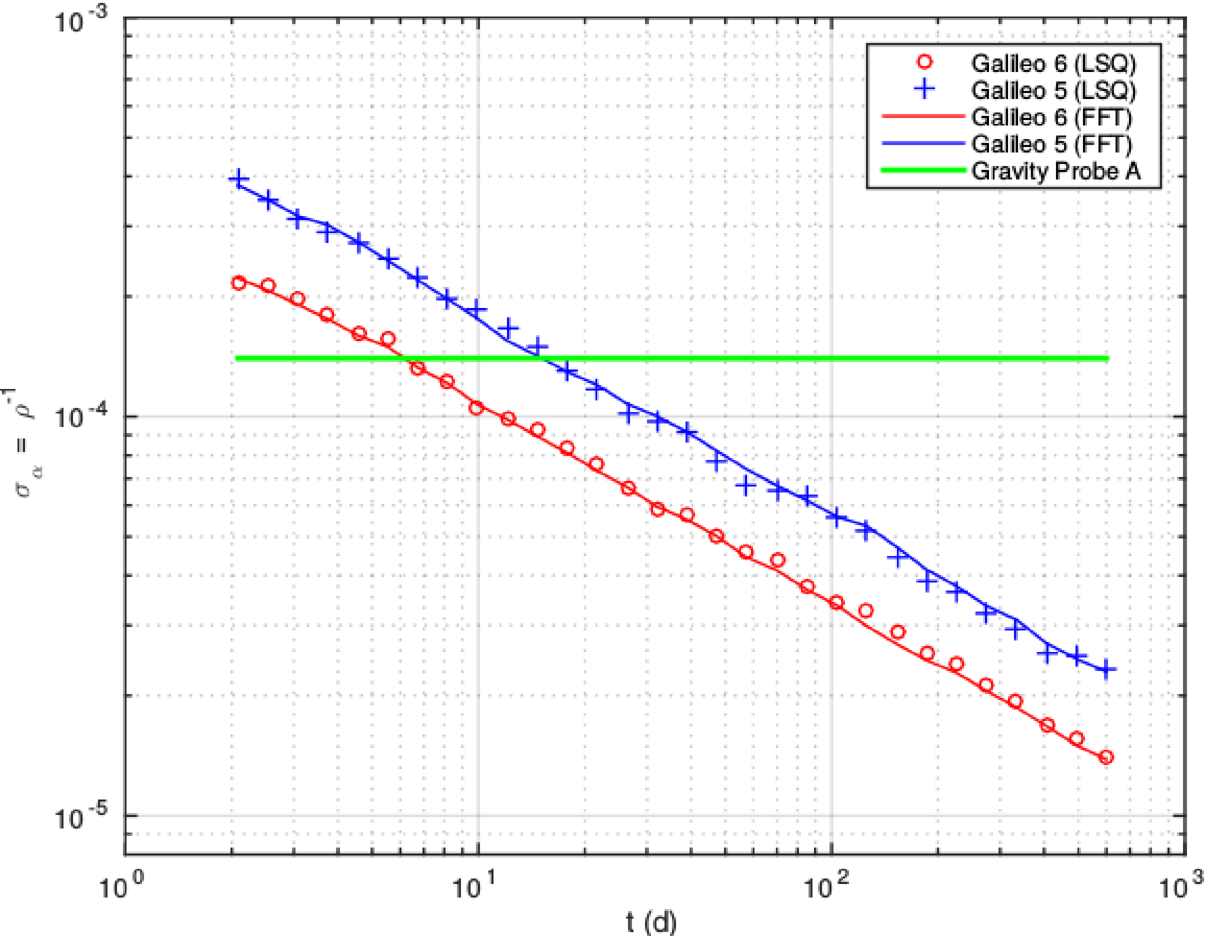
- $\tilde{X}(f)$: Fourier transform of the (ideal) signal
- $S_N(f)$: PSD of the random noise

Linear least-square method

Find the minimum of the merit function χ^2 with respect to alpha

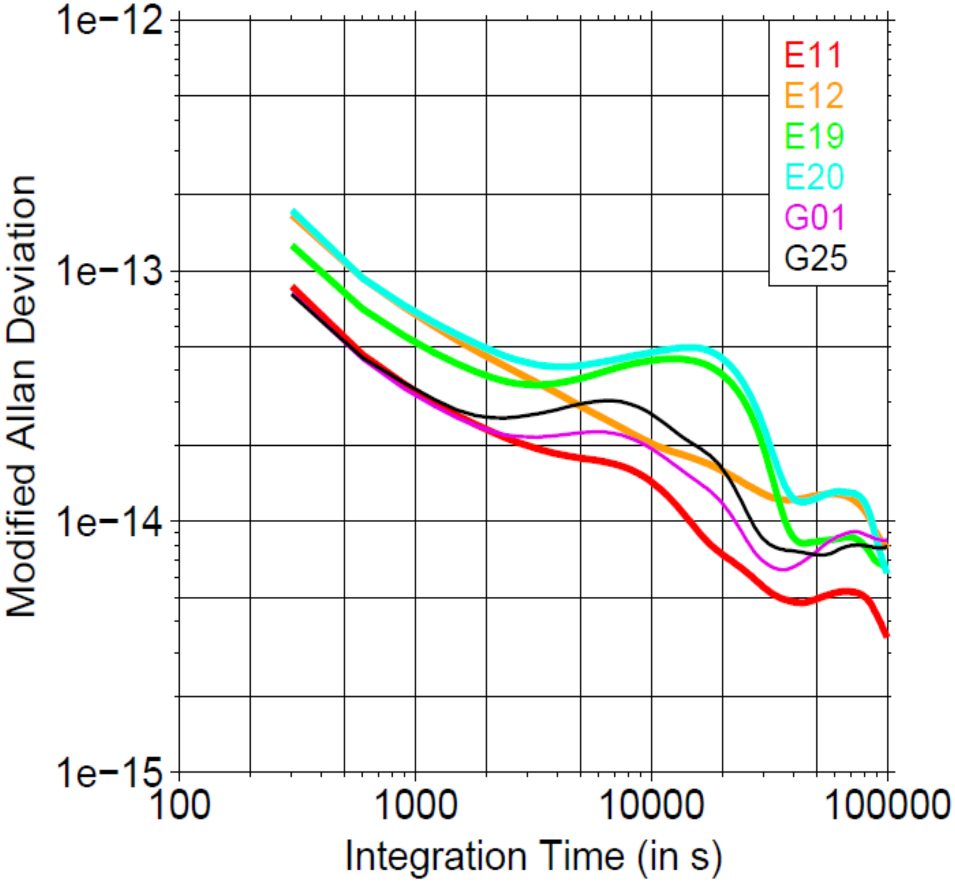
$$\chi^2 = \sum_{i=1}^{N} \left[(y(t_i) + \epsilon_i + \epsilon_{\mathrm{sys}}) - (\tilde{y}(\alpha; t_i) + A) \right]^2$$

Sensitivity of the gravitational redshift test



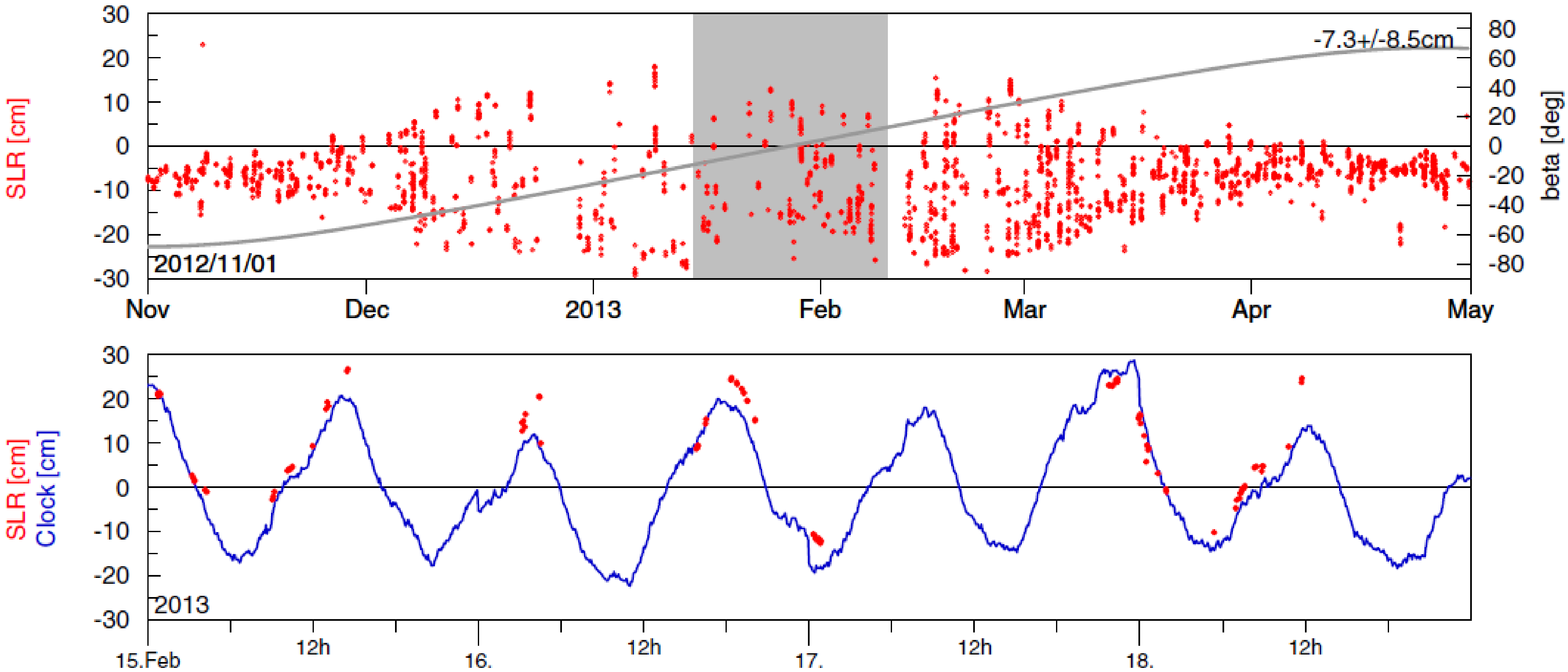
- *The best actual limit on grav. redshift (GP-A) is reached after ~2 weeks with Galileo 5*
- *After one year of integration the sensitivity is ~3x10-5 → a factor of 5 better than GP-A, which was a dedicated experiment (expected sensitivity of ACES-PHARAO is 2-3x10-6)*

- The two very different methods agree on the sensitivity of the test
- We proved mathematically that $\sigma_\alpha = \rho^{-1}$
- **Problem** : all systematic effects that mimic the gravitational redshift signal will induce a bias in the estimation of alpha → fake violation of LPI



Bump in the MDEV → systematic effect at orbital frequency due to a **radial error in the estimated orbit** (Montenbruck et al., J.Geo., 2014)

→ mimick a grav. redshift violation !



- Systematic effect shows a **dependency with the sun elevation angle**, ie. the direction of Sun w.r.t. the satellite orbital plane (Montenbruck et al., J.Geo., 2014)
- At least 75 % of this effect due to **mismodeling of Solar Radiation Pressure** (SRP), other effects could be due onboard temperature variations

- Systematic effect due to mismodelling of the SRP :
  - Effect at orbital frequency with a frequency shift (1/year) (linked to the direction of the Sun
  - Amplitude modulation at frequency (1/year)

$$\epsilon_{\rm sys} = A \sin((n_{\rm sat} + \omega_{\rm year})t + \phi_1)(1 + B(\cos(\omega_{\rm year} t + \phi_2) - 1)),\ \omega_{\rm year} = 2\pi/{\rm year}$$

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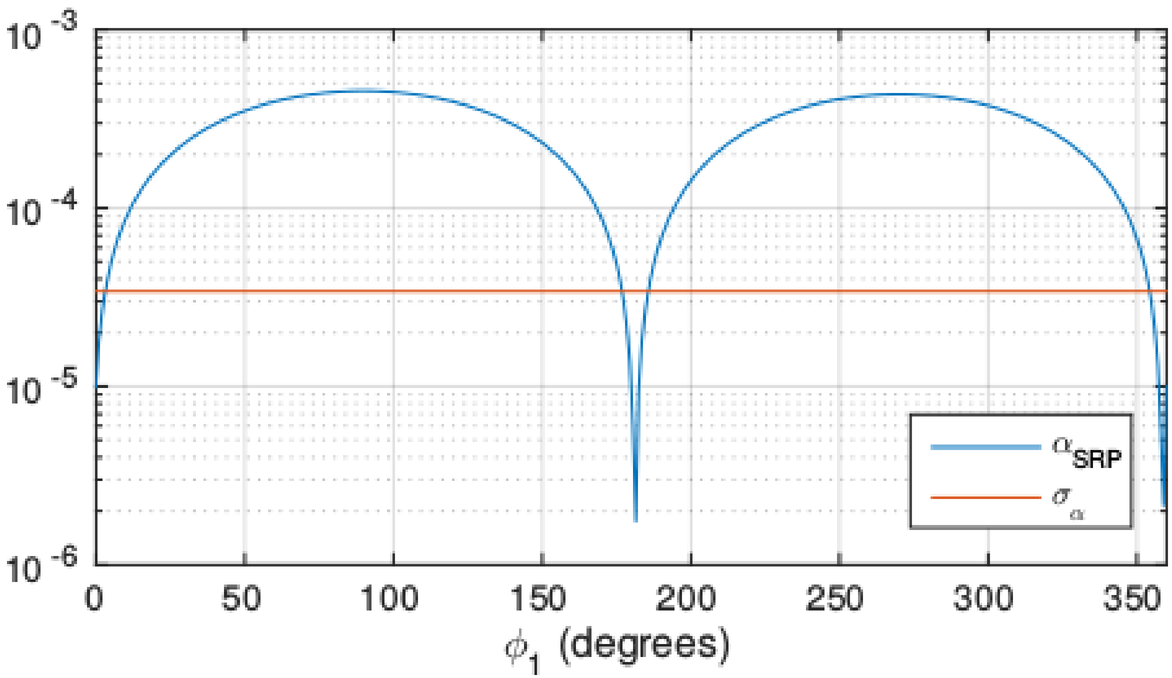
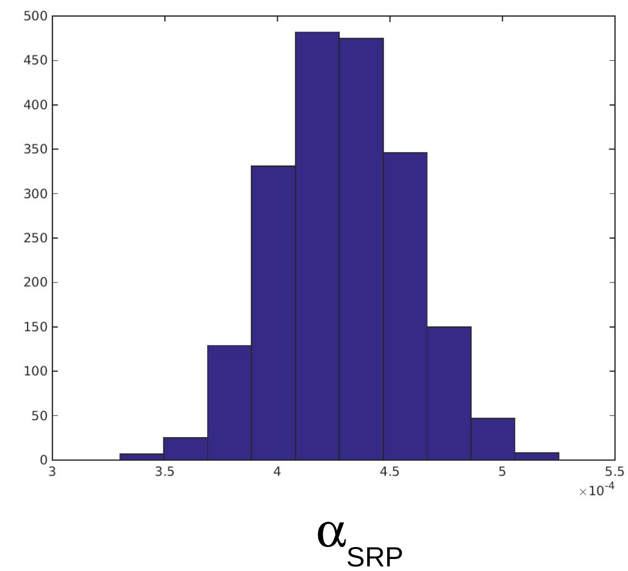
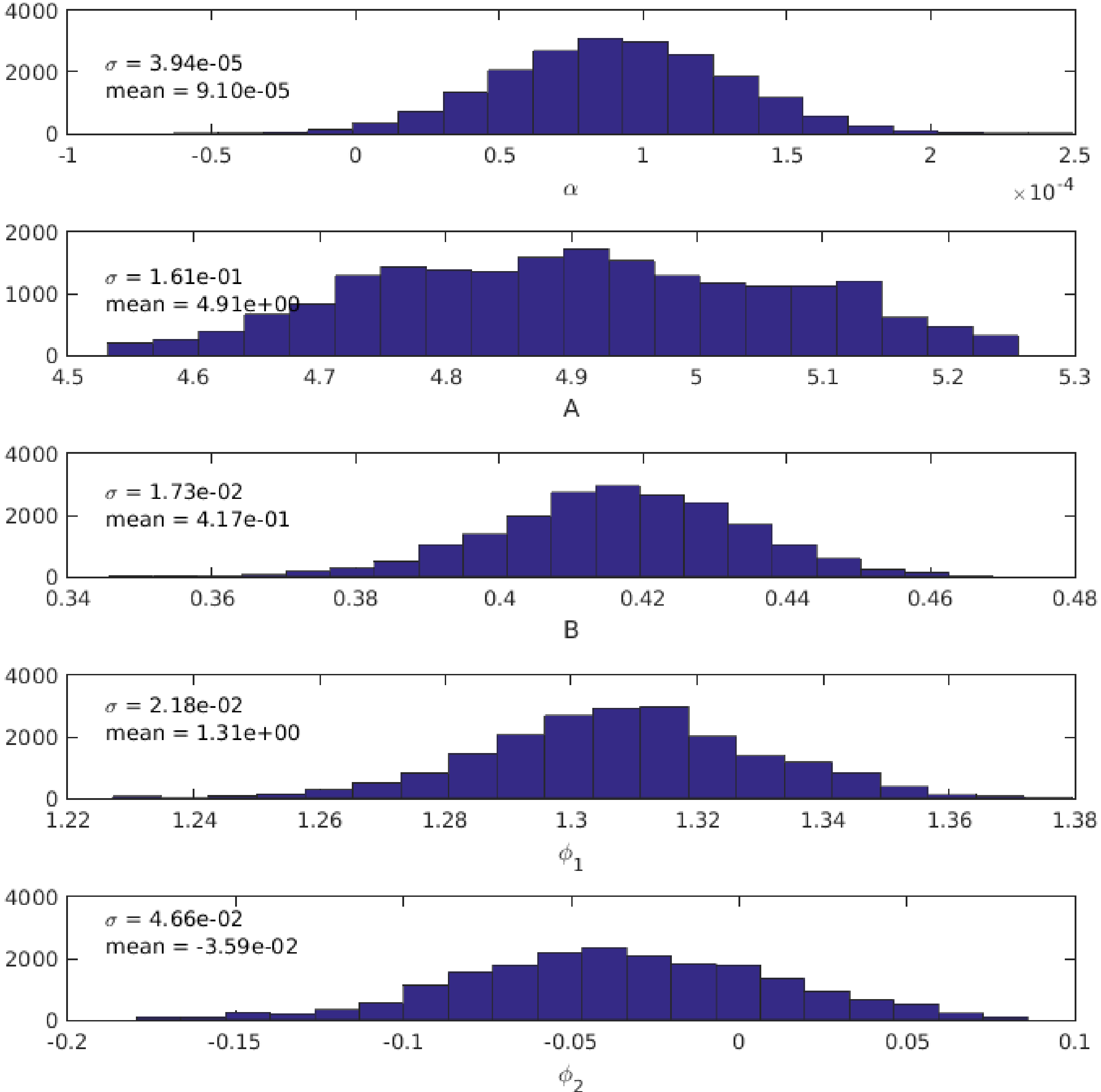


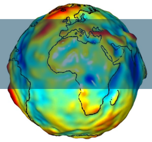


Estimation of parameters wit Bayesian analysis

$$\epsilon_{\text{sys}} = A\sin((n_{\text{sat}} + \omega_{\text{year}})t + \phi_1)(1 + B(\cos(\omega_{\text{year}}t + \phi_2) - 1)), \ \omega_{\text{year}} = 2\pi/\text{year}$$



Decorrelation between fit parameters occurs for 1 year integration time



- **Atomic clocks are rapidly improving** in accuracy and stability

- **Chronometric Geodesy**: directly measure gravity potential differences with clock comparisons (~ 0.6 $m^2.s^{-2}$, ~ 6 cm); and variations of gravitational potential differences (~0.1 $m^2.s^{-2}$ @ 7h, ~ 1 cm @ 7h)

- Several projects linked to chronometric geodesy : ACES, ITOC, applications to geophysics

- **it will be possible, with Galileo satellites 5 and 6, and at least one year of data, to improve on the GP-A (1976) limit on the gravitational redshift test, down to an accuracy around $3\text{-}5 \times 10^{-5}$**