Aspects of 6d Supersymmetric Theories

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String Geometry and BPS state counting
IHP Paris April 2016

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6 dimensions

- QFT in higher dimension is non-renormalizable.
- 6d superconformal field theories (6d SCFTs): local field theory
- 6d little string theories (LSD): non-local and string theory

Outline

- 6d superconformal field theories
  - 6d (2,0) SCFTs : index function
  - 6d (1,0) SCFTs and `enhancement of global symmetry in elliptic genus of selfdual strings’
- 6d little string theories
  - T-duality in type IIA and type IIB LST
- Conclusion

6d (2,0) SCFTs

Nahm 1978

- There are maximally supersymmetric conformal field theories in 6-dimensions. The allowed supersymmetry is chiral (2,0) with selfdual tensor $H=dB={}^*H$, five scalars Φ_I $(I=1,2,\dots 5)$ and symplectic Weyl spinors Ψ for abelian case.

- The source for the tensor is selfdual strings ${}^*d{}^*H=J$.

- A_N type arises as the low energy dynamics of N M5 branes. D_N type arises on N M5+OM5 branes. Selfdual strings are M2 branes connecting M5 branes.

- ADE types can also arises from the decoupling limit of type IIB string theory on C^2/Γ_{ADE} singularity

6d (2,0) SCFTs

Witten'95, Seiberg Witten '96

- The theory has O(2,8) conformal symmetry and SO(5)=Sp(2) R-symmetry. Its conformal supergroup is OSp(2,6|2) ⊃ O(2,8) x $Sp(2)_R$
  - SO(2,6) with P_μ, $M_{\mu\nu}$, K_μ
- There is no Lagrangian description for nonabelian case.
- In A_N theory, selfdual strings are M2 branes connecting M5 branes.
- It has been shown many ways that A_N and D_N type theories have N^3 degrees of freedom.

Compactification to 5-dm

- After a circle compactification to 5-dim, the low energy description becomes 5d N=2 super Yang-Mills theory.
- Instantons are supposed to capture the Kaluza-Klein modes.
- Dyonic instanton dynamics captures the physics of selfdual strings and momentum in 6d (2,0) theory. Many new physics on instantons were predicted and confirmed.
- However the 5d N=2 super Yang-Mills theory is incomplete as it is not UV finite.

Chiral Primary Operators?

- The way to calculate chiral primary operator of SCFT on R^6 is to calculate the Witten index on $S^5 \times R$. (radially quantization) We choose supercharge Q and S so that

$$\mathcal{Q}^2 \sim E - 2(R_1 + R_2) - j_1 - j_2 - j_3$$

- We define the Witten index with $a_1+a_2+a_3=0$ as

$$Z_{S^5\times S^1}(\beta, m, a_i) \equiv \mathrm{Tr}\left[(-1)^F e^{-\beta(E-\frac{R_1+R_2}{2})} e^{-\beta a_i j_i} e^{\beta m \frac{R_1-R_2}{2}}\right]$$

- Express this in a path integral, and evaluate using the localization. There are three fixed points on CP^2 .

$$Z_{S^5\times S^1}(\mu) = \int [d\phi] e^{-S_0(\phi)} Z^{(1)}_{\mathbb{R}^4\times T^2}(\phi,\mu) Z^{(2)}_{\mathbb{R}^4\times T^2}(\phi,\mu) Z^{(3)}_{\mathbb{R}^4\times T^2}(\phi,\mu)$$

Two approaches

- We compactify the Euclidean time circle $\tau \sim \tau + \beta$. The metric for $S^5 x S^1$ is

$$ds^2_{S^1\times S^5} = d\tau^2 + ds^2_{CP^2} + (dy+V)^2,\; J = \frac{1}{2}dV = \text{Kahler form}$$

- Note that S^5 is a circle fibered over CP^2. There are two interesting limits of the index function: small β and large β limits.

- In the small β limit, the index function becomes a S^5 partition function of 5d YM theory and explored extensively. Especially the index vacuum energy and 1/2 BPS operators have been studied extensively as the calculation gets simplified in a certain limit of the mass parameter m.

- In the large β limit, the index function is clearer. The theory can be written only with Z_K-modding along the fiber direction y. There is a natural t'Hooft coupling, N/K and the 5d theory on $S^1 x CP^2$. There is a Yang-Mills + Chern-Simons term $J\wedge$ tr (AdA+…), quantized overall coupling constant $K/4\pi^2$

Physics on $R^{4+1} \times S^1$

	0	1	2	3	4	5	6	7	8	9	10
M5	x	x	x	x	x	x					
M2	x					x					
		ε_1	ε_1	ε_2	ε_2	S^1	v^I	ε_3	ε_3	ε_4	ε_4

- Both approaches need to understand the physics near three fixed points. The partition functions with omega-deformation parameters and the Coulomb phase parameter v^I can be expanded either in electric charge or momentum along the circle.

- One quick way is to calculate both perturbative (massive W-boson contributions) and dyonic instanton contributions.

$$Z(v^I, \tau, \epsilon_{1,2}, m) = Z_{pert}(v_I, \epsilon_{1,2}, m) \sum_{k=0}^{\infty} q^k Z_k(v_I, \epsilon_{1,2}, m)$$

Nekrasov(2004),Nekrasov,Okounkov(200
Bruzzo,Fucito,Morales,Tanzini(2003),
H.Kim,S.Kim,E.Koh,KL,S.Lee(2011)

- Instantons are KK modes along the circle S^1. It is a natural expression as we want to integrate over holonomy variables v^I.

S^1x CP^2 reduction

- For the supercharge $Q = Q^{++}_{---}$ and $S = S^{--}_{+++}$, we redefine the fiber S^1 rotation by twisted rotation $j_1 + j_2 + j_3 + \frac{3}{2}(R_1 + R_2) + n(R_1 - R_2)$ and keep Z_K invariant modes

- Unrefined index with m= 1/2 - a_3, and we get the partition function

$$e^{\beta\omega_3\left(\frac{N(N^2-1)}{6}+\frac{N}{24}\right)}\prod_{s=0}^{\infty}\prod_{d=1}^{N}\frac{1}{1-e^{-\beta\omega_3(d+s)}}\,. \qquad e^{\beta\omega_w(\frac{N(N^2-1)}{6}+\frac{N}{24})}\mathrm{PE}\left(\frac{q+q^2+\cdots q^N}{1-q}\right)$$

- Ground state is F= $2J(s_1,s_2,\ldots,s_N)=2J(N-1,N-3,\ldots,-(N-3),-(N-1))$. Instanton number is $-1/2\ \Sigma_i\ s_i^2 = N(N^2-1)/6$.

- Excited states can be obtained by add instantons in three fixed and reducing the uniform fluxes by $2J(\ldots -1,\ldots,1)$.

- t'Hooft coupling constant N/K

Index function

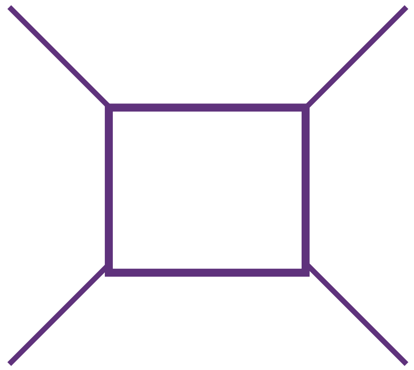
$$Z_{S^5\times S^1} \;=\; 1+qy+q^2\left[2y^2+y(y_1+y_2+y_3)-(y_1^{-1}+y_2^{-1}+y_3^{-1})+y^{-1}\right]+\mathcal{O}(q^3)$$

$$\begin{aligned}
U(2) \;&:\; q^3\Big[2y^3+2y^2(y_1+y_2+y_3)+y\left(y_1^2+y_2^2+y_3^2-\frac{1}{y_1}-\frac{1}{y_2}-\frac{1}{y_3}\right) \\
&\qquad -\left(\frac{y_1}{y_2}+\frac{y_2}{y_1}+\frac{y_2}{y_3}+\frac{y_3}{y_2}+\frac{y_3}{y_1}+\frac{y_1}{y_3}\right)+y^{-1}(y_1+y_2+y_3)\Big] \\
U(3) \;&:\; q^3\Big[3y^3+2y^2(y_1+y_2+y_3)+y\left(y_1^2+y_2^2+y_3^2-\frac{1}{y_1}-\frac{1}{y_2}-\frac{1}{y_3}\right) \\
&\qquad -\left(\frac{y_1}{y_2}+\frac{y_2}{y_1}+\frac{y_2}{y_3}+\frac{y_3}{y_2}+\frac{y_3}{y_1}+\frac{y_1}{y_3}\right)+y^{-1}(y_1+y_2+y_3)\Big]
\end{aligned}$$

6d (1,0) SCFTs

Seiberg'96,Danielsson et.al.'97

- We have supercharge Q (1,0) with ε-spinor parameter (0,1). The gaugino belongs to (0,1) and the hyper and tensor spinor belongs to (1,0)



- The gauge anomaly due to vector and hypermultiplet fermion 1-loop should vanish.

- The gauge anomaly polynomial is made of two pieces. The first one should vanish. The second one can be removed with coupling to tensor multiplet and using the Green-Schwartz mechanism

$$\mathrm{Tr}_R F^4 = \alpha_R \mathrm{tr} F^4 + c_R (\mathrm{tr} F^2)^2$$

$$\alpha_R = 0 \;\; for \;\; SU(2), SU(3), G_2, SO(8), F_4, E_6, E_7, E_8$$

$$c_{\mathrm{tot}} = \left[c_{Ad} - \sum_{R \;\mathrm{matter}} c_R \right] \geq 0$$

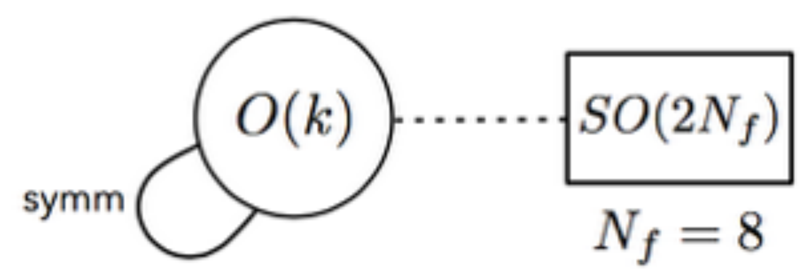
Example I

Heckman,Morrison,Vafa(Heckman('13),Morrison, Rudelius, Vafa ('15)

- Let us consider a simple case where a single M5 brane explores the D_{N+4} type singularity. As a M5 brane gets fractionalized to two and there is a 6d SCFT between them.

- The gauge group is Sp(N) with tensor multiplet coupled and global symmetry is SO(4N+16) with 2N+8 fundamental hypermultiplets. For N=0, the global symmetry is enhanced from SO(16) to E_8.

- Instanton strings are selfdual strings. One can calculate the elliptic genus of selfdual strings with ADHM model with flavors. The gauge group for k strings is O(k).

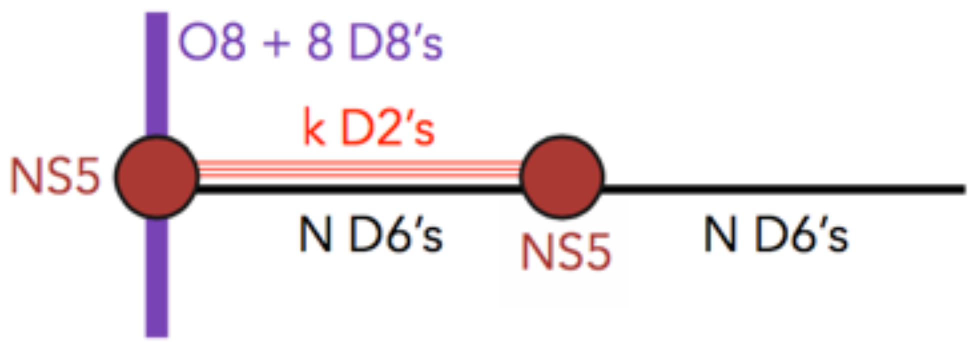


(b) E-string

Example II

SU(N) + Anti+ (N+8)Fund

JKim,S.Kim,KL 1510



	0	1	2	3	4	5	6	7	8	9
NS5	•	•	•	•	•	•	–	–	–	–
D6	•	•	•	•	•	•	•	–	–	–
O8-D8	•	•	•	•	•	•	–	•	•	•
D2	•	•	–	–	–	–	•	–	–	–

- N D6 branes give Z_N ALF space. 6d SCFT has gauge symmetry SU(N). O8+8D8 wall plus N D6 branes gives N+8 fundamental hypermultiplet. Presence of NS5 brane on the wall gives anti-symmetric hyper.

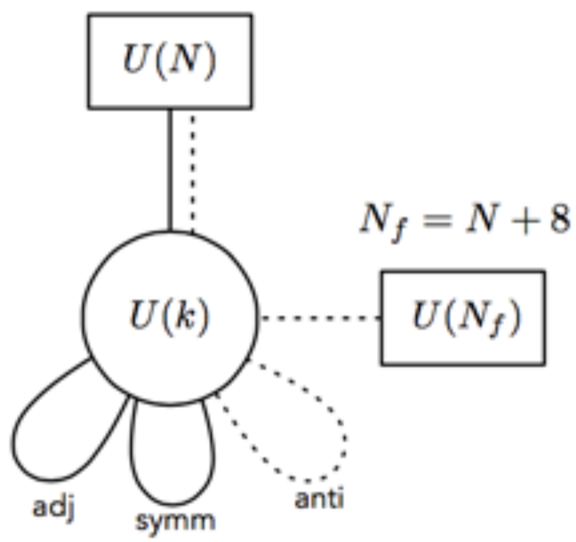
- For SU(0), SU(1),SU(2), anti-symmetric multiplet is trivial. But they affect UV physics on D2 branes.

- For SU(3), anti-symmetric multiplet is equivalent to fundamental representation. Thus it is SU(3) theory with 12 fundamental hypermultiplets.

string dynamics

Gadde and Gukov (13),
Benini,Eager,Hori,Tachikawa I,II(13)

- String dynamics can be written by a quiver-diagram. Elliptic genus can be calculated. The enhancement of the global symmetry can be tested.



(a)

Field	Type	$U(k)$	$U(N)$	$U(N_f)$	$U(1)_A$
$(A_\mu, \lambda^{\dot{\alpha}A})$	vector	**adj**	$-$	$-$	0
$(a_{\alpha\dot{\beta}}, \chi^A_\alpha)$	hyper	**adj**	$-$	$-$	0
$(q_{\dot{\alpha}}, \psi^A)$	hyper	**k**	$\overline{\mathbf{N}}$	$-$	0
(Ξ_l)	Fermi	**k**	$-$	$\overline{\mathbf{N_f}}$	0
$(\varphi_A, \Phi^{\dot{\alpha}})$	twisted hyper	**sym**	$-$	$-$	$+1$
(Ψ_α)	Fermi	**anti**	$-$	$-$	$+1$
(ψ)	Fermi	**k**	**N**	$-$	$+1$

(b)

single string

$$\oint d\phi \, \frac{\eta^3 \theta_1(2\epsilon_+)}{i\theta_1(\epsilon_1)\theta_1(\epsilon_2)} \cdot \prod_{i=1}^{N} \frac{\eta\,\theta_1(\phi + a_i + M)}{\theta_1(\epsilon_+ \pm (\phi - a_i))} \cdot \frac{\eta^2}{\theta_1(-\epsilon_+ \pm (2\phi + M))} \cdot \prod_{l=1}^{N+8} \frac{\theta_1(\phi - m_l)}{\eta}$$

JK prescription: with n>0, we choose the poles of positive charge Q

$$\epsilon_+ + \phi - a_j = 0 \quad (j = 1, \cdots, N), \qquad -\epsilon_+ + 2\phi + M = 0,$$

- $\phi = a_j - \epsilon_+ \quad (j = 1, \cdots, N)$

$$-\sum_{j=1}^{N} \frac{\eta^{-6} \prod_{l=1}^{N+8} \theta_1(a_j - \epsilon_+ - m_l)}{\theta_1(\epsilon_1)\theta_1(\epsilon_2)\theta_1(2a_j - 3\epsilon_+ + M)} \cdot \prod_{i \neq j} \frac{\theta_1(a_i + a_j - \epsilon_+ + M)}{\theta_1(a_j - a_i)\theta_1(2\epsilon_+ - (a_j - a_i))}$$

- $\phi = \frac{\epsilon_+ - M}{2} + \ell_I$ for $\ell = \{0, \frac{1}{2}, \frac{1+\tau}{2}, \frac{\tau}{2}\} \quad (I = 1, 2, 3, 4)$

$$-\frac{1}{2} \frac{\eta^{-6}}{\theta_1(\epsilon_1)\theta_1(\epsilon_2)} \left[\frac{\prod_{l=1}^{N+8} \theta_1(\frac{\epsilon_+ - M}{2} - m_l)}{\prod_{i=1}^{N} \theta_1(\frac{3\epsilon_+ - M}{2} - a_i)} + (-1)^N \sum_{I=2}^{4} \frac{\prod_{l=1}^{N+8} \theta_I(\frac{\epsilon_+ - M}{2} - m_l)}{\prod_{i=1}^{N} \theta_I(\frac{3\epsilon_+ - M}{2} - a_i)} \right]$$

two strings

$$\oint \frac{d\phi_{1,2}}{2} \frac{-\eta^6\theta_1(2\epsilon_+)^2}{\theta_1(\epsilon_1)^2\theta_1(\epsilon_2)^2} \prod_{i\neq j} \frac{\theta_1(\phi_{ij})\theta_1(\phi_{ij}+2\epsilon_+)}{\theta_1(\phi_{ij}+\epsilon_1)\theta_1(\phi_{ij}+\epsilon_2)} \prod_{l=1}^{N+8} \frac{\theta_1(\phi_{1,2}-m_l)}{\eta^2} \prod_{i=1}^{N} \frac{\eta^2\theta_1(\phi_{1,2}+a_i+M)}{\theta_1(\epsilon_+ \pm (\phi_{1,2}-a_i))}$$
$$\times \frac{\eta^4\theta_1(\epsilon_- \pm (\phi_1+\phi_2+M))}{\theta_1(-\epsilon_+ \pm (\phi_1+\phi_2+M))\theta_1(-\epsilon_+ \pm (2\phi_{1,2}+M))}.$$

We adopt the concise notations such as $\phi_{ij} \equiv \phi_i - \phi_j$, $a_{mn} \equiv a_m - a_n$, $\theta_I(\phi_{i,j}+b) \equiv \theta_I(\phi_i+b)\,\theta_I(\phi_j+b)$, $\theta_I(a_{m,n}+b) \equiv \theta_I(a_m+b)\,\theta_I(a_n+b)$, $\theta_{I,J}(b) \equiv \theta_I(b)\,\theta_J(b)$. The Weyl group $W \subset U(2)$ is $\mathbb{Z}_2$. After picking an auxiliary vector $\mathfrak{n}$ to be $(+1,+1)$, we collect all contributing residues given as follows.

Poles

$(\phi_1, \phi_2) = (a_m - \epsilon_+, a_n - \epsilon_+)$ for $m \neq n$.

$(\phi_1, \phi_2) = (\frac{\epsilon_+ - M}{2} + \ell_I, a_m - \epsilon_+)$ and $(\phi_1, \phi_2) = (a_m - \epsilon_+, \frac{\epsilon_+ - M}{2} + \ell_I)$

more…..

testing SO(20) with Sp(1)=SU(2)

Approach 2d O(1) for 6d Sp(1) in Example I

$$-\frac{\eta^2}{\theta_1(\epsilon_{1,2})}\sum_{I=1}^{4}\frac{\eta^2}{\theta_I(\epsilon_+\pm a)}\prod_{l=1}^{10}\frac{\theta_I(m_l)}{\eta}$$

Approach 2d U(1) for 6d SU(2) in Example II

$$-\frac{\eta^{-6}}{\theta_1(\epsilon_{1,2})}\left[\frac{\prod_{l=1}^{10}\theta_1(a-\epsilon_+-m_l)}{\theta_1(2a-3\epsilon_++M)}\frac{\theta_1(-\epsilon_++M)}{\theta_1(2a)\theta_1(2\epsilon_+-2a)}+(\pm a\to\mp a)\right]-\frac{\eta^{-6}}{\theta_1(\epsilon_{1,2})}\sum_{I=1}^{4}\frac{\prod_{l=1}^{10}\theta_I(\frac{\epsilon_+-M}{2}-m_l)}{2\theta_I(\frac{3\epsilon_+-M}{2}\pm a)}$$

Expand in q power

$$t=e^{2\pi i\epsilon_+},\ u=e^{2\pi i\epsilon_-},\ y_i=e^{2\pi i\tilde{m}_i},\ \overline{y}=e^{2\pi i\overline{m}},\ Y=e^{2\pi iM},\ w_i=e^{2\pi i\tilde{a}_i},\ \overline{w}=e^{2\pi i\overline{a}}.$$

$$\frac{t}{(1-tu)(1-tu^{-1})}\Big[q^{-1/2}+\frac{q^{1/2}\cdot t^2}{(1-t^2w_1^2)(1-t^2w_1^{-2})}\Big(-\chi^{\mathrm{SO(20)}}_{\mathbf{512}}\chi^{\mathrm{SU(2)}}_{1/2}(w_1)+\chi^{\mathrm{SO(20)}}_{\mathbf{512}}\chi^{\mathrm{SU(2)}}_{1/2}(t)$$
$$+\chi^{\mathrm{SO(20)}}_{\mathbf{20}}\chi^{\mathrm{SU(2)}}_{1/2}(t)\chi^{\mathrm{SU(2)}}_{3/2}(w_1)-\chi^{\mathrm{SO(20)}}_{\mathbf{20}}\chi^{\mathrm{SU(2)}}_{3/2}(t)\chi^{\mathrm{SU(2)}}_{1/2}(w_1)-\chi^{\mathrm{SO(20)}}_{\mathbf{190}}\chi^{\mathrm{SU(2)}}_{1}(w_1)+\chi^{\mathrm{SU(2)}}_{1/2}(t)\chi^{\mathrm{SU(2)}}_{1/2}(u)$$
$$+\chi^{\mathrm{SU(2)}}_{3/2}(t)\chi^{\mathrm{SU(2)}}_{1/2}(u)-\chi^{\mathrm{SU(2)}}_{1/2}(t)\chi^{\mathrm{SU(2)}}_{1/2}(u)\chi^{\mathrm{SU(2)}}_{1}(w_1)+\chi^{\mathrm{SU(2)}}_{2}(t)\chi^{\mathrm{SU(2)}}_{1}(w_1)-\chi^{\mathrm{SU(2)}}_{1}(t)\chi^{\mathrm{SU(2)}}_{2}(w_1)$$
$$+\chi^{\mathrm{SO(20)}}_{\mathbf{190}}\chi^{\mathrm{SU(2)}}_{1}(t)\Big)+\mathcal{O}(q^{3/2})\Big].$$

$q^{-1/2}$: zero point energy

testing SU(12) with single string for SU(3)

$$
\begin{aligned}
\mathbf{12} &\longrightarrow \mathbf{1}_{-11} + \mathbf{11}_{+1} \\
\overline{\mathbf{12}} &\longrightarrow \mathbf{1}_{+11} + \overline{\mathbf{11}}_{-1} \\
\mathbf{143} &\longrightarrow \mathbf{1}_0 + \mathbf{11}_{12} + \overline{\mathbf{11}}_{-12} + \mathbf{120}_0,
\end{aligned}
$$

Expand in q power

$$
\frac{t^2}{(1-tu)^2(1-tu^{-1})^2}\Big[q^{-1}\cdot\frac{t\,\chi_{1/2}^{\mathrm{SU(2)}}(t)}{(1+tu^{-1})(1+tu)} + q^0\cdot\Big(t^{-2}\chi_{\mathbf{8}}^{SU(3)} + t^{-1}\big(\chi_{1/2}^{\mathrm{SU(2)}}(u) - \chi_{\mathbf{3}}^{SU(3)}\chi_{\overline{\mathbf{12}}}^{\mathrm{SU(12)}} + \chi_{\bar{\mathbf{3}}}^{SU(3)}\chi_{\mathbf{12}}^{\mathrm{SU(12)}}\big) + \chi_{\mathbf{143}}^{\mathrm{SU(12)}} + 1 + \chi_{\mathbf{8}}^{SU(3)} + \mathcal{O}(t^1)\Big) + \mathcal{O}(q^1)\Big]. \quad (3.26)
$$

Little String Theories

Aharony-Berkooz'99,J.Kim,S.Kim,KL'15

- a maximally symmetric LST in 6-dim arises as the low energy dynamics of NS5 branes + fundamental strings in the limit where gravity decouples: fix l_s and take g_s=0
- There is only one scale, the little string tension $1/l_s^2$.
- There are type IIA version with (2,0) supersymmetry and type IIB with (1,1) supersymmetries.
- Low energy dynamics of type IIA LST is the 6d (2,0) SCFT of ADE type symmetry.
- Low energy dynamics of type IIB LST is the 6d (1,1) Yang-Mills theory of any compact group.

T-duality

- type IIA LST on compactified on a circle S with momentum p and winding w are T-dual to type IIB LST compactified on the dual circle S with momentum w and winding p.

- elliptic genus of instanton strings and M-strings are needed to show this.

- type IIB LST on a circle is characterized by the gauge holonomy $(\alpha_1,\alpha_2,\ldots,\alpha_N)$, leading to the fractionalization of momentum.

- type IIA LST has fractionalized strings as NS5 branes on `M-circle' can be connected by M2 branes.

type IIB LST

J.Kim,S.Kim,KL'15

- 6d (1,1) Yang-Mills theory has instanton strings. They are little strings of the theory.
- Instanton string dynamics can be characterized by (4,4) ADHM string model. Its elliptic genus can be obtained by simply generalizing the 5d result to elliptic case. k-instanton string contribution is characterized by Young diagrams
- There is also perturbative contribution from 6d theory which counts the massless modes along the string direction.
- The total contribution is a product of perturbative and stringy contributions.

$$Z_{\rm IIB}(\alpha_i,\epsilon_\pm;q,w) = Z^{\rm IIB}_{\rm KK}(\epsilon_\pm,m;q)Z^{IIB}_{\rm string}(\alpha_i,\epsilon_\pm,m;q,w)$$

$$Z^{IIB}_{\rm string}(\alpha_i,\epsilon_\pm,m;q,w) = \sum_{n=0}^{\infty} w^n Z_n(\alpha_i,\epsilon_\pm,m;q)$$

type IIA LST

Aharony-Berkooz'99,J.Kim,S.Kim,KL'15

- NS5 branes on M-circle= M5 branes at position $(\alpha_1,\alpha_2,\ldots,\alpha_N)$ on M-circle

- M2 branes connecting these M5 branes leads to fractionally winded strings. Elliptic genus for M-strings.

Haghighat,Iqbal,Kozcaz,Lockhart,Vafa (2013)

- There is also perturbative contribution from 6d (2,0) theory in the Coulomb branch.

- The total contribution is a product of perturbative and stringy contributions.

$$Z_{\text{IIA}}(\alpha_i,\epsilon_\pm;q',w') = Z^{\text{IIA}}_{\text{KK}}(\epsilon_\pm,m;q')Z^{IIA}_{\text{frac string}}(\alpha_i,\epsilon_\pm,m;q',w')$$

$$Z^{IIA}_{\text{frac string}}(\alpha_i,\epsilon_\pm,m;q,w) = \sum_{n_I=0}^{\infty}\prod_{I=1}^{n} e^{n_I(\alpha_I-\alpha_{I+1})+\cdots+n_N(\alpha_N-\alpha_1+\ln w')}Z_n(\alpha_i,\epsilon_\pm,m;q)$$

Comparison

Aharony-Berkooz'99,J.Kim,S.Kim,KL'15

- One has to take care of the mode where strings get bounded and unhinged from NS5 branes.

$$\hat{Z}_{\rm IIA}(\alpha_i,\epsilon_{1,2},m,q',w') = \frac{Z_{\rm IIA}(\alpha_i,\epsilon_{1,2},m,q',w')}{Z_{\rm extra}(w')}.$$

- We have checked the T-duality up to three NS5 branes

- For the single NS5 brane, the contributions of type IIA and type IIB are identical, implying the symmetry under the exchange of p' and w'

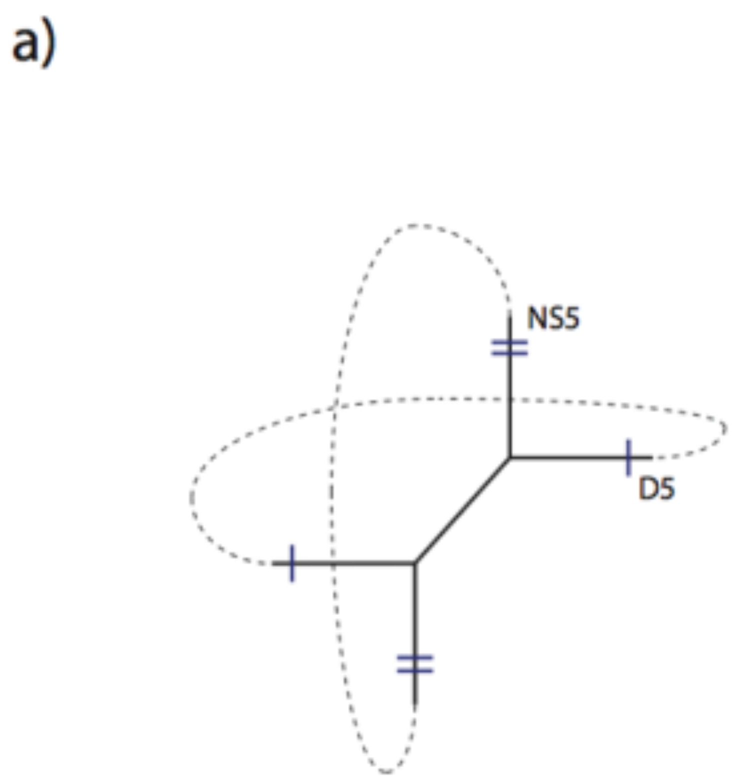
$$Z_{\rm IIA}(\epsilon_\pm,m;q',w') = PE\Big[I_-(\epsilon_\pm,m)z_{\rm sp}(\epsilon_\pm,m,q',w')\Big]$$

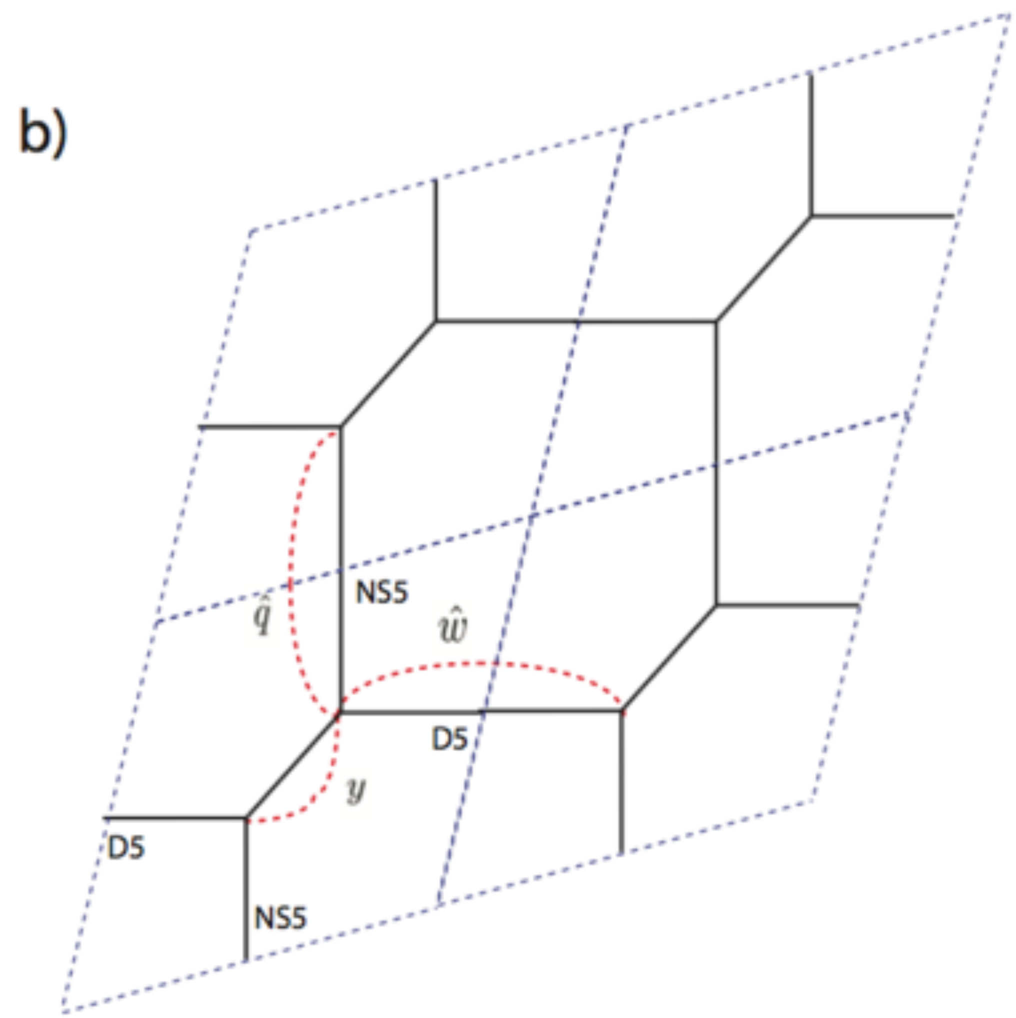
symmetric under q’, w’ exchange

$$
\begin{aligned}
z_{\rm sp}(\epsilon_\pm, m; q', w') &= (q'+w') + (q'^2+w'^2) + (q'w')\left[tu + \frac{t}{u} + \frac{1}{tu} + \frac{u}{t} - uy - \frac{y}{u} - \frac{u}{y} - \frac{1}{uy}\right] \\
&+ q'^3 + w'^3 + (q'^2w' + q'w'^2)\left[t^2u^2 + \frac{t^2}{u^2} + \frac{u^2}{t^2} + \frac{1}{t^2u^2} + t^2 + \frac{1}{t^2} - tu^2y - \frac{ty}{u^2} - \frac{tu^2}{y} - \frac{t}{u^2y}\right. \\
&- \frac{y}{tu^2} - \frac{u^2}{ty} - \frac{1}{tu^2y} - \frac{u^2y}{t} + tu + \frac{t}{u} + \frac{1}{tu} + \frac{u}{t} - 2ty - \frac{2t}{y} - \frac{2}{ty} - \frac{2y}{t} + 2u^2 + \frac{2}{u^2} - uy - \frac{y}{u} \\
&\left. - \frac{u}{y} - \frac{1}{uy} + y^2 + \frac{1}{y^2} + 4\right] + (q'^4 + w'^4) + (q'^3w' + q'w'^3)\left[t^3u^3 + \frac{t^3}{u^3} + \frac{u^3}{t^3} + \frac{1}{t^3u^3} + t^3u + \frac{t^3}{u}\right. \\
&+ \frac{u}{t^3} + \frac{1}{t^3u} - t^2u^3y - \frac{t^2y}{u^3} - \frac{t^2u^3}{y} - \frac{t^2}{u^3y} - \frac{u^3y}{t^2} - \frac{y}{t^2u^3} - \frac{u^3}{t^2y} - \frac{1}{t^2u^3y} + t^2u^2 + \frac{t^2}{u^2} + \frac{u^2}{t^2} \\
&+ \frac{1}{t^2u^2} - 2t^2uy - \frac{2t^2y}{u} - \frac{2t^2u}{y} - \frac{2t^2}{uy} - \frac{2uy}{t^2} - \frac{2y}{t^2u} - \frac{2u}{t^2y} - \frac{2}{t^2uy} + 2t^2 + \frac{2}{t^2} + 2tu^3 + \frac{2t}{u^3}
\end{aligned}
$$

$$t = e^{2\pi i \epsilon_+},\ u = e^{2\pi i \epsilon_-},\ y = e^{2\pi i m}.$$

- There is also triality for the single NS5 brane case. There is a p-q 5 brane web for mass-deformed case





$$\hat{q} = qy^{-1}\ , \quad \hat{w} = wy^{-1}\ .$$

triality under exchange of $y,\ \hat{q},\ \hat{w}$

Hollywood,Iqbal,Vafa(2008)

$$\tilde{Z}(\epsilon_\pm;\hat{q},\hat{w},y) = PE\Big[I_{\rm com}\tilde{z}_{\rm sp}(\epsilon_\pm;\hat{q},\hat{w},y)\Big],$$

$$\begin{aligned}
\hat{z}_{sp}(\epsilon_\pm;\hat{q},\hat{w},y) &= \hat{q}+\hat{w}+y-(u+u^{-1})(\hat{q}\hat{w}+\hat{q}y+\hat{w}y)+\frac{(1+u^2)(t+u+t^2u+tu^2)}{tu^2}\hat{q}\hat{w}y \\
&+(\hat{q}^2\hat{w}+\hat{q}\hat{w}^2+\hat{q}^2y+\hat{q}y^2+\hat{w}^2y+\hat{w}y^2)-(u+u^{-1})(\hat{q}^2\hat{w}^2+\hat{q}^2y^2+\hat{w}^2y^2) \\
&-\frac{(u^2+1)\left(t^2\left(u^2+1\right)+2tu+u^2+1\right)}{tu^2}\hat{q}\hat{w}y(\hat{q}+\hat{w}+y) \\
&+(\hat{q}^3\hat{w}^2+\hat{q}^2\hat{w}^3+\hat{q}^3y^2+\hat{q}^2y^3+\hat{w}^3y^2+\hat{w}^2y^3)+\frac{(1+u^2)(t+u+t^2u+tu^2)}{tu^2}\hat{q}\hat{w}y(\hat{q}^2+\hat{w}^2+y^2) \\
&+\frac{t^4\left(u^5+u^3+u\right)+t^3\left(u^6+4u^4+4u^2+1\right)}{t^2u^3}\hat{q}\hat{w}y(\hat{q}\hat{w}+\hat{q}y+\hat{w}y) \\
&+\frac{t^2\left(3u^4+7u^2+3\right)u+t\left(u^6+4u^4+4u^2+1\right)+u^5+u^3+u}{t^2u^3}\hat{q}\hat{w}y(\hat{q}\hat{w}+\hat{q}y+\hat{w}y) \\
&-(u+u^{-1})(\hat{q}^3\hat{w}^3+\hat{q}^3y^3+\hat{w}^3y^3)
\end{aligned}$$

Conclusion

- There are a lot more to be learned about 6d SCFTs and LST
- UV of 6d (1,1) SYM is completed in LST.
- Elliptic genus for selfdual strings is a powerful tool. But it is not easy to write down the UV theory for some cases, say 6d (1,0) scft with E_8 gauge symmetry.
- More approaches to 6d theories like bootstrap, gravity dual, DLCQ need to be explored.
- There is also a powerful relation to lower dimensional physics (5,4,3,2,1,0).