Do All BPS Black Hole Microstates Carry Zero Angular Momentum?

Ashoke Sen

Harish-Chandra Research Institute, Allahabad, India

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1. Motivation

2. The zero angular momentum conjecture

3. Evidence

Based on

Abhishek Chowdhury, Richard Garavuso, Swapnamay Mondal, A.S., arXiv:1405.0412, arXiv:1512.00026

+ earlier work

Motivation

Existence of smooth horizon of the black hole has been under scanner in recent years.

Alternative scenarios: Fuzzballs, firewalls, etc.

Nevertheless the near horizon geometry has been remarkably successful in giving the correct expression for the BPS black hole entropy via Bekenstein-Hawking-Wald formula

Strominger, Vafa; $\cdots$

– not explained by the alternative scenarios.

Our goal will be to provide further evidence for the existence of smooth near horizon geometry.

Strategy: Work out some of its predictions for the detailed properties of microstates $\cdots$

$\cdots$ **and then check if these predictions are true.**

Our subject of discussion will be BPS black holes in string theory preserving four supersymmetries

– have finite area event horizon and hence finite entropy.

(Solutions with more SUSY have zero area event horizon in supergravity approximation.)

Also for definiteness we shall focus on black holes in 3+1 dimensional string theory, although some of the analysis generalizes to 4+1 dimensions.

We shall argue that existence of smooth near horizon geometry leads to the conclusion that microstates of BPS black holes carry zero angular momentum.

Then we shall try to verify it explicitly by studying the microstates.

This in turn will lead to precise mathematical predictions which we can try to verify by explicit computation.

The geometry around a BPS black hole solution includes an infinite throat separating the horizon from the asymptotic region

$$ds^2 = a^2 \left\{ -e^{2\rho} dt^2 + d\rho^2 \right\} + b^2 (d\theta^2 + \sin^2\theta \, d\phi^2) + ds^2_{compact}$$

– $AdS_2 \times S^2$ factor in the geometry.

$\rho \to -\infty$: horizon, $\rho \to \infty$: asymptotic region

Note: the geometry is spherically symmetric.

– can be shown to be a consequence of the infinite throat AdS_2 geometry and the supersymmetries of the solution.

Spherical symmetry $\rightarrow$ the black hole carries zero angular momentum

Since the black hole describes an ensemble of microstates, this would imply that the average angular momentum carried by the microstates vanish.

As long as all the microstates are shielded from the asymptotic geometry by the infinitely long AdS_2 throat, one can argue that the ensemble is microcanonical

$\Rightarrow$ each microstate must carry zero angular momentum.

A.S.; Dabholkar, Gomes, Murthy, A.S.

A clarification

A BPS black hole preserves only a subset of supersymmetries of the vacuum.

– transforms in some nontrivial representation of the SUSY algebra

– has states of different spin.

How can the black hole microstates have zero angular momentum?

Precise statement

A black hole microstate transforms in the tensor product representation of the basic BPS supermultiplet and singlets of rotational SU(2).

The basic BPS supermultiplet comes from quantization of fermion zero modes with support outside the near horizon region and was not included in our previous argument.

All further reference to the angular momentum of a state will refer to the representation of SU(2) with which the basic BPS supermultiplet is tensored.

Note 1: The argument about zero angular momentum is based on the structure of the near horizon geometry of the black hole

– can be trusted fully only when the string coupling and the curvature at the horizon is small.

Requires

1. Large charges

2. Choosing the string coupling g_s such that ($g_s \times$ charge) is large but g_s is small.

This limits the kind of test we can perform.

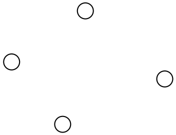
We shall enlarge the scope by assuming that it also holds for

– small charges

– at any generic point in the closed string moduli space including the region of small ($g_s \times$ charge).

This seems reasonable since the arguments rely essentially on symmetries of the near horizon geometry instead of the detailed structure.

Note 2: If we fix the total charge, then there may be both, single centered and multi-centered black hole solutions carrying the same total charge.



The multi-centered black holes do not necessarily carry zero angular momentum

We need a strategy for separating out the contribution from single centered black holes in the microscopic counting.

Strategy

Find a chamber of the closed string moduli space – labelled by asymptotic values of metric, 2-form, dilaton etc – where there are no multi-centered configurations

– not possible for black holes in N=2 SUSY theories but possible for black holes in N=4, 8 SUSY theories in 3+1 dimensions.

We focus on N=4 and N=8 SUSY theories from now on and follow this strategy.

There is an alternate strategy that has had partial success for N=2 SUSY theories

Manschot, Pioline, A.S.; Bena, Berkooz, de Boer, El-Showk, Van den Bleeken; Lee, Wang, Yi

– will not be discussed here.

An indirect test of zero angular momentum conjecture

For BPS states we can define an index

$$\mathrm{Tr}_{\mathrm{BPS}}((-1)^{2J_3})$$

– robust under deformations of the closed string moduli up to wall crossing

Olive, Witten

Since the black hole microstates have been argued to all carry $J_3 = 0$, we are led to the prediction

$$\text{index} = \text{degeneracy} \geq 0$$

– valid in the chamber that admits only single centered black hole solutions.

Prediction from black hole near horizon geometry

$$\text{index} \geq 0$$

in appropriate chamber.

On the other hand in N=4 and N=8 SUSY theories the indices for microstates have been computed exactly by working in the weak coupling regime

– Fourier coeffcients of (Siegel) modular forms

Dijkgraaf, Verlinde, Verlinde; Shih, Strominger, Yin; David, Jatkar, A.S., $\cdots$

The index is positive in all cases tested!

A.S.; Bringmann, Murthy

We now set out to directly test this conjecture for black holes in type II on T^6

In this theory, at any generic point in the moduli space of the theory, only single centered black holes contribute to the index.

For these black holes BPS index has been calculated using a specific duality frame. Shih, Strominger, Yin

The spectrum has many accidental degeneracies.

We use a different duality frame that allows us to work at more general points in the closed string moduli space and lift accidental degeneracies.

Our system:

Consider type IIA on T^6 labelled by $x^4, \cdots x^9$.

Consider N_1 D2-branes along 4-5, N_2 D2-branes along 6-7, N_3 D2-branes along 8-9 and N_4 D6-branes along 4-5-6-7-8-9

When gravity is strong this describes a BPS black hole with entropy

$$2\pi\sqrt{N_1N_2N_3N_4} - 2\ln(4N_1N_2N_3N_4) + \cdots$$

– gives log(index) since

$\text{index} \equiv \text{Tr}_{BPS}((-1)^{2J_3}) = \text{Tr}_{BPS}(1)$=degeneracy

From the counting in the dual theory we independently know the result for the index as long as

$$\mathbf{gcd}\{N_1N_3, N_1N_4, N_2N_3, N_2N_4, N_1N_2, N_3N_4\} = 1$$

$$\text{index} = \sum_{s|s_0} s\, c(4N_1N_2N_3N_4/s^2)$$

$$s_0 = \prod_{\substack{i,j=1 \\ i<j}}^{4} \mathbf{gcd}\{N_i, N_j\}$$

$c(u)$ is defined via the expansion:

$$\vartheta_1(z|\tau)^2\, \eta(\tau)^{-6} \equiv \sum_{k,m} c(4k - m^2)\, e^{2\pi i(k\,\tau + m\,z)}$$

ϑ_1: odd Jacobi theta function

η: Dedekind eta function

For example, for $N_1 = N_2 = N_3 = 1$, $N_4 = N$ the expected index is $c(4N)$

For large N, log(c(4N)) grows as:

$$2\pi\sqrt{N} - 2\ln(4N) + \cdots$$

in agreement with the macroscopic formula.

For low N, explicit calculation gives:

$$c(4) = 12, \quad c(8) = 56, \quad c(12) = 208, \quad \cdots$$

Comparison at finite N

N	Microscopic result $\ln(c(4N))$	Macroscopic result $2\pi\sqrt{N} - 2\ln(4N) + \cdots$
1	2.48	3.51
2	4.03	4.73
3	5.34	5.91

Even for these small values of N, the microscopic and macroscopic results are close.

We want to check if each of the microstates carry zero angular momentum at generic point in the closed string moduli space.

Begin with the configuration where each circle of T^6 has unit radius, the metric is diagonal and 2-form field is zero.

– special point in the closed string moduli space.

Deform it by switching on small constant values of g_{mn} and b_{mn}

– taken to be parametrically small but non-zero.

Task

1. Construct SUSY quantum mechanics describing low energy dynamics of the system

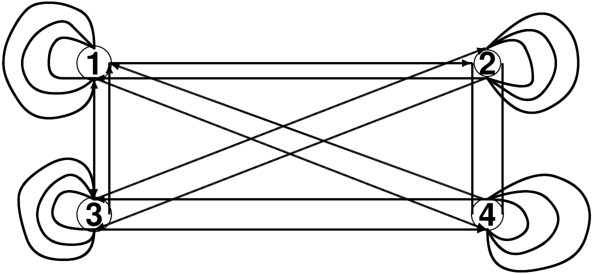
2. Find the spectrum of BPS states by looking for SUSY ground states of this system.

The quantum mechanics has four supercharges corresponding to unbroken SUSY of the black hole

– is described by an N=4 supersymmetric quiver quantum mechanics.

– dimensional reduction of N=1 theory in D=4

Quiver associated with the system



We shall list the chiral superfields and their physical interpretation.

1. For each brane stack labelled by k=1,2,3,4, we have 3 complex scalars $\phi_1^{(k)}$, $\phi_2^{(k)}$, $\phi_3^{(k)}$ in adjoint of $U(N_k)$

Physical interpretation of diagonal components of $\phi_i^{(k)}$

– transverse position along T^6 / Wilson lines on each brane stack

6 real variables for each brane.

2. For each pair of branes $(\mathbf{k}, \ell)$ for $\mathbf{k}, \ell = 1, 2, 3, 4$, we have a complex bifundamental field $\mathbf{Z}^{(\mathbf{k}\ell)}$ forming scalar component of a chiral multiplet.

Comes from open string stretched between brane stack k and brane stack ℓ.

Note: $\mathbf{Z}^{(\ell\mathbf{k})}$ is distinct from $\mathbf{Z}^{(\mathbf{k}\ell)}$

– form conjugate representations of the gauge group.

Superpotential

$$W = W_1 + W_2 + W_3 + W_4$$

$$W_4 = -\Big[\mathrm{Tr}\Big(\phi_1^{(1)}\phi_2^{(1)}\phi_3^{(1)} - \phi_1^{(1)}\phi_3^{(1)}\phi_2^{(1)}\Big) - \mathrm{Tr}\Big(\phi_1^{(2)}\phi_2^{(2)}\phi_3^{(2)} - \phi_1^{(2)}\phi_3^{(2)}\phi_2^{(2)}\Big)$$
$$+\mathrm{Tr}\Big(\phi_1^{(3)}\phi_2^{(3)}\phi_3^{(3)} - \phi_1^{(3)}\phi_3^{(3)}\phi_2^{(3)}\Big) + \mathrm{Tr}\Big(\phi_1^{(4)}\phi_2^{(4)}\phi_3^{(4)} - \phi_1^{(4)}\phi_3^{(4)}\phi_2^{(4)}\Big)\Big].$$

$$\begin{aligned}
W_1 \quad = \quad \mathrm{Tr}\Big[& \Big(\phi_3^{(1)}Z^{(12)}Z^{(21)} - \phi_3^{(2)}Z^{(21)}Z^{(12)}\Big) \\
& + \Big(\phi_1^{(2)}Z^{(23)}Z^{(32)} - \phi_1^{(3)}Z^{(32)}Z^{(23)}\Big) \\
& + \Big(\phi_2^{(3)}Z^{(31)}Z^{(13)} - \phi_2^{(1)}Z^{(13)}Z^{(31)}\Big) \\
& + \Big(\phi_1^{(1)}Z^{(14)}Z^{(41)} - \phi_1^{(4)}Z^{(41)}Z^{(14)}\Big) \\
& + \Big(\phi_2^{(2)}Z^{(24)}Z^{(42)} - \phi_2^{(4)}Z^{(42)}Z^{(24)}\Big) \\
& + \Big(\phi_3^{(3)}Z^{(34)}Z^{(43)} - \phi_3^{(4)}Z^{(43)}Z^{(34)}\Big)\Big],
\end{aligned}$$

$$\mathbf{W_2} = \mathrm{Tr}\Big[Z^{(31)}Z^{(12)}Z^{(23)} + Z^{(13)}Z^{(32)}Z^{(21)} + Z^{(12)}Z^{(24)}Z^{(41)} + Z^{(42)}Z^{(21)}Z^{(14)} - Z^{(13)}Z^{(34)}Z^{(41)} + Z^{(31)}Z^{(14)}Z^{(43)} + Z^{(34)}Z^{(42)}Z^{(23)} + Z^{(43)}Z^{(32)}Z^{(24)} \Big],$$

$$\mathbf{W_3} = \mathrm{Tr}\Big[c^{(12)}\left(\phi_3^{(1)} \otimes I_{N_2} - I_{N_1} \otimes \phi_3^{(2)}\right) + c^{(23)}\left(\phi_1^{(2)} \otimes I_{N_3} - I_{N_2} \otimes \phi_1^{(3)}\right) + c^{(13)}\left(\phi_2^{(3)} \otimes I_{N_1} - I_{N_3} \otimes \phi_2^{(1)}\right) + c^{(14)}\left(\phi_1^{(1)} \otimes I_{N_4} - I_{N_1} \otimes \phi_1^{(4)}\right) + c^{(24)}\left(\phi_2^{(2)} \otimes I_{N_4} - I_{N_2} \otimes \phi_2^{(4)}\right) + c^{(34)}\left(\phi_3^{(3)} \otimes I_{N_4} - I_{N_3} \otimes \phi_3^{(4)}\right) \Big],$$

$c^{(k\ell)}$: determined in terms of background g_{mn} and b_{mn}.

Total potential:

$$\mathbf{V} = \mathbf{V_F} + \mathbf{V_D}$$

$$\mathbf{V_F} = \sum_{\alpha} |\partial \mathbf{V}/\partial \chi_\alpha|^2$$

$\{\chi_\alpha\}$: **set of all chiral multiplet scalars**

$$\mathbf{V_D} = \frac{1}{2}\sum_{k=1}^{4} \mathrm{Tr}\Big[\Big(\sum_{\substack{\ell=1\\ \ell\neq k}}^{4} Z^{(k\ell)}Z^{(k\ell)\dagger} - \sum_{\substack{\ell=1\\ \ell\neq k}}^{4} Z^{(\ell k)\dagger}Z^{(\ell k)} + \sum_{i=1}^{3}[\phi_i^{(k)}, \phi_i^{(k)\dagger}] - c^{(k)} I_{N_k}\Big)^2\Big],$$

$c^{(k)}$: **FI-terms determined in terms of background b_{mn}.**

Shift symmetries of V:

$$\phi_{m}^{(k)} \to \phi_{m}^{(k)} + \xi_{m} I_{N_k}, \quad \text{for} \quad 1 \leq k, m \leq 3, \quad k \neq m,$$

$$\phi_{k}^{(k)} \to \phi_{k}^{(k)} + \zeta_{k} I_{N_k}, \quad \phi_{k}^{(4)} \to \phi_{k}^{(4)} + \zeta_{k} I_{N_4}, \quad \text{for} \quad 1 \leq k \leq 3,$$

ξ_i, ζ_i **are complex parameters**

They represent translation along T^6 and dual T^6.

In order to look for supersymmetric ground states of the system we have to first find the classical supersymmetric configurations

– configurations satisfying

$$V_F = V_D = 0$$

$\Rightarrow$ the vacuum manifold (moduli space of quiver representations).

Low energy dynamics of the system is described by that of a superparticle moving on this manifold

– need to quantize this motion to find SUSY ground states

Shift symmetries $\Rightarrow$ flat directions of the vacuum manifold.

Motion along these flat directions generate momentum / winding charge along T^6

– will be ignored by requiring these quantum numbers to be zero.

Superpartners of the flat directions lead to fermionic zero modes

Quantization of these zero modes $\Rightarrow$ BPS supermultiplet with which every state is to be tensored.

For analyzing the spectrum of states with which the BPS supermultiplet is tensored, we can work with

$$\mathbf{M} \equiv \text{the vacuum manifold / shift symmetries}$$

M is given by the space spanned by $Z^{(k\ell)}$ and $\phi_i^{(k)}$ satisfying

$$V_F = 0, \quad V_D = 0$$

up to gauge equivalence and equivalence under shift symmetries.

Our conjecture implies that quantization of a superparticle moving on M should generate only zero angular momentum states.

What does this imply for M?

Upon quantization, the SUSY ground states are in one to one correspondence with harmonic forms on M

Witten

The J_3 eigenvalue carried by a p-form is $(p - d)/2$.

d: complex dimension of M

$\Rightarrow$ (p-d) must vanish for all harmonic p-forms.

$$J_3 = (p - d)/2$$

Now every manifold of complex dimension d has a harmonic zero form and a harmonic 2d-form.

$\Rightarrow$ states with J_3 eigenvalues $\pm d/2$.

Therefore zero angular momentum conjecture implies that we must have d=0

M must be a collection of points!

Furthermore, since the index is known from the counting in a dual description we have a prediction the number of points!

To summarize, the zero angular momentum conjecture implies that

The vacuum manifold (moduli space of quiver representation), modulo shift symmetries, must be a collection of points, and for

$$\gcd\{N_1N_3, N_1N_4, N_2N_3, N_2N_4, N_1N_2, N_3N_4\} = 1$$

the number of points is given by

$$\sum_{s|s_0} s\, c(4N_1N_2N_3N_4/s^2)$$

$$s_0 = \prod_{\substack{i,j=1 \\ i<j}}^{4} \gcd\{N_i, N_j\}$$

We proceed to test this by explicitly solving the $V_F = V_D = 0$ equations for

$$N_1 = N_2 = N_3 = 1, \quad N_4 = 1, 2, 3$$

1. Numerical analysis of the Hilbert series using ‘Macaulay2’ and ‘Singular’

2. NSolve in Mathematica

Result: M is a collection of points in each of these three cases.

Number of points:

12 for N_4=1, 56 for $N_4 = 2$, 208 for $N_4 = 3$

– perfect agreement with the dual counting!

Conclusion

The existence of a smooth near horizon geometry can not only predict statistical properties of the collection of microstates, but detailed property of individual microstates.

Any attempt to replace the near horizon AdS_2 geometry by horizonless solutions must explain why the near horizon geometry of the black hole is so efficient in deriving properties of microstates.