Orbit and optics distortion in nonscaling FFAG - tracking study of 10 to 20 GeV muon ring -

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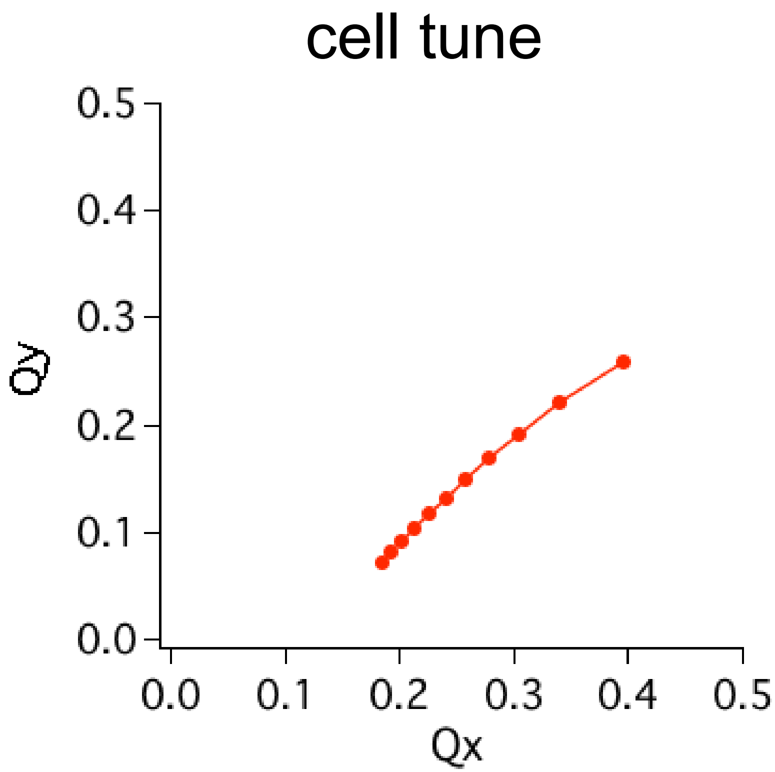
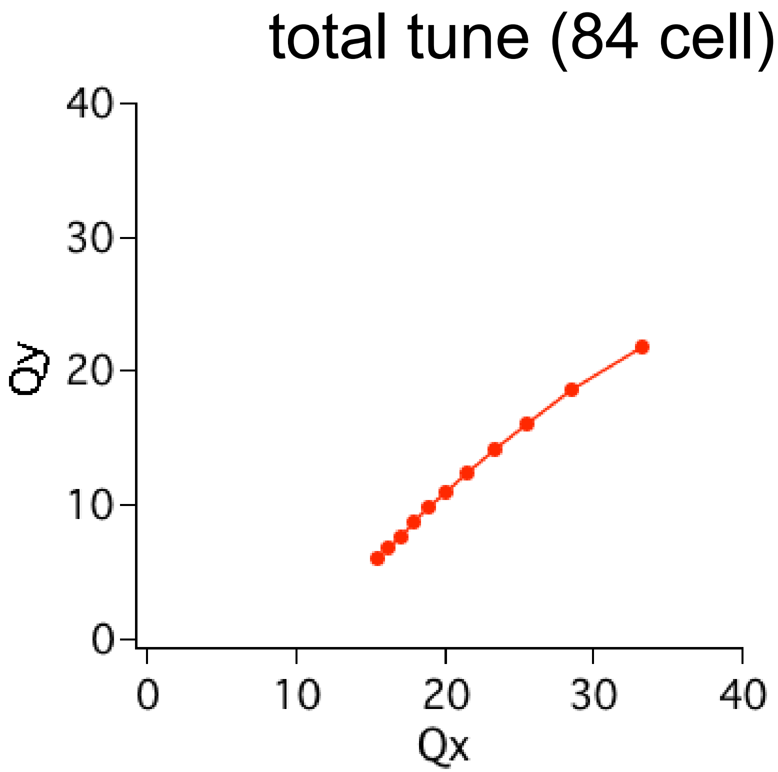
http://www.astec.ac.uk/intbeams/users/machida/doc/nufact/ffag/machida_20070414.pdf & ppt

Introduction (1)
tune excursion

- Linear nonscaling FFAG keeps natural chromaticity.
- Example of muon 10 to 20 ring.
- Tune becomes integer and half-integer many times.





Introduction (2)

study of "resonance"

- Analytical and tracking study exist on rapid betatron "resonance crossing".
- Those studies, particularly analytical one, examine the growth of action variable (e.g. Koscielniak and Baartman, PAC05).

- We know dipole errors introduce orbit distortion and quadrupole errors introduce optical distortion.
- In practice, it is important to know how much orbit shifts and how much beam size grows (not emittance).

Introduction (3)
"*correction*"

- Betatron phase between cells changes a lot during acceleration.
- We want to keep the DC operation for corrector as well.

- It is not easy to put correction elements for orbit and optics in a nonscaling FFAG (Kelliher's talk).

Contents

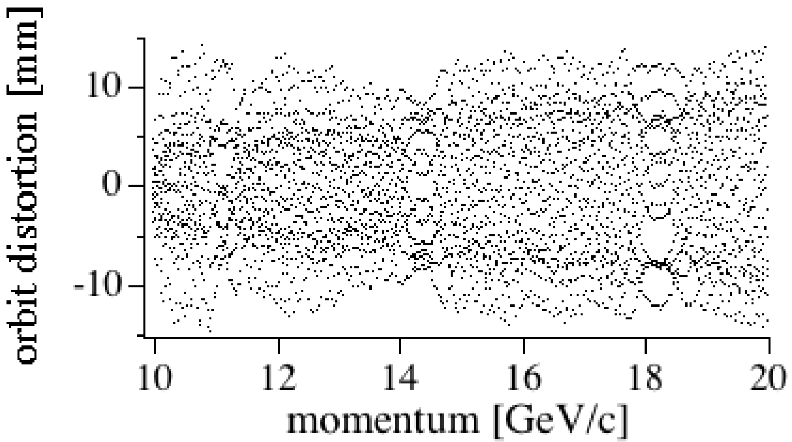
- Orbit distortion (7 slides)
  – Misalignment
  – quadrupole errors
  – rf cavity
- Optical distortion (7 slides)
  – Quadrupole errors
- A beam with momentum spread (2 slides)

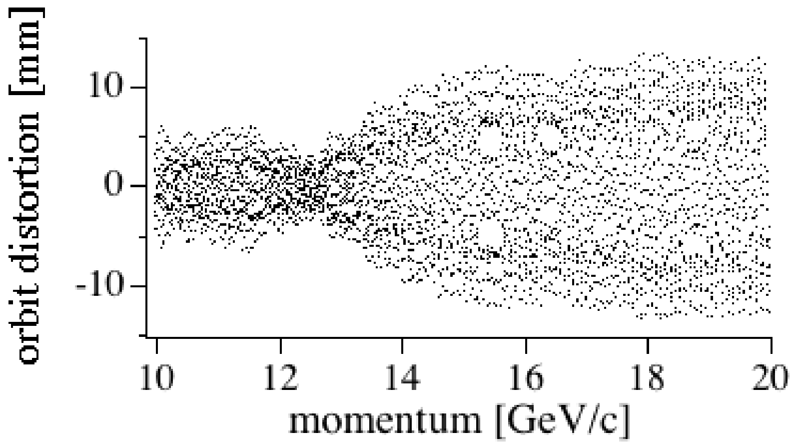
- Conclusions

Orbit distortion (1)

alignment errors

- Orbit distortion due to alignment errors.
- One example with σ=0.1mm and 2σ cut.





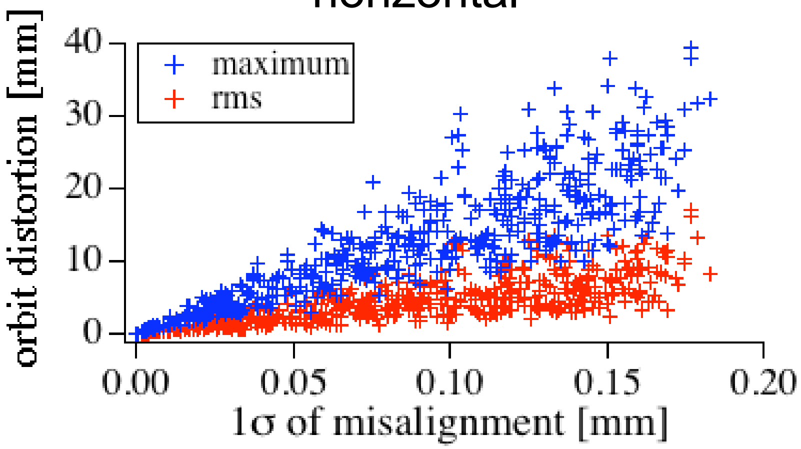
- “Amplification factor” = orbit distortion/alignment error

Orbit distortion (2)

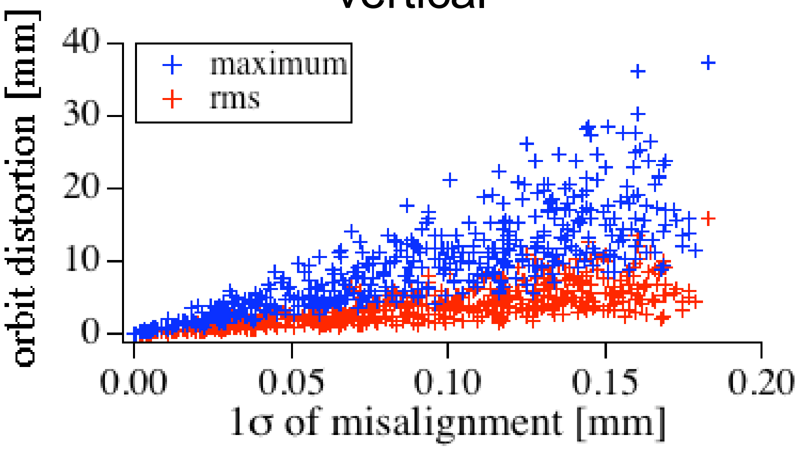
statistics with 501 seeds

- Maximum and rms orbit distortion with various magnitude of alignment errors.
- 10 to 20 muon ring.
- Acceleration is finished within 17 turns.

horizontal



vertical

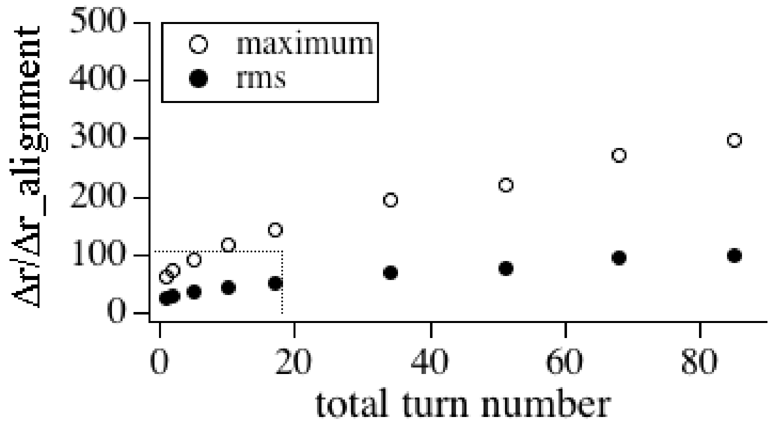


Orbit distortion (3)

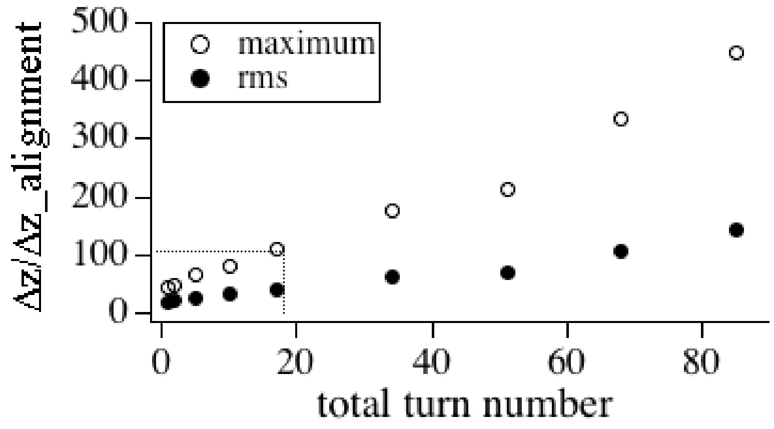
acceleration rate

- More accurate “amplification factor” is obtained by the linear fit of the previous figures.
- Slower acceleration (larger turn number to finish acceleration) gives larger amplification factor.
- Dependence is almost linear.

horizontal



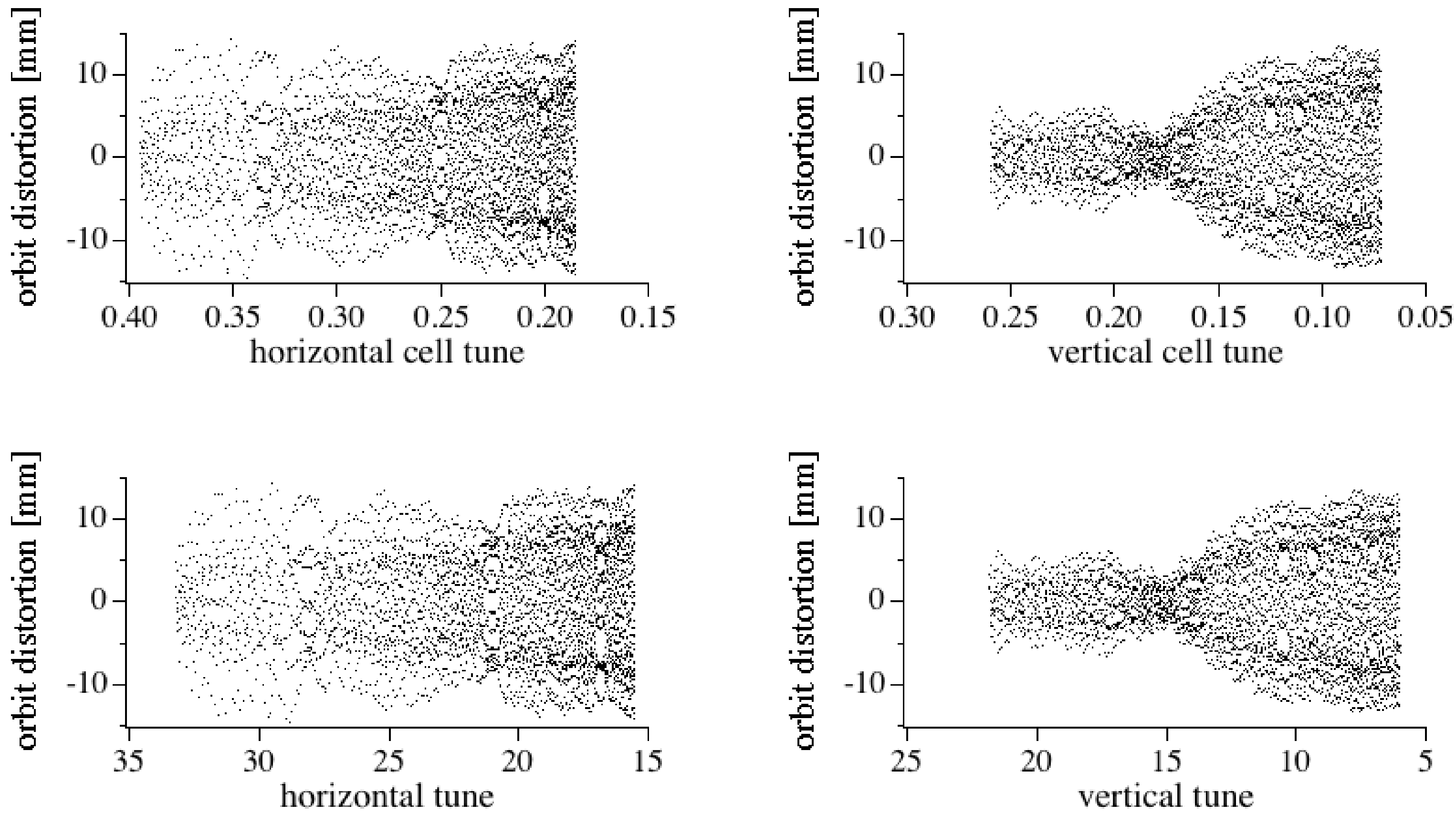
vertical



Orbit distortion (4)

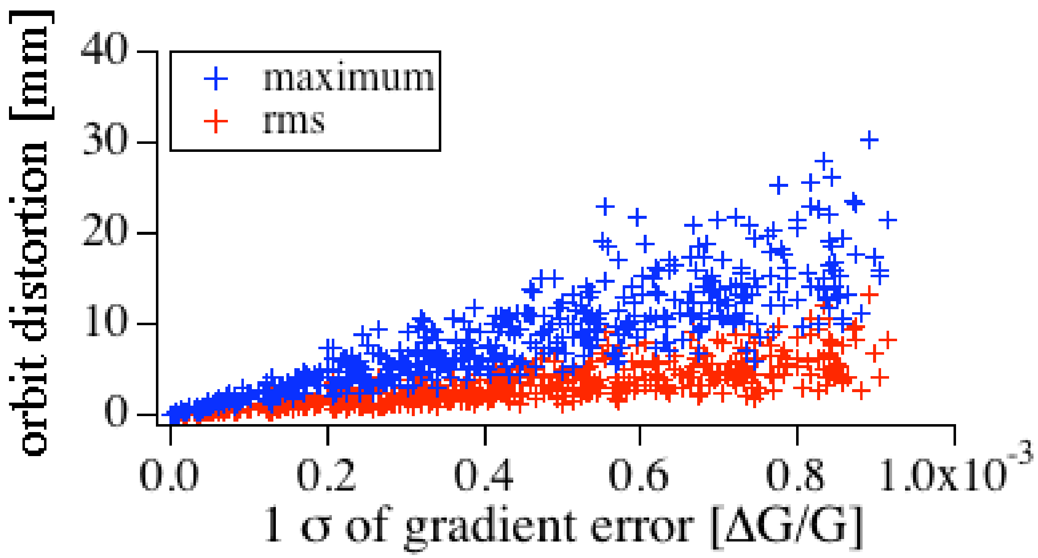
as a function of tune

- Dependence of either cell or total tune is not clear.



Orbit distortion (5)
gradient errors

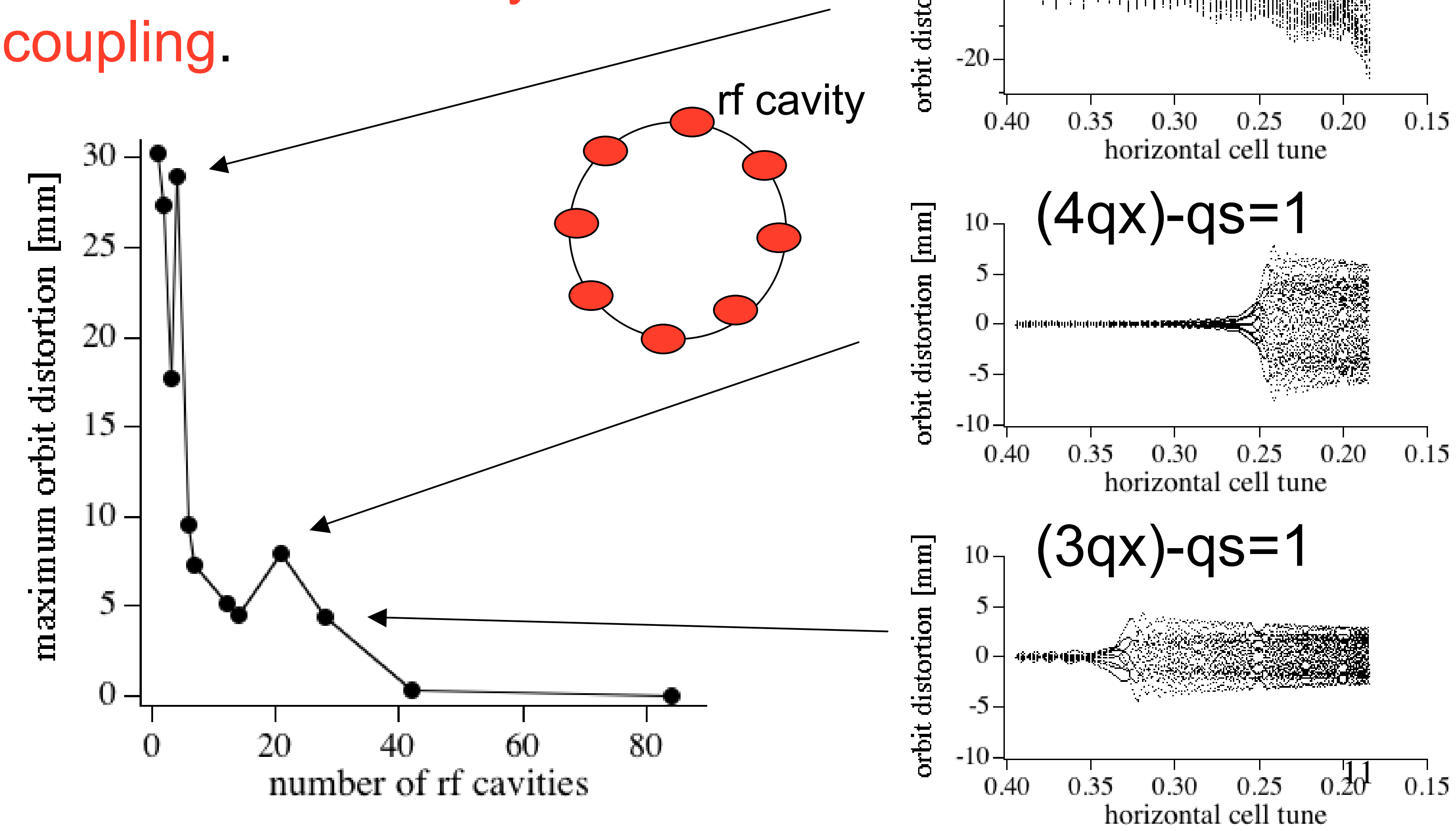
- Gradient errors in nonscaling FFAG is another source of orbit distortion.
- Gradient error of 0.1% gives almost same orbit distortion due to alignment error of 0.2 mm.
- That is consistent with offset of 0.2 m and gradient of 30 T/m at QD.



Orbit distortion (6)

pattern of rf location

- Assume that rf cavities are installed uniformly.
- rf cavities induces synchro-beta coupling.



Orbit distortion (7)

summary

- Amplification factor is about 100 with nominal acceleration rate.
- It is not integer "resonance" which makes distortion.
- Orbit is distorted by quasi-random kicks, instead.
- Pattern of rf cavity introduces "resonance" condition which depends on cell tune not total tune.

Optics distortion (1)
emittance and beam size

- In order to separate emittance growth and beam size growth, we introduce fixed and fitted beta functions.
- Fixed beta
  - First, track a beam in error-free lattice. Then, define beta function at each cell by

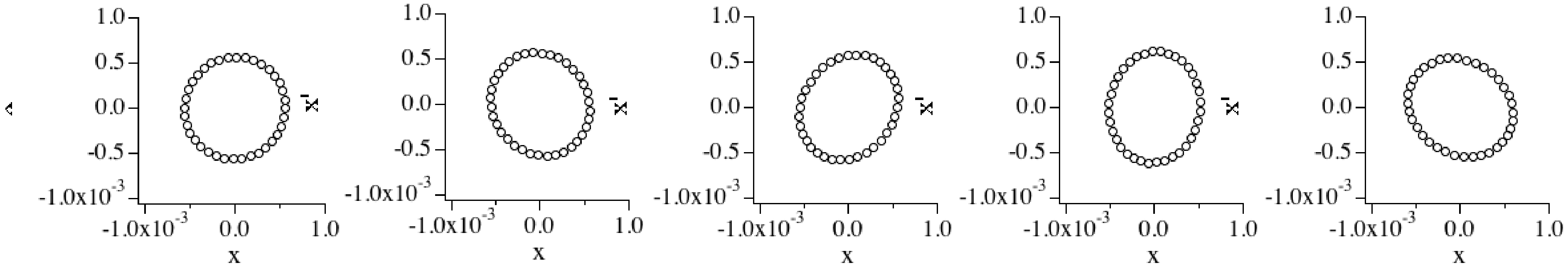
$$\beta_x = \frac{\langle x_i^2 \rangle}{\varepsilon_{x,rms}} \qquad \varepsilon_{x,rms} = \sqrt{\langle x_i^2 \rangle \langle p_{x,i}^2 \rangle - \langle x_i p_{x,i} \rangle^2}$$

- Fitted beta
  - For each lattice with errors, define beta function individually by the same equations.
- In both cased, initial beta is a solution of periodic function.

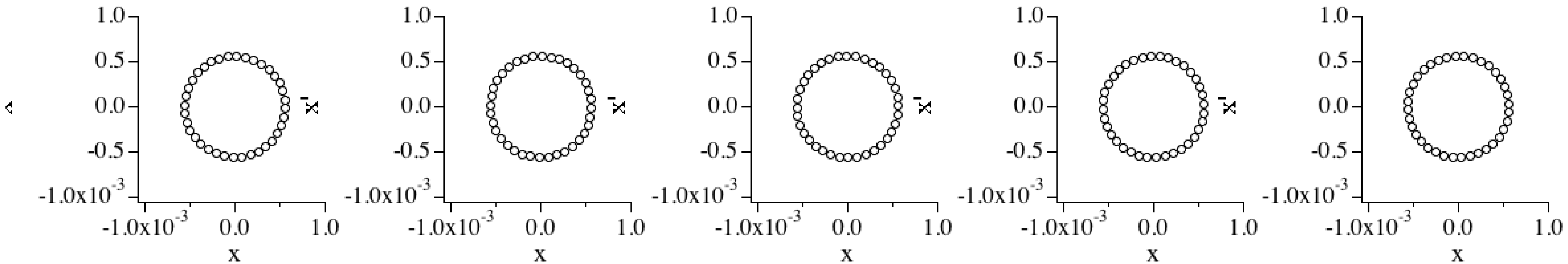
- C-S invariant with fixed beta tells beam size growth and the one with fitted beta tells emittance growth.

Optics distortion (2)
gradient errors

- Normalized with beta without errors (fixed beta)

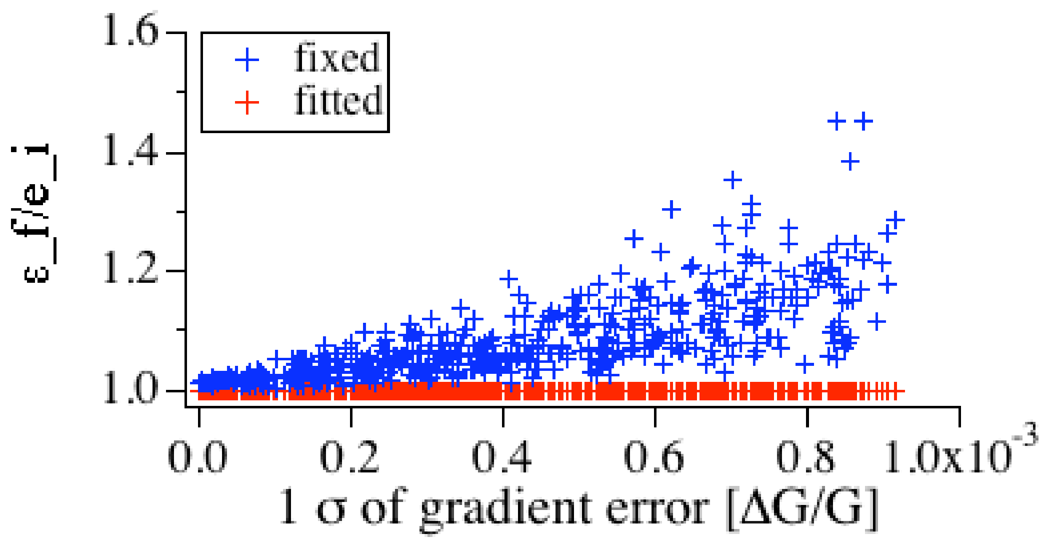


- Normalized with beta with errors (fitted beta)



Optics distortion (3)
statistics with 501 seeds

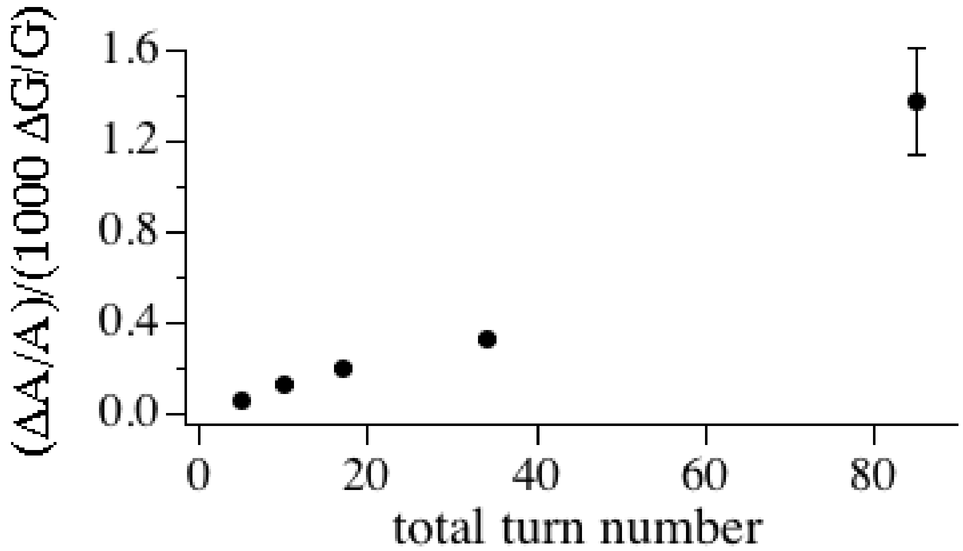
- Growth of C-S invariant with fixed and fitted beta.
  – With fitted beta, there is no growth, meaning that emittance is preserved.
  – With fixed beta, C-S invariant grows almost linearly with gradient error, meaning that tumbling is excited and beam size becomes larger.



Optics distortion (4)

acceleration rate

- "Growth factor" is obtained by the linear fit of the fixed beta points on the previous figure.
- Slower acceleration (larger turn number to finish acceleration) gives larger growth factor.
- Dependence is almost linear.

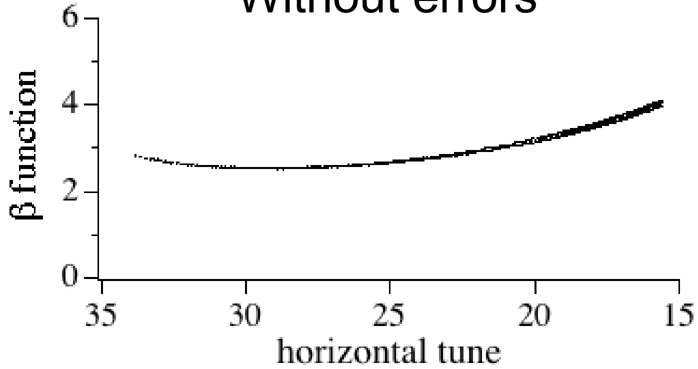


Optics distortion (5)

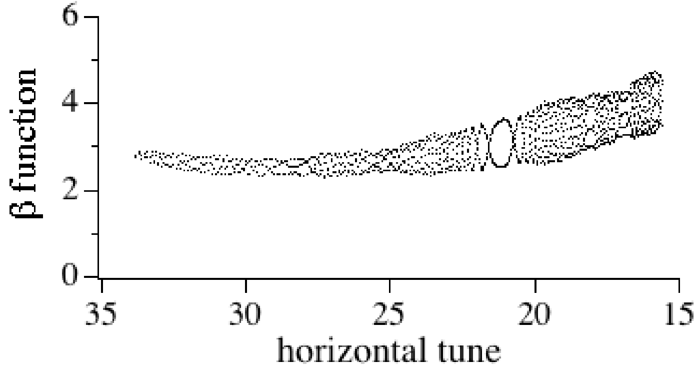
fitted beta as a function of tune

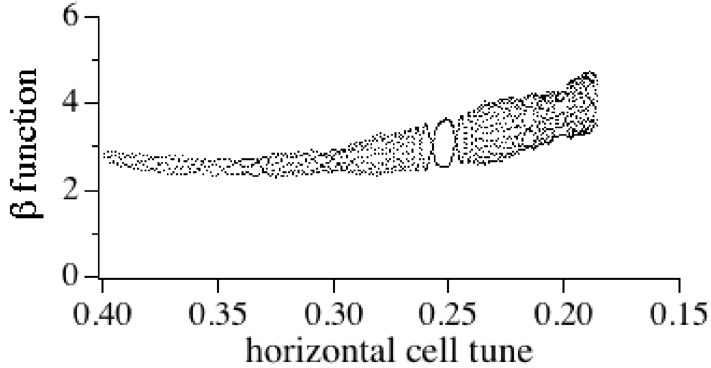
- Fitted beta function does not have clear dependence of either cell or total tune.

Without errors



With errors



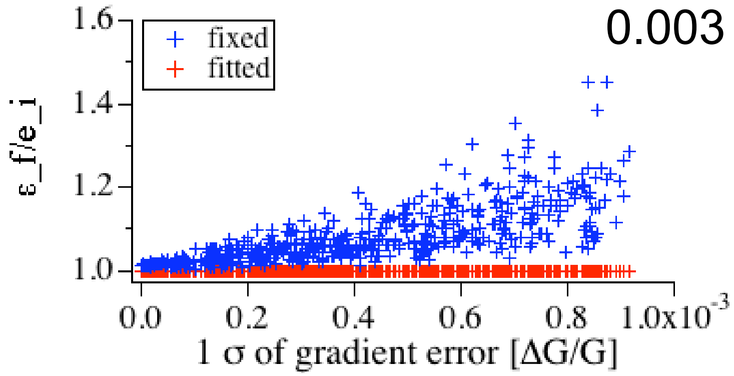


Optics distortion (6)

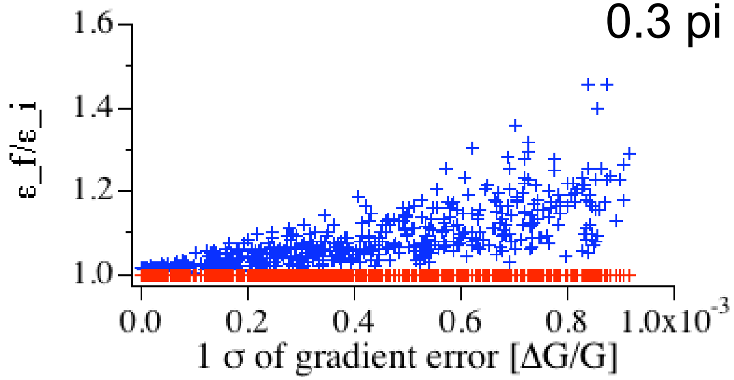
different emittance

- All tracking so far assumed 0.003 pi mm.
- The dependence is the same for 0.3 pi mm and 30 pi mm.

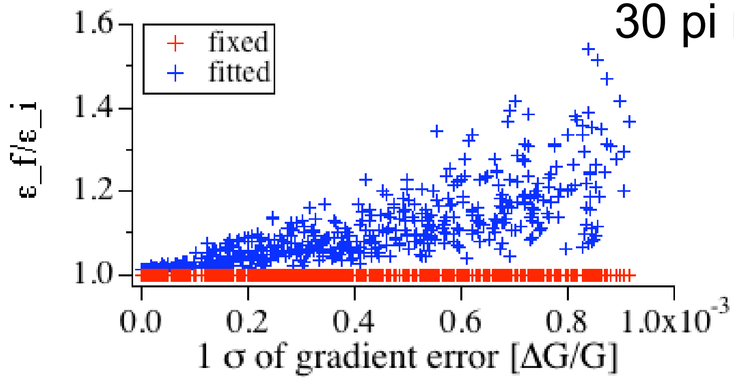
0.003 pi mm



0.3 pi mm



30 pi mm



Optics distortion (7)

summary

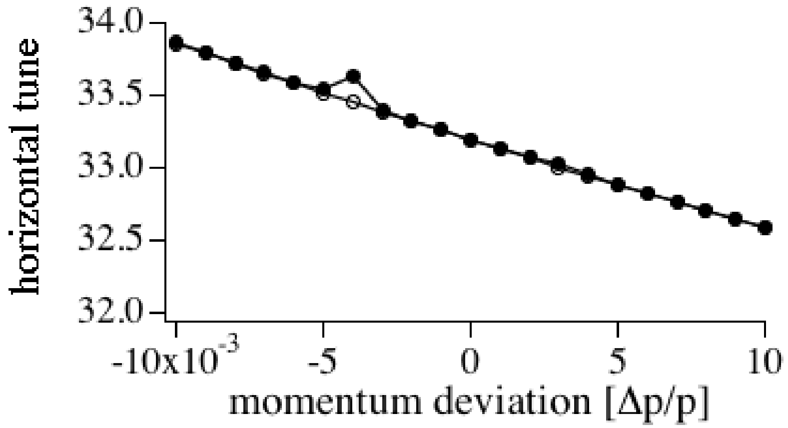
- Growth factor is about 200 with nominal acceleration rate.
- It is not half-integer “resonance” which makes distortion.
- Quasi-random mismatch induces tumbling.

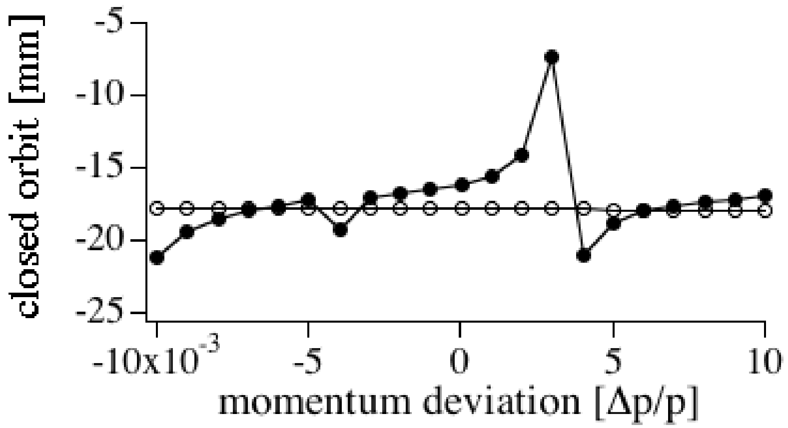
Beam with momentum spread (1)

orbit

- Within +-1%, the tune changes more than a unit.
- Dispersion function is small, but closed orbit strongly depends on momentum a lot.

empty circle: without error
filled circle: with error



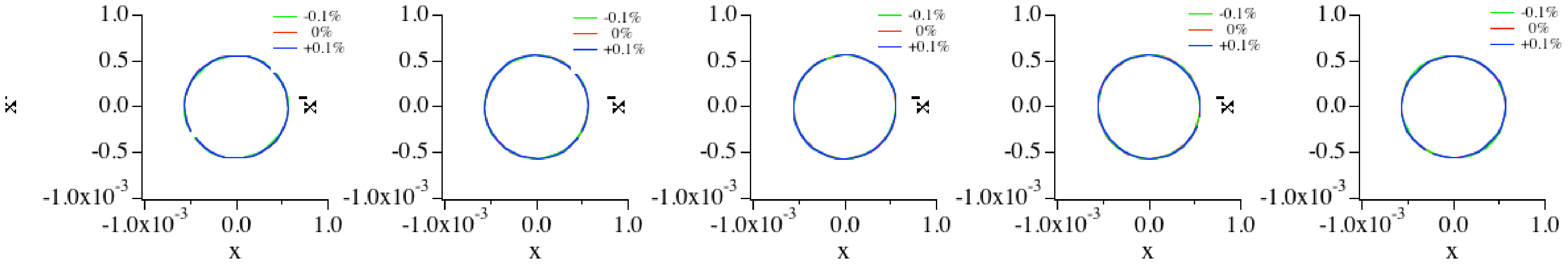


Beam with momentum spread (2)

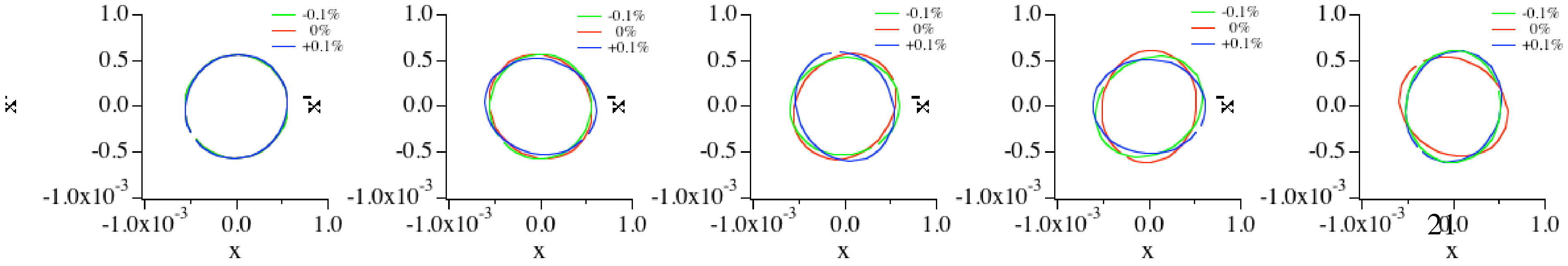
optics

- Beams with different momentum end up with different tumbling.
- As a superposition, it causes emittance growth.

Without errors



With errors



Conclusions

- In practice, it is hard to correct orbit and optics distortion for the entire momentum range. Some orbit and optics distortion are left.
- Distortion in nonscaling FFAG is due to quasi-random kicks of dipole and quadrupole, not due to resonance.
- Resonance can be seen smaller structure introduced by rf cavity.
- It is impossible to makes dispersion matching for a beam with momentum spread.
- Final emittance becomes larger because of tumbling depending on momentum spread.