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Tracking studies regarding the electron model 3-5.4463 MeV of a non-linear, non-scaling proton driver FFAG 3-10 GeV.

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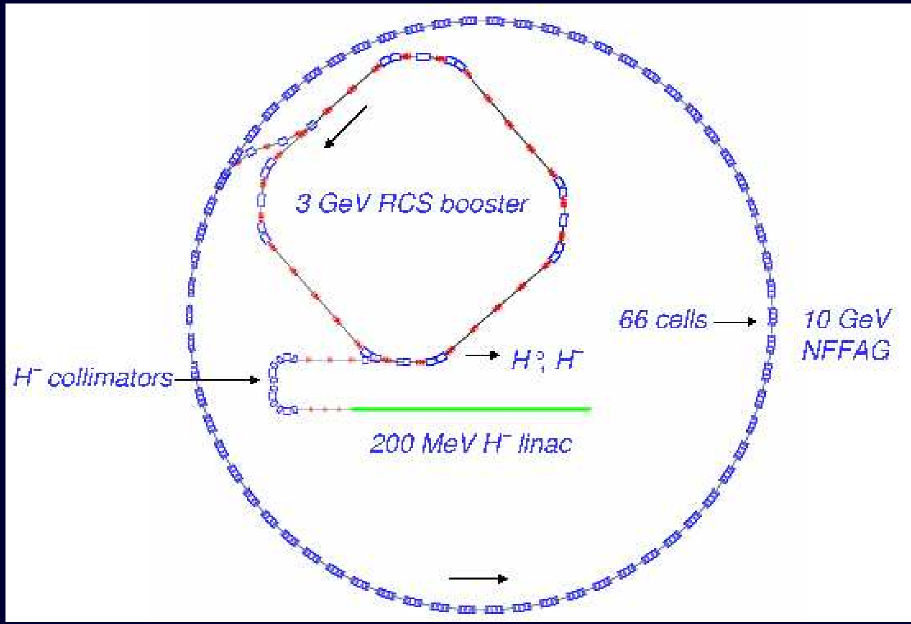
Introduction

■ The 50 Hz, 3-10 GeV, proton driver uses a new type of FFAG, with a non-linear, non-scaling and non-isochronous, cell focusing structure.

■ An electron model 3-5.4463 MeV has been designed to test various aspects of the optics.



■ We describe the setting up of the optics data files and produce preliminary beam dynamics studies concerning in this electron model, using the ray-tracing code ZGOUBI.

■ Similar investigations were carried out for a non-scaling, non-linear, isochronous muon ring proposed for the 8-20 GeV fast acceleration of muons in a neutrino factory[†] and for its associated 11-20 MeV electron model[‡].

[†] 6-D beam dynamics in an isochronous FFAG ring, F. Lemuet,F. Méot,G. Rees, proceedings of 2005 Particle Accelerator Conference, Knoxville, Tennessee.
[‡]6-D beam dynamics in an electron model lattice of an isochronous FFAG, F. Lemuet,F. Méot,G. Rees, Nuclear physics B,155 (2006) pp 330-331.

Ray-tracing method ZGOUBI

■ Solve the Lorentz equation $\vec{u}\,' = \vec{u} \times \vec{B}$ by Taylor developments of speed and position

$$\vec{R}(M_1) \approx \vec{R}(M_0) + \vec{u}(M_0)\,\Delta s + \vec{u}\,'(M_0)\,\frac{\Delta s^2}{2!} + ... + \vec{u}\,'''''(M_0)\,\frac{\Delta s^6}{6!} \quad [+...]$$

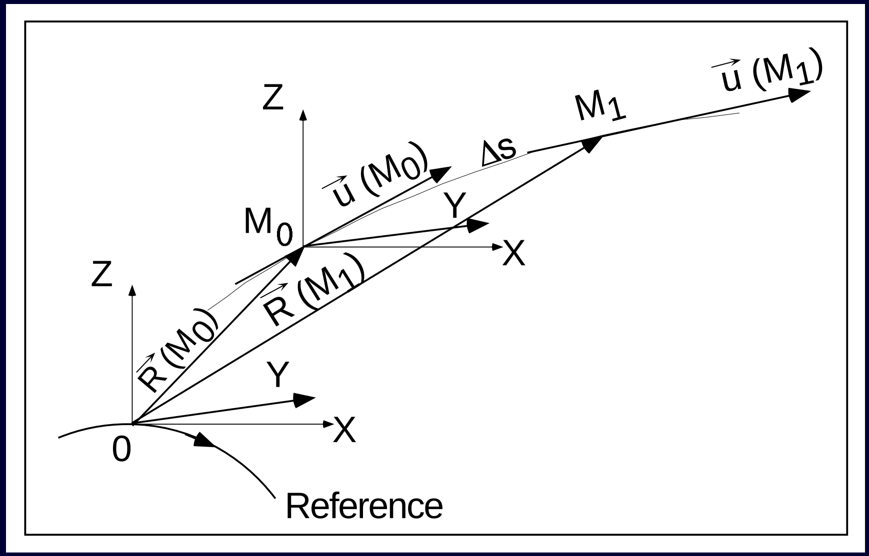
$$\vec{u}(M_1) \approx \vec{u}(M_0) + \vec{u}\,'(M_0)\,\Delta s + \vec{u}\,''(M_0)\,\frac{\Delta s^2}{2!} + ... + \vec{u}\,'''''(M_0)\,\frac{\Delta s^5}{5!} \quad [+...]$$

■ Coefficients are calculated by successive derivations

$$\vec{u}\,'' = \vec{u}\,' \times \vec{B} + \vec{u} \times \vec{B}\,' \ , \qquad \vec{u}\,''' = ... \qquad etc.$$

■ A field model is needed to calculate the field and its derivatives

$$\vec{B}(X,Y,Z), \qquad \partial^{i+j+k}\vec{B}/\partial X^i \partial Y^j \partial Z^k$$



■ Derivatives $d^n\vec{B}/ds^n$ are calculated ($X_{i,j,...},\ i,j,... = 1,3$ for X,Y or Z). using

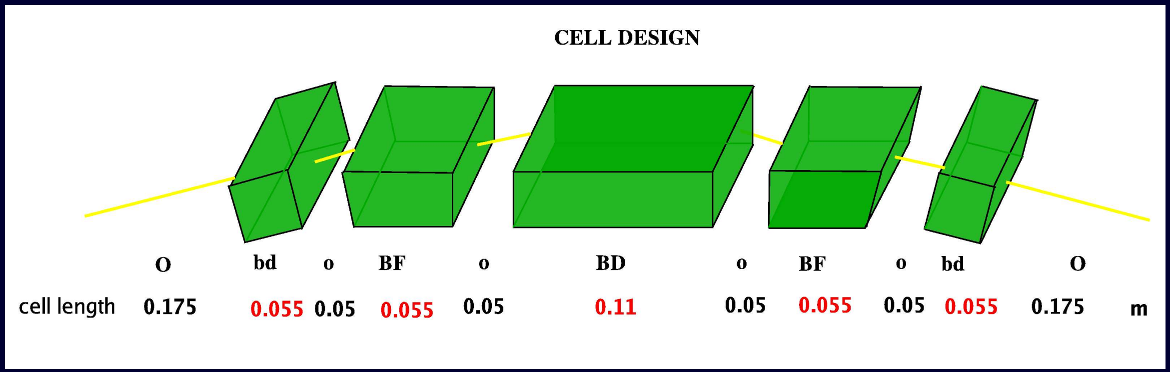
$$\vec{B}\,'(s) = \sum_i \frac{\partial \vec{B}(X,Y,Z)}{\partial X_i}\, u_i(s)\ , \qquad \vec{B}\,''(s) = \sum_{ij} \frac{\partial^2 \vec{B}(X,Y,Z)}{\partial X_i \partial X_j}\, u_i(s)\, u_j(s) + \sum_i \frac{\partial \vec{B}(X,Y,Z)}{\partial X_i}\, u_i'(s)$$

$$\vec{B}\,''' = \sum_{ijk} \frac{\partial^3 \vec{B}}{\partial X_i \partial X_j \partial X_k} \qquad etc.$$

Fields and cell parameters

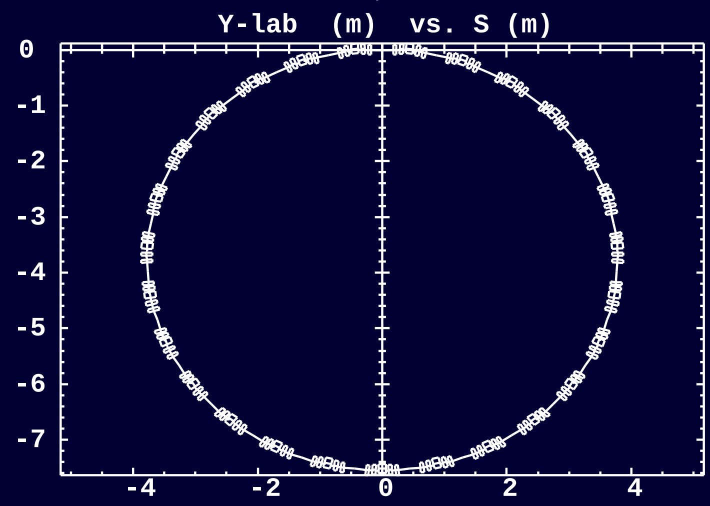
■ Geometrical lattice design

■ The design is based on a O-bd-o-BF-o-BD-o-BF-o-BD-O cell, symmetric wrt. the center of BD, built from magnets with a straight optical axis and rectangular $B(x)$ dependence.



■ The bd and BD magnets have parallel faces, while the second magnet, BF, has its upstream (resp. downstream) edge parrallel to the downstream (resp. upstream) edge of bd (resp. BD).

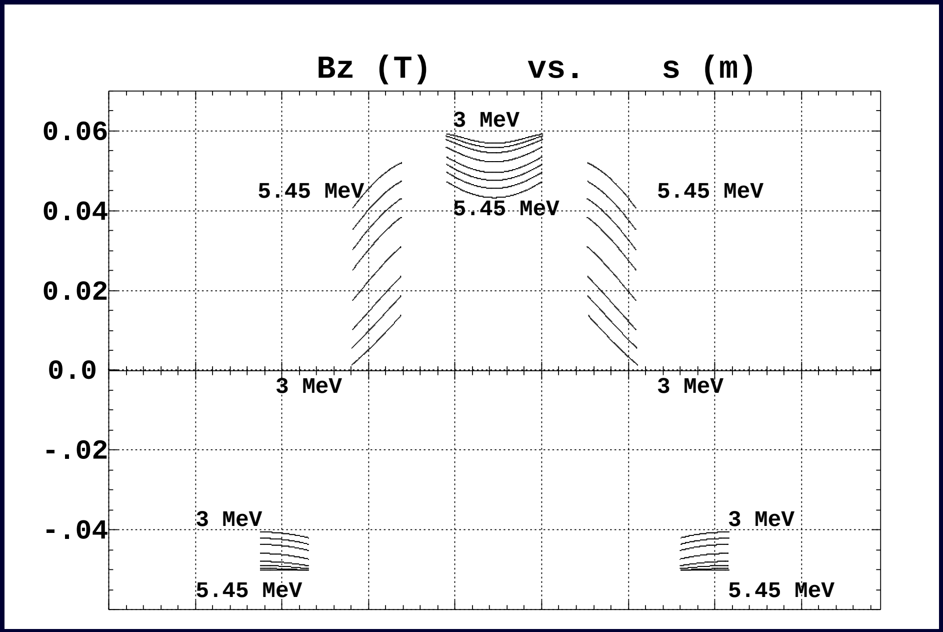
■ The rings consists of 27 cells yielding a circumference of 23.76 m.



■ Gradients and fields

■ The magnets are non-linear and combined function :

- bd, BD are horizontally defocusing, respectively reverse and positive bend.
- BF is horizontally focusing with positive bend.



■ In our simulations we use rectangular multipole with straight axis for all 3 types of magnets, and a hard-edge model.

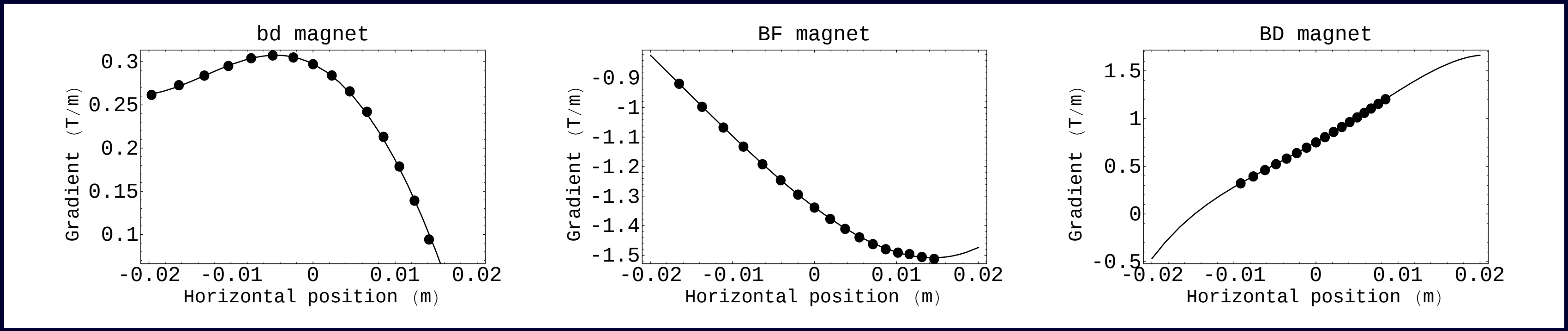
■ First order effect of fringe field extent on vertical motion is accounted for via a correction vertical kick of the form $-\tan(\alpha)/\rho + FR/(6*\rho^2*\cos(\alpha))$ with ρ being the local curvature radius and α the angle between the trajectory of the particle and the normal to the magnet face.

■ The local magnet strengths, given by the design data, yield the local gradients (dots) using the relation,

$$g[T/m] = \frac{p[GeV/c]K[m^{-2}]}{0.2998}$$

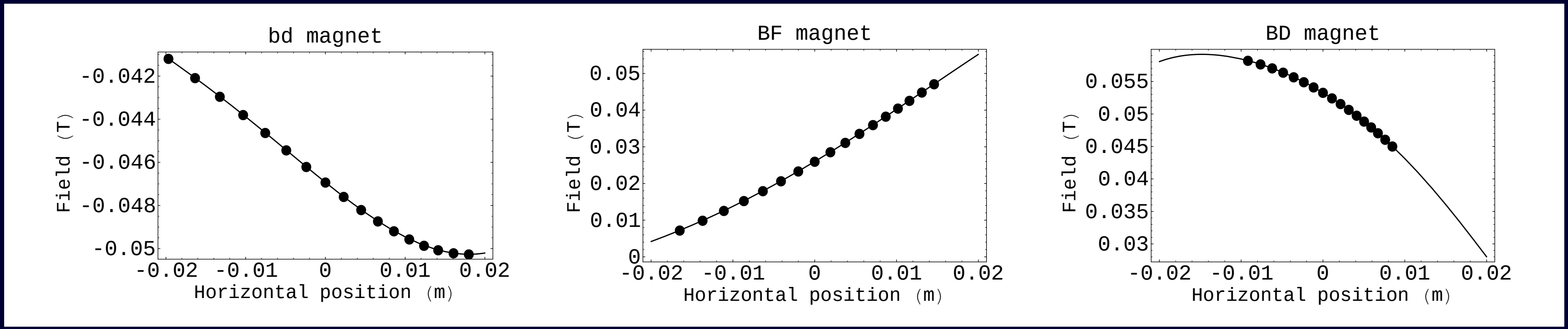
these gradients are approximated using three 4th degree polynomials (lines),

$$g(x) = g_0 + g_1\ x + g_2\ x^2 + g_3\ x^3 + g_4\ x^4$$



which are next integrated so to obtain the magnetic field law in each magnet

$$b(x) = b_0 + b_1\ x + b_2\ x^2 + b_3\ x^3 + b_4\ x^4 + b_5\ x^5 \quad (1)$$



■ The simulation of the rectangular dependence $B(x)$ in bd, BF, BD magnets uses a classical multipole modelling of the form

$$\vec{B} = \vec{\text{grad}} V_n$$

$$V_n(s,x,z) = (n!)^2 \left(\sum_{q=0}^{\infty} \frac{(-)^q G^{(2q)}(s)(x^2+z^2)^q}{4^q q!(n+q)!} \right) \left(\sum_{m=0}^{n} \frac{sin\left(m\frac{\pi}{2}\right) x^{n-m} z^m}{m!(n-m)!} \right)$$

with coefficient values $G^{(2q)}(center)$ derived from equation 1.

■ The s-dependence $G(s)$ allows the simulation of field fall-offs at magnet ends to be included when desired, using a Enge's fall-off model.

■ Geometrical matching procedure

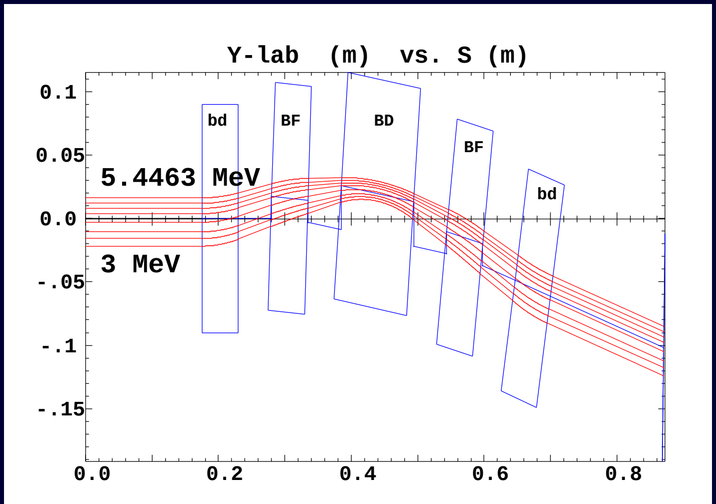
■ The constraints imposed are that the closed orbit angles in the drifts between the magnets be as close as possible to the design ones.

■ The parameters to be varied are the closed orbit coordinates at the start of the cell, and the positioning of the magnets via rotation and horizontal translation.

■ Satisfactory c.o. angle values have been obtained for three energies 3 MeV, 4.05 MeV, 5.4463 MeV with the consequence that c.o. angles at the other energies considered in the original design take a value that agrees with the design data to the level of a percent and better.

First order results

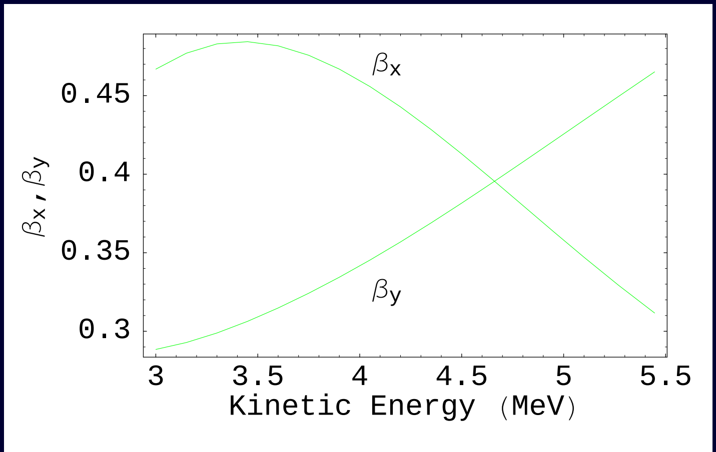
We present first order tracking results in a cell adjusted with the matching procedure.



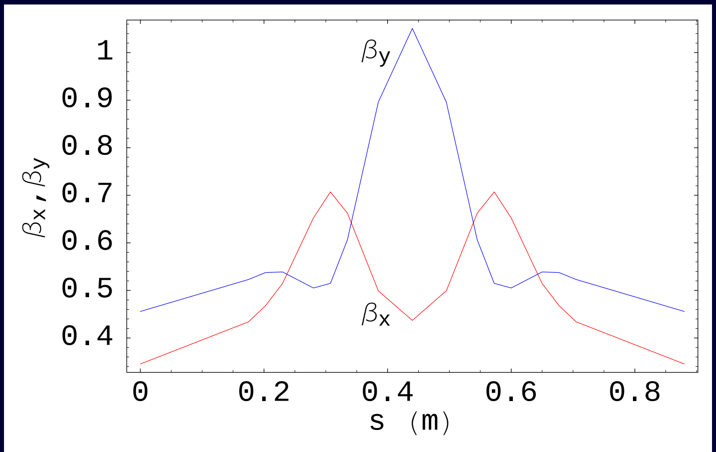
■ Closed orbits

■ Closed orbits for the electron model 3-5.4463 MeV, as the energy increases the trajectories move from the inner to the outer part of the magnets.

■ Focusing



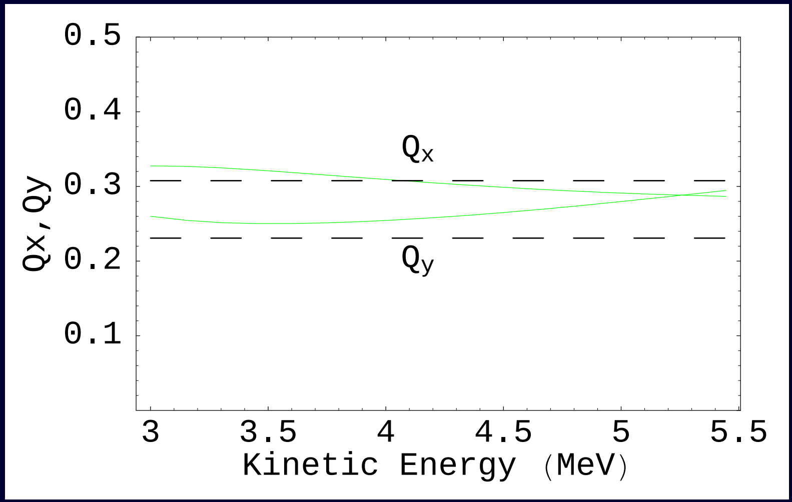
■ Periodic β values at the cell end as a function of energies.

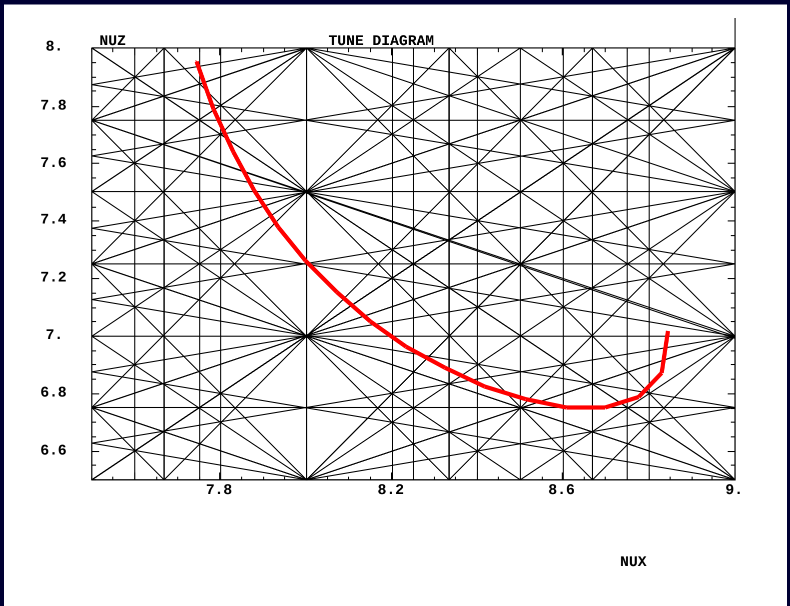


■ β functions as a function of s for the reference energy 4.05 MeV.

■ The design tunes per cell are constant with $Q_x = \frac{4}{13}$ and $Q_y = \frac{3}{13}$ (dashed lines).

■ The tunes per cell calculated with ray-tracing (green lines) are slighty different with a small variation with energy.





■ Total tunes ($27 \times Q_{x,y}$) variations in the tune diagram for E : 3 $\rightarrow$ 5.4463 MeV in a systematic resonance lines up to the fifth order.

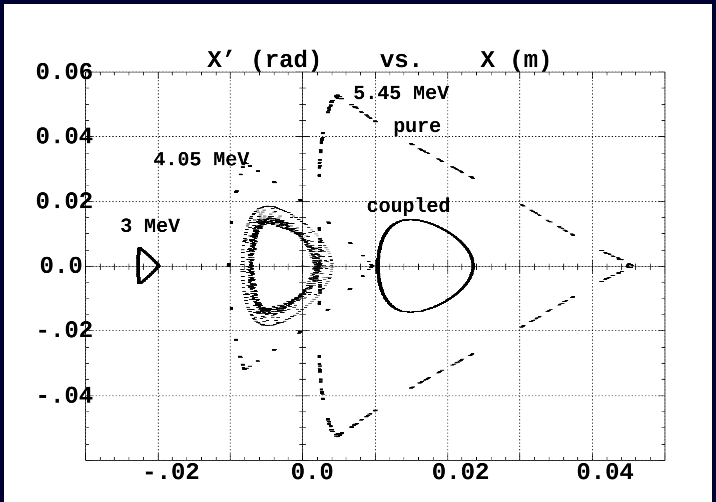
■ If a better agreement is needed, a matching procedure could be performed in ZGOUBI to match the tunes, which allows an automatic adjustment of the b_i coefficients in the multipole expansion

$$b(x) = \sum_{i=0}^{n} b_i \ x^i$$

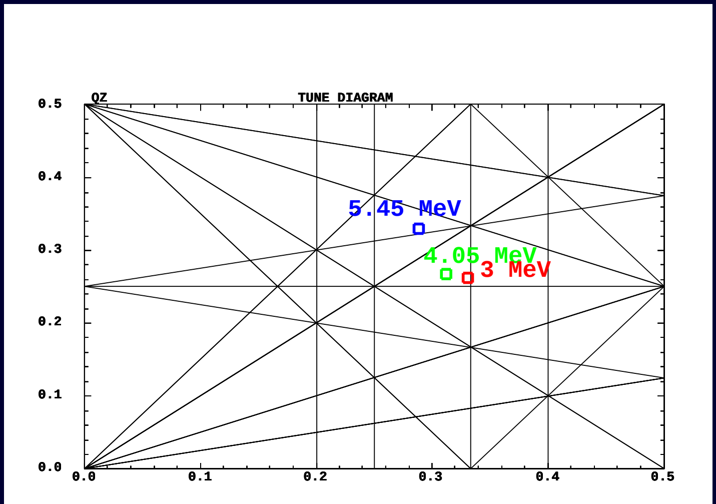
Dynamical aperture

■ Stability limits

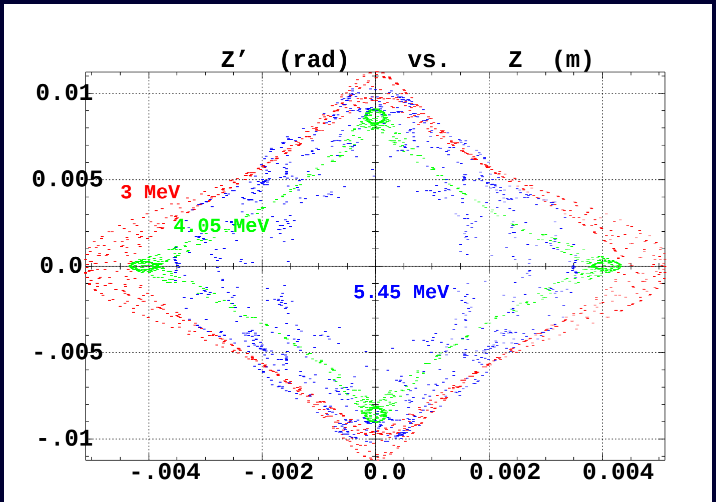
■ Limit phase space trajectories obtained by multiturn tracking at injection, reference and extraction energies in case of a pure horizontal motion and when we introduce a small z motion.



■ In that case the limits decrease sensibly, except for the injection energy for which the limit is inchanged.



■ Total tunes corresponding to the coupled motion are displayed in a fifth order tune diagram.

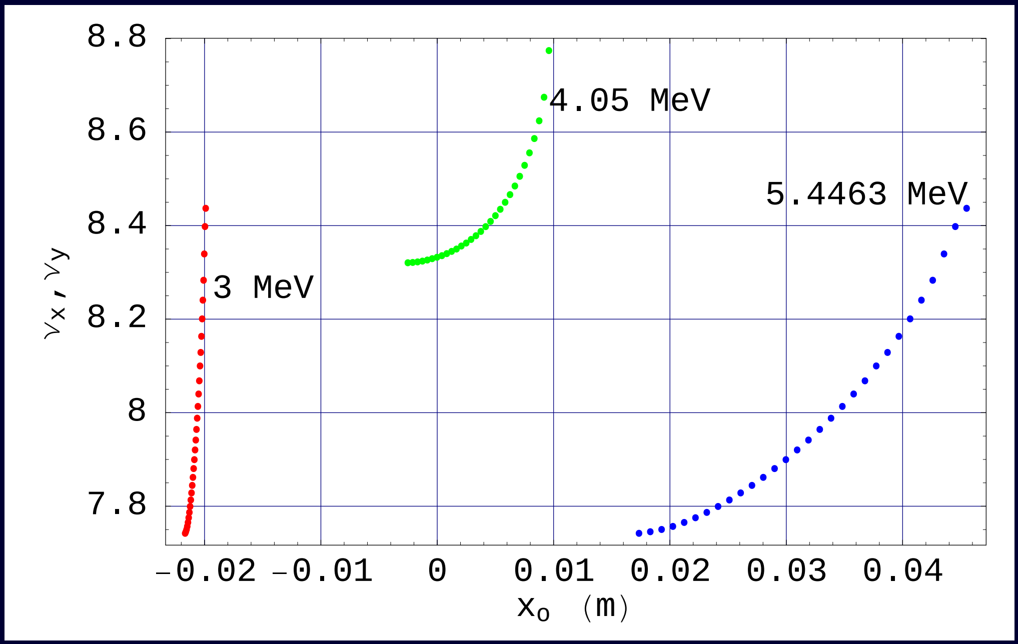


■ Vertical stability limits for the particles launched on the horizontal closed orbits.

■ Amplitude Detuning

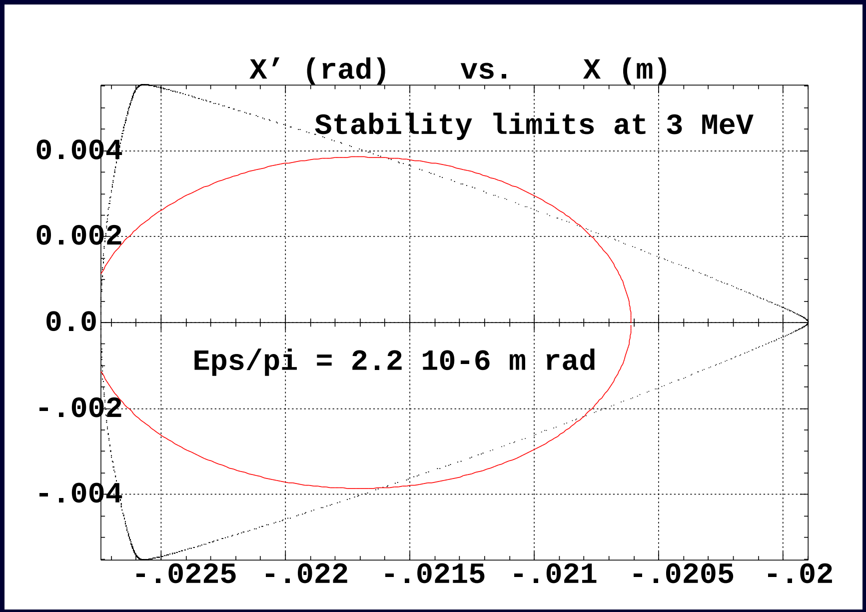
■ Amplitude detuning in case of pure x motion for the 3, 4.05, 5.4663 MeV energies.

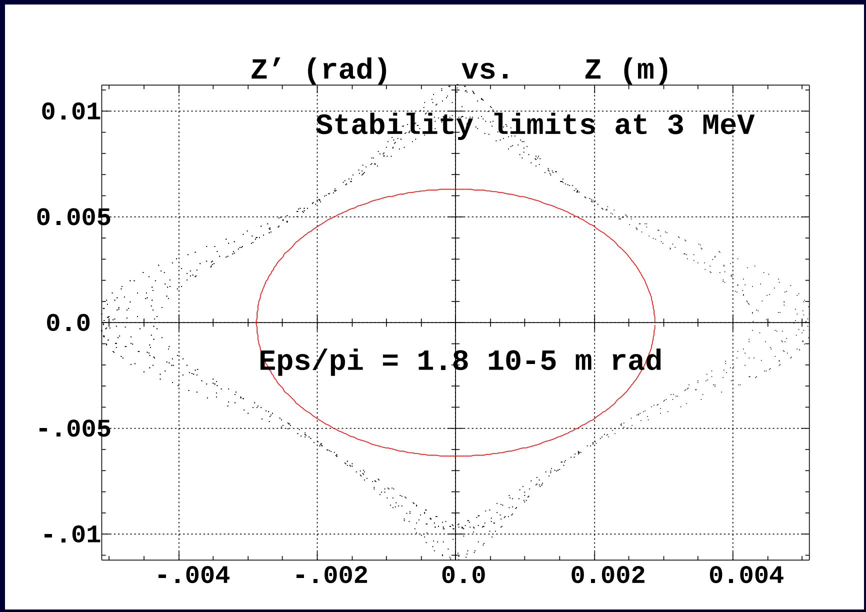
■ The total tunes have been calculated for the three energies starting from the closed orbits x_{co} up to the x_{limits} calculated previously.



■ Acceptance

■ Horizontal and vertical dynamical apertures at 3 MeV calculted previously and their corresponding matched ellipse.





■ The acceptances as the surfaces of these ellipses are :

- $\frac{\epsilon_x}{\pi} = 2.2\ 10^{-6}$ m rad
- $\frac{\epsilon_z}{\pi} = 1.8\ 10^{-5}$ m rad.

Conclusions

- Preliminary tracking studies have been performed in the non-scaling, non-linear electron model of a 3-10 GeV proton driver.

- A more exhaustive investigation could now be carried out.

- Further steps would include acceleration and full transmission studies as it has been performed in the isochronous muon ring and its electron model.