I.Spiral FFAG median plane magnetic field modeling for Zgoubi ray-tracing code.

1) Field law model.

2) Field fall-off function.

3) Results. Zgoubi input data file.

- II. Automatic dynamic parameters computation with tracking.
	- 1) Closed orbits.
	- 2) Tunes.
	- 3) Stability limits.
- III.Determination of possible working points.

1) Scan in the tune diagram by varying (k, ξ).

2) Dynamic aperture study, choice of parameters for 3D Field Map development.

IV.Ray-Tracing Simulations with SigmaPhi TOSCA 3D Field Maps.

> Spiral FFAG magnetic field modeling for Zgoubi: FFAG-SPI

- Goals of RACCAM: study FFAGs for medical applications, build a FFAG magnet ED. prototype for proton acceleration.
- Energy range. Injection: $6 17$ MeV; Extraction: $60 180$ MeV. \Rightarrow
- Focus on scaling FFAG magnets with spiral edges. \Rightarrow
- \Rightarrow Ring would be more compact than synchrotrons with encouraging beam dynamics properties: constant tunes, large dynamic aperture, high intensity and repetition rate.3 Y(m)

 \overline{a}

 $1\,$

 $\pmb{0}$

 -1

 -2

 -3

 -3

 -2

 -1

 $X(m)$

Principle scheme of a spiral FFAG ring.

Field Law Model.

$$
B_Z(r,\theta,z=0) = B_{Z0}\left(\frac{r}{R_0}\right)^k F_e(d_e) F_s(d_s)
$$

 \Rightarrow R₀: reference radius, radius of particle at 180MeV in magnet center. $\mathsf{B}_{\mathsf{z} 0}$: reference magnetic field, magnetic field at R_{0} in magnet center. k: field index. $B_7(T)^1$ F(d): field fall-off function.

Field along the central spiral (blue dots).

 \Rightarrow Field Fall-off function F(d) describes azimuthal evolution of B_z, especially for fringe field region.

 \Rightarrow d: distance from the calculation point to the entrance / exit magnetic face, depends on the spiral angle ξ. *1* $F(d) = -$

$$
= \frac{1}{1+exp}\left[C_0 + C_1\frac{d}{g} + C_2\left(\frac{d}{g}\right)^2 + C_3\left(\frac{d}{g}\right)^3 + C_4\left(\frac{d}{g}\right)^4 + C_5\left(\frac{d}{g}\right)^5\right]
$$

 $\mathsf{C}_0 \, ... \, \mathsf{C}_5$: Enge Coefficients g: full magnetic gap

 \Rightarrow We have a tool for simulating particle trajectories in a theoritical FFAG magnet. We can vary many parameters in the data file shown below such as R_0 , k, ξ, g, C₀...C₅

Part of Zgoubi data file describing a spiral FFAG.

Automatic dynamic parameters determination by tracking.

- \Rightarrow From a complete set of parameters, we want to determine dynamic parameters with multiturn tracking: closed orbits, tunes, horizontal and vertical dynamic apertures.
- \Rightarrow Simulations long and iterative: need automated simulations.

1) Closed orbits in the median plane.

• A particle with a stable motion around the ring will draw a trajectory in the horizontal phase space (an ellipse in general), we define the center of that trajectory as the closed orbit.

Scheme of study for closed orbits computation.

2) Tunes Calculation with Tracking.

- 2 methods: \Rightarrow
	- $\sum_{i=1}^{n}$ Determine the 1st order parameters from Twiss Matrix, calculation from a set of paraxial rays centered on the closed orbit.
	- $\sum_{i=1}^{n}$ Multiturn tracking, ellipse matching and Fourier Analysis of betatron oscillations.

3) Stability limits.

- \Rightarrow Definition:
	- \mathbf{r} Maximum horizontal and vertical dimensions of the beams that can circulate inside the ring.
	- \mathbf{r} Can be assimilated as the farthest stable trajectory of a single particle form the closed orbit.

Horizontal Stability Limit: \Rightarrow

- $\mathbf{2}$ From CO, small vertical motion given to the particle (to let eventual coupling phenomena appear).
- Initial horizontal position of the trajectory slightly shifted. \mathbf{E}
- $\sum_{i=1}^n$ Particle tracked over few hundred cells.

Vertical Stability Limit: \Rightarrow

- $\mathbf{2}$ Same operation as before.
- \mathbf{E} Initial vertical position of the CO trajectory slightly shifted from median plane.
- \mathbf{E} Particle tracked over few hundred cells.
- \mathbf{E} Last stable trajectory defined as the vertical stability limit.

Closed Orbit

Particle shifted from median plane.

Stability limit

•• All these studies can be done at ≠ energies from injection to extraction in order to study dynamic parameters wrt. energy.

Examples of parameters study wrt. energy. Left: tunes for \neq gap shapes; Right: Horizontal Stability Limits

Determination of Possible working points.

1) Scan of (k,ξ) parameters.

- •Objective: find one or several appropriate working points for a ring, i.e. find (Q_{x}, Q_{z}) couples far from dangerous resonances and which give large dynamic aperture.
- \Rightarrow We fixed the main parameters except the field index and the spiral angle. We can then explore the tune diagram by changing (k,ξ) as:

$$
Q_X \approx \sqrt{1+k} \quad Q_Z \approx \sqrt{-k+F(1+tan^2(\xi))}
$$

F: magnetic flutter

 \Rightarrow Automatically done by changing (k,ξ) in Zgoubi data file and running previously shown parameter calculations with tracking.

Parameters.

- E = 17 180 MeV protons
- N = 8 cells, cell opening angle = $2\pi/N \sim 45^{\circ}$
- $R0 = 3.54688$ m (radius of reference)

 $B0 = 1.5$ T (maximum magnetic field at R0)

pf = 0.38 (packing factor), magnet opening angle = $(2\pi/N)$.pf $\sim 17.1^{\circ}$

 $g = 4$ cm at 180 MeV (full gap), parallel gap

$$
C_0 = 0.1455 \; ; \; C_1 = 2.267 \; ; \; C_2 = -0.6395 \; ; \; C_3 = 1.1558 \; ; \; C_4 = C_5 = 0
$$

(k,ξ) varying

Studies done at 180 MeV

Stability regions in (k, ξ) and (Q_x, Q_z) diagrams. Red: BeamOptics matrix code ; Blue: Zgoubi ray-tracing code.

• Matrix and Ray-tracing codes give equivalent shapes in the diagrams but discrepancies appear, especially for large k and ξ that can be explained by the fringing field modeling in the 1st order matrix code.

2) Dynamic aperture study. Choice of parameters for 3D Field Map Development.

- • Multiturn injection could require to have a fractional tune close to 0.2 or 0.8 (cf Jaroslaw Pasternak's talk)
- \Rightarrow We choose a region close to $\rm Q_{x}\rm =2.82$.
- Ð Scan of (k,ξ) parameters in that region during which tunes and stability limits are computed and plotted in the tune diagram.

- \Rightarrow Influence of the sextupolar resonance is dramatic if we are sitting to close from it but we can at least choose points of study with large dynamic aperture.
- \Rightarrow We are still working on the choice of the final working points but interesting study points have been chosen for TOSCA 3D Field Map modeling from SigmaPhi.
- \Rightarrow First Maps have been calculated (cf T. Planche's talk) and tracking within these maps is on the way.

> Ray-Tracing Simulations with SigmaPhi TOSCA 3D Field Maps.

Parameters. E = 17 – 180 MeV protons $N = 8$ cells $R0 = 3.54688$ m (radius of reference) $B0 = 1.5$ T (maximum magnetic field at R0) $pf = 0.38$ (packing factor) $g = 4$ cm at 180 MeV (full gap), gap shaping k = 4.415 ξ = 50.36°

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Spiral FFAG magnet with TOSCA 3D.

- \Rightarrow 3D TOSCA Field Maps calculated by SigmaPhi.
- \Rightarrow Tracking done with a median plane 2D Field Map extracted from the 3D.
- \Rightarrow Tracking with the whole 3D Field Map is on the way:
	- \gg need to fix discrepancies between 2D and 3D.
	- \mathcal{P} need to optimize number of steps in the map.

FFAG-SPI

SUMMARY.

- \Rightarrow Development of a Zgoubi routine modeling Spiral FFAG magnetic field for tracking.
- \Rightarrow Automatic dynamical parameters search.
- \Rightarrow Scan in tune diagram by varying (k, ξ) for working points studies.
- \Rightarrow 8 cells configuration is under investigation with TOSCA 3D.
- \Rightarrow Still investigating other sets of parameters (more cells)