I. Spiral FFAG median plane magnetic field modeling for Zgoubi ray-tracing code.
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   2) Dynamic aperture study, choice of parameters for 3D Field Map development.

IV. Ray-Tracing Simulations with SigmaPhi TOSCA 3D Field Maps.

Spiral FFAG magnetic field modeling for Zgoubi: FFAG-SPI

- Goals of RACCAM: study FFAGs for medical applications, build a FFAG magnet prototype for proton acceleration.
- Energy range. Injection: 6 – 17 MeV; Extraction: 60 – 180 MeV.
- Focus on scaling FFAG magnets with spiral edges.
- Ring would be more compact than synchrotrons with encouraging beam dynamics properties: constant tunes, large dynamic aperture, high intensity and repetition rate.

Principle scheme of a spiral FFAG ring.

Field Law Model.

\[ B_z(r, \theta, z = 0) = B_{Z0} \left( \frac{r}{R_0} \right)^k F_e(d_e)F_s(d_s) \]

- \( R_0 \): reference radius, radius of particle at 180 MeV in magnet center.
- \( B_{Z0} \): reference magnetic field, magnetic field at \( R_0 \) in magnet center.
- \( k \): field index.
- \( F(d) \): field fall-off function.

Field along the central spiral (blue dots).

Field Fall-off function $F(d)$ describes azimuthal evolution of $B_z$, especially for fringe field region.

d: distance from the calculation point to the entrance / exit magnetic face, depends on the spiral angle $\xi$.

$$
F(d) = \frac{1}{1 + \exp \left[ C_0 + C_1 \frac{d}{g} + C_2 \left( \frac{d}{g} \right)^2 + C_3 \left( \frac{d}{g} \right)^3 + C_4 \left( \frac{d}{g} \right)^4 + C_5 \left( \frac{d}{g} \right)^5 \right]}
$$

$C_0 \ldots C_5$: Enge Coefficients

$g$: full magnetic gap

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ENSPG

FFAG 2007 Workshop

J. Fourrier LPSC / IN2P3 / CNRS

We have a tool for simulating particle trajectories in a theoretical FFAG magnet. We can vary many parameters in the data file shown below such as $R_0$, $k$, $\xi$, $g$, $C_0$...$C_5$

```
'FFAG-SPI'
0
1 90. 354.687988
20. 0. 15. 4.41500 0.02 0.05
9. 0.
6 0.1455 2.267 -0.6395 1.1550 0. 0. 0.
8.55 50.36 1000000. -1000000. 1000000. 1000000.
9. 0.
6 0.1455 2.267 -0.6395 1.1550 0. 0. 0.
-8.55 50.36 1000000. -1000000. 1000000. 1000000.
6 -1
0 0 0 0 0 0 0
0 0 0 0 0 0
2 125.
0.5
2
0. -0.741764932 0. 0.0436332313
```

Part of Zgoubi data file describing a spiral FFAG.

Automatic dynamic parameters determination by tracking.
From a complete set of parameters, we want to determine dynamic parameters with multiturn tracking: closed orbits, tunes, horizontal and vertical dynamic apertures.

Simulations long and iterative: need automated simulations.

1) Closed orbits in the median plane.

- A particle with a stable motion around the ring will draw a trajectory in the horizontal phase space (an ellipse in general), we define the center of that trajectory as the closed orbit.

Scheme of study for closed orbits computation.

Particle with stable motion
Closed orbit

Scheme of study for closed orbits computation.
2) Tunes Calculation with Tracking.

- 2 methods:
  - Determine the 1st order parameters from Twiss Matrix, calculation from a set of paraxial rays centered on the closed orbit.
  - Multiturn tracking, ellipse matching and Fourier Analysis of betatron oscillations.

3) Stability limits.

- Definition:
  - Maximum horizontal and vertical dimensions of the beams that can circulate inside the ring.
  - Can be assimilated as the farthest stable trajectory of a single particle form the closed orbit.
Horizontal Stability Limit:
- From CO, small vertical motion given to the particle (to let eventual coupling phenomena appear).
- Initial horizontal position of the trajectory slightly shifted.
- Particle tracked over few hundred cells.
- Last stable trajectory defined as the horizontal stability limit.

Closed Orbit
Particle shifted from closed orbit.
Stability limit
Vertical Stability Limit:

- Same operation as before.
- Initial vertical position of the CO trajectory slightly shifted from median plane.
- Particle tracked over few hundred cells.
- Last stable trajectory defined as the vertical stability limit.

- All these studies can be done at $\neq$ energies from injection to extraction in order to study dynamic parameters wrt. energy.

Examples of parameters study wrt. energy.
Left: tunes for $\neq$ gap shapes; Right: Horizontal Stability Limits.

Determination of Possible working points.

1) Scan of \((k, \xi)\) parameters.
   - Objective: find one or several appropriate working points for a ring, i.e. find \((Q_x, Q_z)\) couples far from dangerous resonances and which give large dynamic aperture.
   - We fixed the main parameters except the field index and the spiral angle. We can then explore the tune diagram by changing \((k, \xi)\) as:

\[
Q_x \approx \sqrt{1 + k} \quad Q_z \approx \sqrt{-k + F(1 + \tan^2(\xi))}
\]

F: magnetic flutter

- Automatically done by changing \((k, \xi)\) in Zgoubi data file and running previously shown parameter calculations with tracking.

**Parameters.**

- $E = 17 - 180$ MeV protons
- $N = 8$ cells, cell opening angle $= 2\pi/N \sim 45^\circ$
- $R_0 = 3.54688$ m (radius of reference)
- $B_0 = 1.5$ T (maximum magnetic field at $R_0$)
- $pf = 0.38$ (packing factor), magnet opening angle $= (2\pi/N).pf \sim 17.1^\circ$
- $g = 4$ cm at 180 MeV (full gap), parallel gap
- $C_0 = 0.1455; C_1 = 2.267; C_2 = -0.6395; C_3 = 1.1558; C_4 = C_5 = 0$
- $(k,\xi)$ varying
- Studies done at 180 MeV

- Stability regions in \((k, \xi)\) and \((Q_x/Q_z)\) diagrams.
  Red: BeamOptics matrix code; Blue: Zgoubi ray-tracing code.

- Matrix and Ray-tracing codes give equivalent shapes in the diagrams but discrepancies appear, especially for large \(k\) and \(\xi\) that can be explained by the fringing field modeling in the 1st order matrix code.

2) Dynamic aperture study. Choice of parameters for 3D Field Map Development.

- Multiturn injection could require to have a fractional tune close to 0.2 or 0.8 (cf Jaroslaw Pasternak’s talk)
- We choose a region close to $Q_x = 2.82$. 
- Scan of $(k, \xi)$ parameters in that region during which tunes and stability limits are computed and plotted in the tune diagram.

Horizontal (left) and vertical (right) dynamic apertures in tune diagram.

$DA_{X\text{max}} = 1866 \text{ mm.mrad}$

$DA_{Y\text{max}} = 206 \text{ mm.mrad}$
Influence of the sextupolar resonance is dramatic if we are sitting too close from it but we can at least choose points of study with large dynamic aperture.

We are still working on the choice of the final working points but interesting study points have been chosen for TOSCA 3D Field Map modeling from SigmaPhi.

First Maps have been calculated (cf T. Planche’s talk) and tracking within these maps is on the way.

Ray-Tracing Simulations with SigmaPhi TOSCA 3D Field Maps.

Parameters.
- $E = 17 - 180$ MeV protons
- $N = 8$ cells
- $R_0 = 3.54688$ m (radius of reference)
- $B_0 = 1.5$ T (maximum magnetic field at $R_0$)
- $p_f = 0.38$ (packing factor)
- $g = 4$ cm at 180 MeV (full gap), gap shaping
- $k = 4.415$
- $\xi = 50.36^\circ$

By: T. Planche, D. Neuvéglise - SigmaPhi.

- 3D TOSCA Field Maps calculated by SigmaPhi.
- Tracking done with a median plane 2D Field Map extracted from the 3D.
- Tracking with the whole 3D Field Map is on the way:
  - need to fix discrepancies between 2D and 3D.
  - need to optimize number of steps in the map.

FFAG-SPI

TOSCA

SUMMARY.

- Development of a Zgoubi routine modeling Spiral FFAG magnetic field for tracking.
- Automatic dynamical parameters search.
- Scan in tune diagram by varying \((k, \xi)\) for working points studies.
- 8 cells configuration is under investigation with TOSCA 3D.
- Still investigating other sets of parameters (more cells)