Meditations on End Fields

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Outline

- Symmetry of the end fields
- Vector potentials vs. scalar potentials
- Enge end field models
- Hard edge limit

Computing End Fields

- Field known in magnet body, outside
- Determine field in-between
- Maxwell's equations
- Must specify how field goes to zero
    - Constants of integration

End Field Symmetries

- Maintain field form in midplane

$$B_y(x, y = 0, s) = f(s)B_{y0}(x)$$

  - Wide magnets, small vertical aperture
- Multipole symmetry

$$rB_\phi(r, \phi, s) = f(s)r^m \cos(m\theta) + O(r^{m+2})$$

  - Multipole symmetric magnets
- Real magnets: something between

End Fields and Magnet Symmetry What's the difference?

- Dipole case, sharp field change
- Midplane symmetry

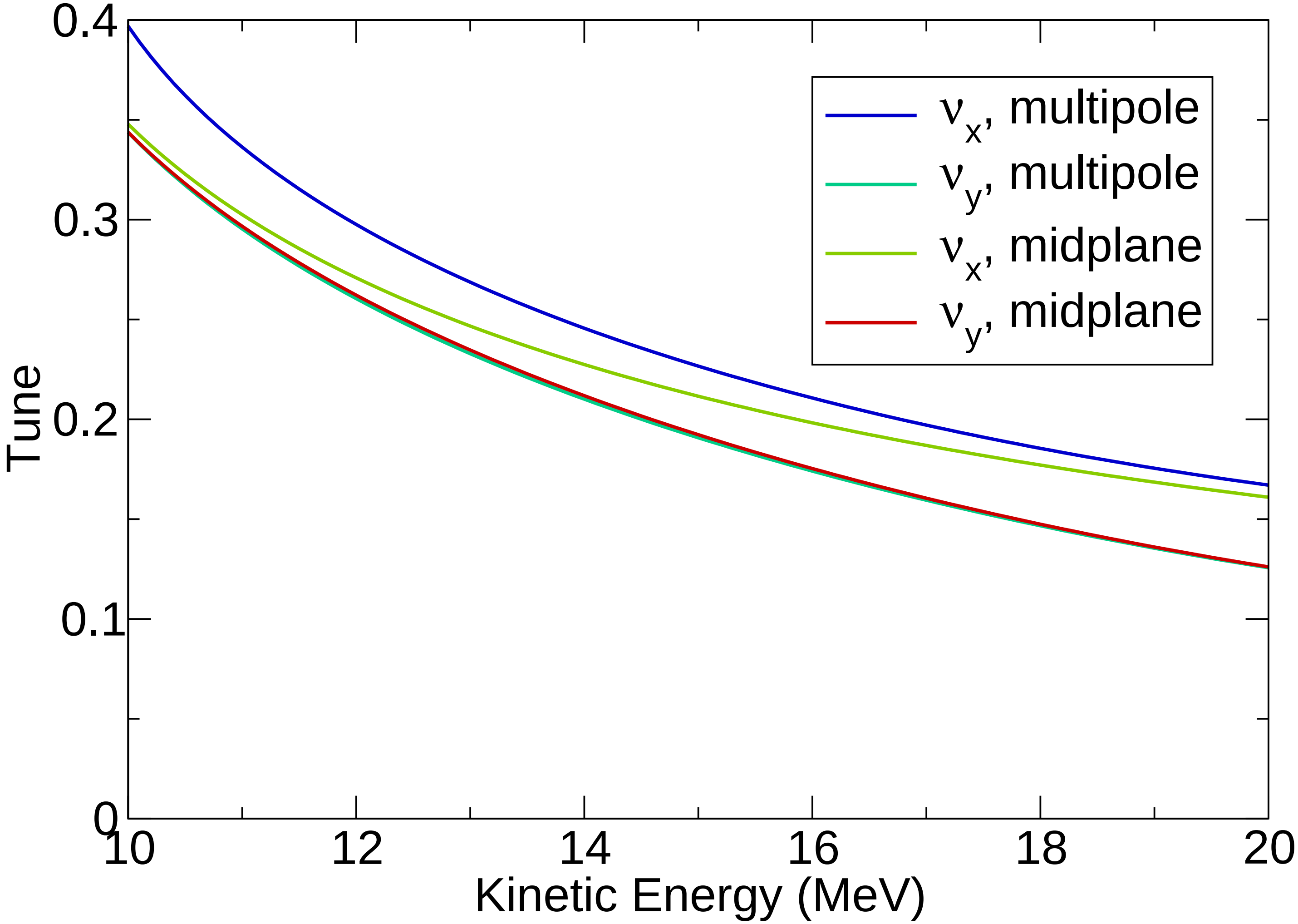
$$B_x = 0 \quad B_y = B_0 \left[1 - \frac{1}{2} y^2 \delta'(s) \right] \quad B_s = B_0 y \delta(s)$$

- Multipole symmetry

$$B_x = -\frac{1}{4} x y \delta'(s) \qquad B_s = B_0 y \delta(s)$$

$$B_y = B_0 \left[1 - \frac{1}{8} (x^2 + 3y^2) \delta'(s) \right]$$

Baseline EMMA Tunes
Different End Field Symmetries



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Vector Potentials Symplecticity

- Computation symplectic when
  - Vector field (motion) derived from Hamiltonian
  - Integration sufficiently accurate
- Field not represented perfectly: truncated series
- Hamiltonian requires vector potential
  - Fields from scalar potential: not symplectic

Field Expansions
Terms in Series

- Scalar potential:

$$\Psi = \sum_n B^{(2n)}(s)\psi_n(x,y)$$

- Vector potential

$$A_s = \sum_n B^{(2n)}(s)A_{sn}(x,y)$$

$$A_\perp = \sum_n B^{(2n+1)}(s)A_{\perp n}(x,y)$$

  - $B^{(2n+1)}$ and $B^{(2n+2)}$ combine, partially cancel

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Field Expansions
Use of Scalar Potential

$$B_\perp = \partial_\perp \Psi = \hat{s} \times (\partial_s A_\perp - \partial_\perp A_s)$$
$$B_s = \partial_s \Psi = \nabla_\perp \times A_\perp$$

- Using Ψ, $B^{(2n)}$ generates $B^{(2n)}$ in $B_\perp$, $B^{(2n+1)}$ in B_s
- Using A, $B^{(2n+1)}$ and $B^{(2n+2)}$ generate $B^{(2n+2)}$ in $B_\perp$ and $B^{(2n+1)}$ in B_s
- Single Ψ term doesn't generate both A terms

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Longitudinal Dependence of End Fields

- Real fields fall off like power law
- Typically use Enge functions instead

$$\frac{1}{1+e^{p(x)}}$$

- Field goes to 0/1 too rapidly
  - Field values don't converge?
  - Integrated effect OK?
- Integrating power law impractical

Hard Edge Limit

- Limit as length of ends goes to zero
- Currently: lowest order term in transverse variables
- Hard edge differs significantly from soft edge
  - Vertical tune diff limit: 0.07!
    - EMMA: prepare excess field
  - Higher order in transverse?

Conclusions

- Use appropriate model for end symmetry
  - Field maps correct
- Expand fields with vector potentials
  - Symplecticity
  - Correct terms in solution expansion
- Enge functions possibly lead to divergent fields
  - May not matter
- Hard edge far off in EMMA