Muon Acceleration with Scaling FFAG using Harmonic Number Jump

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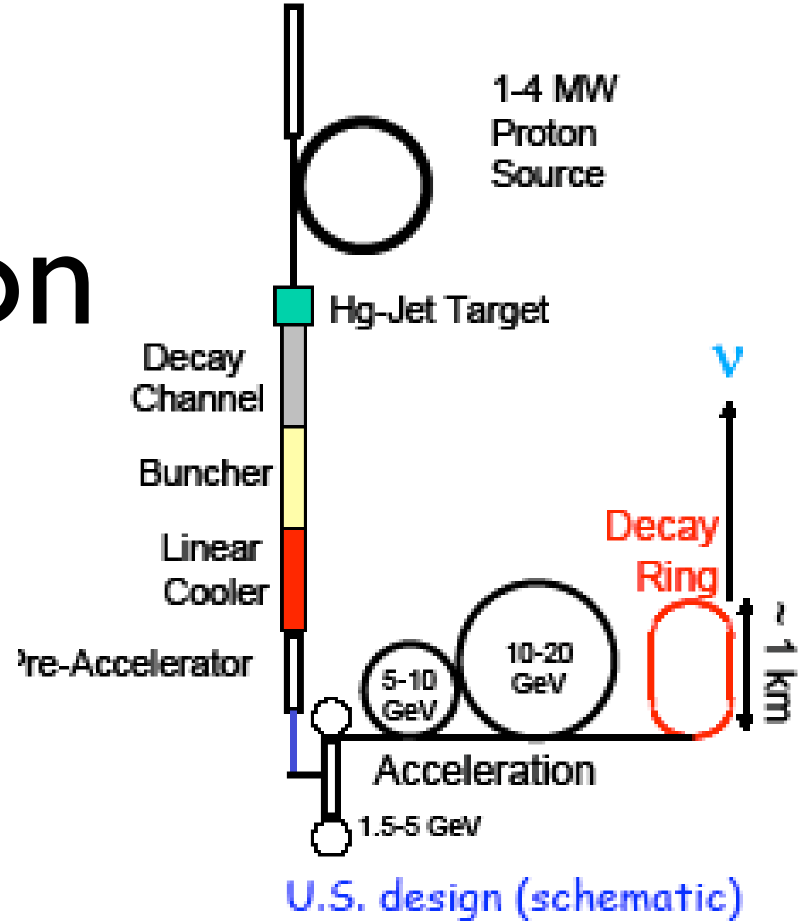
Chain of FFAGs for muon acceleration



- Non-scaling FFAGs: advantages
  - small aperture
  - const. rf frequency (high frequency & field)
- problem
  - non-linear time of flight (path length) for large amplitude causes large phase slip.

    cf. cascade rings
- The problem could be solved with scaling FFAG for 5-10GeV muon ring.
  - cf. slippage factor is constant. $1/\gamma^2 - \alpha \sim -1/k+1$

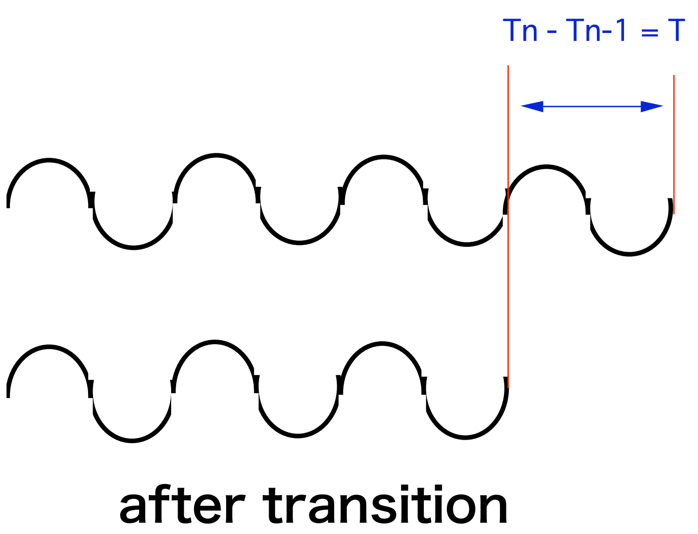
Scaling FFAG with HNJ

- Scaling FFAG with HNJ for low energy (5 -10GeV) ring
  - Higher frequency (~200MHz) rf cavity : good matching -> Phase Rotation & non-scaling FFAG
  - Harmonic Number Jump(HNJ) acceleration

    const. rf frequency --> high frequency & high field
- Scaling FFAG with HNJ for high energy (10 -20GeV) ring
- Scaling FFAG proton driver with HNJ
  - POP experiment with KURRI FFAG

Harmonic number jump (HNJ) acceleration

- HNJ-acceleration (Kolomenski, Fujisawa, Ruggiero)
  - Difference of revolution period between n-th and (n-1)-th turn equals m(integer) times rf period.
- $T_n - T_{n-1} = T_{rf} \times m$
  - Tn : revolution period for n-turn
  - Trf : rf period
  - m : integer (<0: before, >0: after transition)



HNJ Acceleration

- Revolution period for n-th turn

$$\left(\frac{T_n}{T_1}\right) = \left(\frac{C_n / v_n}{C_1 / v_1}\right)$$

  - C: circumference, v: particle velocity

- Scaling FFAG $\frac{C_n}{C_1} = \left(\frac{p_n}{p_1}\right)^{\frac{1}{k+1}}$

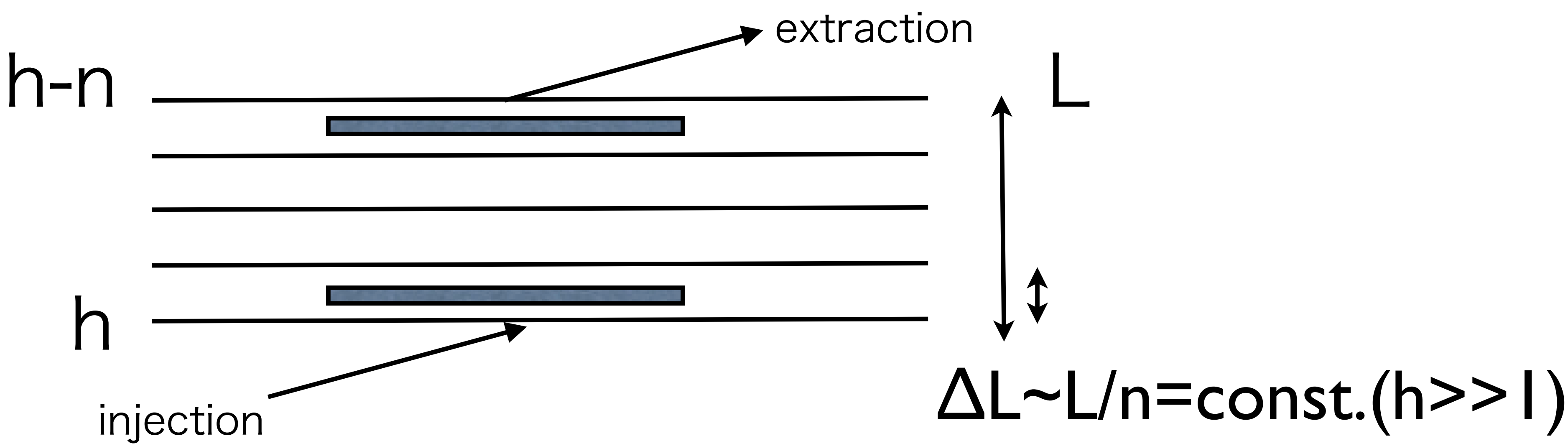
- For muon acceleration (γ>>1,v~c)
  - When k increases, or ring size decreases,

    No. of turns decreases.

    Energy gain/turn increases.
  - Turn separation(h>>1) for each orbit is almost constant.
  - Need optimization!

$$\frac{C_n}{C_1} = \frac{h_n}{h_1}, \quad p_n = p_1\left(\frac{h_n}{h_1}\right)^{k+1}, h_n = h_1 + n \times m$$

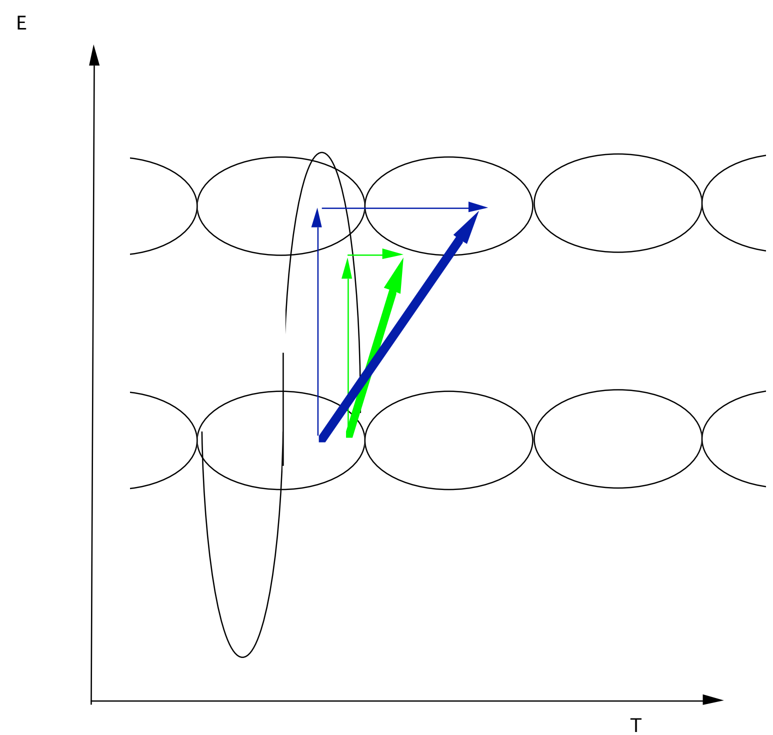
HNJ (γ >>1) with scaling FFAG



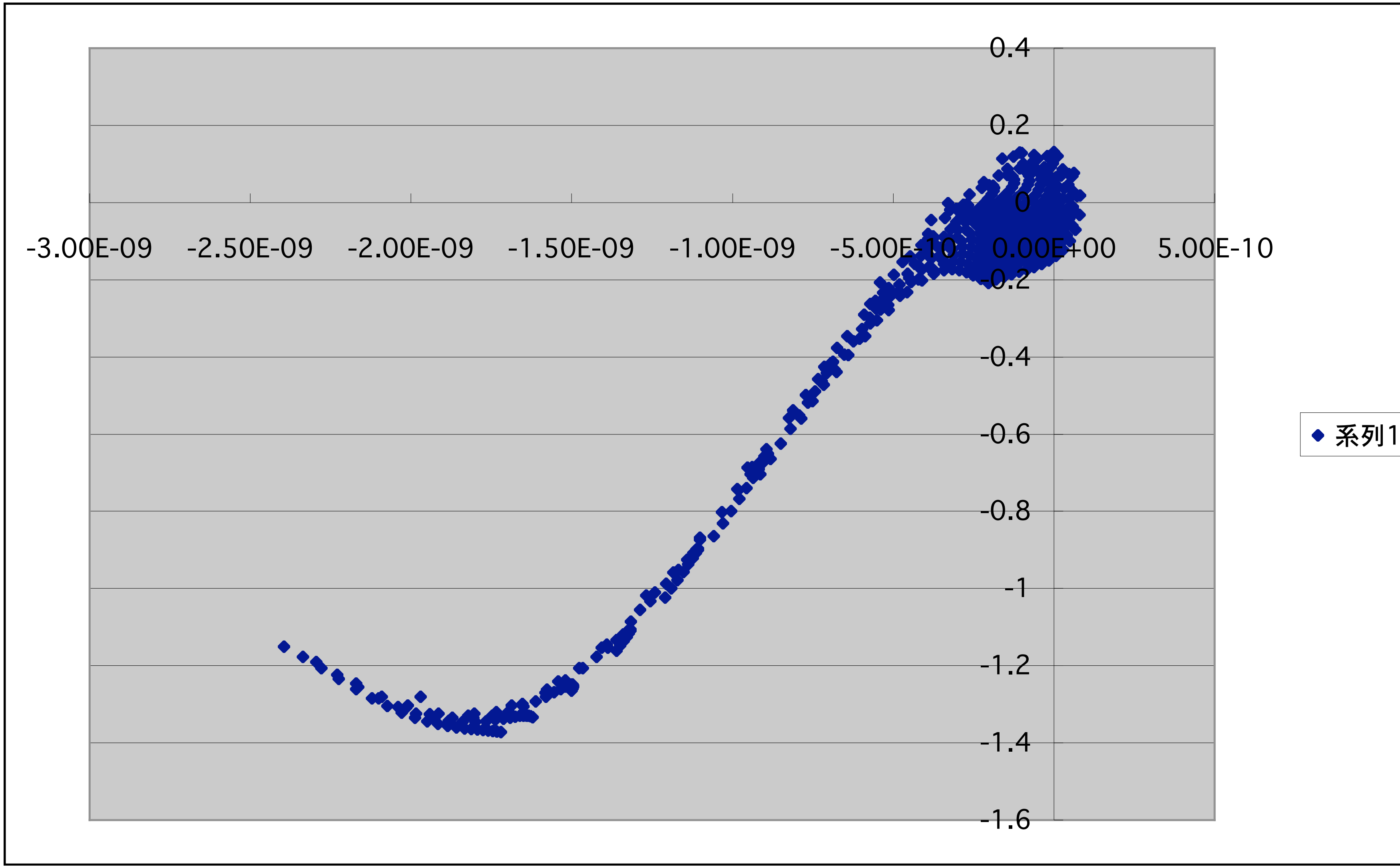
- In scaling FFAG for γ>>1, momentum compaction is constant.
  - linear behaviour in flight path(orbit excursion) ,TOF etc.
- Injection/extraction : DC septum magnet
  - Large orbit turn separation
- Multi-cavity system :possible (difficult for non-scaling, scaling/low beta)

Issues of HNJ

- Phase acceptance
  - Smaller for HNJ cf. synchronized acceleration
  - Because energy gain/turn is so large for HNJ that phase slip/turn should be 2π. If stable phase is away from $\pi/2$, phase slip/turn should be much less than 2π.



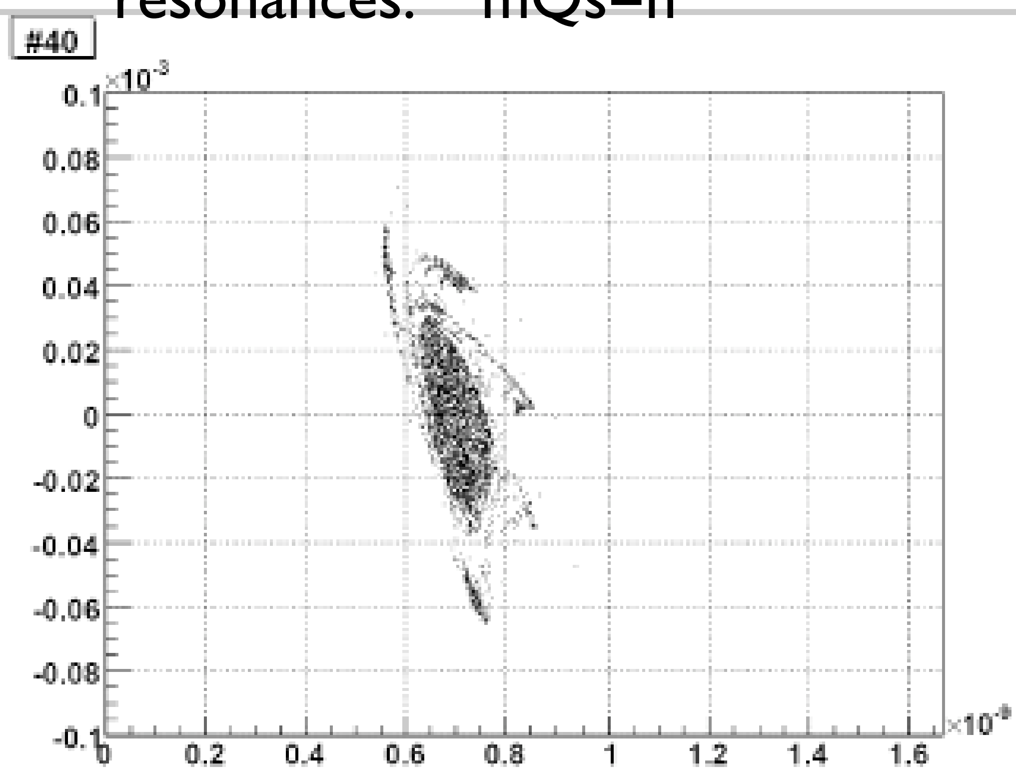
Phase acceptance

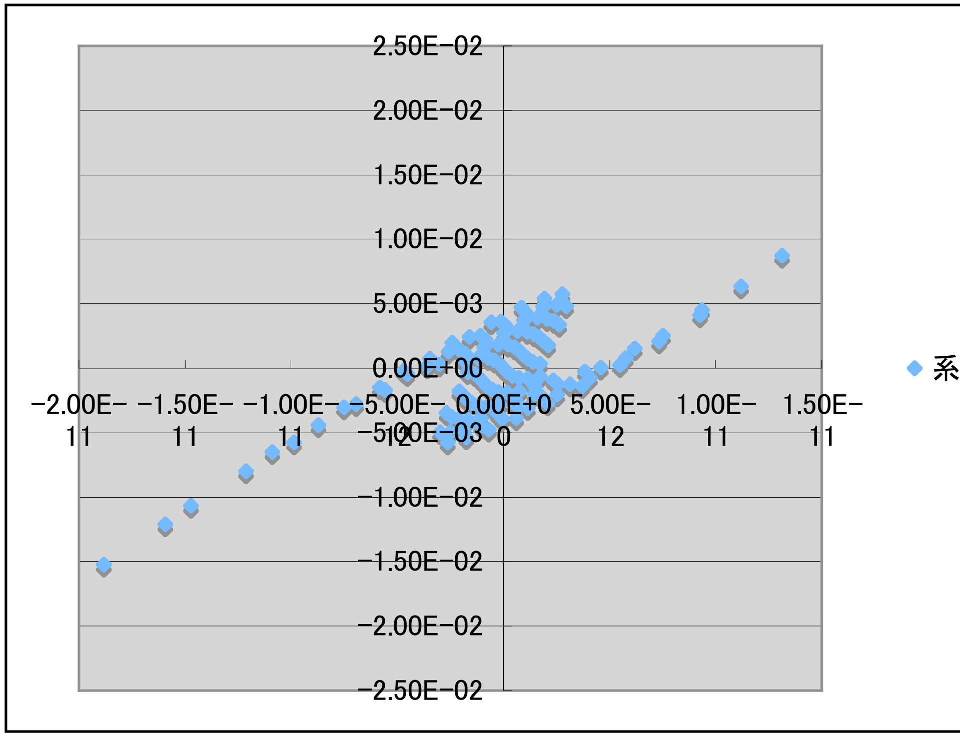


Non-linear behaviors

proton driver : by T. Planche

- Non-linear source dynamic aperture problems in longitudinal direction
  - Sinusoidal rf field contains non-linear components.
  - Synchroton tune is high enough to see non-linear resonances. mQs=n





Scaling FFAG

- Focusing
  - Spiral sector

    Focusing: body + edge

    Small ring size

    Rather large edge angle > 60 degree

  - Radial sector

    Negative bend

    doublet, triplet (DFD, FDF)

$$B = B_0\left(\frac{r}{r_0}\right)^k F\left(\theta - \varsigma \ln\frac{r}{r_0}\right)$$

- Muon acceleration with Scaling FFAG
  - Energy P=5-10GeV
  - Bmax < 2T (Iron magnet :NC or super ferric)
  - Orbit excursion <0.5m
  - Beam size : full aperture@10GeV < ~15cm
  - RF frequency : 200-400MHz
  - RF field : ~30MV/m, Energy gain/m >1.5MeV/m

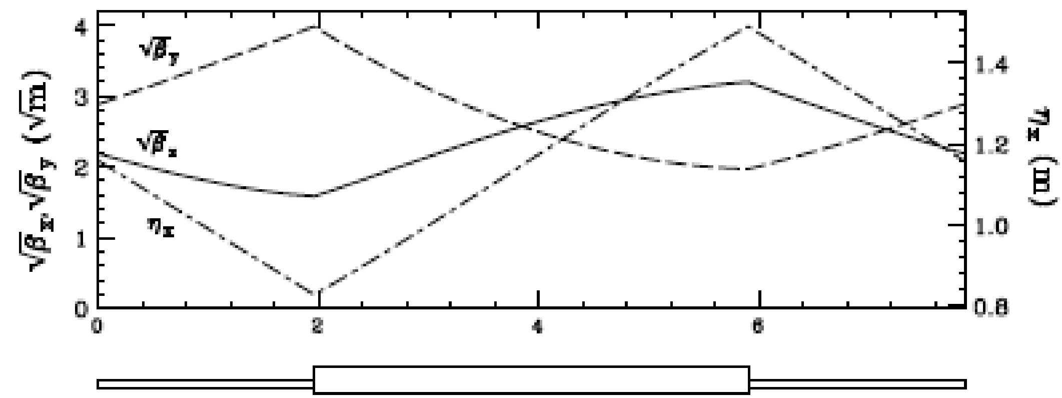
5-10GeV scaling FFAG spiral sector

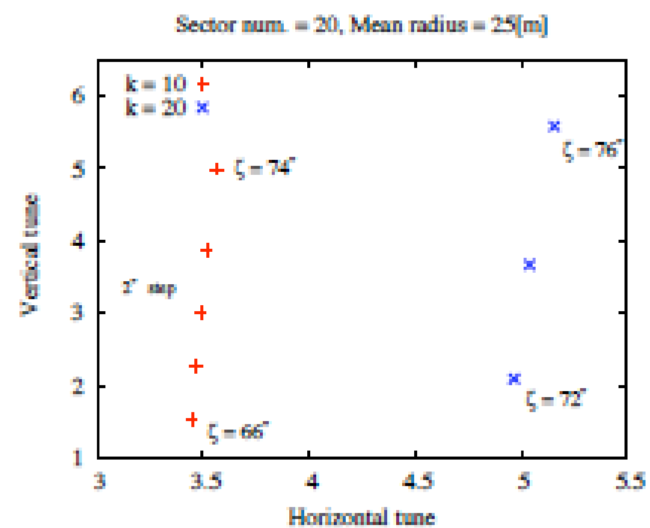
- Ring parameters
  - r=25m
  - N=20cells
  - spiral angle: 72degree
  - Bmax ~2.6T (p.f.=0.5)
  - k=20
  - Orbit excursion

    83.9cm
  - Beam size(half) at 10GeV

    H: 3.8cm+1.2cm=5.0cm,V=5.0cm @s.s.

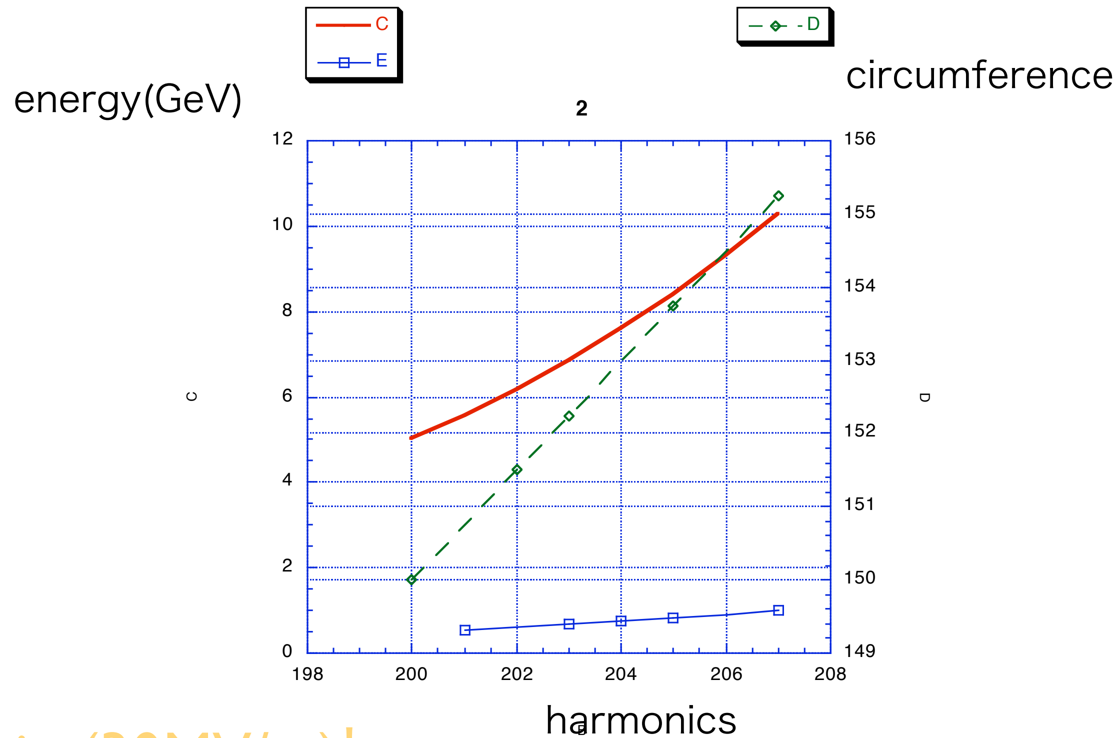
    H: 5.2cm+1.5cm=6.7cm,V=6.9cm @magnet

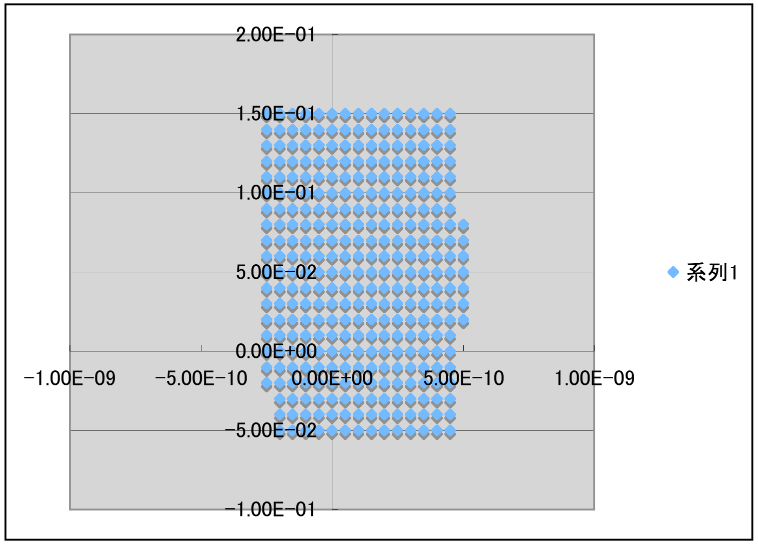


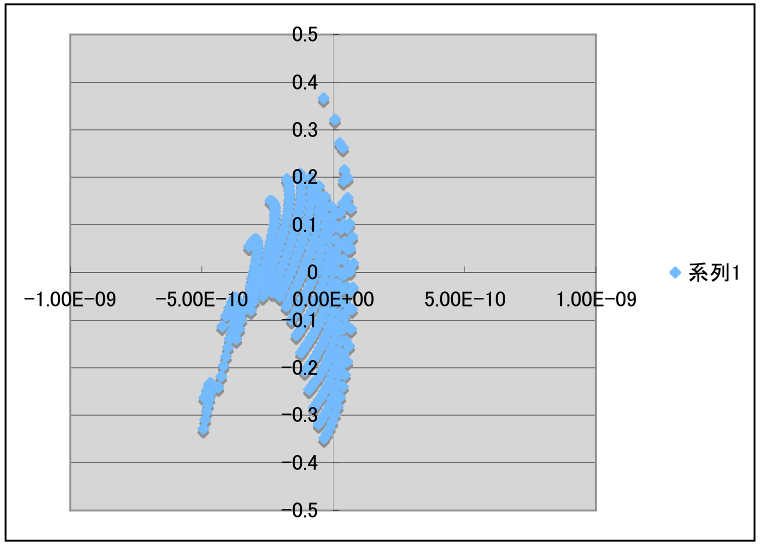


Spiral FFAG 5-10GeV

- rf parameters
  - h=200
  - f=400MHz
  - fai_s=2π/3
  - 4-cell cavity 45MV/cavity(30MV/m)!







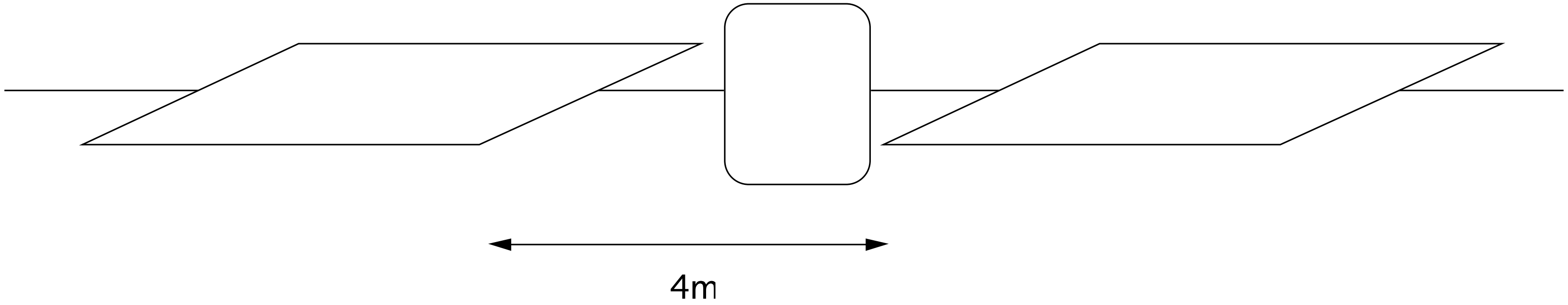
Spiral FFAG 5-10GeV

- ## Lattice
  - almost satisfied but need more optimaization

    k-value:lower, Bmax:lower, packing factor:smaller, circumference:larger
- ## HNJ acceleration
  - seems to have enough acceptance
  - frequency of rf cavity

    400MHz --->200MHz (depend on lattice design)
  - No. of turns: should be larger   >10 turns (now 7turns)

    reduce rf voltage   30MV/m --> 20MV/m
  - Increase ring radius!



5-10GeV Scaling FFAG radial sector - FDF lattice

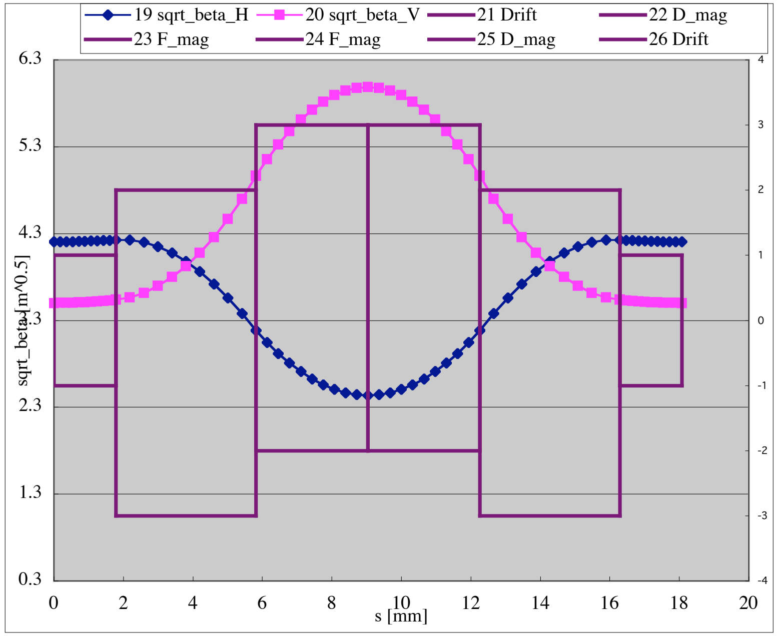
- Ring parameters
  - r=200m
  - N=70cells
  - Bmax ~F:1.6T ,D:1.5T
  - k=150
  - Orbit excursion

    62cm
  - Beam size(half) at 10GeV

    H: 7.4cm+1.2cm=8.6cm,V:6.1cm @s.s.

    H: 7.4cm+1.2cm=8.6cm,V:6.1cm @F-magnet

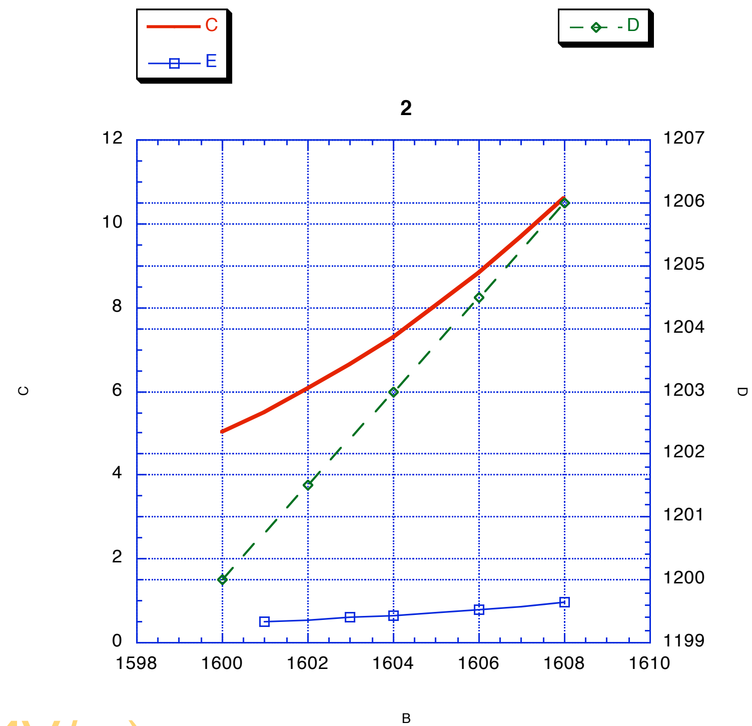
    H: 4.0cm+1.2cm=5.2cm,V:7.0cm @D-magnet

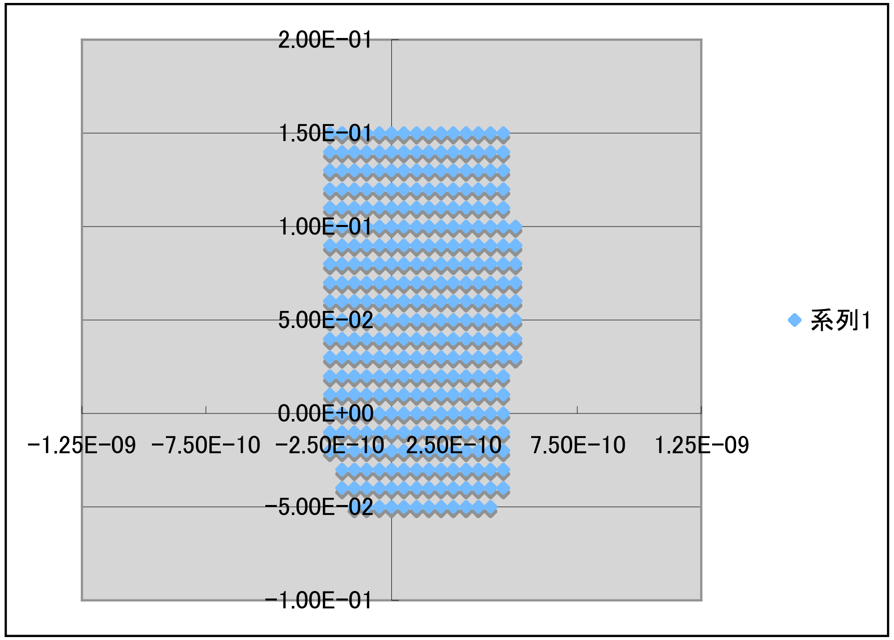


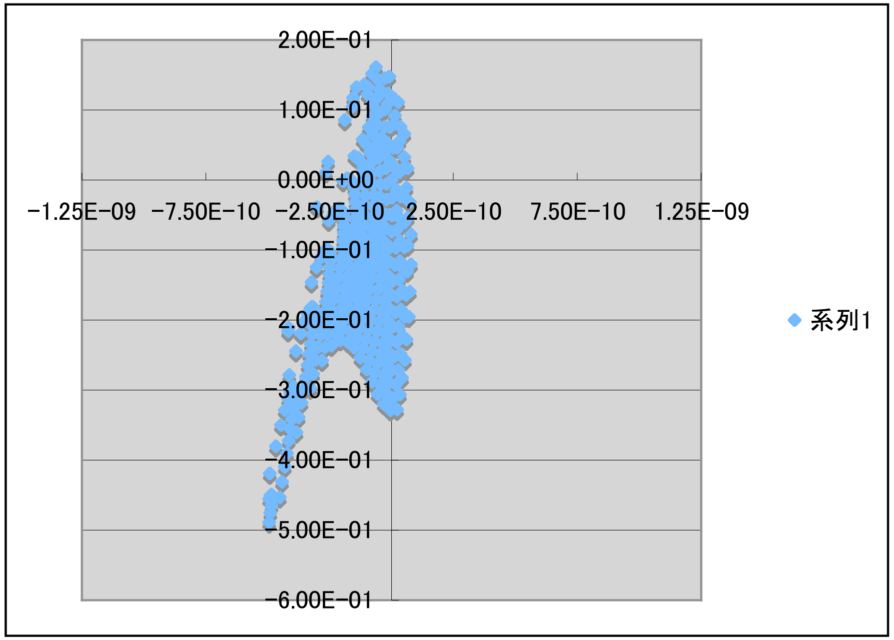
FD ratio @ BL	F/D	1.3	
number of cell	N	70	
field index (k-value)	k	150	
opening angle of F with respect to machine center	β_F	0.02	[rad]
opening angle of D/2 with respect to machine cente	β_D	0.016	[rad]
bending angle of F	θ_F	11.14285714	[deg]
bending angle of D/2	θ_D	8.571428571	[deg]
orbit radius of D center (@ ext.)	r0	200.31	[m]
kinetic energy (@ ext.)	Ek	9.10E+09	[eV]
momentum (@ ext.)	p	9.994E+09	[eV/c]
	Bρ	3.331E+01	[Tm]
B field of F(@ ext.)	B_F	1.606E+00	[T]
B field of D (@ ext.)	B_D	1.547E+00	[T]
drift length (half)	L_s	1.783871159	[m]
drift length (full size)		3.567742318	[m]
bending radius of F	ρ_F	20.73903955	[m]
bendign radius of D	ρ_D	21.531353	[m]
path length of F/2	L_F	4.03331898	[m]
path length of F		8.066637961	[m]
path length of D	L_D	3.221082877	[m]
pulse drift length (half) [for race track FFAG]	L_pulse	0	[m]
total drift length (half)	L_ts	1.783871159	[m]
betatron tune	ν_h	**18.4424482**	
	ν_v	**11.19264951**	
phase advance per cell	ϕ_h	**94.84687648**	[deg]

Radial FFAG 5-10GeV

- rf parameters
  - h=1600
  - f=400MHz
  - fai_s=2 π /3
  - 1-cell cavity 15MV/cavity(15MV/m)







Summary

- Scaling FFAG with HNJ acceleration for Muon 5-10GeV (10-20GeV) seems promising
  - B field <2.6T : Superferric or NC magnet
  - Ring size

    Spiral: r=25m

    Radial(FDF): r=200m

    Spiral is rather compact.
  - rf frequency : 400MHz need optimization for lower frequency: ex. increase of ring size
  - Longitudinal acceptance : large enough
- Flight time problem of non-scaling FFAG may be cured by scaling FFAG
- Subjects
  - asymmetric rf cavity

Proof-of-principle Experiment of HNJ Acceleration

- Proton FFAG @KURRI (20-150MeV)
- HNJ rf cavity
  - f=300MHz
  - 1MV(peak) ---> 30 turns (20-40MeV)
  - P=100kW(peak)
- β<1(single cavity)

1- Harmonic Jump Acceleration

Main Characteristics

Particles	Proton	
Injection Energy [MeV]	20	Initial harmonic number = 141
RF frequency [MHz]	300	
Injection radius [m]	4.55	
Extraction Energy [MeV]	150	
Extraction radius [m]	4.16	
K value	7.6	
Δh per turn	-1	(Below transition)
Number of turn	79	
Synchronous phase [deg]	75