Computation of transient gravity signals with earth normal modes : overview and open problems.

E-GRAAL : Earthquake Gravity Alerts

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1. Computation of the Earth normal modes

2. Synthetic seismograms

3. Response of the recording instrument

The eigenfrequencies ω and associated eigenfunctions $\boldsymbol{s}$ of a non-rotating, hydrostatic Earth are found by solving :

- The momentum equation

$$\rho^0 \, \frac{d^2\boldsymbol{s}}{dt^2} \;=\; \boldsymbol{\nabla}.(\boldsymbol{\sigma}^{\boldsymbol{0}} + \boldsymbol{\sigma}^{\boldsymbol{E_1}}) + (\rho^0 + \rho^{E_1})\boldsymbol{\nabla}(\phi^0 + \phi^{E_1}) \quad (1)$$

$$-\omega^2\rho^0\boldsymbol{s} - \boldsymbol{\nabla}.\boldsymbol{\sigma}^{\boldsymbol{PK_1}} + \rho^0\boldsymbol{\nabla}\phi^{E_1} + \rho^0\boldsymbol{s}.\boldsymbol{\nabla}\boldsymbol{\nabla}\phi 0 = 0 \quad (2)$$

- The Poisson equation

$$\nabla^2\phi \;=\; -4\pi G\nabla.(\rho\boldsymbol{s}) \quad (3)$$

- + some boundary conditions

Eigensolutions of equation $(2) + (3)$ of the form

$$\boldsymbol{s} = U(r)\boldsymbol{P}_{lm} + V(r)\boldsymbol{B}_{lm} + W(r)\boldsymbol{C}_{lm} \tag{4}$$

$$\phi = P(r)\mathcal{Y}_{lm} \tag{5}$$

with

- $\boldsymbol{P}_{lm} = \mathcal{Y}_{lm}(\Theta, \Phi)\boldsymbol{r}$
- $\boldsymbol{B}_{lm} = k\boldsymbol{\nabla}\mathcal{Y}_{lm}(\Theta, \Phi)$ and
- $\boldsymbol{C}_{lm} = k(\boldsymbol{r}x\boldsymbol{\nabla})\mathcal{Y}_{lm}(\Theta, \Phi)$.

4 equations of second order $\rightarrow$ equivalent system of coupled scalar equations

$$\begin{aligned}
\dot{U} &= -2(\kappa + \frac{4}{3}\mu)^{-1}(\kappa - \frac{2}{3}\mu)r^{-1}U + k(\kappa + \frac{4}{3}\mu)^{-1}(\kappa - \frac{2}{3}\mu)r^{-1}V + (\kappa + \frac{4}{3}\mu)^{-1}R \\
\dot{V} &= -kr^{-1}U + r^{-1}V + \mu^{-1}S \\
\dot{P} &= -4\pi G\rho U - (l+1)r^{-1}P + B \\
\dot{R} &= [-\omega^2\rho - 4\rho g r^{-1} + 12\kappa\mu(\kappa + \frac{4}{3}\mu)^{-1}r^{-2}]U + [k\rho g r^{-1} - 6k\kappa\mu(\kappa + \frac{4}{3}\mu)^{-1}r^{-2}]V \\
&\quad -4\mu(\kappa + \frac{4}{3}\mu)^{-1}r^{-1}R + kr^{-1}S - (l+1)\rho r^{-1}P + \rho B \\
\dot{S} &= [k\rho g r^{-1} - 6k\kappa\mu(\kappa + \frac{4}{3}\mu)^{-1}r^{-2}]U - [\omega^2\rho + 2\mu r^{-2} - 4k^2\mu(\kappa + \frac{1}{3}\mu)(\kappa + \frac{4}{3}\mu)^{-1}r^{-2}]V \\
&\quad -k(\kappa - \frac{2}{3}\mu)(\kappa + \frac{4}{3}\mu)^{-1}r^{-1}R - 3r^{-1}S + k\rho r^{-1}P \\
\dot{B} &= -4\pi G(l+1)\rho r^{-1}U + 4\pi Gk\rho r^{-1}V + (l-1)r^{-1}B
\end{aligned} \tag{6}$$

$$\begin{aligned}
\dot{W} &= r^{-1}W + \mu^{-1}T \\
\dot{T} &= [-\omega^2\rho + (k^2 - 2)\mu r^{-2}]W - 3r^{-1}T
\end{aligned} \tag{7}$$

Numerical integration with the normal mode code MINEOS.

The kinematic response of our model to a linear source function is given by

- for $t < t_{rupt}$:

$$\boldsymbol{s} = \sum_{modes} \frac{1}{\nu^2} \mathcal{A} \left[\frac{t}{t_{rupt}} + \frac{1}{i\nu t_{rupt}} \left(1 - e^{-\gamma t} e^{i\omega t}\right) \right] \tag{8}$$

$$\boldsymbol{v} = \sum_{modes} \frac{1}{\nu^2} \frac{1}{t_{rupt}} \mathcal{A} \left(1 - e^{-\gamma t} e^{i\omega t}\right) \tag{9}$$

$$\boldsymbol{a} = \sum_{modes} \frac{-i}{\nu} \frac{1}{t_{rupt}} \mathcal{A} \left(e^{-\gamma t} e^{i\omega t}\right) \tag{10}$$

with $\nu = \omega + i\gamma$ is the complex angular frequency of the mode, ω the real frequency, and $\gamma = \frac{\omega}{2Q}$ the decay rate.

- for $t > t_{rupt}$:

$$\boldsymbol{s} = \sum_{modes} \frac{1}{\nu^2} \mathcal{A} \left[1 - e^{-\gamma t} e^{i \omega t} \right] \tag{11}$$

$$\boldsymbol{v} = \sum_{modes} \frac{-i}{\nu} \mathcal{A} \, e^{-\gamma t} e^{i \omega t} \tag{12}$$

$$\boldsymbol{a} = \sum_{modes} \mathcal{A} \, e^{-\gamma t} e^{i \omega t} \tag{13}$$

$\mathcal{A}$ contains U(r), V(r) ... and informations on the source and the radiation pattern.

We want to simulate the response of an idealized seismometer !
= mass contained in a housing attached to the Earth surface.

Electro-mechanical restoring force $\boldsymbol{F}$

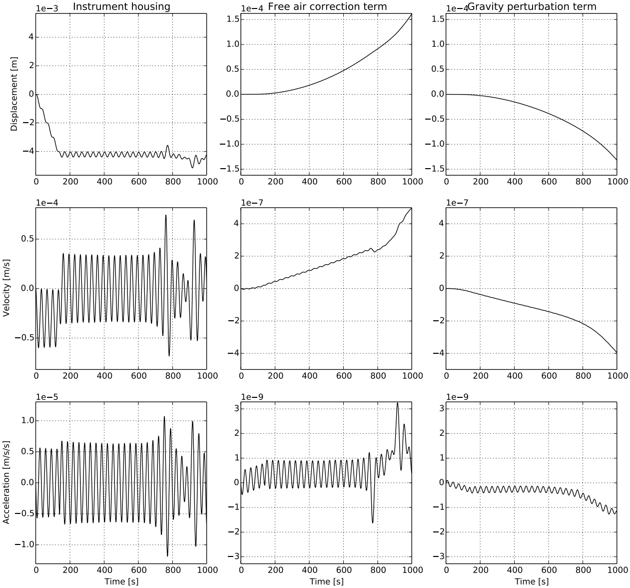
$$\boldsymbol{F} = \delta_t^2 \boldsymbol{s} + \boldsymbol{s}.\boldsymbol{\nabla}\boldsymbol{\nabla}\phi^0 + \boldsymbol{\nabla}\phi^{E_1} \tag{14}$$

- $\delta_t^2 \boldsymbol{s}$: inertial acceleration of the ground,
- $\boldsymbol{s}.\boldsymbol{\nabla}\boldsymbol{\nabla}\phi^0$: free-air contribution,
- $\boldsymbol{\nabla}\phi^{E_1}$: gravity potential perturbation contribution.

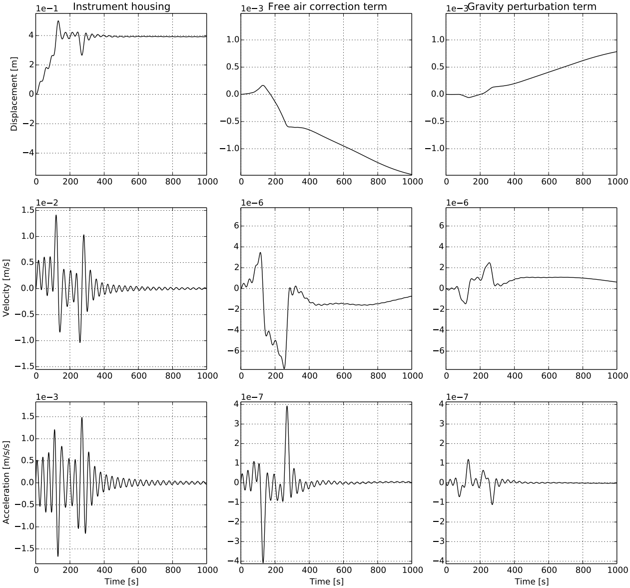
Gravitationally modified eigenfunctions, to be inserted into $\boldsymbol{\mathcal{A}}$:

$$\begin{aligned} U(r) \rightarrow \quad U_\star(r) &= U(r) + U_{free} + U_{pot} \\ &= U(r) + \omega^{-2} \left[\; 2ga^{-1}U(r) \; + \; (l+1)\, a^{-1}P(r) \; \right] \\ V(r) \rightarrow \quad V_\star(r) &= V(r) + V_{tilt} + U_{pot} \\ &= V(r) - \omega^{-2} \left[\; kga^{-1}U(r) \; + \; ka^{-1}P(r) \; \right] \end{aligned} \tag{15}$$

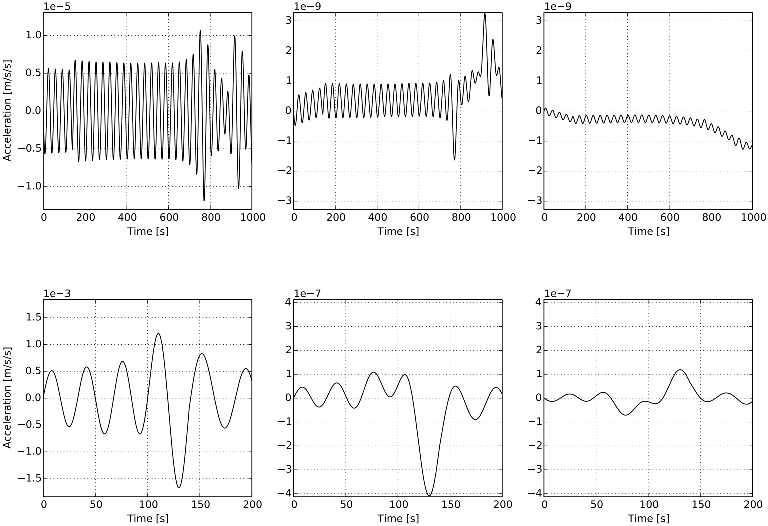
For high-frequency modes, $U_\star \sim U$ and $V_\star \sim V$.



Synthetics computed at the Virgo station (Italy) after the Tohoku event.



Synthetics computed at the Kamioka observatory (Japan) after the Tohoku event.



Acceleration response at the Virgo station (Italy) (top) Kamioka observatory (Japan) (bottom) after the Tohoku event.

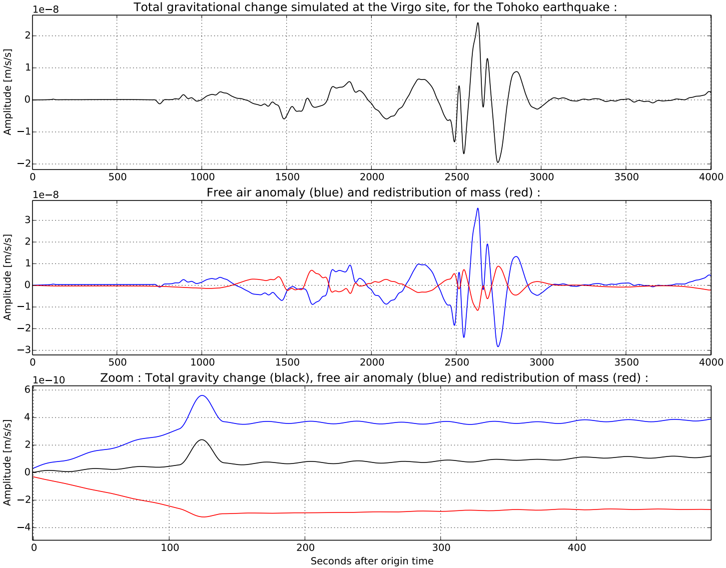
Ongoing problems :

- Ringing phenomenon : truncation in the mode summation. $\rightarrow$ need of a more complete catalog.
- What happens if we shut down the self gravitation in our model ?
- Consistency with analytical models ?

Perspectives :

- Comparison with David Al Attar's code.

Thank you for your attention.


Total gravitational change simulated at the Virgo site, for the Tohoko earthquake :
1e−8
Amplitude [m/s/s]
2
1
0
−1
−2
0
500
1000
1500
2000
2500
3000
3500
4000
Free air anomaly (blue) and redistribution of mass (red) :
1e−8
Amplitude [m/s/s]
3
2
1
0
−1
−2
−3
0
500
1000
1500
2000
2500
3000
3500
4000
Zoom : Total gravity change (black), free air anomaly (blue) and redistribution of mass (red) :
1e−10
Amplitude [m/s/s]
6
4
2
0
−2
−4
0
100
200
300
400
Seconds after origin time