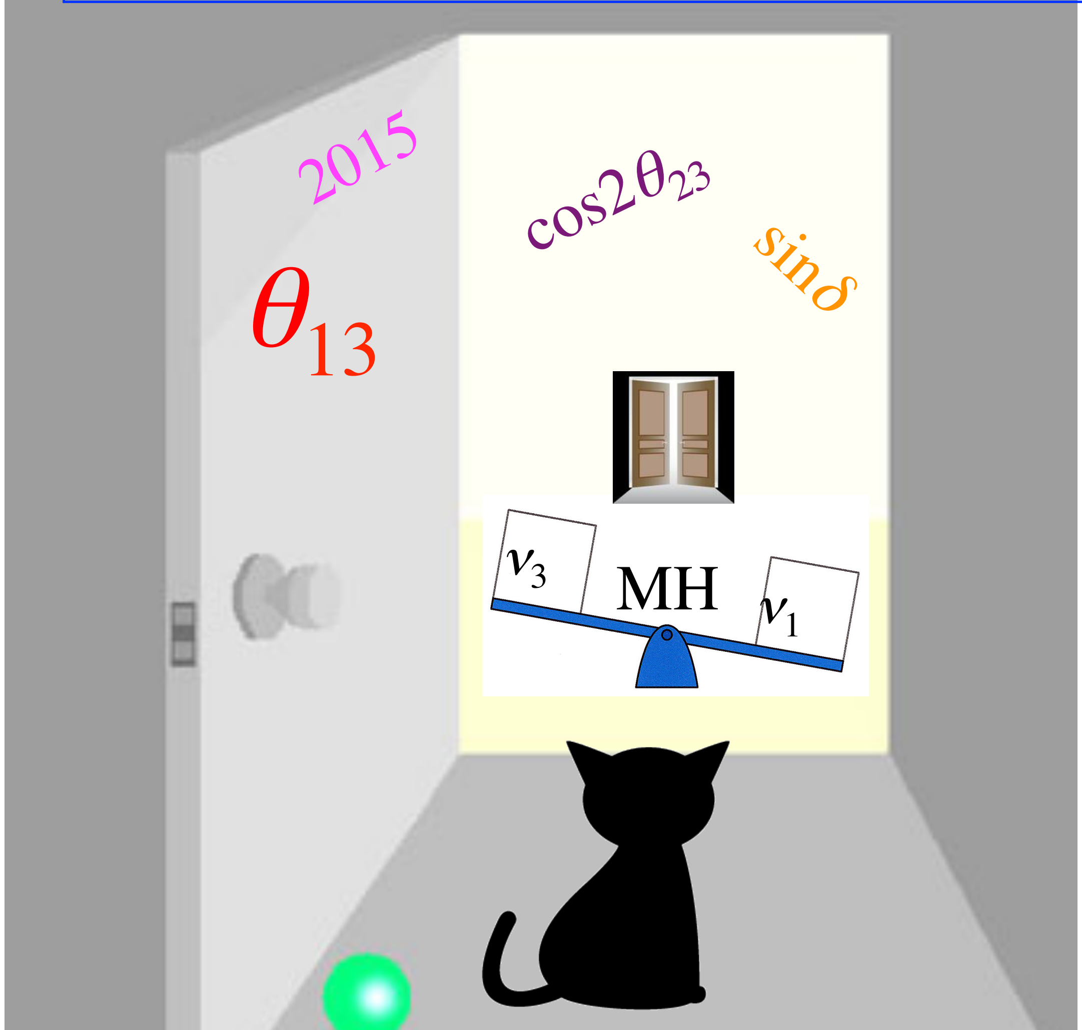
Neutrino Oscillations

Where are we now and where to go?



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02/04/2015

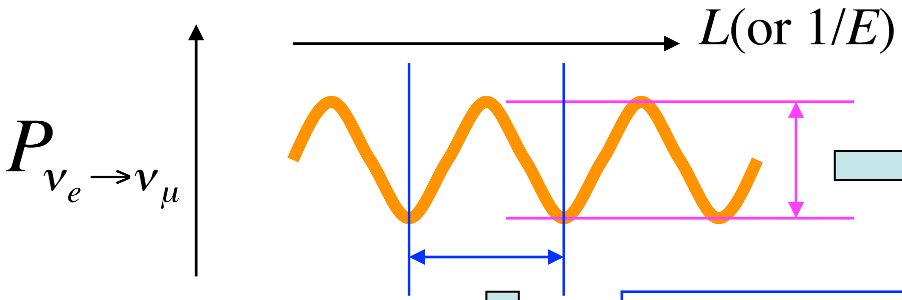
Flow of the talk

* What is neutrino oscillation?
* Why we measure neutrino oscillation?
* What we know now?
* What we need to know in the future?
* How we can make it?
* Summary

Neutrino Oscillation

=Phenomenon for ν to Changes its Flavor Periodically

$$P(\nu_\mu \rightarrow \nu_e) = \sin^2 2\theta \sin^2 \frac{\Delta m^2}{4E} L$$



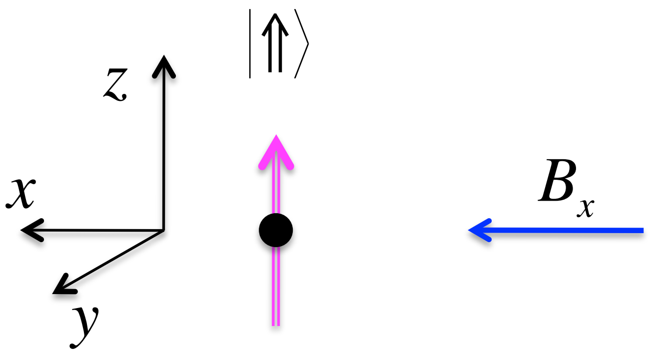
Amplitude ==> $\sin^2 2\theta$

Frequency => $\Delta m^2 = \left| m_2^2 - m_1^2 \right|$

$$\left.\begin{array}{c}\Delta m^2 \\ \theta\end{array}\right\}$$ ➔ Information of transition amplitudes of neutrinos

N.O. and Spin Precession

Analogy:
Spin under Magnetic field



* Spin wave function:

$$|\psi(t)\rangle = \alpha(t)|\Uparrow\rangle + \beta(t)|\Downarrow\rangle = \begin{pmatrix}\alpha\\ \beta\end{pmatrix}$$

* Pauli equation:

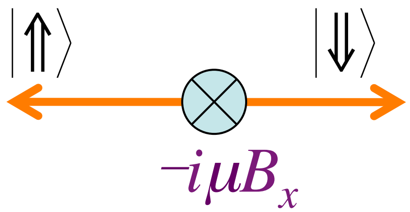
$$\dot{\psi} = -i\mu\left(\vec{B}\cdot\vec{\sigma}\right)\psi$$

* For $\vec{B} = (B_x, 0, 0)$

$$\begin{pmatrix}\dot{\alpha}\\ \dot{\beta}\end{pmatrix} = -i\mu B_x \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = -i\mu B_x \begin{pmatrix}\beta\\ \alpha\end{pmatrix}$$

➔ The Magnetic field transforms
|⇧> ←→ |⇩>

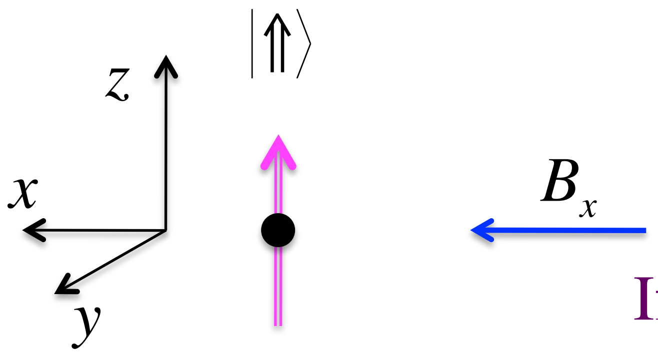
$$\begin{cases}\dot{\alpha} = -i\mu B_x \beta\\ \dot{\beta} = -i\mu B_x \alpha\end{cases}$$



Transition amplitude = $-i\mu B_x$

N.O. and Spin Precession

Analogy:
Spin under Magnetic field



General Solution:

$$\ddot{\alpha} = -i\mu B_x \dot{\beta} = -(\mu B_x)^2 \alpha$$

$$\Rightarrow \begin{cases} \alpha(t) = C_+ e^{-i\mu B_x t} + C_- e^{i\mu B_x t} \\ \beta(t) = C_+ e^{-i\mu B_x t} - C_- e^{i\mu B_x t} \end{cases}$$

If we start from $|\psi(0)\rangle = |\Uparrow\rangle$, the initial condition

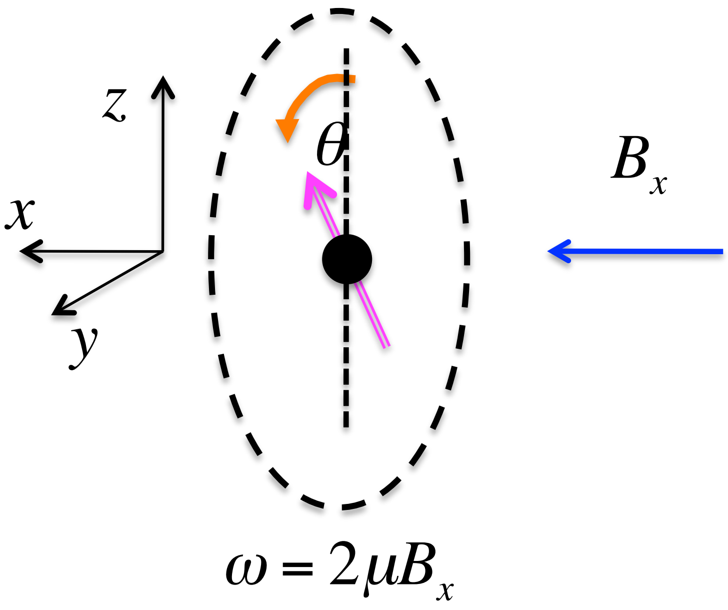
$$\begin{cases} \alpha(0) = 1 = C_+ + C_- \\ \beta(0) = 0 = C_+ - C_- \end{cases} \Rightarrow \quad C_+ = C_- = \frac{1}{2}$$

$$\begin{cases} \alpha(t) = \frac{1}{2}\left(e^{-i\mu B_x t} + e^{i\mu B_x t}\right) = \cos(\mu B_x t) \\ \beta(t) = \frac{1}{2}\left(e^{-i\mu B_x t} - e^{i\mu B_x t}\right) = -i\sin(\mu B_x t) \end{cases}$$

N.O. and Spin Precession

The wave function becomes

$$|\psi(t)\rangle = \cos(\mu B_x t)|\Uparrow\rangle - i\sin(\mu B_x t)|\Downarrow\rangle = |\Uparrow(\theta = 2\mu B_x t)\rangle_{yz}$$



The magnetic field makes the spin precession in z-y plane.
The probability to be $|\Downarrow\rangle$ state at time t is

$$P_{\Uparrow\to\Downarrow}(t) = \left|\langle\Downarrow|\psi(t)\rangle\right|^2 = \underline{\sin^2(\mu B_x t)}$$

Oscillation

➔ Neutrino Oscillation can be considered as a precession in flavor space.

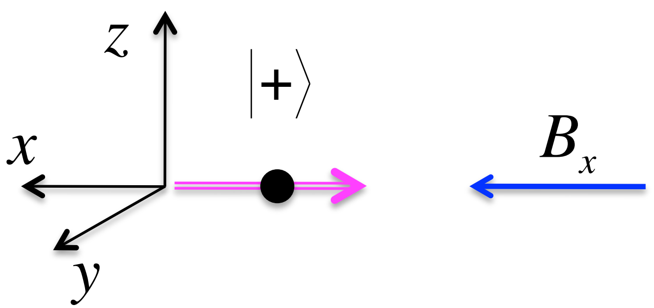
N.O. and Spin Precession

Again, from general solution, $\begin{cases} \alpha(t) = C_+ e^{-i\mu B_x t} + C_- e^{i\mu B_x t} \\ \beta(t) = C_+ e^{-i\mu B_x t} - C_- e^{i\mu B_x t} \end{cases}$

If the initial condition is

$$C_+ = \frac{1}{\sqrt{2}}, \quad C_- = 0 \quad \Rightarrow \quad \begin{cases} \alpha(t) = e^{-i\mu B_x t} / \sqrt{2} \\ \beta(t) = e^{-i\mu B_x t} / \sqrt{2} \end{cases}$$

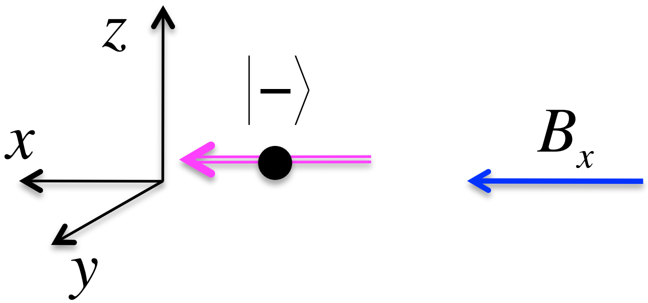
$$\left|\psi_+(t)\right\rangle = \frac{\left|\Uparrow\right\rangle + \left|\Downarrow\right\rangle}{\sqrt{2}} e^{-i\mu B_x t} \equiv \left|+\right\rangle e^{-iE_+ t}$$



$\left|+\right\rangle$ is energy eigenstate with energy

$$E_+ = \mu B_x$$

N.O. and Spin Precession



Another energy eigenstate

$$|\psi_-(t)\rangle = \frac{|\Uparrow\rangle - |\Downarrow\rangle}{\sqrt{2}} e^{i\mu B_x t} \equiv |-\rangle e^{-iE_- t}$$

$$E_- = -\mu B_x$$

Energy eigenstate is a mixture of original states

$$\begin{pmatrix} |+\rangle \\ |-\rangle \end{pmatrix} = \underline{\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}} \begin{pmatrix} |\Uparrow\rangle \\ |\Downarrow\rangle \end{pmatrix}$$

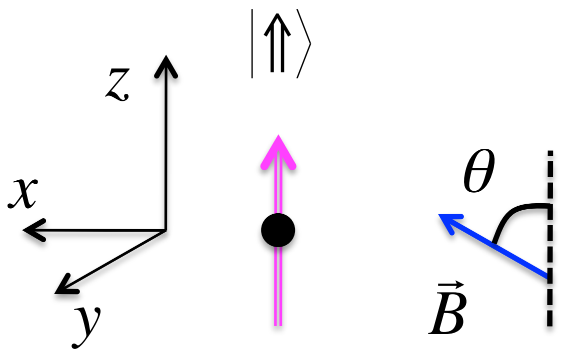
Mixing matrix

Relation between angular velocity of the precession and energy

$$\omega = 2\mu B_x = E_+ - E_- = \Delta E$$

N.O. and Spin Precession

Analogy:
Spin under Magnetic field

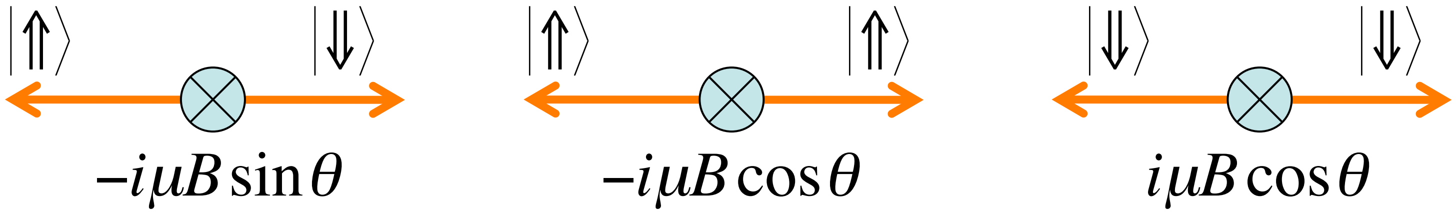


If the magnetic field is inclined,

* For $\vec{B} = (B_x, 0, B_z) = B(\sin\theta, 0, \cos\theta)$

* Pauli Eq.

$$\begin{pmatrix} \dot{\alpha} \\ \dot{\beta} \end{pmatrix} = -i\mu B \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

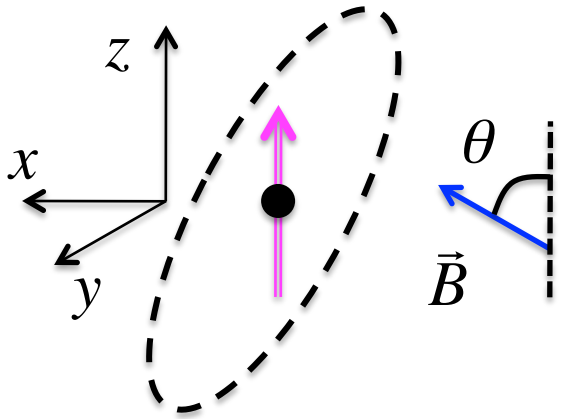


* Energy eigenstates

$$\begin{pmatrix} |+\rangle \\ |-\rangle \end{pmatrix} = \begin{pmatrix} \sin(\theta/2) & \cos(\theta/2) \\ \cos(\theta/2) & -\sin(\theta/2) \end{pmatrix} \begin{pmatrix} |\Uparrow\rangle \\ |\Downarrow\rangle \end{pmatrix} \qquad E_\pm = \pm\mu B$$

N.O. and Spin Precession

Analogy:
Spin under Magnetic field



* Wave function at time t

$$\left|\psi(t)\right\rangle = C_+\left|+\right\rangle e^{-i\mu Bt} + C_-\left|-\right\rangle e^{i\mu Bt}$$

If the initial condition is

$$\left|\psi(0)\right\rangle = C_+\left|+\right\rangle + C_-\left|-\right\rangle = \left|\Uparrow\right\rangle$$

$$\rightarrow \quad C_+ = \sin(\theta/2), \quad C_- = \cos(\theta/2)$$

The wave function is determined to be

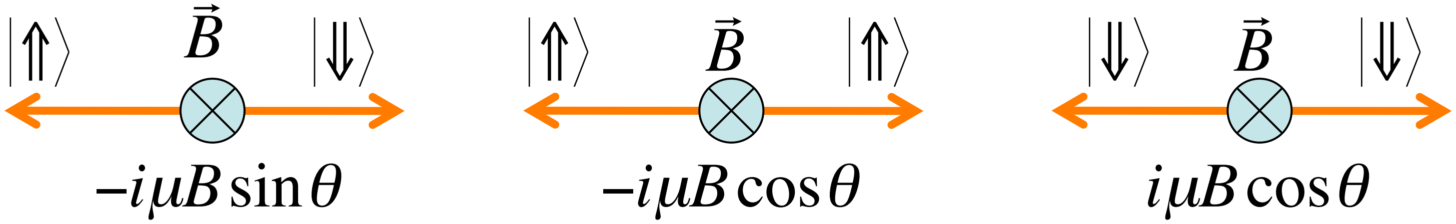
$$\left|\psi(t)\right\rangle = \left(\cos(\mu Bt) + i\sin(\mu Bt)\cos\theta\right)\left|\Uparrow\right\rangle - i\sin\theta\sin(\mu Bt)\left|\Downarrow\right\rangle$$

Spin transition probability is

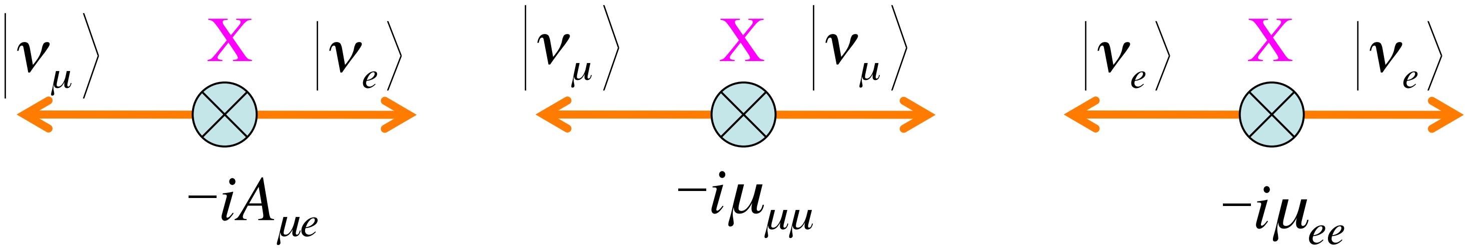
$$P_{\Uparrow\to\Downarrow}(t) = \left|\left\langle\Downarrow\middle|\psi(t)\right\rangle\right|^2 = \underline{\sin^2\theta}\,\sin^2\left(\frac{\Delta E}{2}t\right)$$

This comes from the direction (ratio) of the transition amplitudes

Turn to ν oscillation

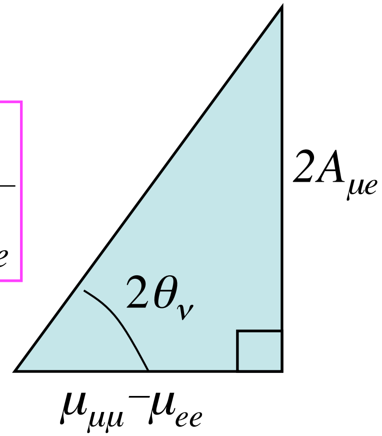


We assume some effect X generates transition amplitude between ν flavors

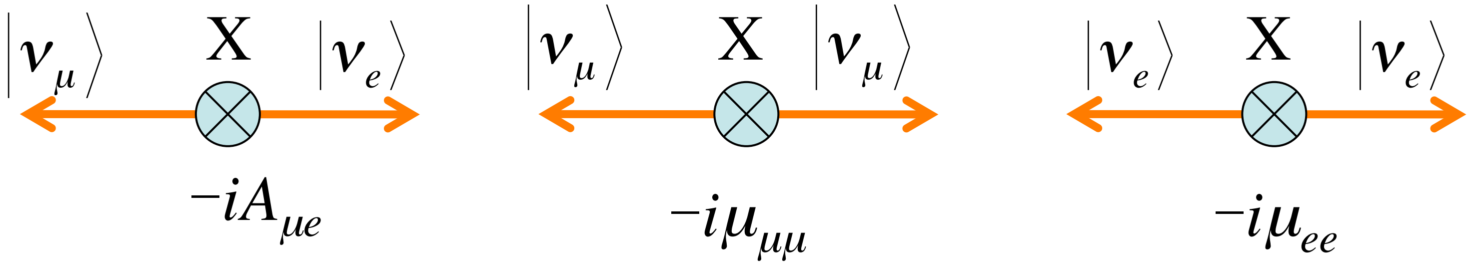


From the Analogy, Mass eignenstate is

$$\begin{pmatrix} \nu_- \\ \nu_+ \end{pmatrix} = \begin{pmatrix} \cos\theta_\nu & \sin\theta_\nu \\ -\sin\theta_\nu & \cos\theta_\nu \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} \qquad \boxed{\tan 2\theta_\nu \equiv \frac{2A_{\mu e}}{\mu_{\mu\mu} - \mu_{ee}}}$$



Turn to ν oscillation



Masses

$$\boxed{\begin{cases} m_- = E_- = \dfrac{\mu_{\mu\mu}+\mu_{ee}}{2} - \dfrac{1}{2}\sqrt{\left(\mu_{\mu\mu}-\mu_{ee}\right)^2 + 4A_{\mu e}^2} \\ m_+ = E_+ = \dfrac{\mu_{\mu\mu}+\mu_{ee}}{2} + \dfrac{1}{2}\sqrt{\left(\mu_{\mu\mu}-\mu_{ee}\right)^2 + 4A_{\mu e}^2} \end{cases}}$$

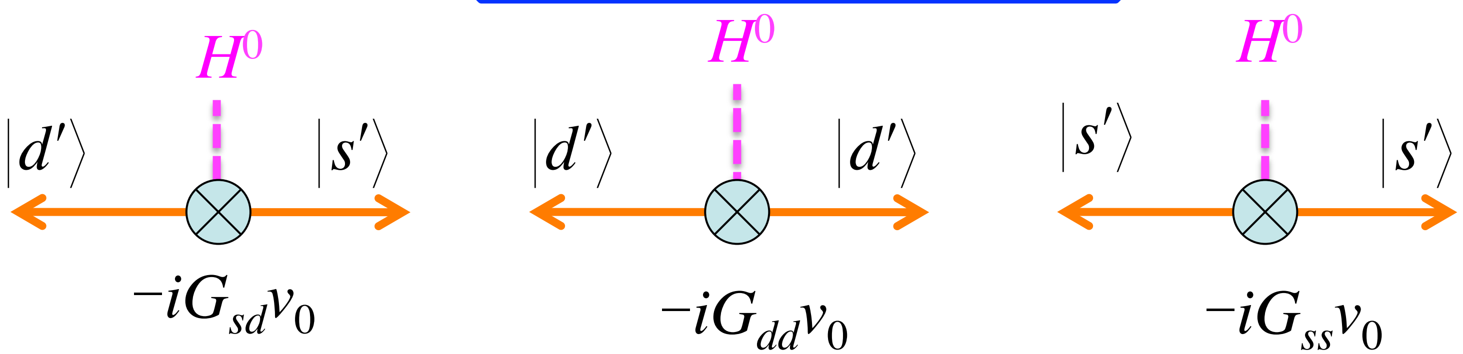
If the initial condition is $|\psi_\nu(0)\rangle = |\nu_\mu\rangle$, the wave function at *t* is

$$|\psi_\nu(t)\rangle = \left(\cos\left(\frac{\Delta m}{2}t\right) + i\sin\left(\frac{\Delta m}{2}t\right)\cos 2\theta_\nu\right)|\nu_\mu\rangle - i\sin 2\theta_\nu \sin\left(\frac{\Delta m}{2}t\right)|\nu_e\rangle$$

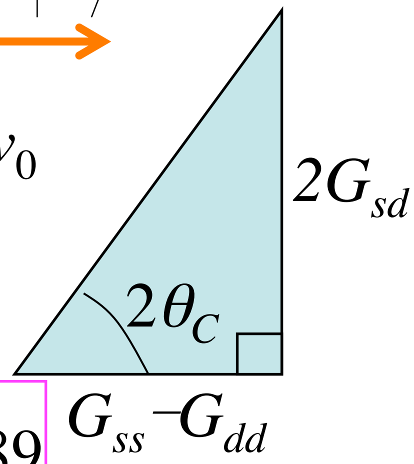
The flavor transition probability is $\boxed{P\left(\nu_\mu \to \nu_e; t\right) = \sin^2 2\theta_\nu \sin^2\left(\frac{\Delta m}{2}t\right)}$

N.O. of neutrino at rest

Quark oscillation



(v_0: Higgs vaccum expectation value ~ 246GeV)



Mass eignenstate

$$\begin{pmatrix} |d\rangle \\ |s\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} |d'\rangle \\ |s'\rangle \end{pmatrix}$$

$$\boxed{\tan 2\theta_C \equiv \frac{2G_{ds}}{G_{ss}-G_{dd}} = 0.489}$$

$$\boxed{m_{d/s} = \frac{v_0}{2}\left(G_{ss}+G_{dd} \mp \sqrt{(G_{ss}-G_{dd})^2 + 4G_{ds}^2}\right) \sim 5\,MeV/100\,MeV}$$

$$\boxed{\Rightarrow \quad G_{dd} = 4.0\times 10^{-5}, \quad G_{ss} = 3.9\times 10^{-4}, \quad G_{ds} = 8.5\times 10^{-5}}$$

Cabbibo angle is generated by the quark flavor transition

Oscillation length ~1/95MeV~0.5fm; too quick to observe

In this case, "Magnetic Field" = Higgs Field

Why we mesure ν oscillations?

There are many oscillations (Irrespective to it is observable or not)

- $K^0 \Leftrightarrow \overline{K^0}$, $B^0 \Leftrightarrow \overline{B^0}$ **Oscillation.** → CP violation
- spin precession by B (=$|\Uparrow\rangle \Leftrightarrow |\Downarrow\rangle$ oscillation) → Formation of Q.M.
- $|u\bar{u}\rangle \Leftrightarrow |d\bar{d}\rangle$ oscillation in π^0, η → Hadron mass, quark model, QCD,
- $d \Leftrightarrow s$ oscillation →Cabbibo angle, Higgs-quark coupling.
- $B \Leftrightarrow W_3$ oscillation → Weinberg angle, Higgs-GB coupling.

→ **We have learned a lot from these "Oscillations"**

We can expect to learn more from ν Oscillations;

$$\nu_\alpha \Leftrightarrow \nu_\beta$$

"Magnetic field-X" for the Neutrino Oscillation

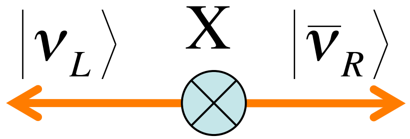
What causes X? (origin)

Why X is so small? (ν mass)

What is the ratio between off-diagonal to
the difference of diagonal amplitude? (mixing angle)

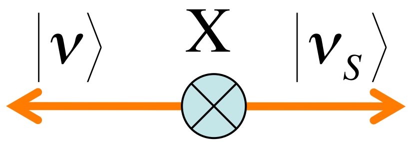
Is X complex number? (CP violation)

Can X change particle to antiparticle? (Majorana neutrino?)



?

Can X connect our ν and sterile ν? (Sterile ν)



?

To answer these question, we need to measure X.

Relativistic oscillation

In experimental condition, ν travels ultrarelativistically

Method−I (a simple and often used method)

$$P \propto \sin^2 \frac{m_2 - m_1}{2} t \xrightarrow{\text{Lorentz Boost}} \sin^2\left(\frac{E_2 - E_1}{2} t - \frac{p_2 - p_1}{2} x\right)$$

Assume $p_1 = p_2 = p$, then $E_i \sim p + \frac{m_i^2}{2p}$ and

$$\frac{1}{2}(\Delta E t - \Delta p x) = \frac{\Delta m^2}{4p} t = \frac{\Delta m^2}{4E} x$$

However, in actual experimental condition, neutrinos are produced in π-decay, μ-decay or β−decay and the neutrino momenta are different for different mass eigenstate

Method-II (more realistic case)

Assume ν is produced in the decay of particle X,

$$X \rightarrow \nu_{\pm} + Y$$

Where Y can be a single particle or multi particle system,
In this case, the energy and momentum of the neutrino is,

$$E_{\pm} \sim E_0\left(1-\frac{m_{\pm}^2}{2M_X E_0}\right), \quad p_{\pm} \sim E_0\left(1-\frac{1}{2}\left(1+\frac{E_0}{M_X}\right)\frac{m_{\pm}^2}{E_0^2}\right)$$

E_0: neutrino energy in case neutrino is massless

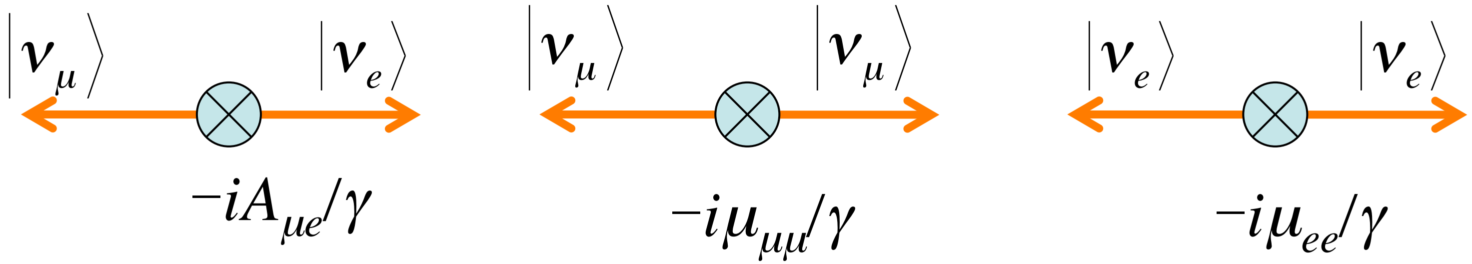
$$\frac{1}{2}\left(\Delta E t - \Delta p x\right) = \frac{\Delta m^2}{4E_0}x \qquad (x=t \text{ used})$$

A similar expression is obtained.

Method-III (general and useful)

$$e^{-imt} \xrightarrow{\text{Lorentz Boost}} e^{-i(Et-\vec{p}\vec{x})} \xrightarrow{\text{On the particle: } \vec{x}=(\vec{p}/E)t} e^{-i(m/\gamma)t}$$

Divide all the transition amplitude by γ

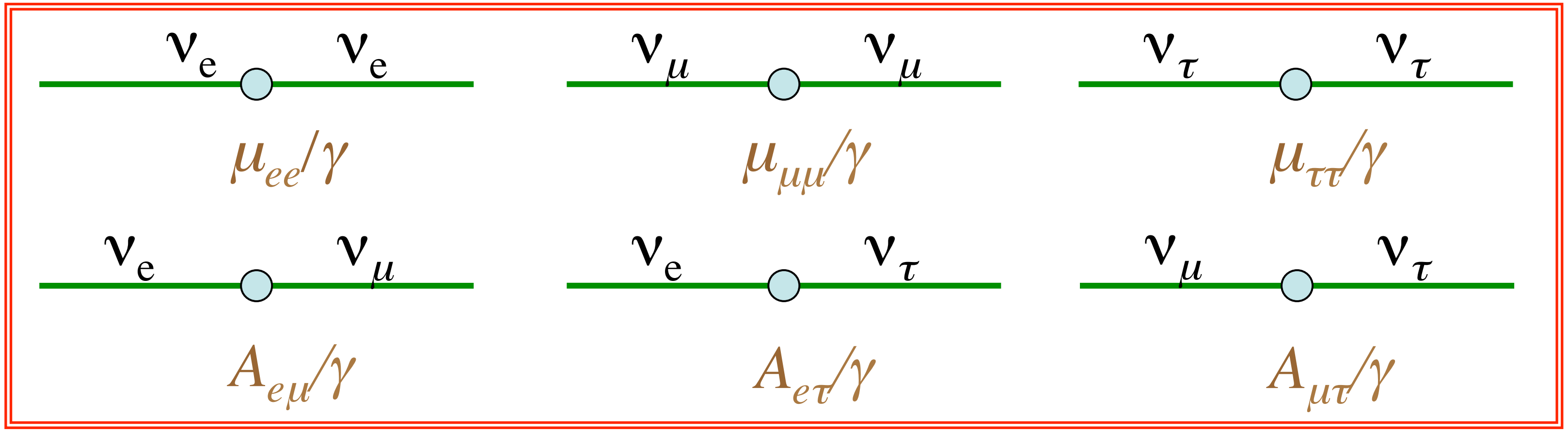


$$\boxed{\tan 2\theta_\nu \xrightarrow{L.B.} \frac{2(A_{\mu e}/\gamma)}{(\mu_{\mu\mu}/\gamma)-(\mu_{ee}/\gamma)} = \frac{2A_{\mu e}}{\mu_{\mu\mu}-\mu_{ee}} = \tan 2\theta_\nu}$$ No change

$$\frac{m_+ - m_-}{2} t \xrightarrow{L.B.} \frac{m_+ - m_-}{2\gamma\beta} L \sim \frac{m_+^2 - m_-^2}{4\overline{E}} L \equiv \frac{\Delta m^2 L}{4\overline{E}}$$

$$\gamma \sim \frac{\overline{E}}{\overline{m}} \sim \frac{2\overline{E}}{m_+ + m_-}$$ is used

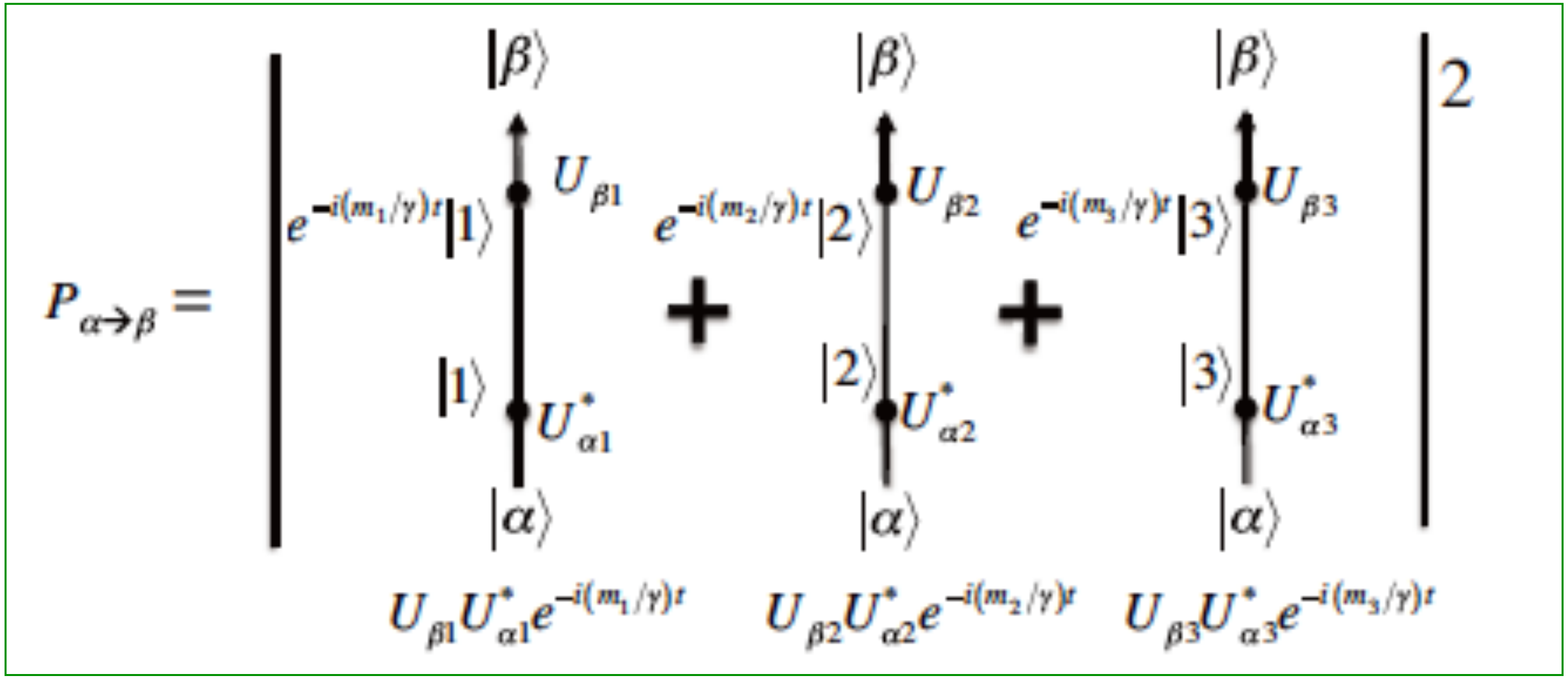
Relativistic 3 Flavor Neutrino Case



$A_{\alpha\beta}$ can be complex number

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

The oscillation probability can be obtained directly from the Feynman Diagram, after substituting $m_i \rightarrow m_i/\gamma$



$$P\left(\nu_\alpha \rightarrow \nu_\beta\right) = \left|U_{\beta 1}U^*_{\alpha 1}e^{-i(m_1/\gamma)t} + U_{\beta 2}U^*_{\alpha 2}e^{-i(m_2/\gamma)t} + U_{\beta 3}U^*_{\alpha 3}e^{-i(m_3/\gamma)t}\right|^2$$

$$= \delta_{\alpha\beta} - 4\sum_{i>j}\mathrm{Re}\left[\Omega^{\alpha\beta}_{ij}\right]\sin^2\Phi_{ij} \mp 2\sum_{i>j}\mathrm{Im}\left[\Omega^{\alpha\beta}_{ij}\right]\sin 2\Phi_{ij}$$

$$\Omega^{\alpha\beta}_{ij} \equiv U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j} \qquad \Phi_{ij} \equiv \frac{\Delta m^2_{ij}L}{4E_\nu}, \quad \Delta m^2_{ij} \equiv m^2_j - m^2_i$$

Transition Amplitudes can be Calculated

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4\sum_{i>j} \mathrm{Re}\left[\Omega_{ij}^{\alpha\beta}\right]\sin^2\Phi_{ij} \mp 2\sum_{i>j} \mathrm{Im}\left[\Omega_{ij}^{\alpha\beta}\right]\sin 2\Phi_{ij}$$

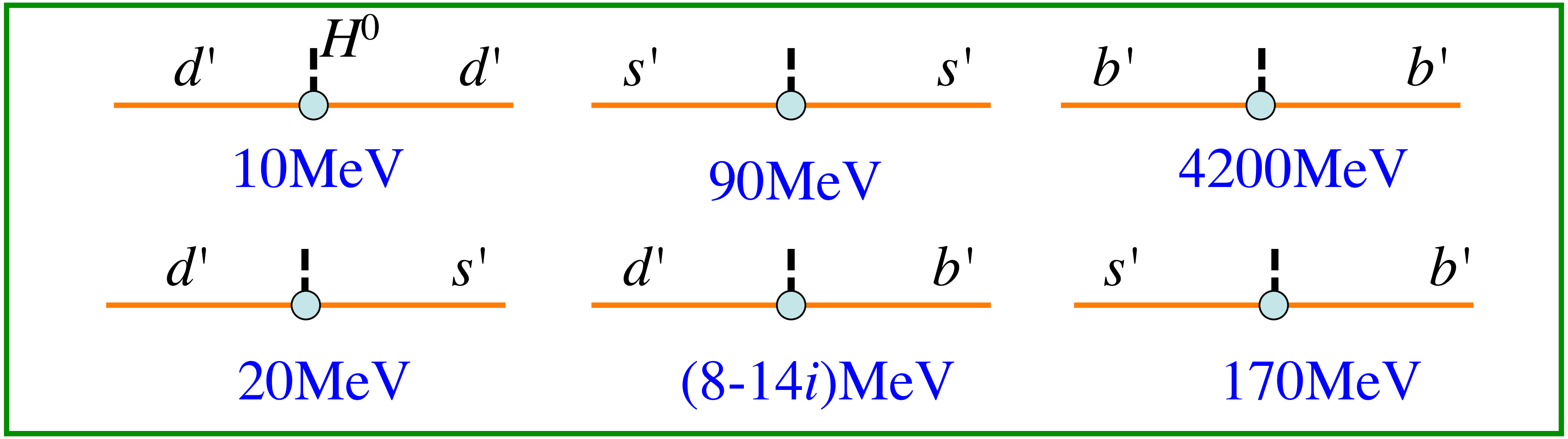
$$\Omega_{ij}^{\alpha\beta} \equiv \underline{\underline{U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*}} \qquad \Phi_{ij} \equiv \frac{\underline{\underline{\Delta m_{ij}^2}} L}{4E_\nu}, \quad \Delta m_{ij}^2 \equiv m_j^2 - m_i^2$$

$$\begin{pmatrix} m_{ee} & A_{e\mu} & A_{e\tau} \\ A_{e\mu}^* & m_{\mu\mu} & A_{\mu\tau} \\ A_{e\tau}^* & A_{\mu\tau}^* & m_{\tau\tau} \end{pmatrix} = \overbrace{\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}^\dagger \underbrace{\begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}} \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}}^{\text{Observable}}$$

Δm^2 & Direct mass measurement are necessary.
(➔ ν Oscillation & direct mass exp. are complementary)

Quark case

Quark Mass + CKM Mixing Matrix



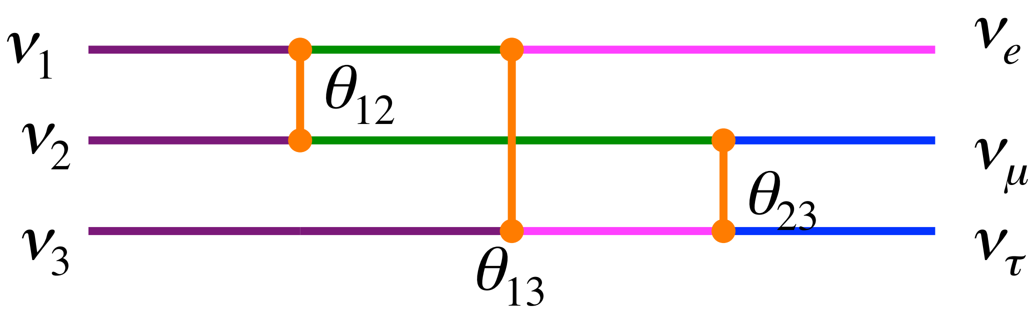
The Flavor Transition is understood to be caused by the Higgs-quark Yukawa coupling

We would like to do the same thing for neutrinos and study the origin of the neutrino flavor transition.

A Good Parametrisation of the Mixing Matrix

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$s_{ij} = \sin\theta_{ij}, c_{ij} = \cos\theta_{ij} \qquad \Delta m_{ij}^2 = m_i^2 - m_j^2$$



Correspondence between (1,2,3) and (e, μ, τ)

If mixing angles (θ_{ij}) are small,

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \sim \begin{pmatrix} 1 & s_{12} & s_{13}e^{-i\delta} \\ -s_{12} & 1 & s_{23} \\ -s_{13}e^{i\delta} & -s_{23} & 1 \end{pmatrix}$$

The transition amplitude becomes

$$\begin{pmatrix} \mu_{ee} & A_{\mu e}^{*} & A_{\tau e}^{*} \\ A_{\mu e} & \mu_{\mu\mu} & A_{\tau\mu}^{*} \\ A_{\tau e} & A_{\tau\mu} & \mu_{\tau\tau} \end{pmatrix} = U_{\nu} \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} U_{\nu}^{\dagger} \sim \begin{pmatrix} m_1 & \Delta m_{21} s_{12} & \Delta m_{31} s_{13} e^{-i\delta} \\ \Delta m_{21} s_{12} & m_2 & \Delta m_{32} s_{23} \\ \Delta m_{31} s_{13} e^{i\delta} & \Delta m_{32} s_{23} & m_3 \end{pmatrix}$$

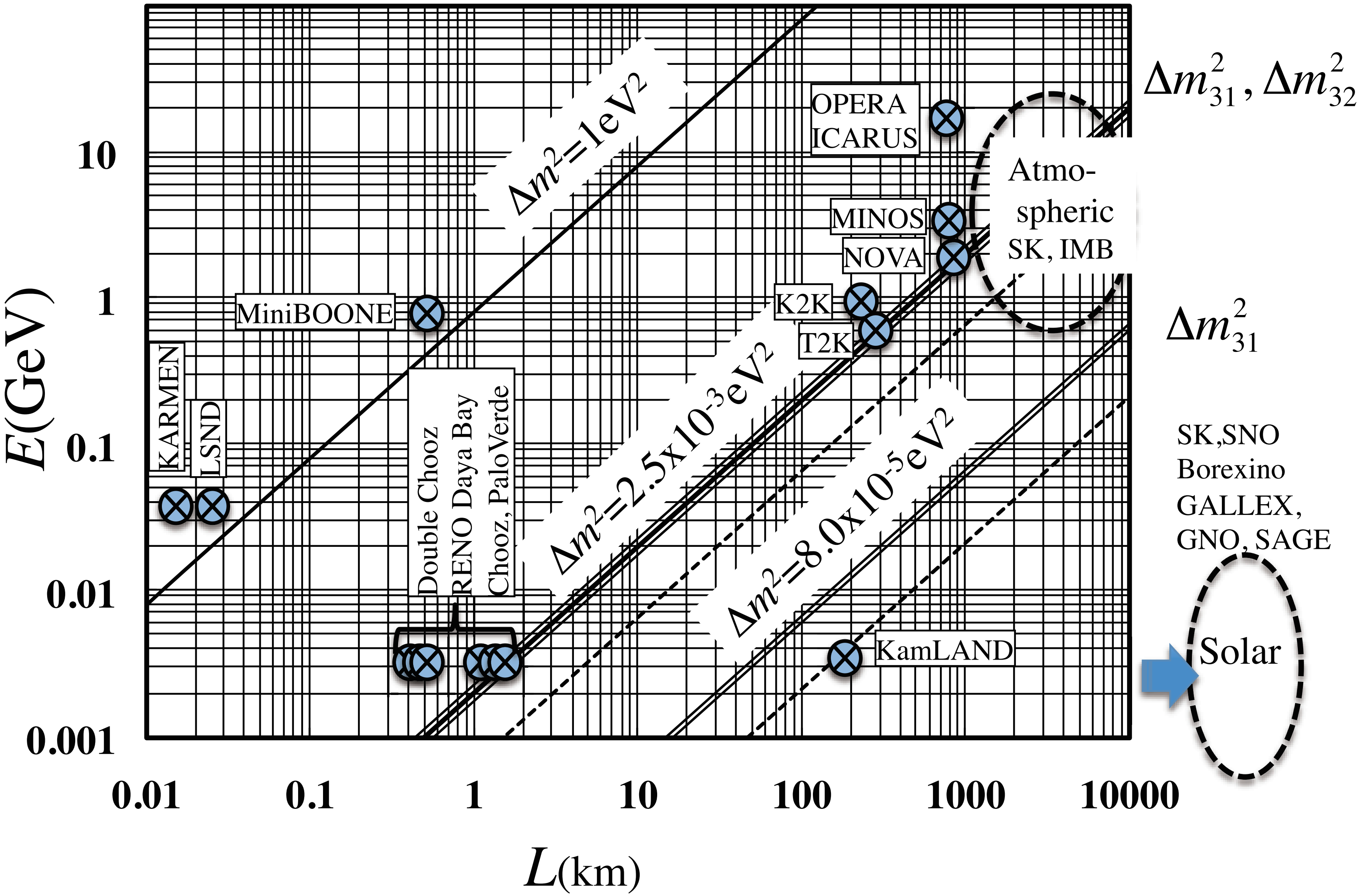
Then $\quad m_1 \sim \mu_{ee}, \quad m_2 \sim \mu_{\mu\mu}, \quad m_3 \sim \mu_{\tau\tau}$

$$s_{12} \sim \frac{A_{\mu e}}{\mu_{\mu\mu} - \mu_{ee}}, \quad s_{23} \sim \frac{A_{\tau\mu}}{\mu_{\tau\tau} - \mu_{\mu\mu}}, \quad s_{13} e^{i\delta} \sim \frac{A_{\tau e}}{\mu_{\tau\tau} - \mu_{ee}}$$

The ordering ν_1, ν_2, ν_3 is such that for small mixing,
$\nu_1 = \nu_e$, $\nu_2 = \nu_\mu$ and $\nu_3 = \nu_\tau$

How neutrino oscillation parameters have been measred

L-E relation of Neutrino Oscillation Experiments



Summary of measurements

	Experiment	Mode	Δm^2 [eV^2]	P_{OSC}	neutrino source
(1)	IMB, Kamiokande, SK, K2K, MINOS, T2K	$\nu_\mu \to \nu_\mu$ $\bar{\nu}_\mu \to \bar{\nu}_\mu$	$\sim \pm 2.5 \times 10^{-3}$	~ 1	Atmospheric/ Accelerator
(2)	T2K, MINOS	$\nu_\mu \to \nu_e$	$\sim \pm 2.5 \times 10^{-3}$	~ 0.05	Accelerator
(3)	Double Chooz Daya Bay, RENO	$\bar{\nu}_e \to \bar{\nu}_e$	$\sim \pm 2.5 \times 10^{-3}$	~ 0.1	Reactor
(4)	Homestake, GNO, GALLEX, SAGE, SK, SNO, Borexino	$\nu_e \to \nu_e$	$\sim +8 \times 10^{-5}$	~ 0.4	Solar
(5)	KamLAND	$\bar{\nu}_e \to \bar{\nu}_e$	$\sim \pm 8 \times 10^{-5}$	~ 0.8	Reactor
(6)	OPERA	$\nu_\mu \to \nu_\tau$	$\sim 10^{-3}$	-	Accelerator

two distinct Δm^2

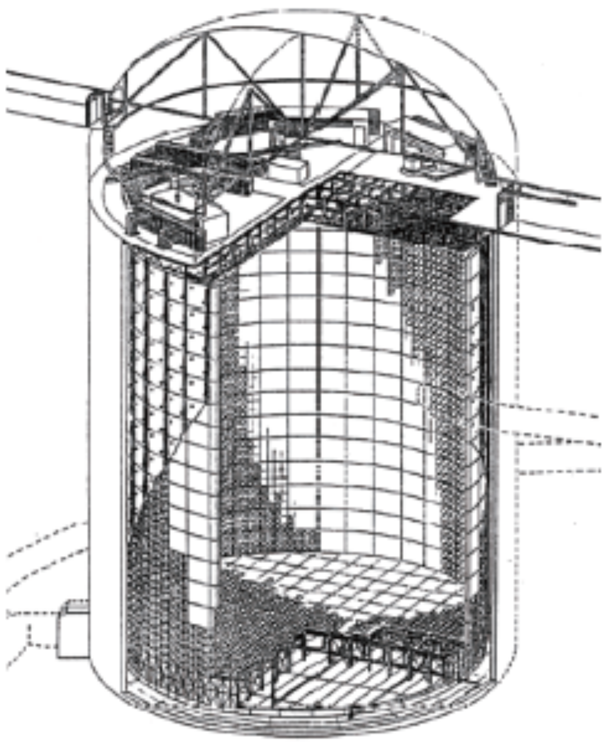
$$\Delta m_{12}^2 + \Delta m_{23}^2 + \Delta m_{31}^2 = 0$$

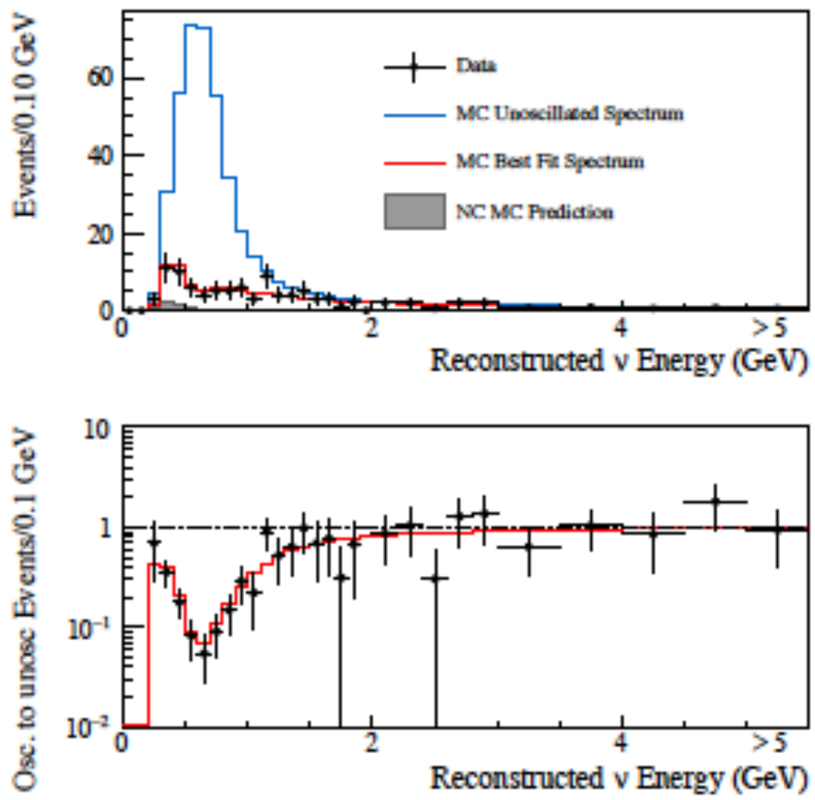
$$\begin{cases} \left|\Delta m_L^2\right| \sim 2.5 \times 10^{-3} \left[eV^2\right] \to \left|\Delta m_{32}^2\right|, \quad \left|\Delta m_{31}^2\right| \\ \left|\Delta m_S^2\right| \sim 8 \times 10^{-5} \left[eV^2\right] \to \left|\Delta m_{21}^2\right| \end{cases}$$

$$\boxed{\theta_{23}, \quad \Delta m_{32}^2} \qquad P(\nu_\mu \rightarrow \nu_\mu) \sim 1 - \sin^2 2\theta_{23} \sin^2 \frac{\Delta m_{32}^2}{4E} L$$

SK Atmospheric, T2K, MINOS

arXiv:1502.01550v1

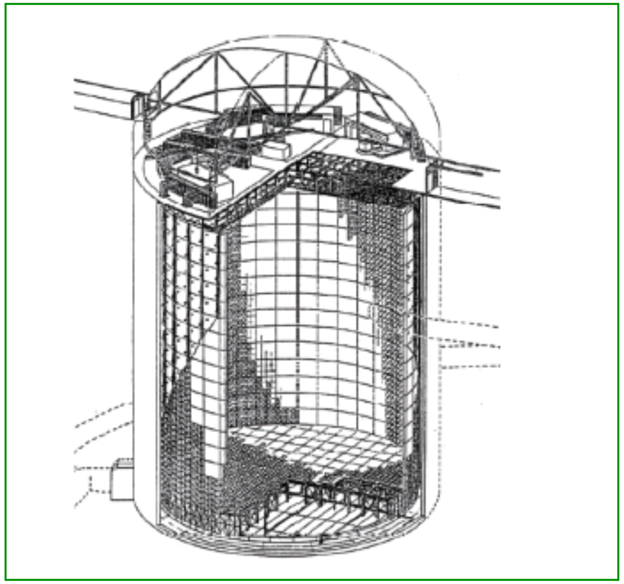




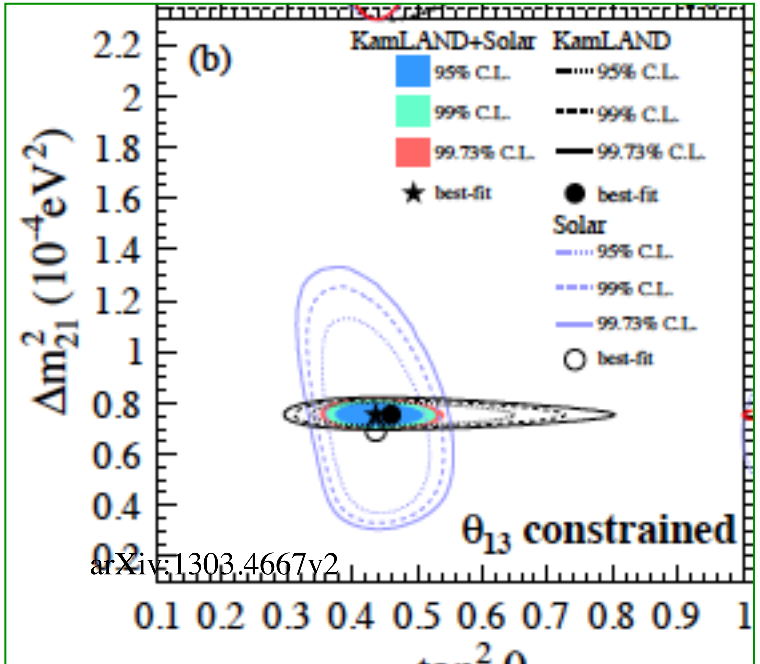
$$\sin^2 2\theta_{23} \sim 1, \quad \Delta m_{32}^2 \sim 2.5 \times 10^{-3} \left[eV^2 \right]$$

$$\boxed{\theta_{12}, \quad \Delta m_{12}^2} \qquad P\left(\nu_e \to \nu_e; @\Delta m_{21}^2\right) \sim 1 - \sin^2 2\theta_{12} \sin^2 \frac{\Delta m_{12}^2}{4E} L$$

Solar Neutrino Experiments



etc.



arXive:1303.4667v2

$$\text{Solar } \nu: \quad \tan^2\theta_{12} = 0.468^{+0.031}_{-0.044}, \quad \Delta m_{21}^2 = 5.4^{+1.7}_{-1.1} \times 10^{-5} eV^2$$

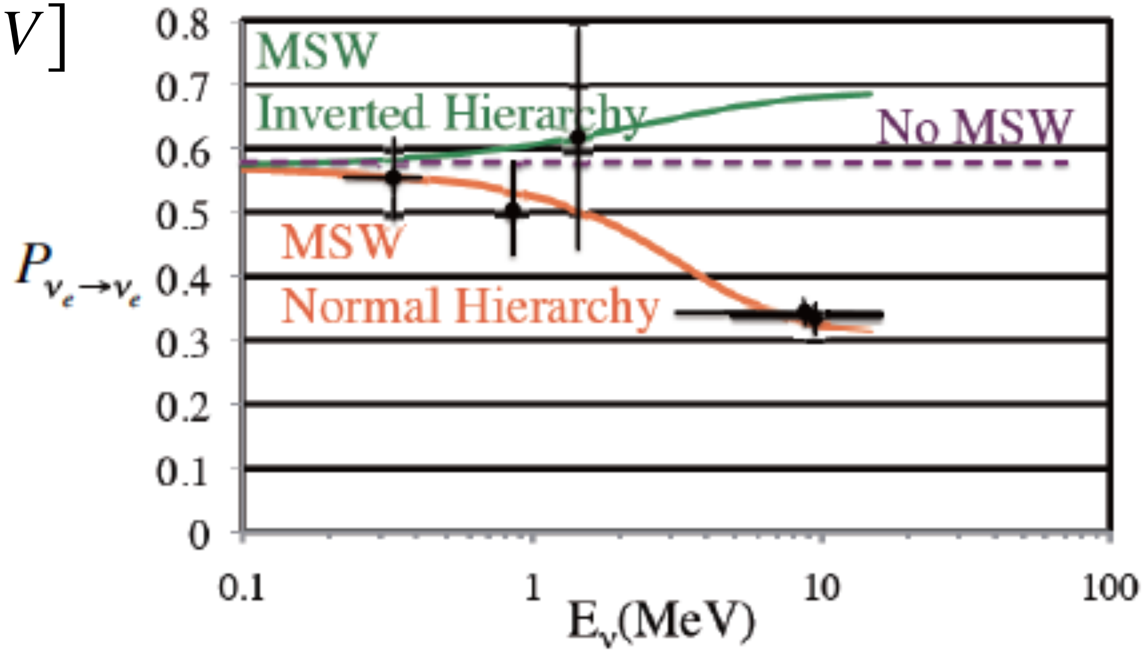
Δm_{21}^2 mass hierarchy

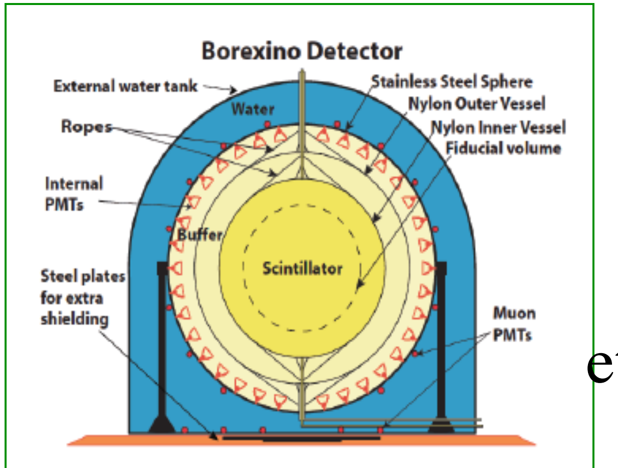
← Matter Effect of Solar Neutrinos

$$P(\nu_e \to \nu_e; @solar) \sim \frac{1}{2}\left(1 + \frac{\cos 2\theta_{12}(\cos 2\theta_{12} - a)}{\sqrt{(\cos 2\theta_{12} - a)^2 + \sin^2 2\theta_{12}}}\right)$$

$$a = \frac{2\sqrt{2}EG_F n_e}{\Delta m_{21}^2} \sim 0.25E[MeV]$$

It changes sign depending on the mass hieararchy



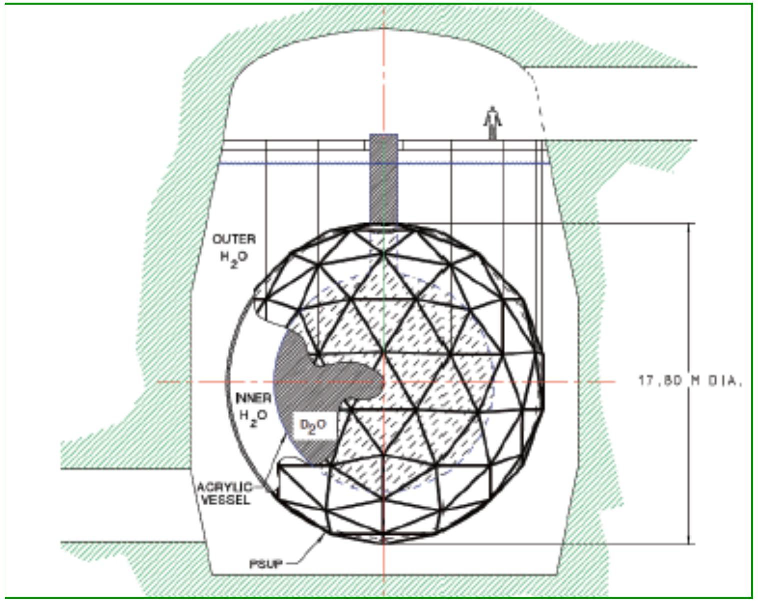


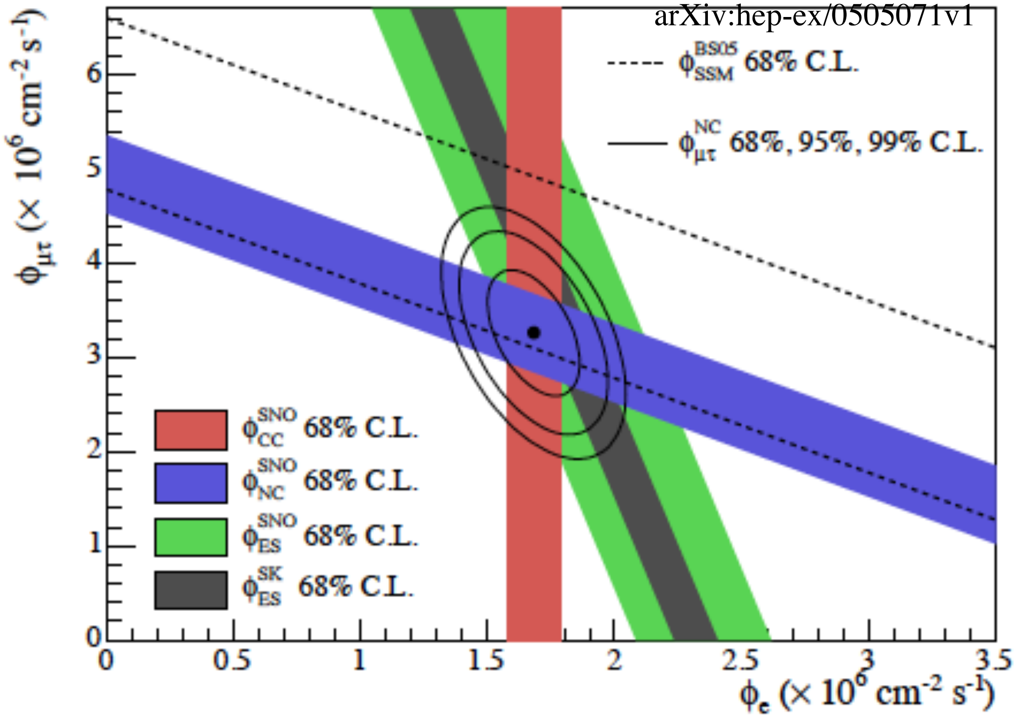
etc.

$$a > 0 \Rightarrow m_2 > m_1$$

Evidence of Flavor Transmutation : SNO experiment

$$\nu_x + D \to \nu_x + p + n$$ NC interaction



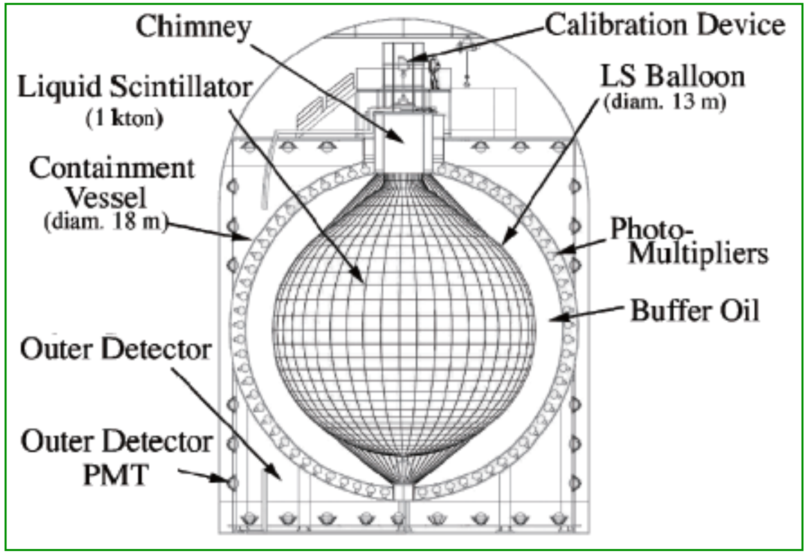


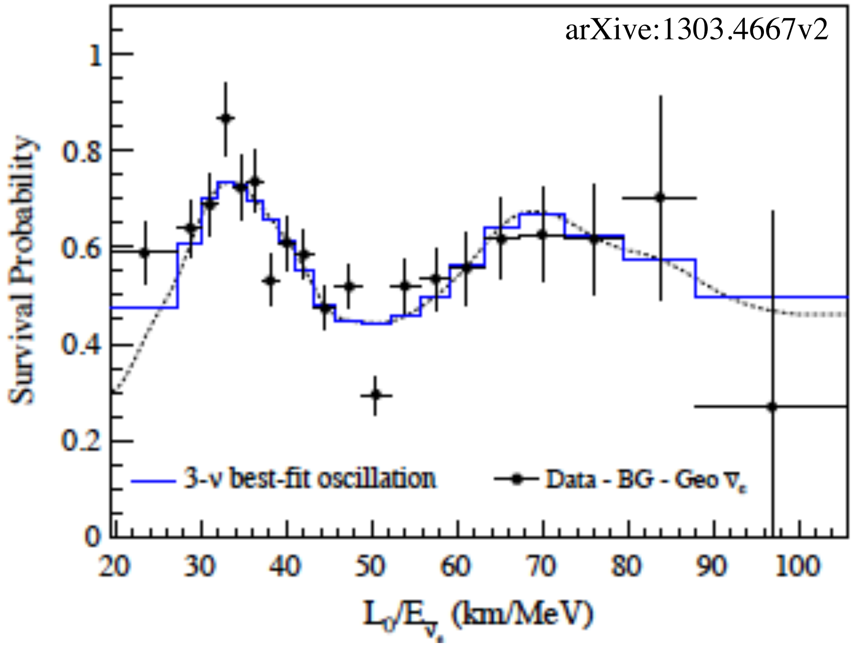
Although $\Phi(\nu_e) < \Phi(SSM)$, $\quad \Phi(\nu_e) + \Phi(\nu_\mu) + \Phi(\nu_\tau) = \Phi(SSM)$

$$\boxed{\theta_{12}, \quad \Delta m_{12}^2}$$

$$P\left(\overline{\nu}_e \rightarrow \overline{\nu}_e ; @\Delta m_{21}^2\right) \sim 1-\sin^2 2\theta_{12} \sin^2 \frac{\Delta m_{12}^2}{4E} L$$

KamLAND Reactor Neutrino Oscillation

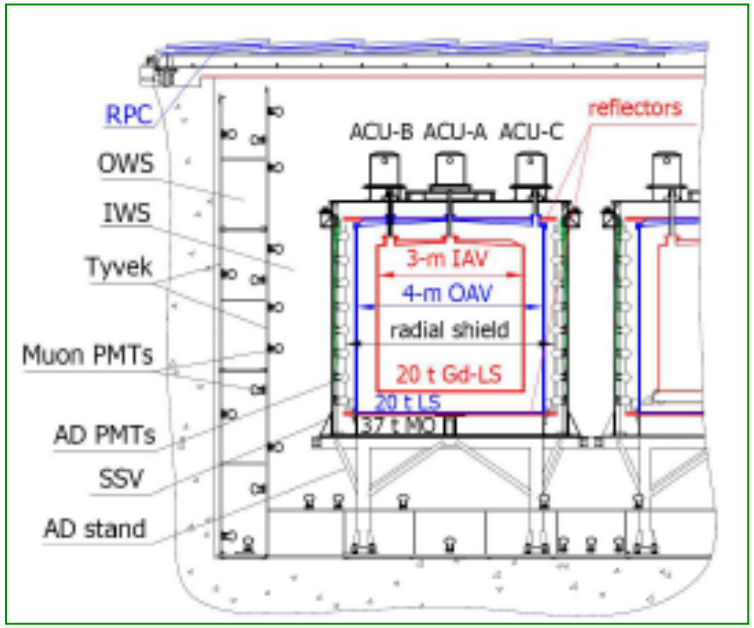


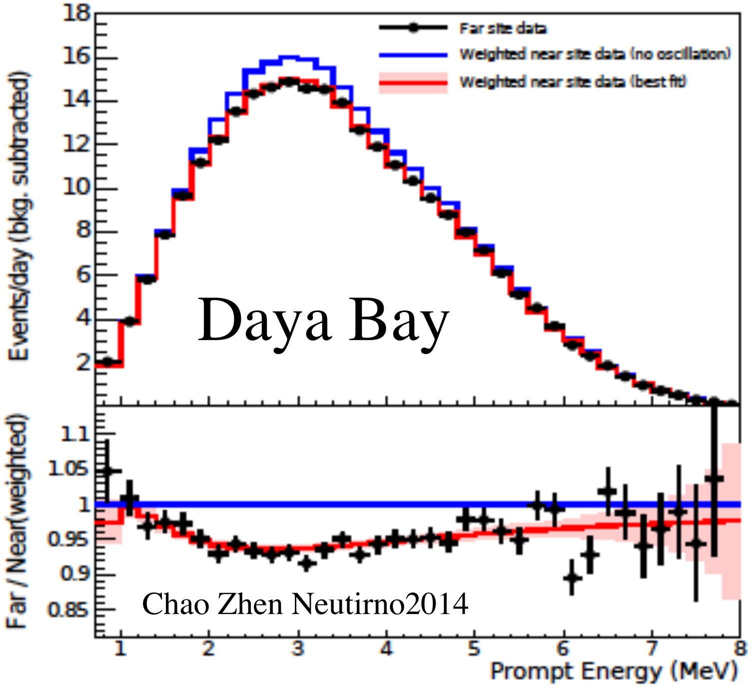


$$\text{KamLAND}: \quad \tan^2\theta_{12} = 0.436^{+0.029}_{-0.025}, \quad \left|\Delta m_{21}^2\right| = 7.53^{+0.18}_{-0.18} \times 10^{-5}\, eV^2$$

$$\boxed{\theta_{13},\quad \Delta m^2_{31}:}\qquad P\left(\bar{\nu}_e \to \bar{\nu}_e; @\Delta m^2_{31}\right) \sim 1 - \sin^2 2\theta_{13} \sin^2 \frac{\Delta m^2_{31}}{4E} L$$

Daya Bay, RENO, Double Chooz Reactor Neutrino experiments

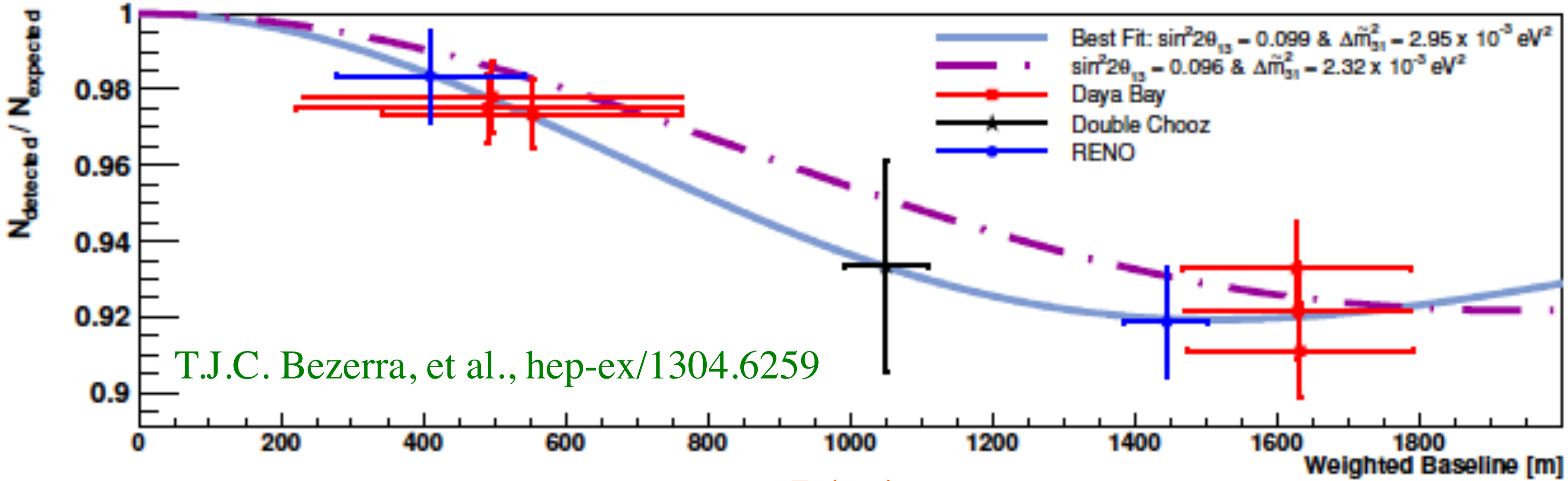




$$\sin^2 2\theta_{13} = 0.084 \pm 0.005, \quad \left|\Delta m^2_{31}\right| = 2.44^{+0.10}_{-0.11} \times 10^{-3}\, eV^2$$

$$\boxed{\Delta m_{31}^2 :} \qquad P\left(\bar{\nu}_e \to \bar{\nu}_e ; @\Delta m_{31}^2\right) \sim 1 - \sin^2 2\theta_{13} \sin^2 \frac{\Delta m_{31}^2}{4E} L$$

Baseline Dependence of Reactor ν Oscillation



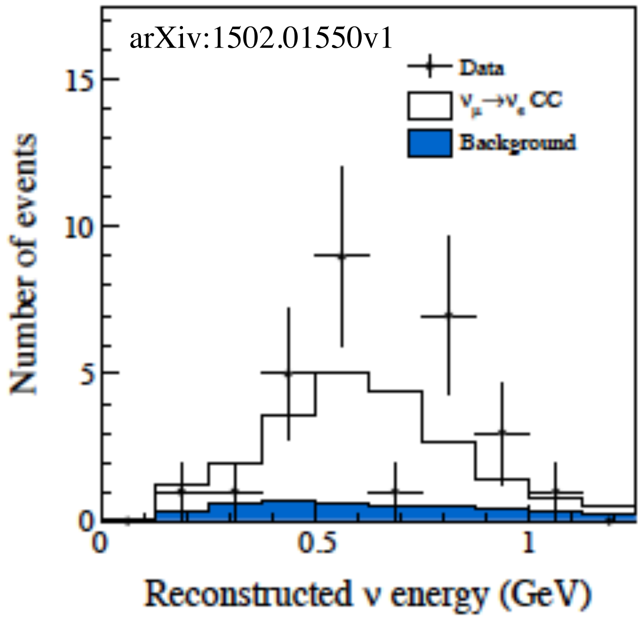
L(m)

$$\left|\Delta \tilde{m}_{31}^2\right| = 2.95_{-1.07}^{+0.59} \times 10^{-3}\, eV^2$$

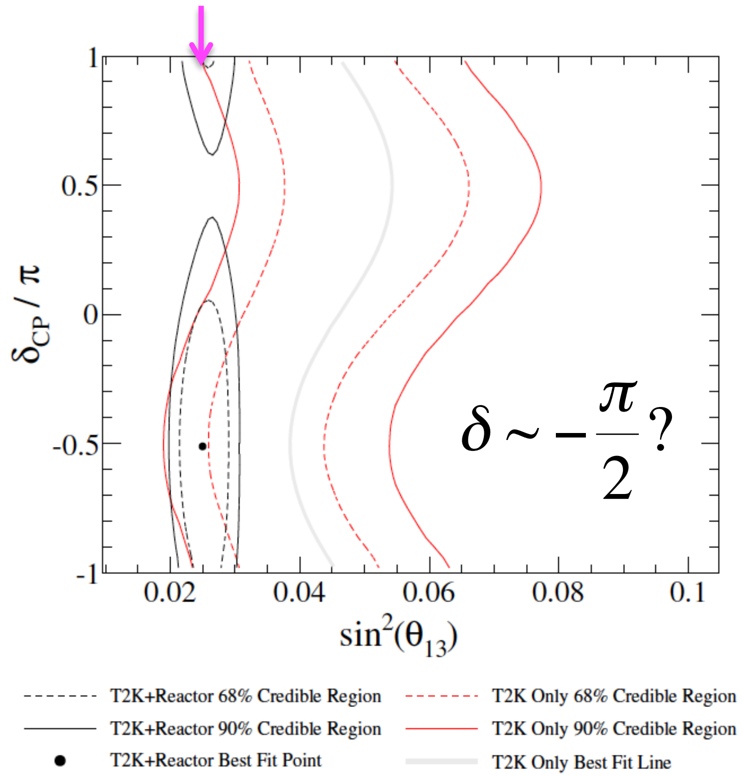
$$\boxed{\delta:} \quad P_{31}\left(\nu_\mu \to \nu_e\right) \sim 0.5\sin^2 2\theta_{13}\frac{\sin^2\left((1-a)\Phi_{32}\right)}{(1-a)^2}$$

$$-0.043\sin 2\theta_{13}\frac{\sin\left((1-a)\Phi_{32}\right)}{1-a}\cos\left(\Phi_{32}+\delta\right)$$

T2K



Reactor-θ13

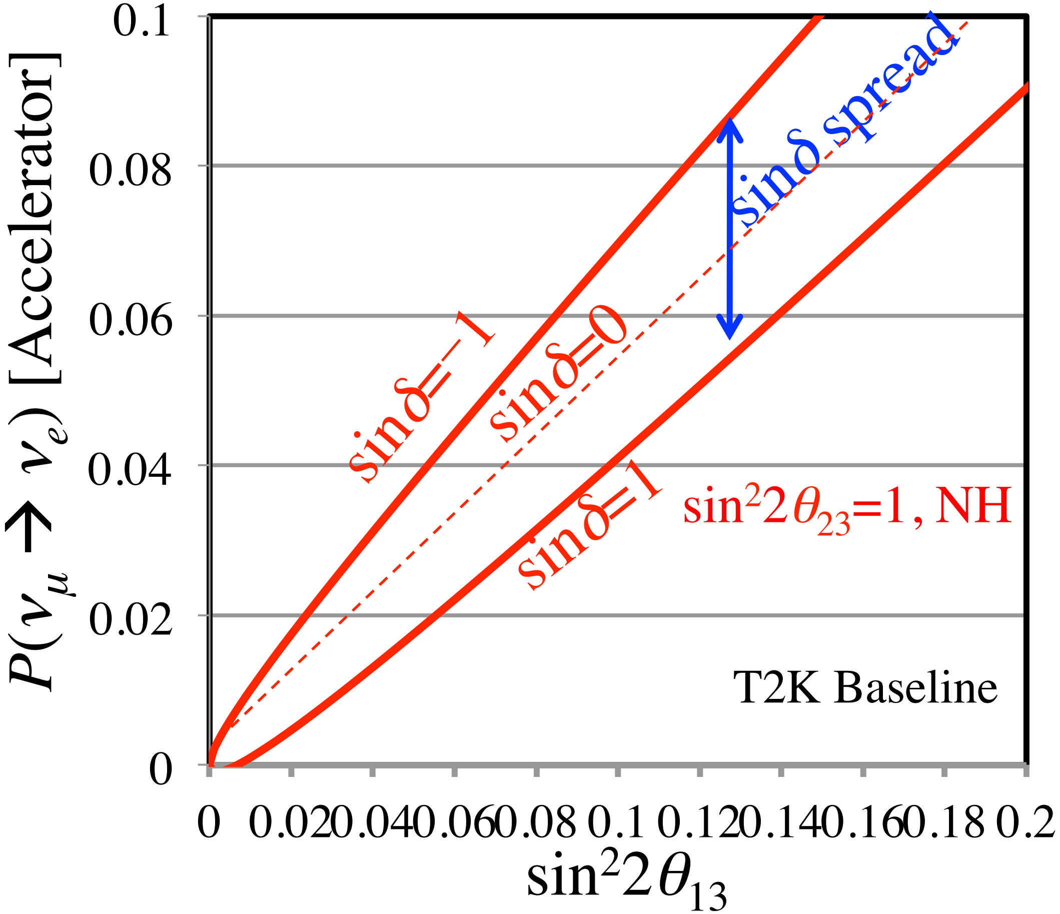


Reactor-Accelerator Complementarity

PRD68, 03317 (2003)

$$P_{31}\left(\nu_\mu \to \nu_e;\Phi_{31}=\pi/2\right) \sim \frac{\sin^2 2\theta_{13}}{2\left(1-\left(L/L_0\right)\right)^2} - 0.043\frac{\sin 2\theta_{13}}{1-\left(L/L_0\right)}\underbrace{\sin\delta}_{\text{unknown}}$$

MH MH



$a=L/L_0$

L_0: Typical matter effect distance~5,800km.

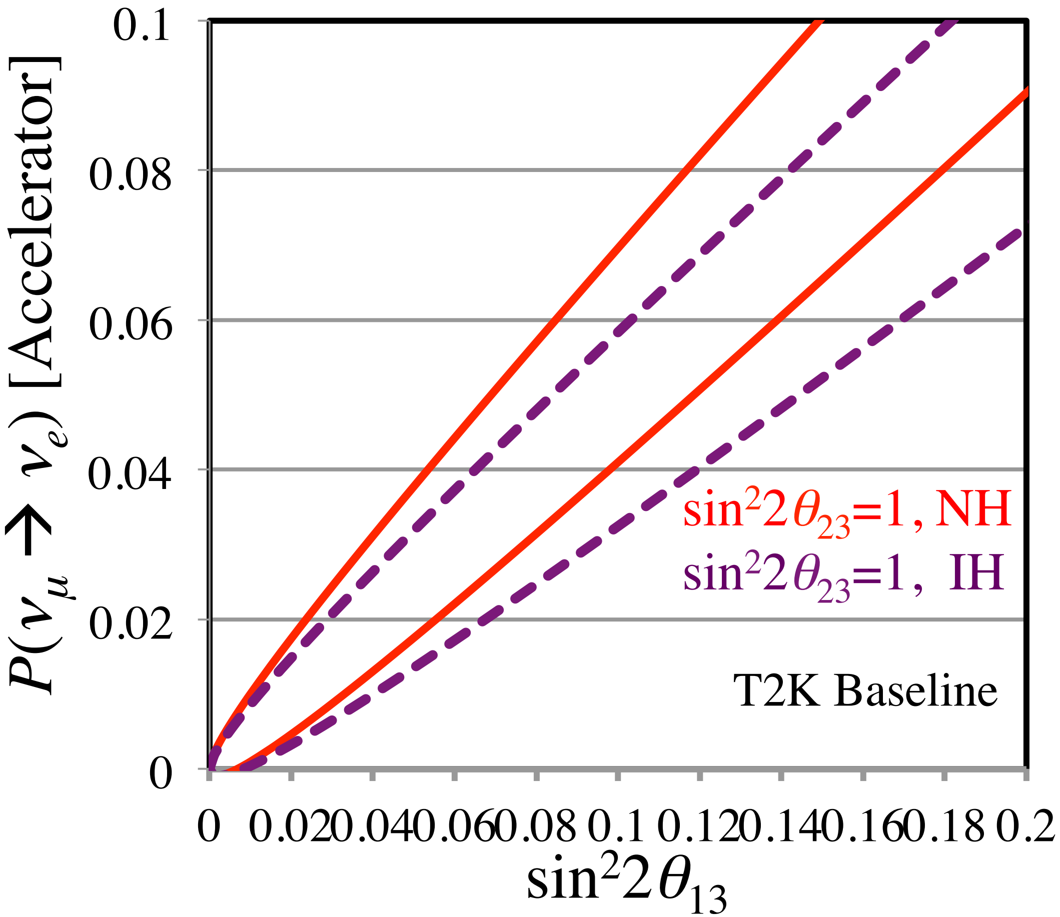
(L/L_0~0.05 for T2K)

L_0>0 for NH

L_0<0 for IH

Reactor-Accelerator Complementarity

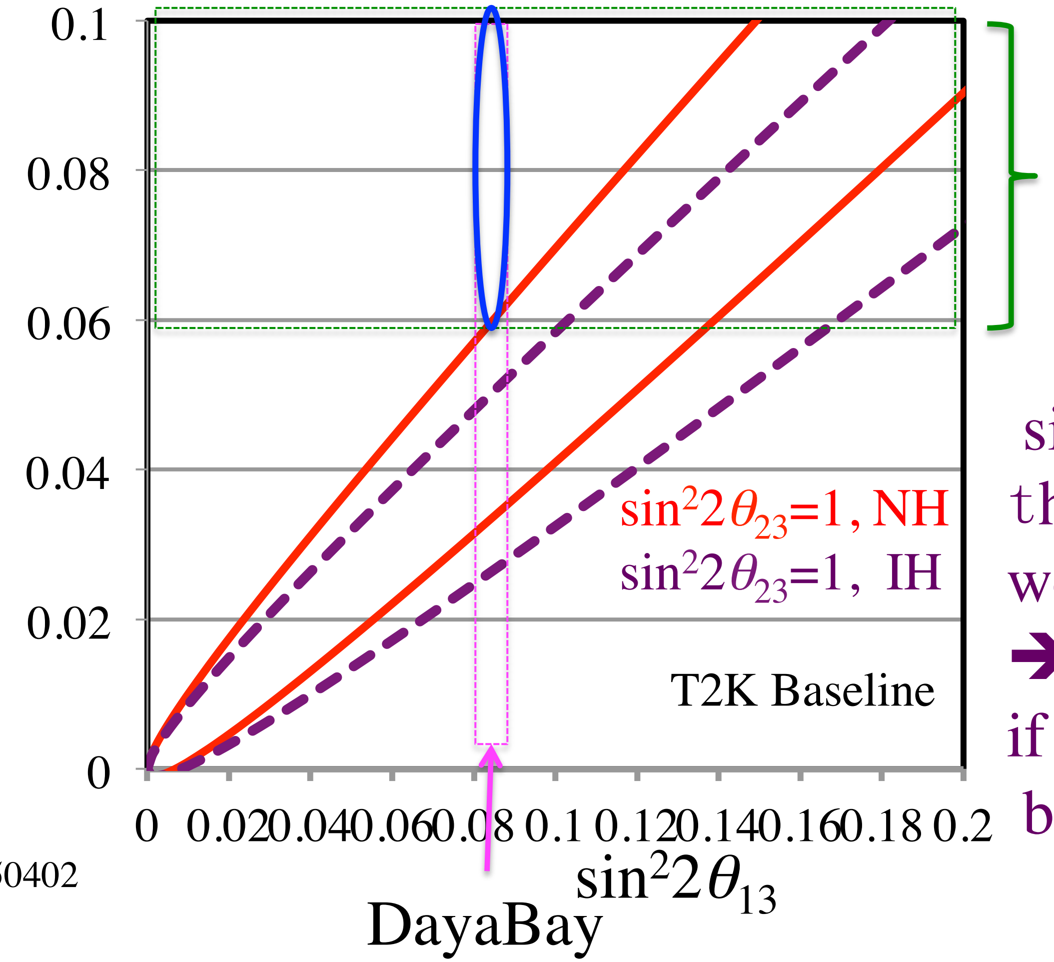
$$P_{31}\left(\nu_\mu \to \nu_e;\Phi_{31}=\pi/2\right) \sim \frac{\sin^2 2\theta_{13}}{2\left(1-\left(L/L_0\right)\right)^2} - 0.043\frac{\sin 2\theta_{13}}{1-\left(L/L_0\right)}\sin\delta$$



For Inverted Mass Hierarchy (L_0<0)

Current Status

$$P_{31}\left(\nu_\mu \to \nu_e; \Phi_{31} = \pi/2\right) \sim \frac{\sin^2 2\theta_{13}}{2\left(1-\left(L/L_0\right)\right)^2} - 0.043\frac{\sin 2\theta_{13}}{1-\left(L/L_0\right)}\sin\delta$$



150402

$\sin\delta$=-1, NH favored but the central value has a weak tension.

➔ It is important to check if this tension is true or not by T2K and DC

Global parameter fit

www.nu-fit.org

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v1.3: Three-neutrino results after the 'Neutrino 2014' conference

Menu

- Summary of data included
- Parameter ranges
- Leptonic mixing matrix
- One-dimensional χ^2 projections
- Two-dimensional allowed regions
- Contributions to the determination of θ_{13}
- Role of atmospheric neutrinos
- Correlation between δ_{CP} and other parameters
- CP violation: Jarlskog invariant and unitarity triangles
- Reactor fluxes

If you are using these results please refer to JHEP 12 (2012) 123 [arXiv:1209.3023]

NuFIT 1.3 (2014)

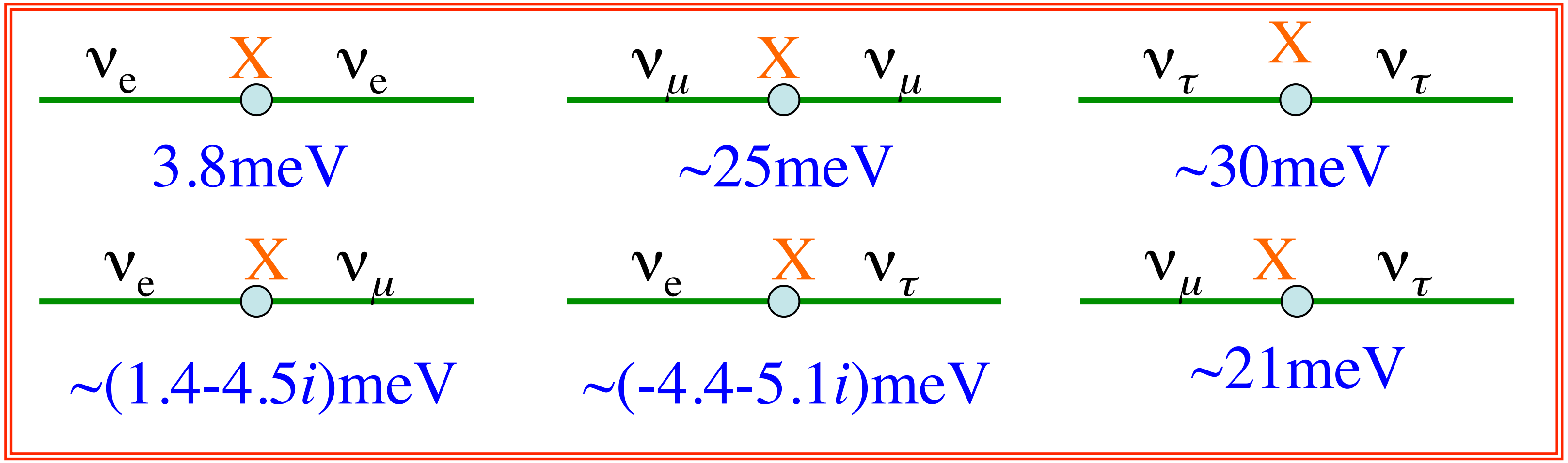
	Free Fluxes + RSBL		Huber Fluxes, no RSBL	
	bfp $\pm1\sigma$	3σ range	bfp $\pm1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.304^{+0.012}_{-0.012}$	$0.270 \to 0.344$	$0.311^{+0.013}_{-0.012}$	$0.276 \to 0.352$
$\theta_{12}/^\circ$	$33.48^{+0.77}_{-0.74}$	$31.30 \to 35.90$	$33.91^{+0.80}_{-0.76}$	$31.67 \to 36.41$
$\sin^2\theta_{23}$	$[0.451^{+0.001}_{-0.001}] \oplus 0.577^{+0.027}_{-0.035}$	$0.385 \to 0.644$	$[0.451^{+0.026}_{-0.020}] \oplus 0.580^{+0.024}_{-0.039}$	$0.383 \to 0.644$
$\theta_{23}/^\circ$	$[42.2^{+0.1}_{-0.1}] \oplus 49.4^{+1.6}_{-2.0}$	$38.4 \to 53.3$	$[42.2^{+1.5}_{-1.1}] \oplus 49.6^{+1.4}_{-2.2}$	$38.2 \to 53.4$
$\sin^2\theta_{13}$	$0.0219^{+0.0010}_{-0.0011}$	$0.0188 \to 0.0251$	$0.0223^{+0.0011}_{-0.0010}$	$0.0192 \to 0.0255$
$\theta_{13}/^\circ$	$8.52^{+0.20}_{-0.21}$	$7.87 \to 9.11$	$8.60^{+0.20}_{-0.20}$	$7.97 \to 9.19$
$\delta_{CP}/^\circ$	251^{+67}_{-59}	$0 \to 360$	259^{+76}_{-69}	$0 \to 360$
$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.50^{+0.19}_{-0.17}$	$7.03 \to 8.09$	$7.55^{+0.18}_{-0.17}$	$7.07 \to 8.12$
$\frac{\Delta m^2_{31}}{10^{-3}\ \mathrm{eV}^2}$ (N)	$[+2.458^{+0.002}_{-0.002}]$	$+2.325 \to +2.599$	$[+2.462^{+0.033}_{-0.033}]$	$+2.326 \to +2.608$
$\frac{\Delta m^2_{32}}{10^{-3}\ \mathrm{eV}^2}$ (I)	$-2.448^{+0.047}_{-0.047}$	$-2.590 \to -2.307$	$-2.453^{+0.047}_{-0.047}$	$-2.596 \to -2.312$

Our Current Knowledge of Neutrino Transition Amplitude

If NH and δ=-π/2,

$$U_{NH} \sim \begin{pmatrix} 0.82 & 0.55 & -0.09+0.13i \\ -0.36+0.07i & 0.65+0.05i & 0.67 \\ 0.43+0.08i & -0.53+0.05i & 0.73 \end{pmatrix}$$

Assumption: m_1~0, ➔ m_2 =8.7meV, m_3=50meV

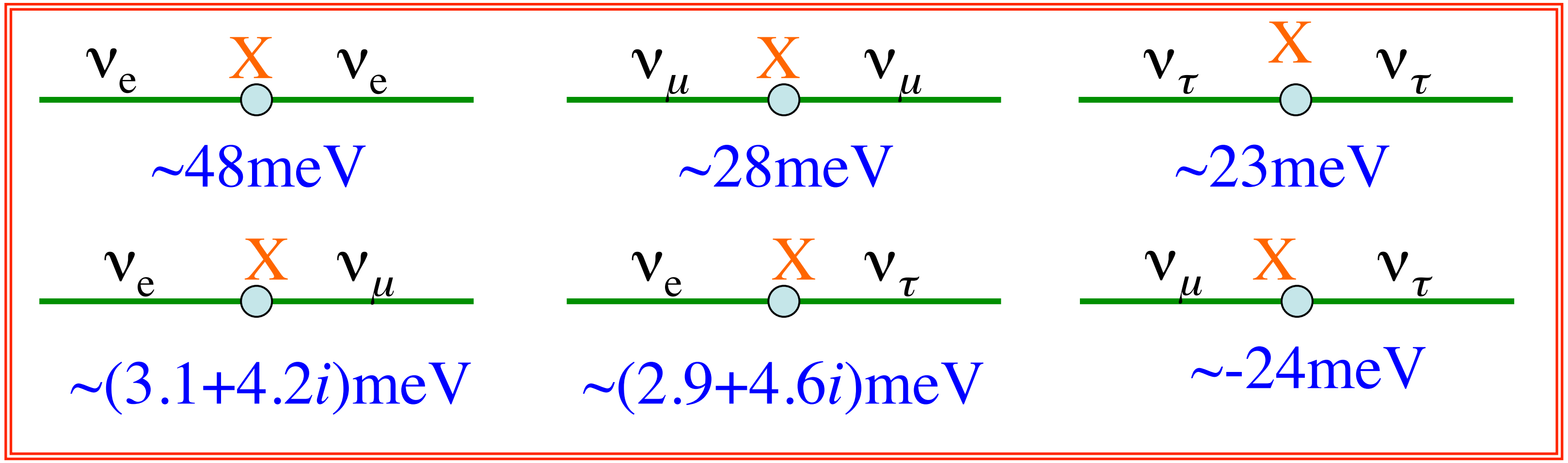


Our Current Knowledge of Neutrino Transition Amplitude

If IH and $\delta=-\pi/2$,

$$U_{IH} \sim \begin{pmatrix} 0.82 & 0.55 & -0.05+0.14i \\ -0.39+0.08i & 0.63-0.05i & 0.65 \\ 0.40+0.09i & -0.53+0.06i & 0.74 \end{pmatrix}$$

Assumption: $m_3 \sim 0$, ➔ m_2 =50meV, m_1=49meV



If we assume the transitions are caused by the Yukawa coupling to the Higgs field, the coupling constants is for NH case are

$\nu_e \quad H^0 \quad \nu_e$ $\qquad$ $\nu_\mu \quad H^0 \quad \nu_\mu$ $\qquad$ $\nu_\tau \quad H^0 \quad \nu_\tau$

G_{ee}=1.5x10^{-14} $\qquad$ $G_{\mu\mu}$=1.0x10^{-13} $\qquad$ $G_{\tau\tau}$=1.2x10^{-13}

$\nu_e \quad H^0 \quad \nu_\mu$ $\qquad$ $\nu_e \quad H^0 \quad \nu_\tau$ $\qquad$ $\nu_\mu \quad H^0 \quad \nu_\tau$

$G_{e\mu}$~(0.6-1.8i)x10^{-14} $\qquad$ $G_{e\tau}$~(-1.8-2.1i)x10^{-14} $\qquad$ $G_{\mu\tau}$~8.1x10^{-14}

The coupling constants are extremely small.

Theorists say this is unnatural

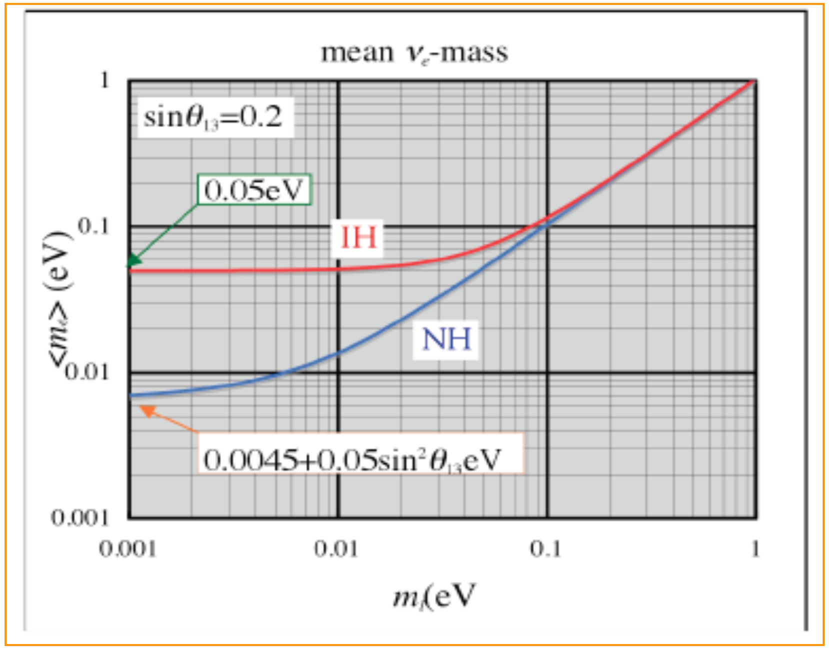
➔ We need to look for other origin.

Relation to the ν_e mass

Impact to absolute ν_e mass measurement

$$\left\langle m_{\nu_e}^2 \right\rangle = |U_{e1}|^2 m_1^2 + |U_{e2}|^2 m_2^2 + |U_{e3}|^2 m_3^2 \sim \begin{cases} m_1^2 + (10meV)^2 & for \quad NH \\ m_3^2 + (48meV)^2 & for \quad IH \end{cases}$$

$<m_{\nu e}>$ [eV]



minimum neutrino mass [eV]

There is minimum neutrino mass.
If IH, the mass will be definitely observed at > 0.048eV.
(KATRIN Sensitivity ~0.2eV)

(I hope it is IH.)

Once $\langle m_{\nu_e}^2 \rangle$ is measured, all the neutrino masses can be determined

For N.H.
$$\begin{cases} m_1 = \sqrt{\langle m_{\nu_e}^2 \rangle - 8.1\times 10^{-5}[eV^2]} \\ m_2 = \sqrt{\langle m_{\nu_e}^2 \rangle - 5.1\times 10^{-6}[eV^2]} \\ m_3 = \sqrt{\langle m_{\nu_e}^2 \rangle + 2.4\times 10^{-3}[eV^2]} \end{cases}$$

For I.H.
$$\begin{cases} m_1 = \sqrt{\langle m_{\nu_e}^2 \rangle + 3.5\times 10^{-5}[eV^2]} \\ m_2 = \sqrt{\langle m_{\nu_e}^2 \rangle + 1.1\times 10^{-4}[eV^2]} \\ m_3 = \sqrt{\langle m_{\nu_e}^2 \rangle - 2.4\times 10^{-3}[eV^2]} \end{cases}$$

And all the transition amplitude can be determined

➔ Now $\langle m_{\nu_e}^2 \rangle$ measurement becomes all the more important

Relation to ν_μ, ν_τ masses

$$\langle m^2_{\nu_\mu} \rangle = |U_{\mu 1}|^2 m_1^2 + |U_{\mu 2}|^2 m_2^2 + |U_{\mu 3}|^2 m_3^2 = \langle m^2_{\nu_e} \rangle \pm (30 meV)^2$$

$$\langle m^2_{\nu_\tau} \rangle = |U_{\tau 1}|^2 m_1^2 + |U_{\tau 2}|^2 m_2^2 + |U_{\tau 3}|^2 m_3^2 = \langle m^2_{\nu_e} \rangle \pm (36 meV)^2$$

Since $\sqrt{\langle m^2_{\nu_e} \rangle} < 2eV$, $\sqrt{\langle m^2_{\nu_\mu} \rangle}$, $\sqrt{\langle m^2_{\nu_\tau} \rangle} < 2eV$

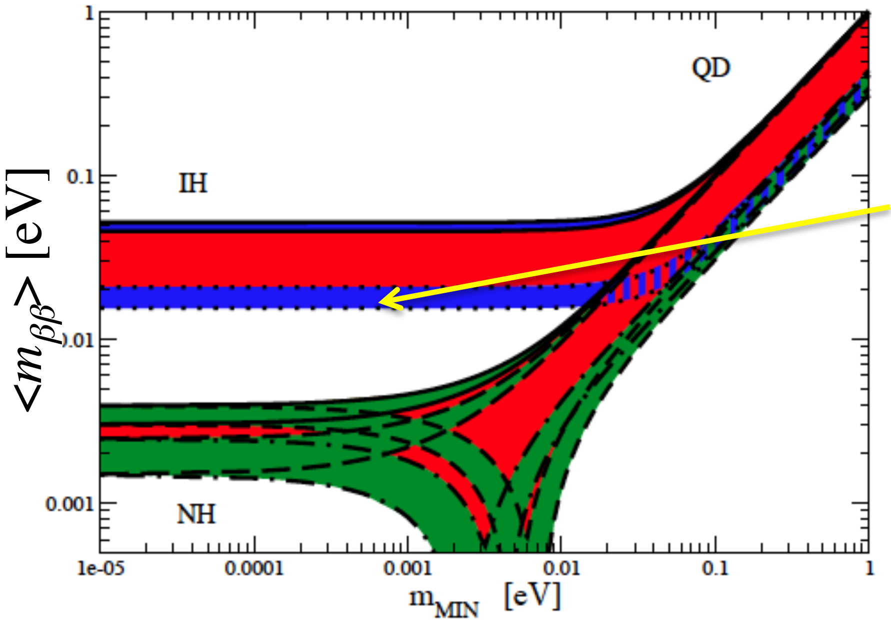
No practical way to measure m_{ν_μ} and m_{ν_τ} with this precision ....

only m_{ν_e} measurement has hope .....

Relation to the Majorana mass

Double Beta Decay mass: $m_{\beta\beta}$

$$|m_{\beta\beta}|^2 = |c_{12}^2 c_{13}^2 m_1 + s_{12}^2 c_{13}^2 m_2 e^{-2i\alpha} + s_{13}^2 m_3 e^{-2i(\beta+\delta)}|^2$$



minimum neutrino mass [eV]

If IH, there is lower limit of $m_{\beta\beta}$~15meV.

=> Either ν is Dirac or Majorana can be definitely determined with experiment with sensitivity 15meV.

(I hope it is IH.)

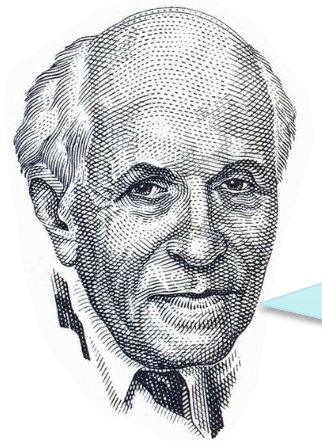
Very Near Future

T2K, NOVA ν_e appearance

Measurement of CPV δ

In order to realize the matter dominance of the current universe,

The Sakharov conditions for Baryogenesis



(1) Baryon number non-conservation.
(2) C and CP violation
(3) Thermal non-equilibrium.

wikipedia

However, CPV effect of quark interactions is very small.

$$\text{CPV effect} \propto J_q = \frac{1}{8} c_{13}^q \sin 2\theta_{12}^q \sin 2\theta_{23}^q \sin 2\theta_{13}^q \sin \delta_q \quad \sim 3\text{x}10^{-5}$$

If quarks can not explain it, leptons should be responsible for it.

CPV effect of ν can be x1,000 times larger: $J_\nu \sim 0.04 \sin \delta_\nu$

Probability Formulas of Available Oscillations

$$P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \sin^2 2\theta_{13}$$

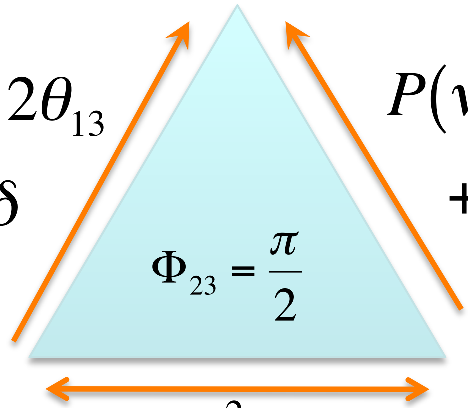
Reactors

ν_e

$$P(\nu_\mu \to \nu_e) \sim s_{23}^2 \sin^2 2\theta_{13} - 0.043 \sin 2\theta_{13} \sin 2\theta_{23} \sin\delta$$

T2K, MINOS, NOVA

$$P(\nu_\tau \to \nu_e) = c_{23}^2 \sin^2 2\theta_{13} + 0.043 \sin 2\theta_{13} \sin 2\theta_{23} \sin\delta$$



ν_μ $\quad$ ν_τ

$$\sin^2 2\theta_{23}$$

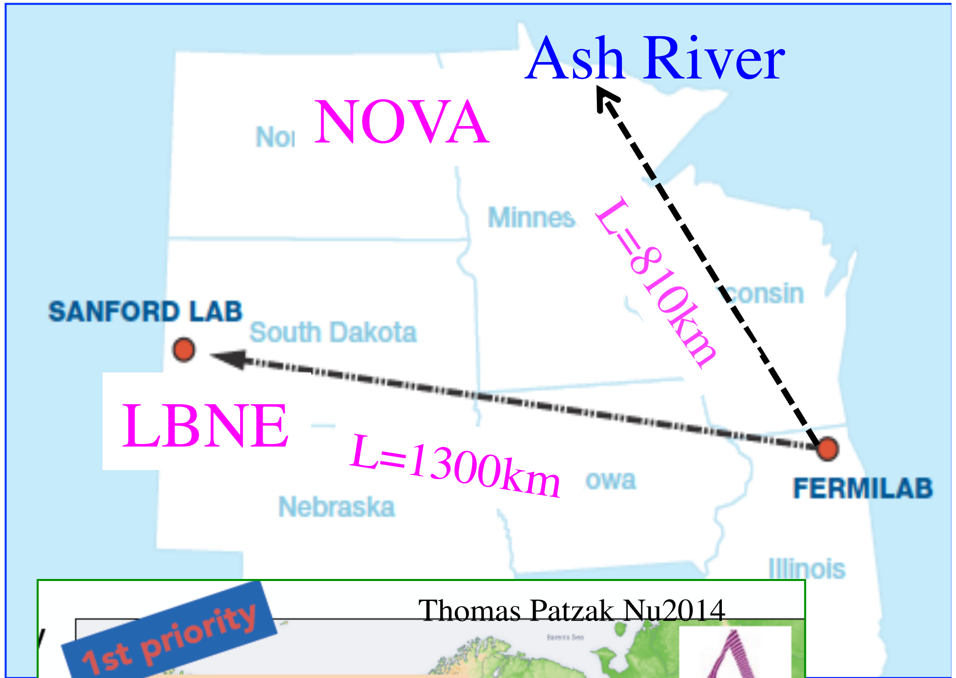
OPERA

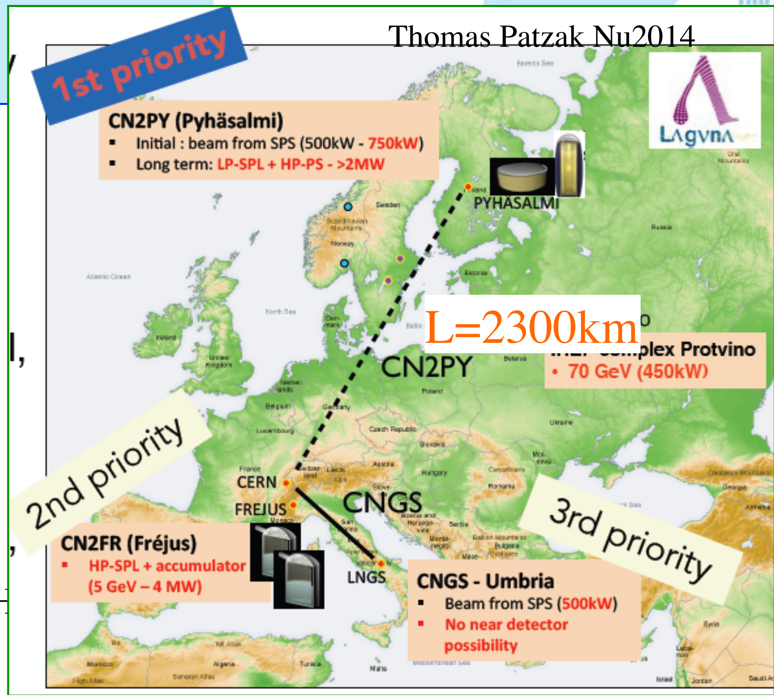
Then we can measure the CPV-δ (?)

$$A_{CP} = \frac{P(\nu_\mu \to \nu_e) - P(\bar{\nu}_\mu \to \bar{\nu}_e)}{P(\nu_\mu \to \nu_e) + P(\bar{\nu}_\mu \to \bar{\nu}_e)} = \frac{0.1 \cot\theta_{23} \sin\delta}{\sin 2\theta_{13}} \sim 0.3 \sin\delta \quad ☺$$

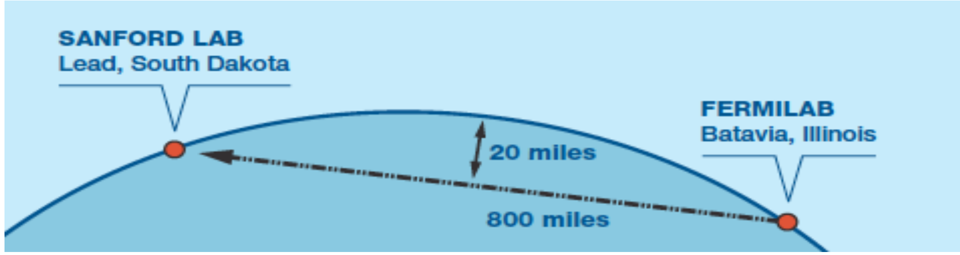
Present and Future long baseline experiments

$\nu_\mu \rightarrow \nu_e$

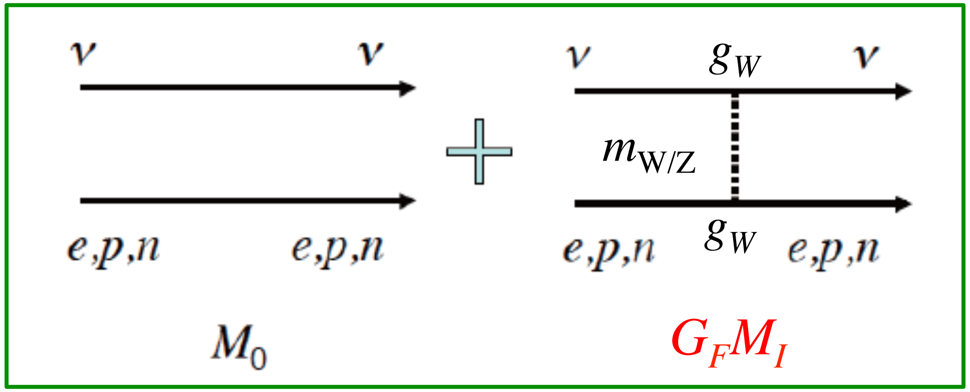




Matter Effect Affects



Actually, ν-beams go through earth and there is matter effect.



scattering

$$P \propto \left| M_0 + G_F M_I \right|^2$$
$$= \left| M_0 \right|^2 + \underline{\underline{2 G_F \,\mathrm{Re}\left[M_0^* M_I \right]}} + G_F^2 \left| M_I \right|^2$$

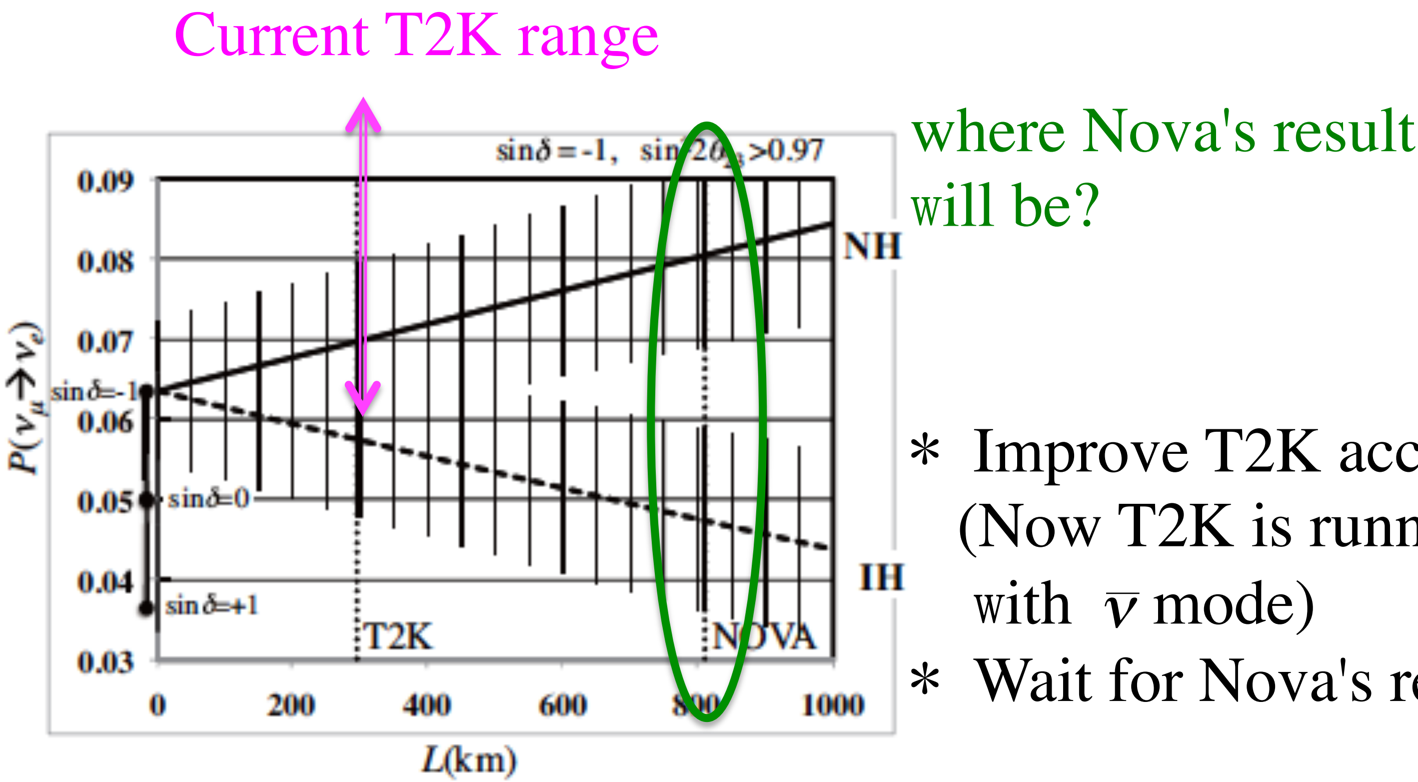
Forward scattering
by Potential

Potential for ultra relativistic neutrino is $V_W \sim \gamma \rho G_F \sim 0.01$ eV ~sqrt(Δm^2)

➔ Matter does affect on neutrino oscillation.

Oscillation probability with matter effect

$$\begin{cases} P\left(\nu_\mu \to \nu_e ; \Phi_{31} = \pi/2\right) \sim \dfrac{0.05}{\left(1+(L/L_0)\right)^2} - \dfrac{0.014}{\left(1+(L/L_0)\right)} \sin\delta \\ P\left(\bar{\nu}_\mu \to \bar{\nu}_e ; \Phi_{31} = \pi/2\right) \sim \dfrac{0.05}{\left(1-(L/L_0)\right)^2} + \dfrac{0.014}{\left(1-(L/L_0)\right)} \sin\delta \end{cases}$$



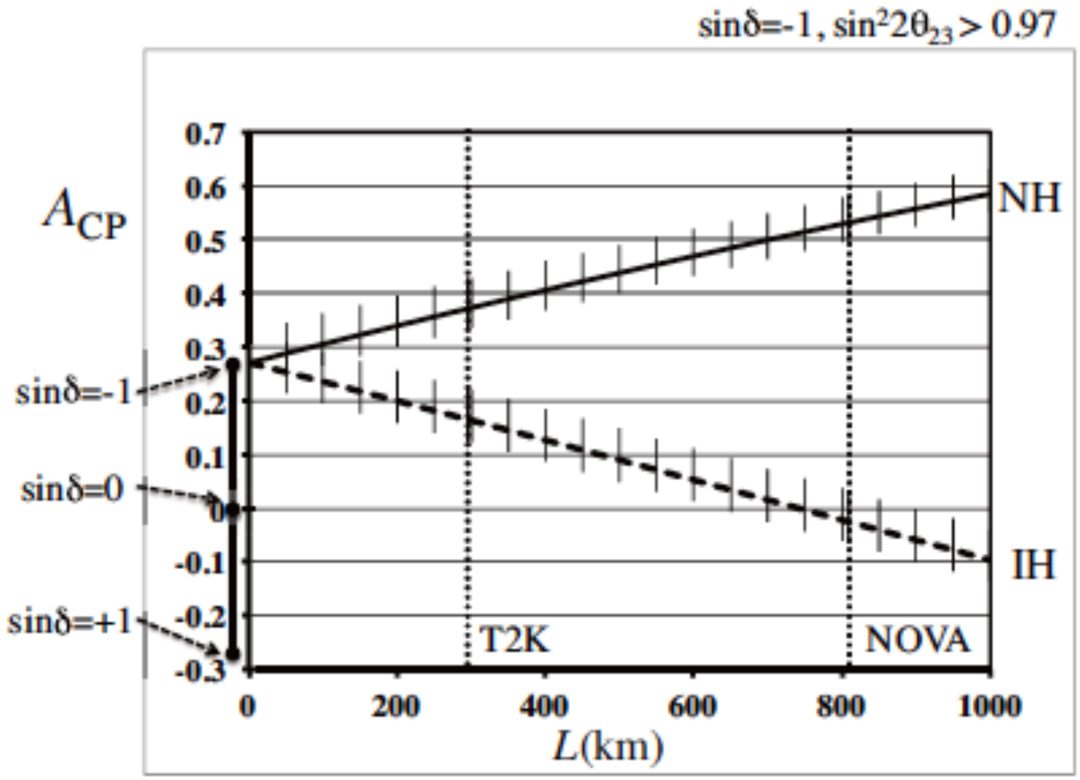
* Improve T2K accuracy (Now T2K is running with $\bar{\nu}$ mode)

* Wait for Nova's result

Affects of the matter effect

$$A_{CP} = \frac{P(\nu_\mu \to \nu_e) - P(\bar{\nu}_\mu \to \bar{\nu}_e)}{P(\nu_\mu \to \nu_e) + P(\bar{\nu}_\mu \to \bar{\nu}_e)} \sim -0.27\sin\delta \pm \frac{L[km]}{2,800}$$

Even if an asymmetry is found, we can not conclude δ is finite.
We need to know Mass Hierarchy

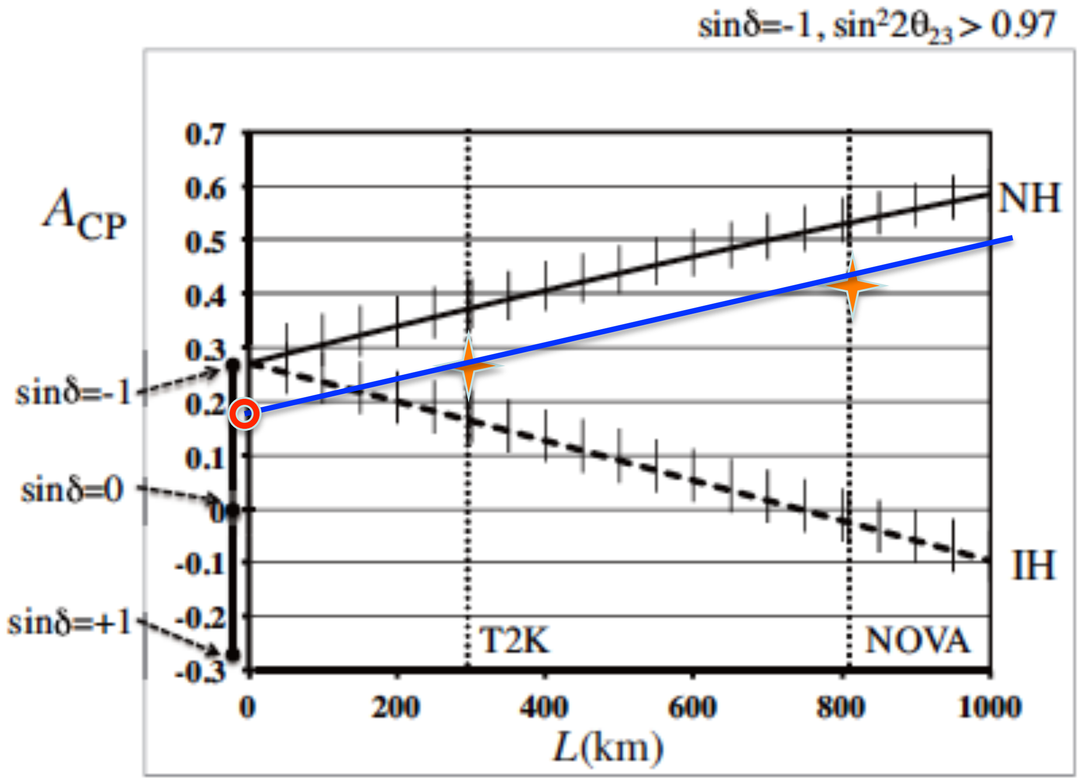


* What T2K and Nova will show??

Affects of the matter effect

$$A_{CP} = \frac{P(\nu_\mu \to \nu_e) - P(\bar{\nu}_\mu \to \bar{\nu}_e)}{P(\nu_\mu \to \nu_e) + P(\bar{\nu}_\mu \to \bar{\nu}_e)} \sim -0.27 \sin\delta \pm \frac{L[km]}{2{,}800}$$

Even if an asymmetry is found, we can not conclude δ is finite.
The sensitivity of δ is limited by Mass Hierarchy



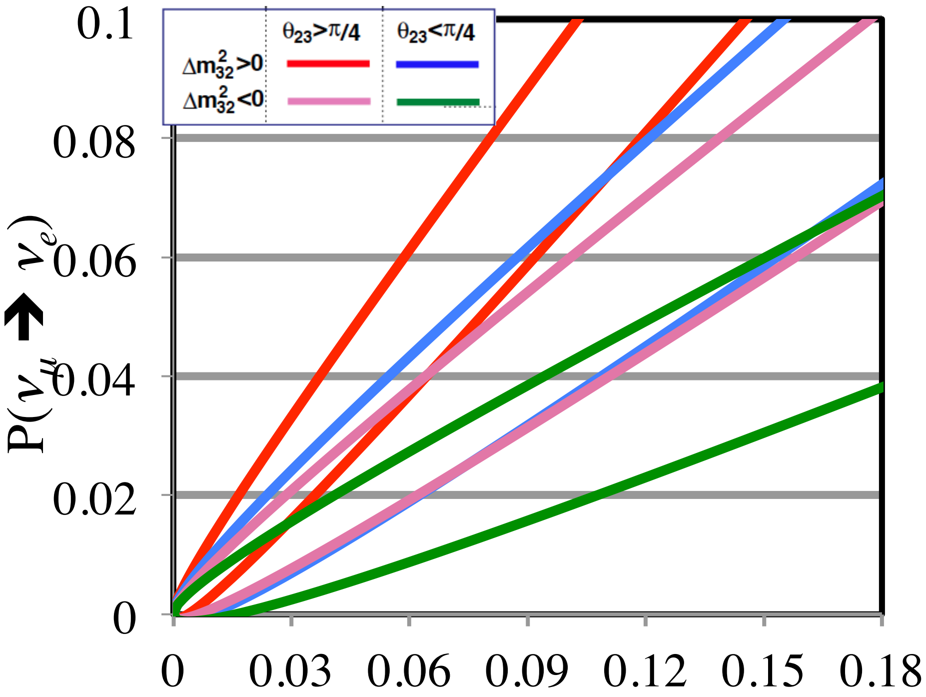
extrapolate two measurements.

inclination shows MH and intercept shows sinδ

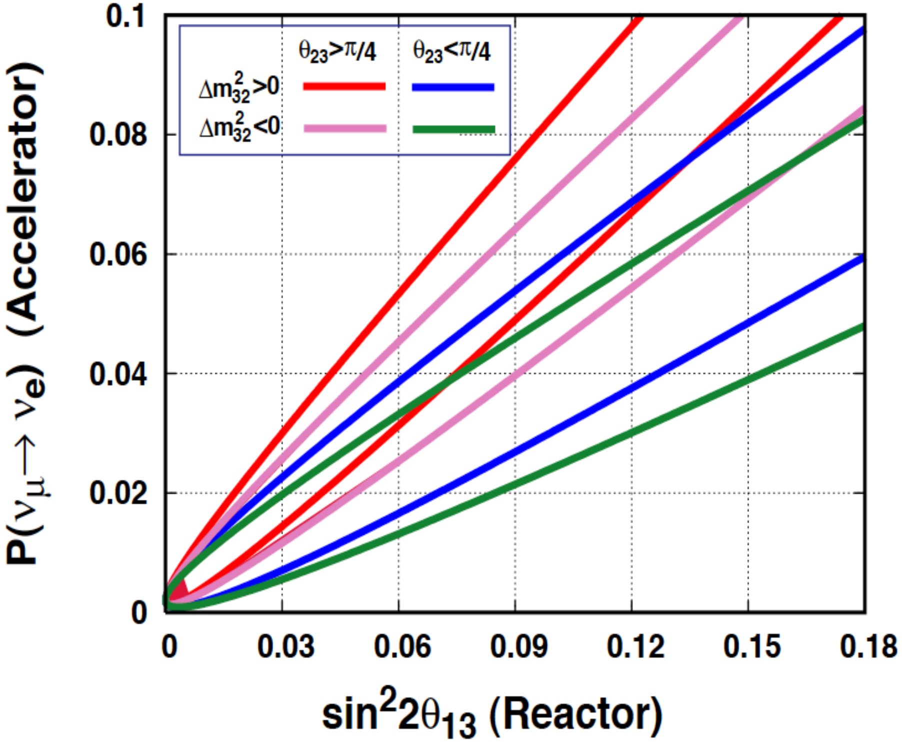
Wait for T2K and Nova's A_{CP} result

Comparison of Nova (L=810km) & T2K(295km)

NOVA



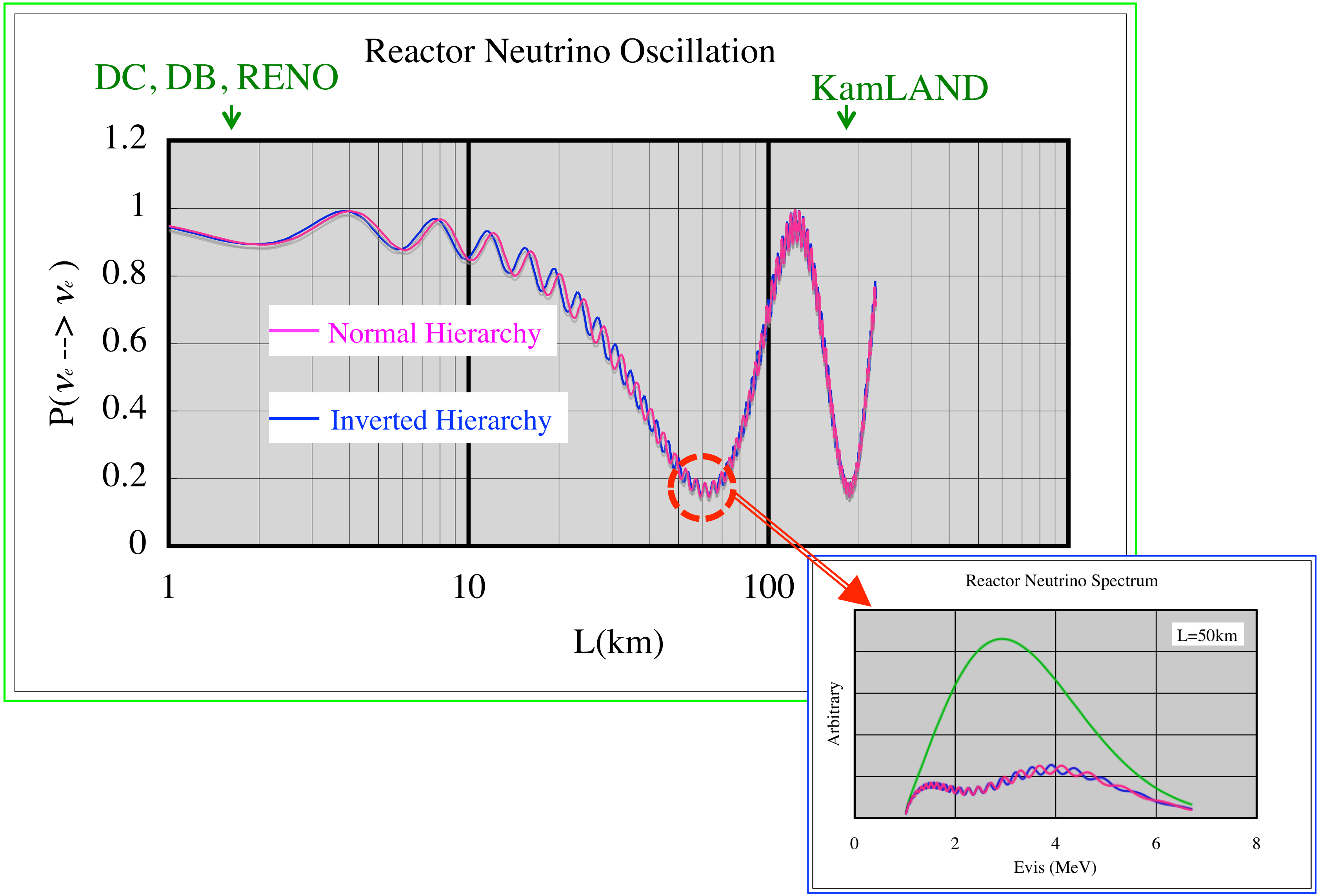
T2K



The spread due to MH is large ➔ easier to determine MH.

A Bit Future Experiments

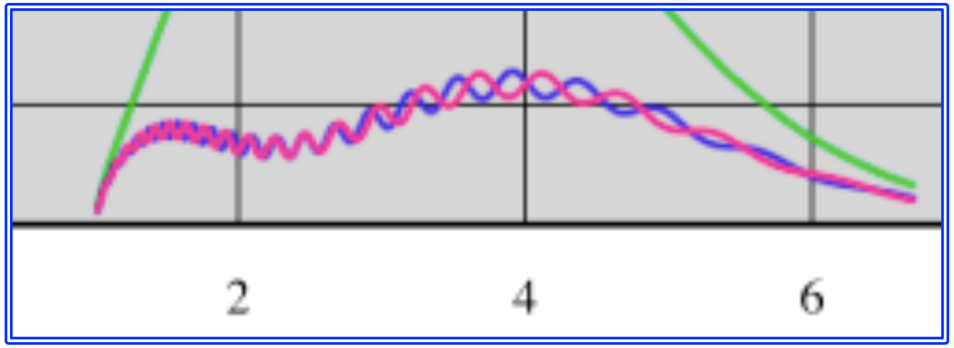
M.H. by medium baseline reactor experiment



Reactor Neutrino Oscillation @L~50km

Principle

Petcov et al., Phys. Lett. B 533, 94 (2002)
S.Choubey et al., Phys. Rev. D 68,113006 (2003)
J. Learned et al., hep-ex/062022
L.Zhan et al., hep-ex/0807.3203
M.Batygov et al., hep-ex/0810.2508



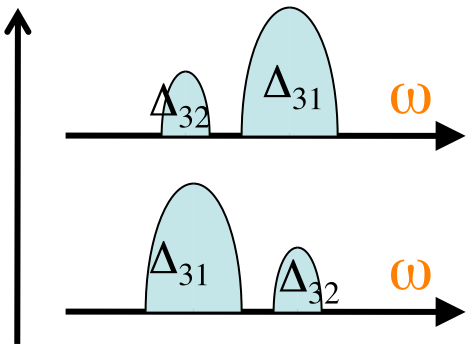
$$\text{Ripple} \propto \sin^2 2\theta_{13}\left(\sin^2\Delta_{31} + \underline{\tan^2\theta_{12}}\sin^2\Delta_{32}\right)$$

It is essential that θ_{12} is not maximum ($\tan^2\theta_{12}$~0.4)

Fourier Trans. => peaks at $\omega = \left|\Delta m^2_{31}\right|, \quad \left|\Delta m^2_{32}\right|$

Smaller peak corresponds to $\left|\Delta m^2_{32}\right|$ larger peak corresponds to $\left|\Delta m^2_{31}\right|$,

Power



: Normal Hierarchy

: Inverted Hierarchy

From L.Wen Nu2014

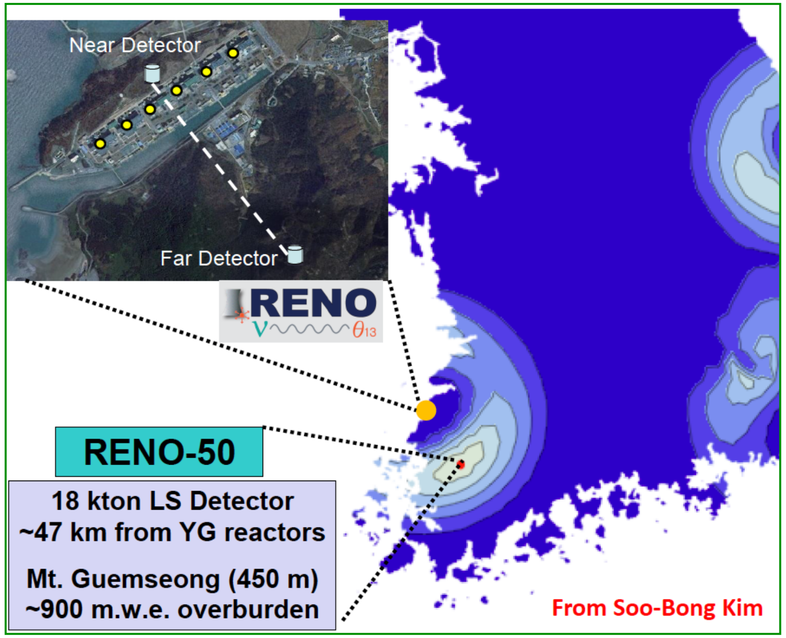
$\geq$ 10ktons of LS detector with energy resolution $\leq 3\%/\sqrt{E(MeV)}$ is required.

2 proposals

JUNO in China

NPP	Daya Bay	Huizhou	Lufeng	Yangjiang	Taishan
Status	Operational	Planned	Planned	Under construction	Under construction
Power	17.4 GW	17.4 GW	17.4 GW	17.4 GW	18.4 GW

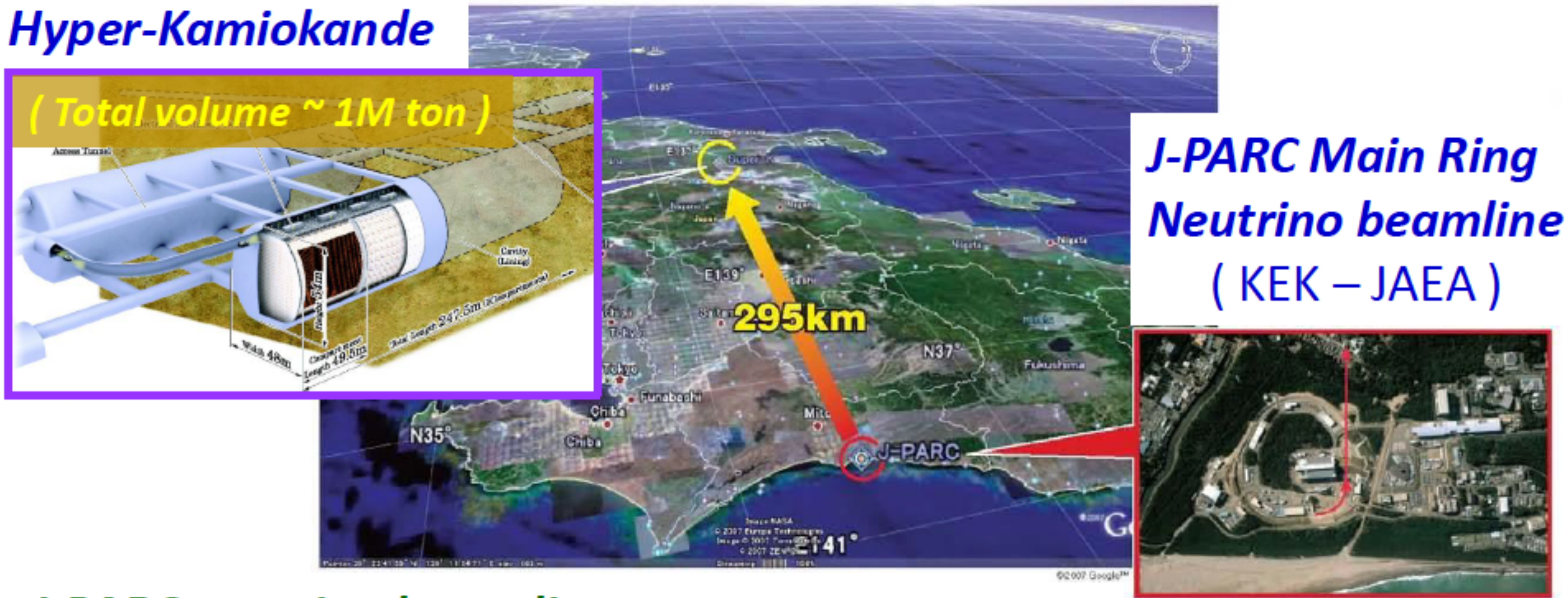
RENO50 in Korea



	KamLAND	JUNO	RENO-50
LS mass	~1 kt	20 kt	18 kt
Energy Resolution	$6\%/\sqrt{E}$	~$3\%/\sqrt{E}$	~$3\%/\sqrt{E}$
Light yield	250 p.e./MeV	1200 p.e./MeV	>1000 p.e./MeV

a bit Future

Hyper-Kamiokande with J-PARC neutrino beam



J-PARC neutrino beam line

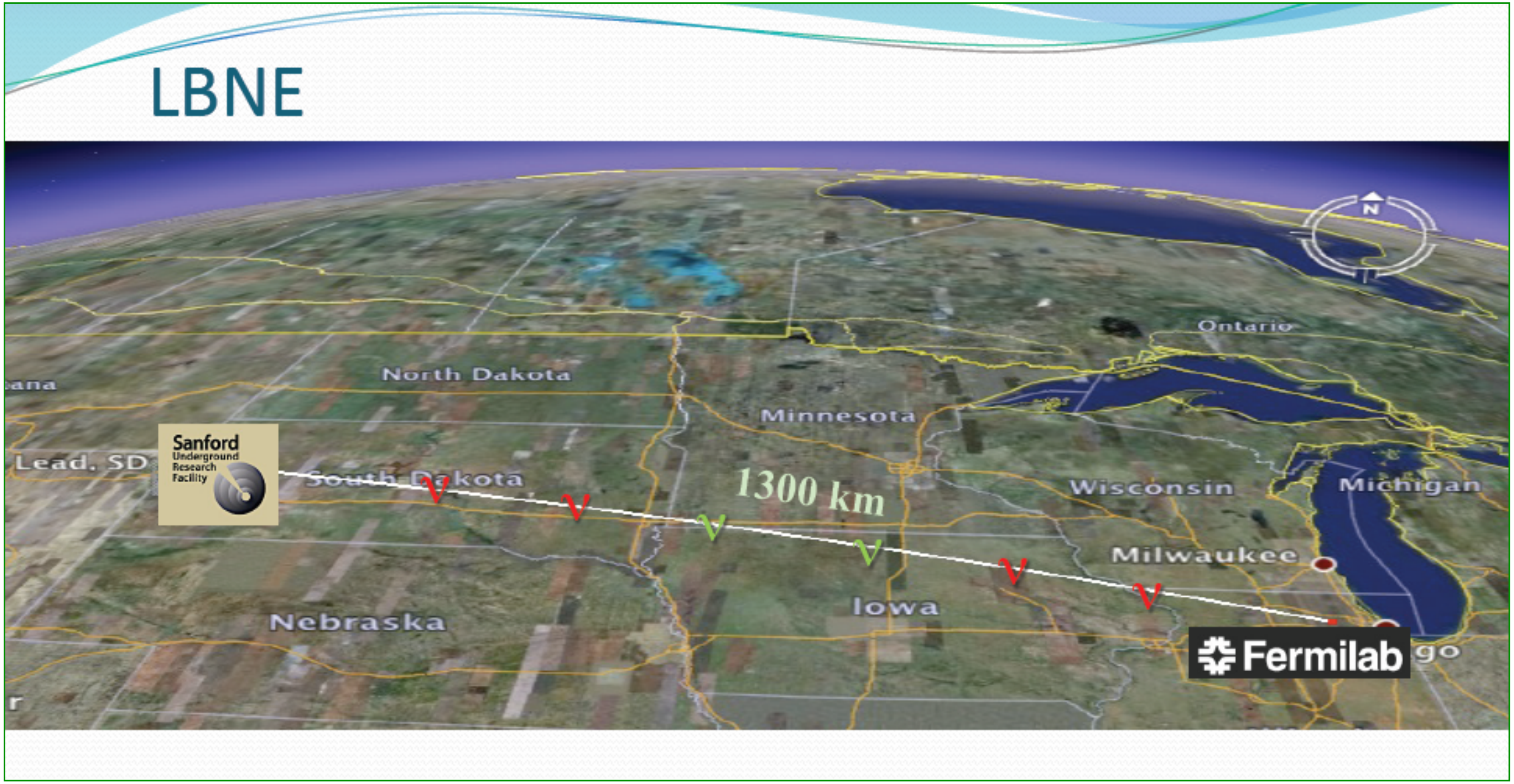
One of the most powerful beamlines in operation and further intensity upgrade (>750kW) is undergoing.

Hyper-Kamiokande

World largest water Cherenkov detector (fid. vol. 560 kt.)

Powerful combination to search for the lepton sector CP violation!

a bit Future

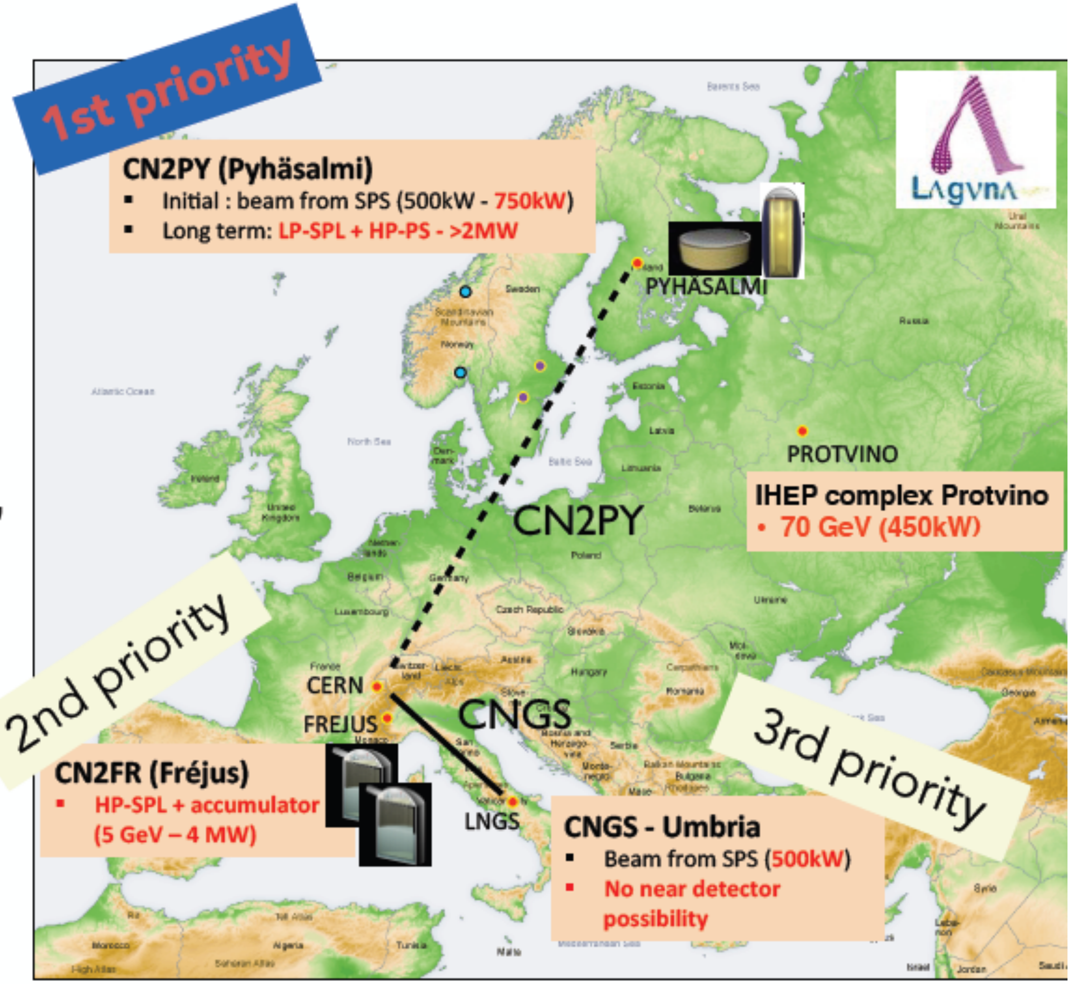

LBNE
North Dakota
Minnesota
Ontario
Lead, SD
Sanford
Underground
Research
Facility
South Dakota
1300 km
Wisconsin
Michigan
Milwaukee
Iowa
Nebraska
Fermilab

a bit Future

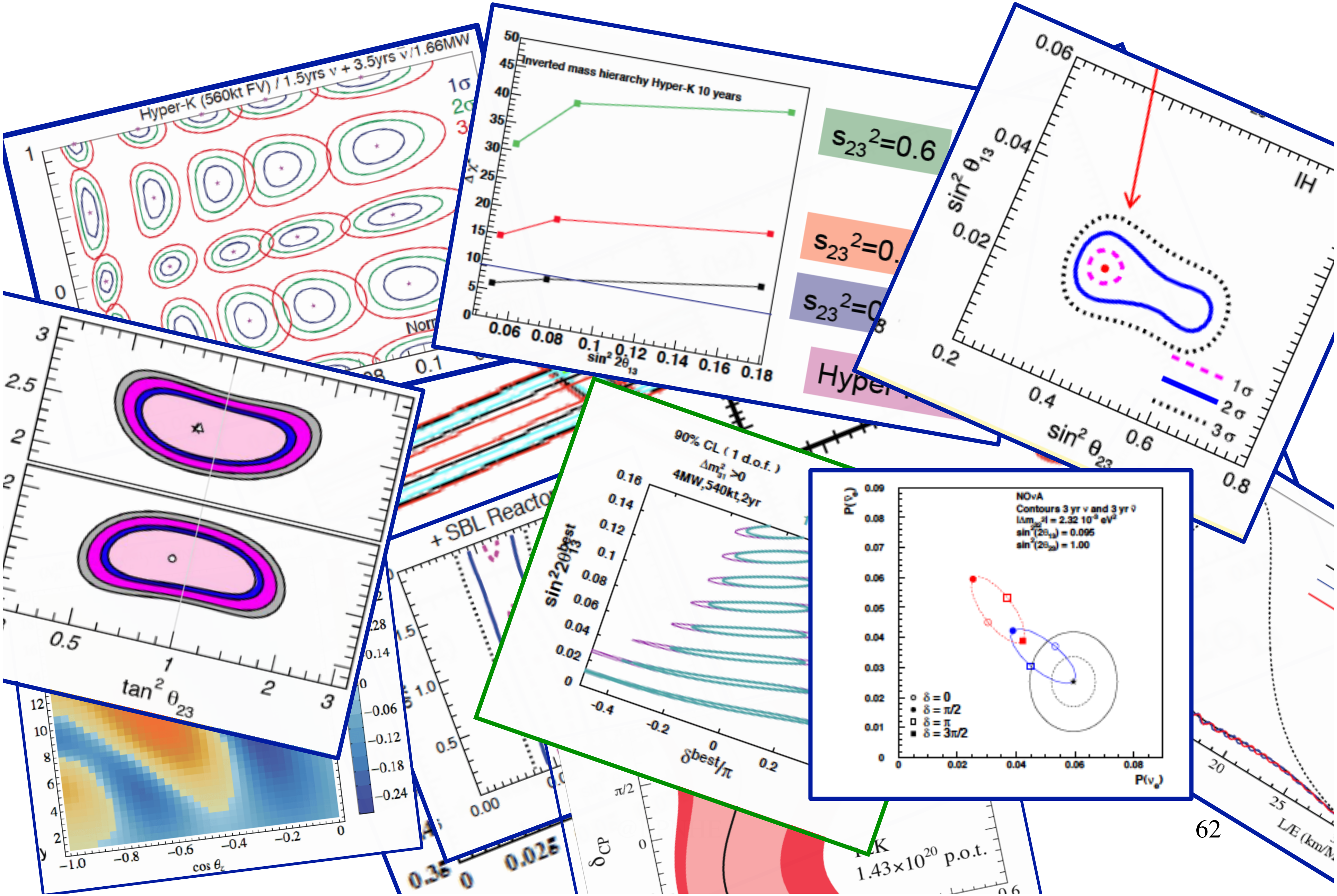
Site prioritisation

Several sites considered in details

- **Pyhäsalmi mine** (privately owned), 4000 m.w.e overburden, excellent infrastructure for deep underground access
- **Fréjus**, nearby road tunnel, 4800 m.w.e. overburden, horizontal access
- **Umbria** (LNGS extension), green site with horizontal access, 2000 m.w.e., CNGS off-axis beam

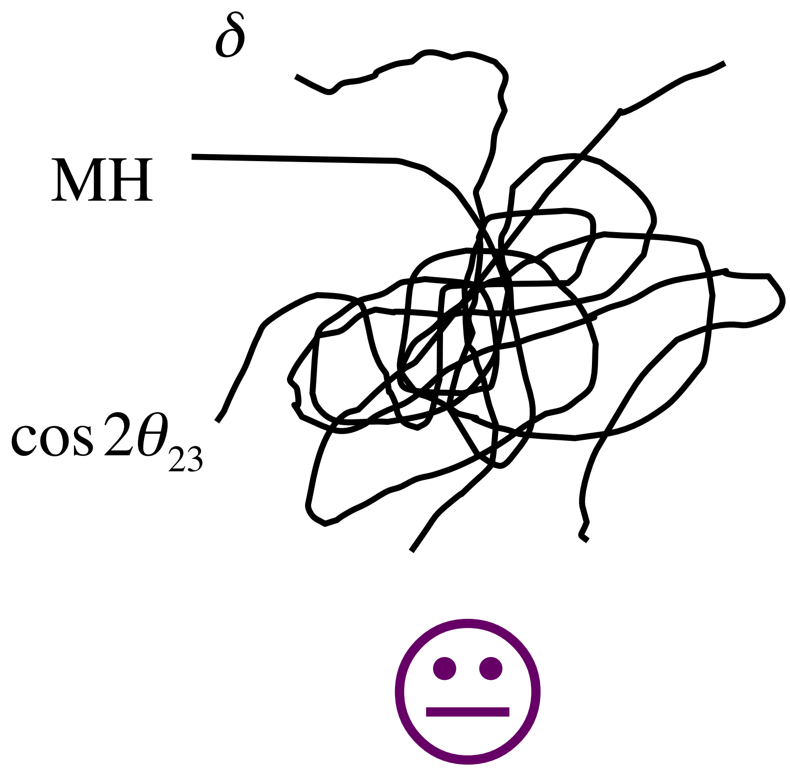


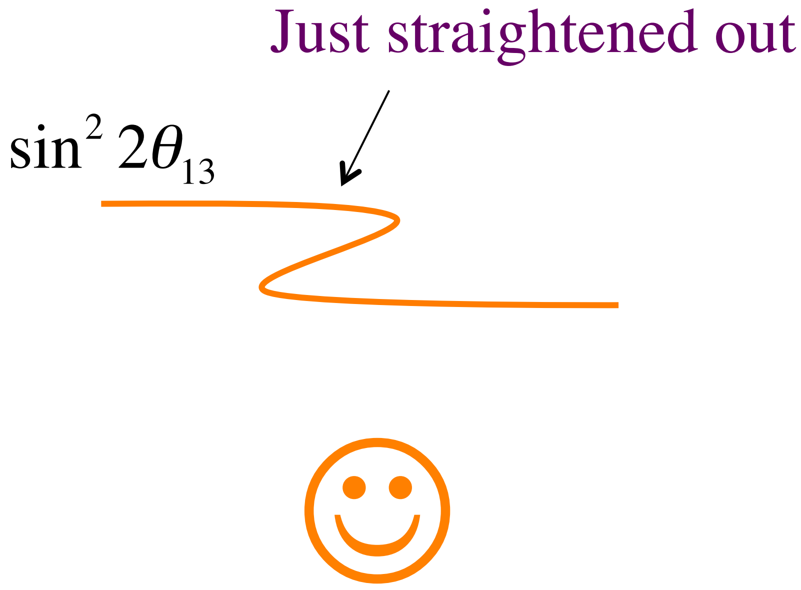
There are many ideas



A view of the ν oscillation study

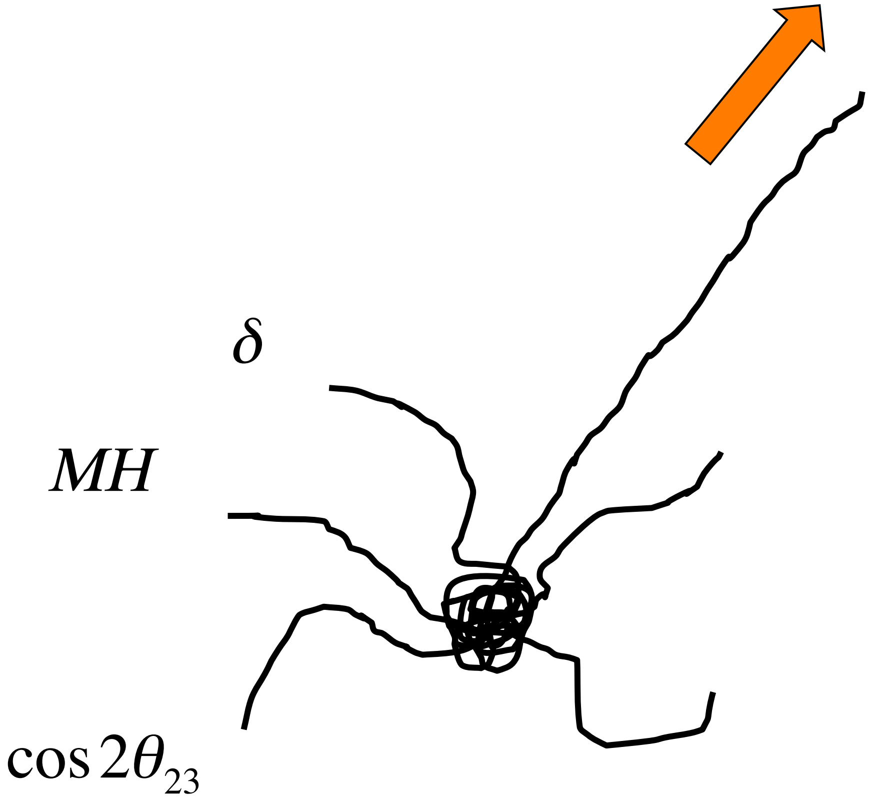
.. It is like releasing tangled threads





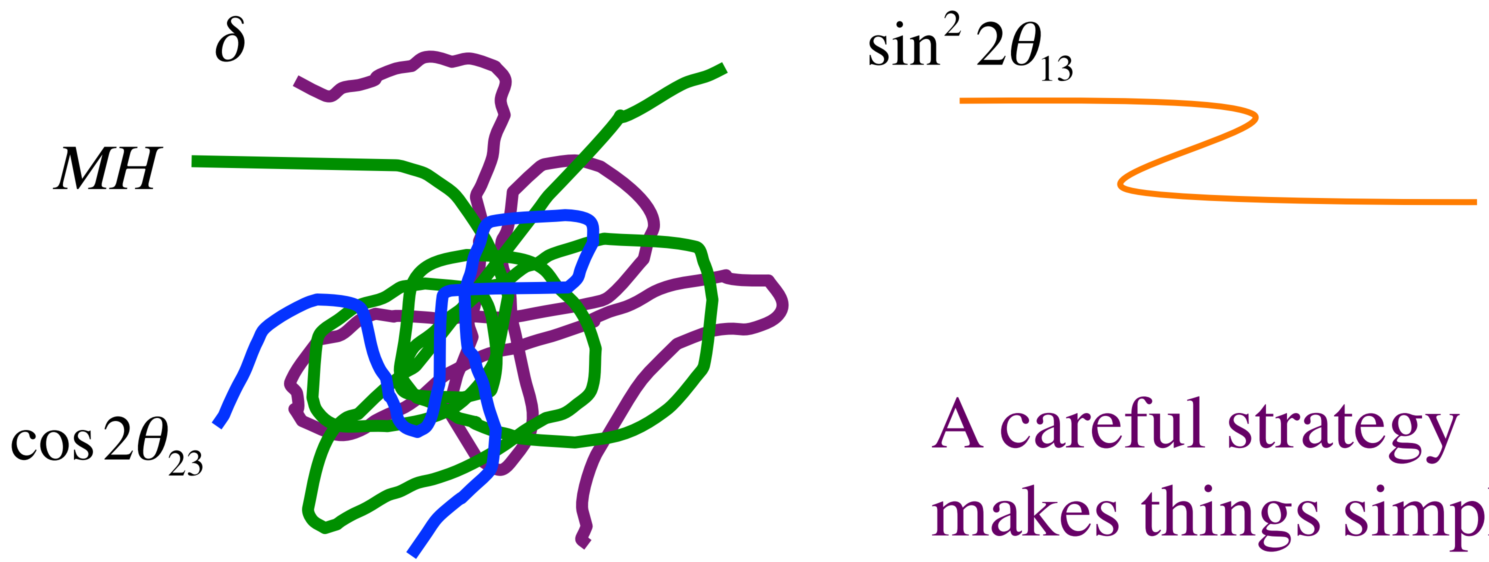
A view of the ν oscillation study

If you pull only one thread,
it makes the tangle tighter.



☹

A view of the ν oscillation study



A careful strategy
makes things simpler

T2K, NOVA, HK, LBNE,
LBNO, Reactor ....

A Golden Scenario

The nature has been amazingly kind to us. ☺
Let's assume she will be kind to us in the future also. ☺

Precise meas. of $\sin^2 2\theta_{13}$ by Reactor (done)

Identification of $\nu\mu \rightarrow \nu e$ Osci. by Accelerator (done)

MH by T2K + Nova +Reactor

$\sin\delta$ by HK / LBNE / LBNO

Neutrino Oscillation Industry

If IH ☺

Determination of All transition amplitudes

Definite m_β measurement @ >50meV

Definite determination of Dirac or Majorana with 15meV sensitivity

Absolute Mass Industry

Summary

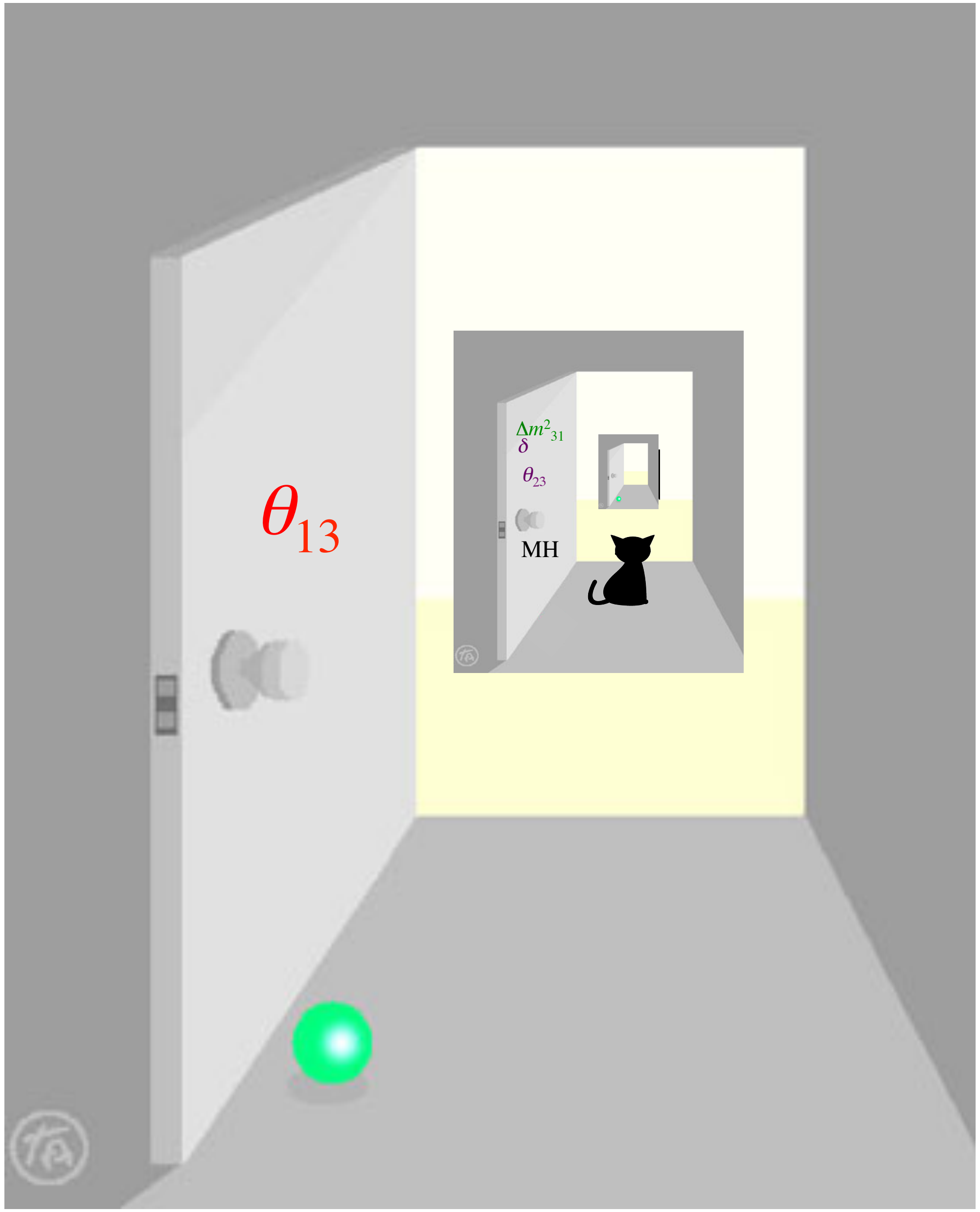
* An important purpose of the N.O. experiments is to measure the transition amplitudes

* θ_{12}, θ_{23}, θ_{13}, Δm^2_{12}, $\left|\Delta \tilde{m}^2_{32}\right|$, $\left|\Delta \tilde{m}^2_{31}\right|$ have been measured.

* Measurements of δ, M.H. become realistic.

* sinδ~-1, N.H. are slightly favored but the central values of short baseline reactor and accelerator ($\nu_\mu \rightarrow \nu_e$) data have slight tension.

* A strategy on how to combine the different experiments is important to efficiently solve the remaining issues.



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