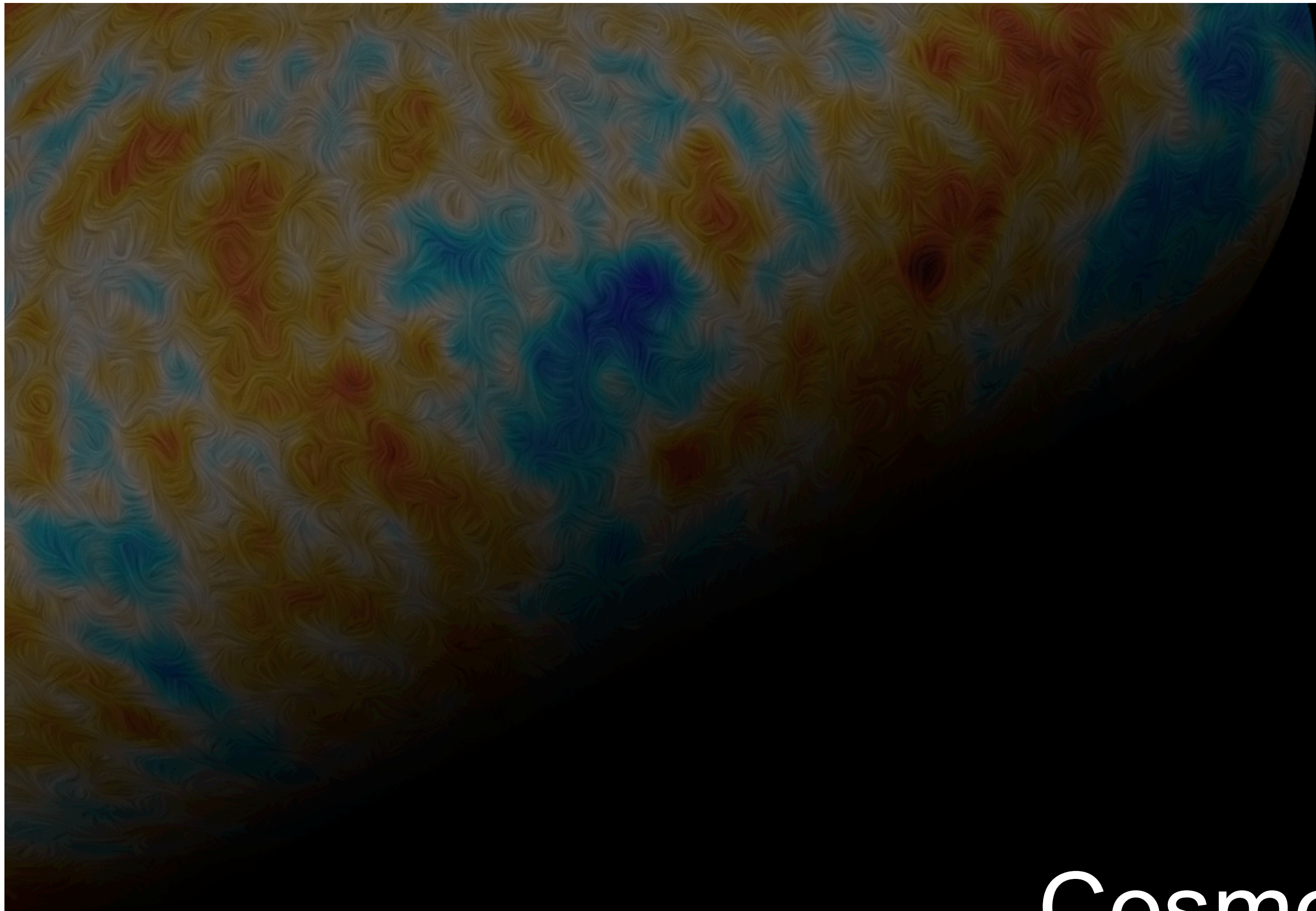
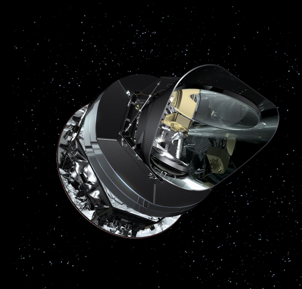




Cosmology

Laurence Perotto

Annecy-Le-Vieux, 2015, 16-22 July

Index

- Introduction
- Part I: The Big Bang Theory
  - Homogeneous Universe
    - (Metric, Friedmann Eqs, Distances, Hubble law, SNIa)
  - Hot Big Bang Model
    - (thermal history, BBN)

1rst course

2nd course

- Part II: The Standard Model of the Cosmology
  - Cosmic Inflation
  - A pinch of phenomenology of the cosmological perturbations
  - observables (LSS, CMB)

- Part III: Recent results
  - Planck 2013
  - BICEP (2014-2015)

3rd course

What is Cosmology ?

- to describe the global properties of the Universe

  matter-energy contents, age, origins, future, …
  description of matter distribution at large scales

- trailer: a panorama with an historical point-of-view

A recent story

In 1920's, an essentially theoretical activity based on:

- General Relativity : geometry <-> contents
- Cosmological Principle: the Universe is the same everywhere and in all directions

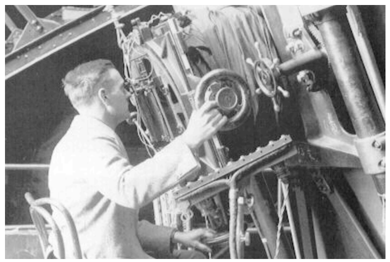
-> Predictions of the Universe evolution (Alexander Friedmann, Georges Lemaître)

Several models are possible and considered:

- stationnary models (cf initial argument for the cosmological constant)
- expanding models

First observations of the «nebulae reddening» (Henrietta Leavitt, Edwin Hubble)

H. Leavitt in Harvard, Massachusetts (vers 1920)



E. Hubble at the Mont Wilson Obs., Californy (1922)

The Hubble's law

Doppler effect : $$z \equiv \Delta\lambda/\lambda = v_r/c$$

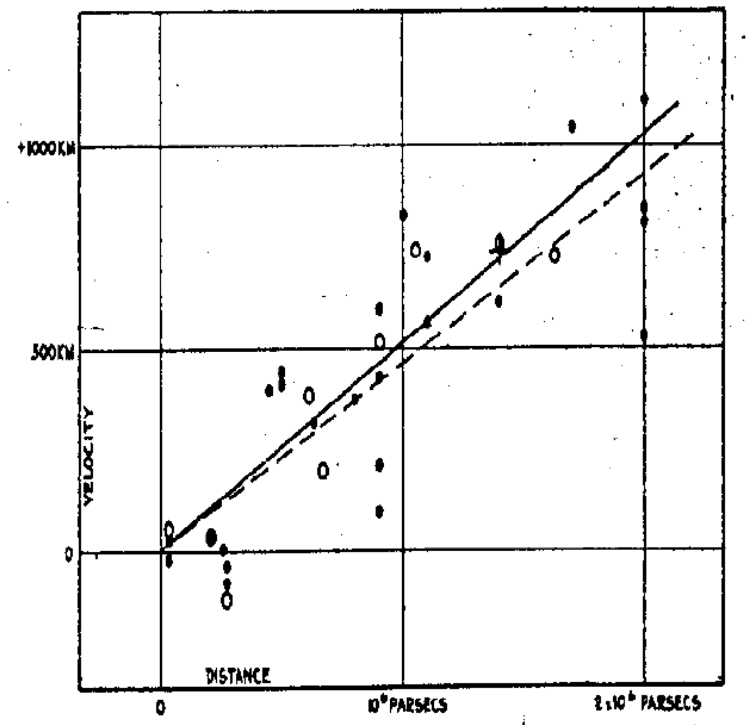
$$z << 1$$

Standard candles (known absolute luminosity)

$$L = l \times (4\pi r^2)$$

Hubble Law $$\vec{v} = H \ \vec{r}$$

$$H = 100 \ h \ \mathrm{km \ s^{-1} Mpc^{-1}}$$

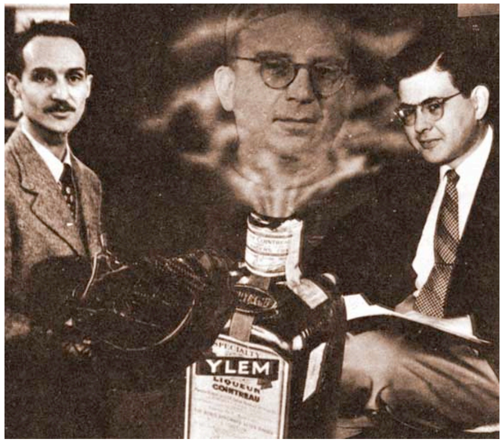


The Big Bang Theory

• expanding universe models are sustained by the observations of galaxy redshifts
• Ralph Alpher, (Hans Bethe), Georges Gamow described the primordial nucleosynthesis (1948) : light nuclei (Hydrogen, Deuterium, Helium, …) forms in the first minutes after the Big Bang

G. Gamow

R. Herman



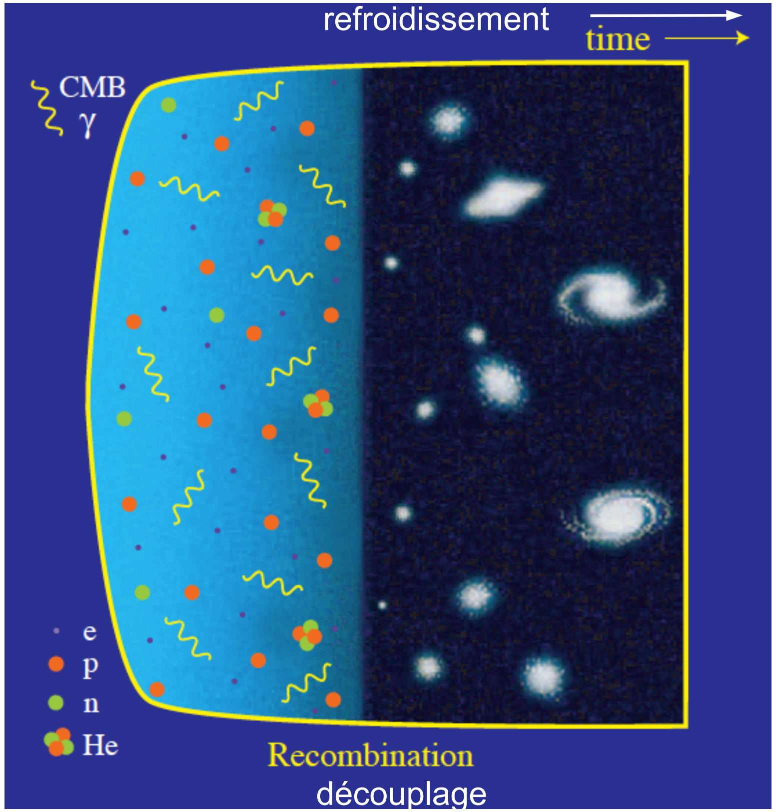
R. Alpher

• Ralph Alpher, Robert Herman, observational consequencies:

Thermal echo of the Big Bang (hot dense primordial plasma) should be seen as a radiation in the microwave frequency domain.

The first light of the Universe

Early Universe: dense and hot plasma, opaque
expansion -> cooling
recombination: atoms form
The Universe becomes transparent: this is the photon decoupling

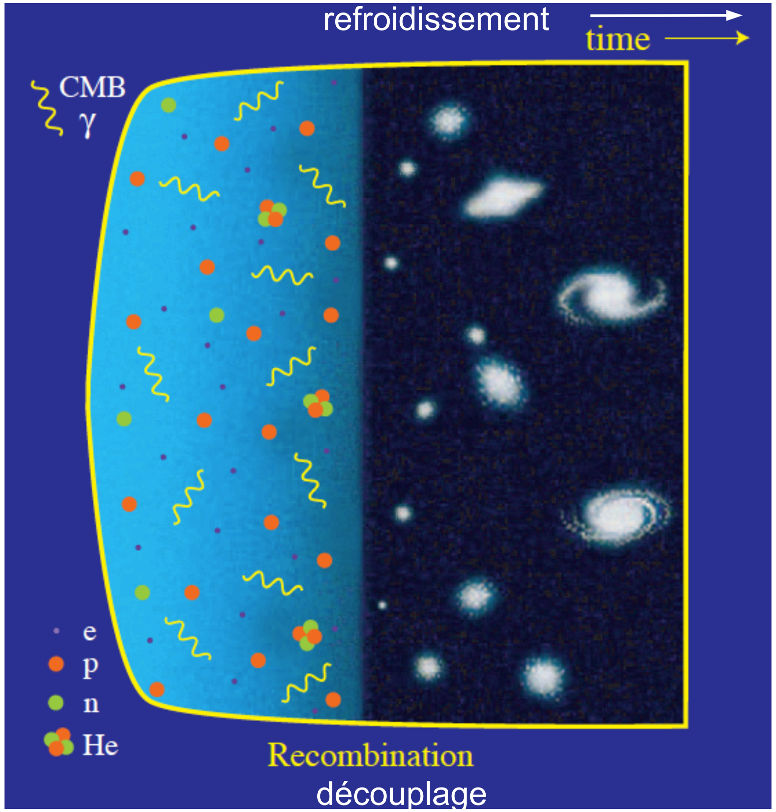


W. Hu

A volume of the primordial Universe…

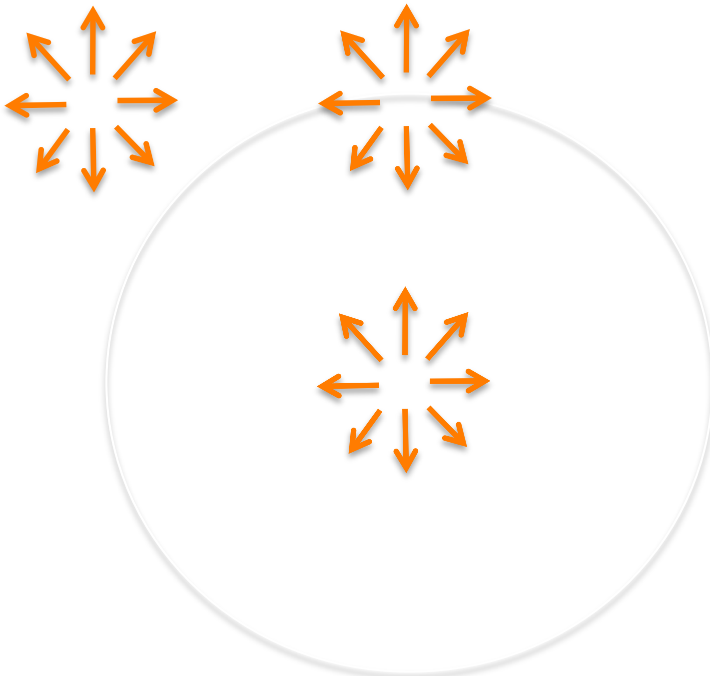
The first light of the Universe

Early Universe: dense and hot plasma, opaque
expansion -> cooling
recombination: atoms form
The Universe becomes transparent: this is the photon decoupling



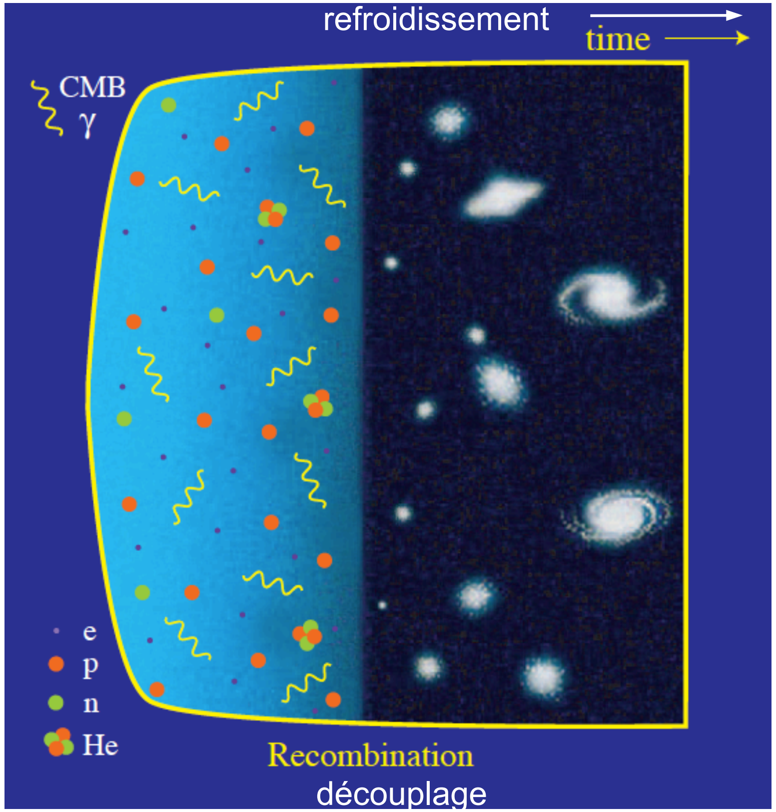
W. Hu

After 380 000 years, photon decouples…



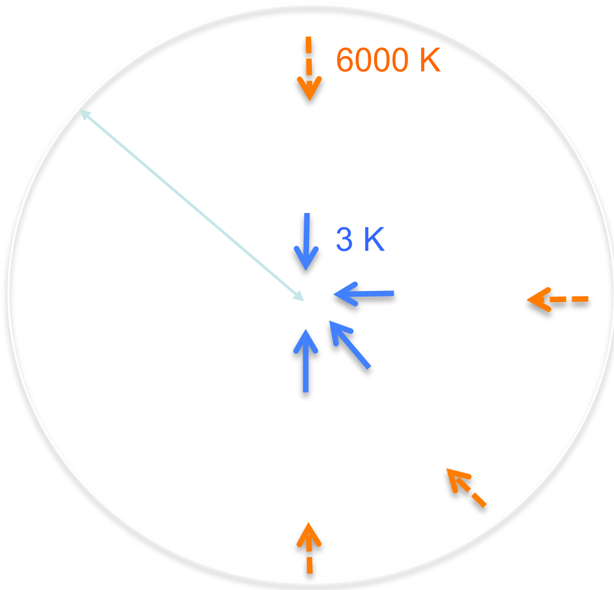
The first light of the Universe

Early Universe: dense and hot plasma, opaque
expansion -> cooling
recombination: atoms form
The Universe becomes transparent: this is the photon decoupling



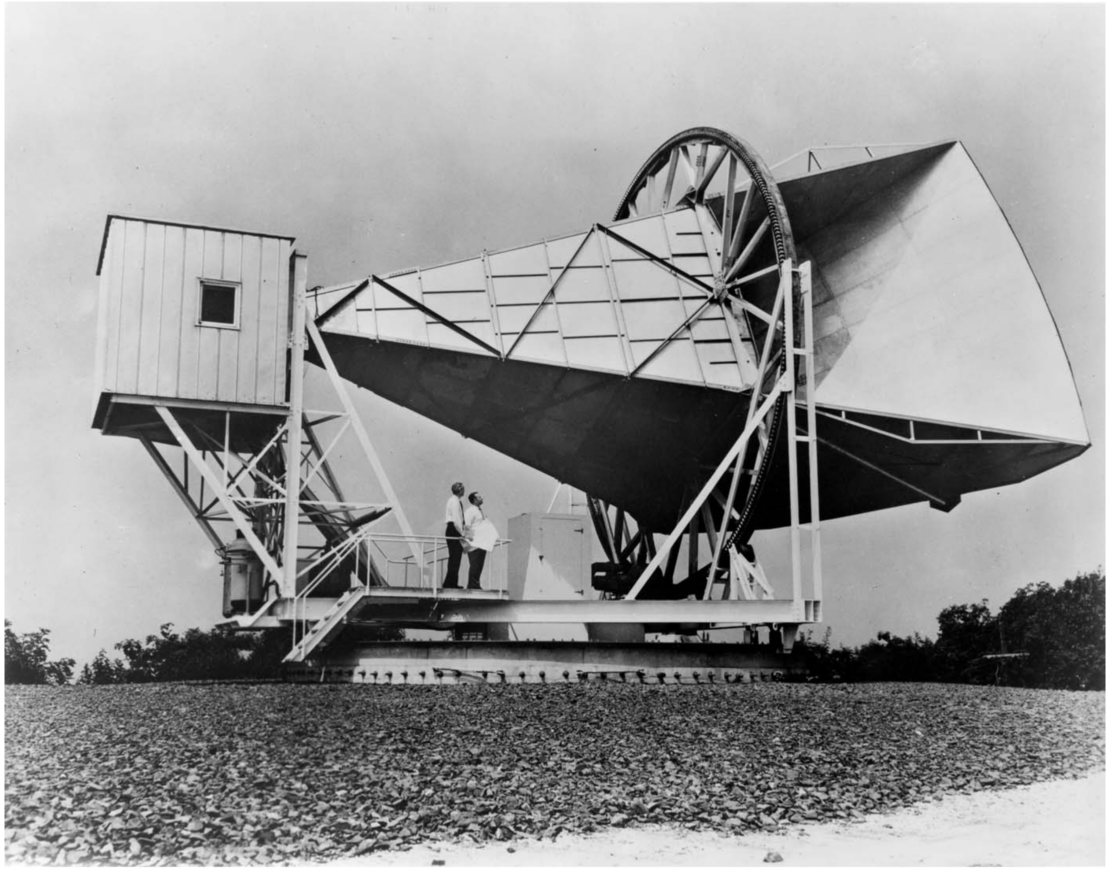
W. Hu

Then only extremely cooled…



Fifteen years later, in the county…

1964 : A. Penzias et A. W. Wilson, radio-astronoms at Bell lab., measured a « noise »



1965 : R. Dicke, P.J.E. Peebles, P.G. Roll, D.T. Wilkinson (MIT) recalculed the Alpher & Herman prediction and planned on the construction of an antenna for testing their hypothesis.

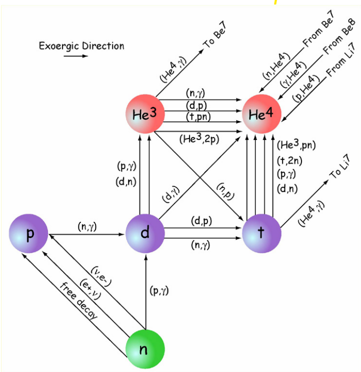
1978 : Nobel price

Observational cosmology

Since 90's, cosmology has become «observational».
Comparing models to more and more precise observations, the Standard model of the Cosmology emerged. Also called Concordance Model as it fully describes all observations

The founding observations

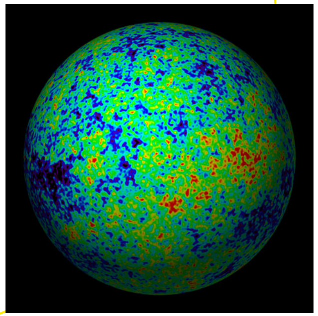
Universe expansion measures (galaxy redshifts survey, Supernovea luminosity-distance measurements)



Big Bang Nucleosynthesis

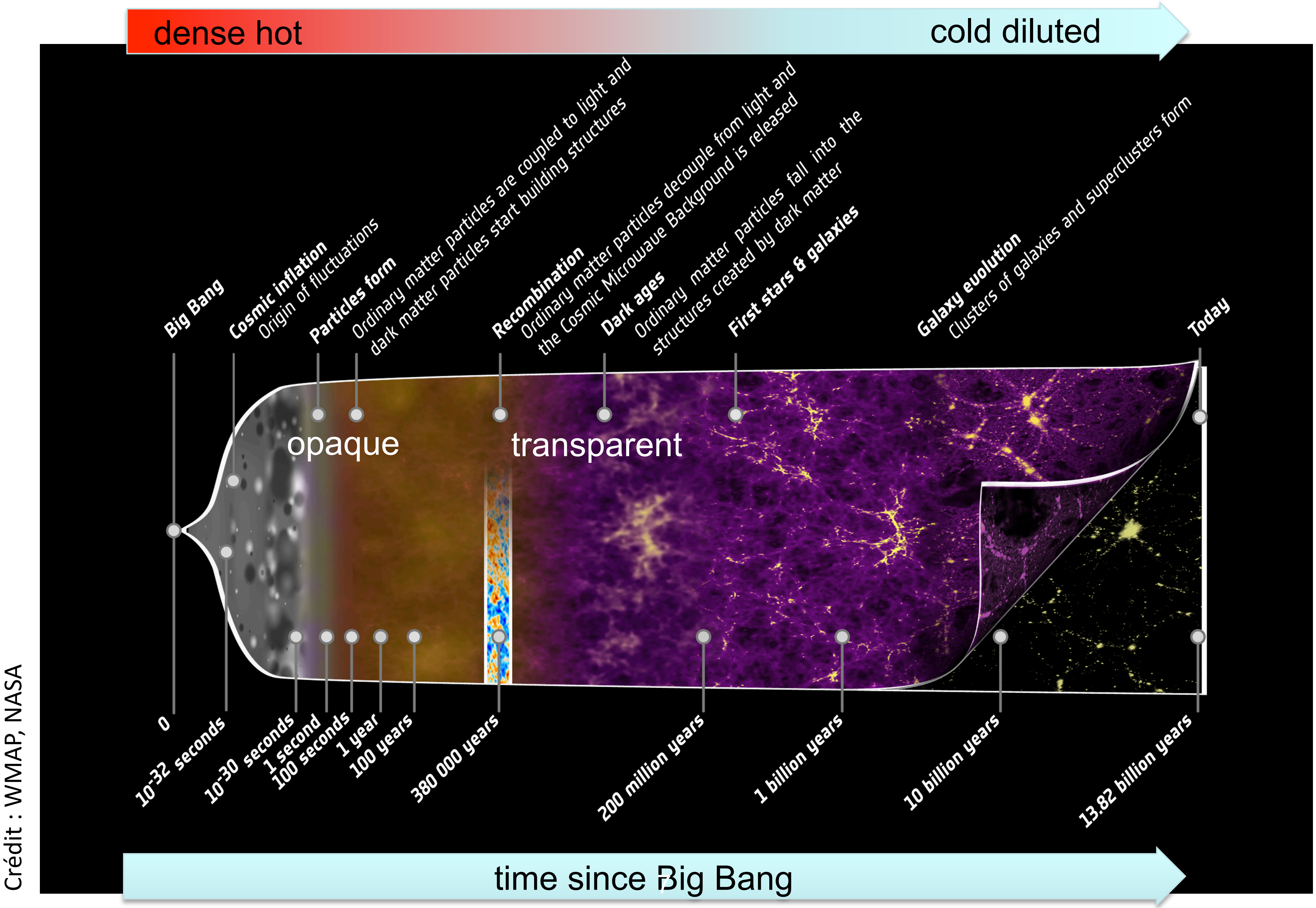
Standard model of the Cosmology :

Hot Big Bang
& growth of the Large Scale Structures (LSS) by gravitational collapse
in an expanding Universe



Cosmic Microwave Background
Crédit : WMAP, NASA

The Universe has an History



Introduction

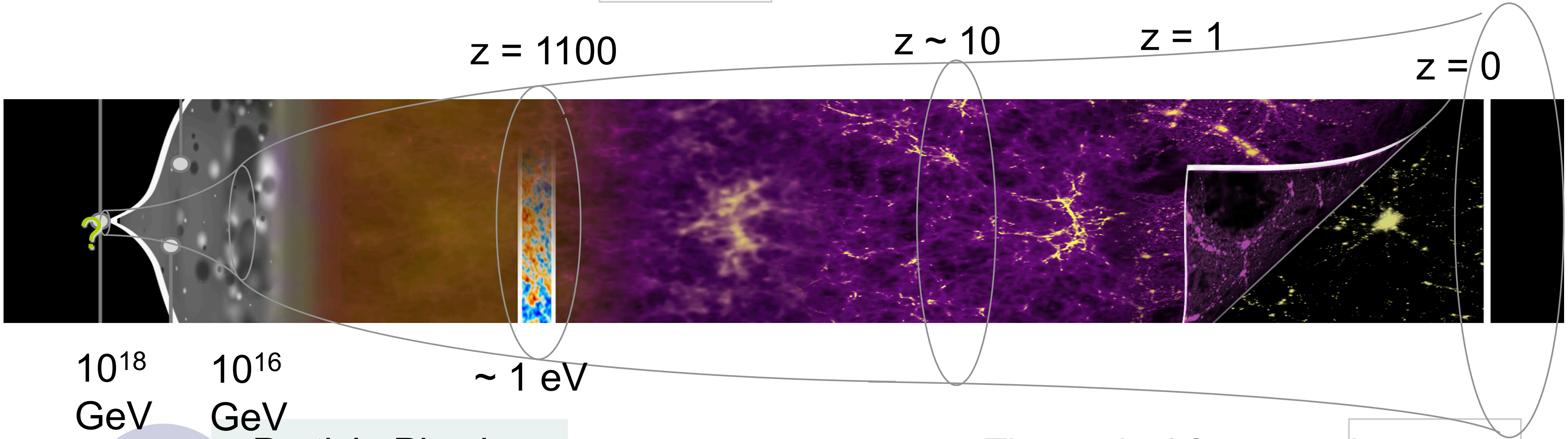
Panorama

Cosmological probes

LSS probes

SNIa

Cosmic Microwave background



Theoretical frameworks

Particle Physics

Q. Gravity
String theory
…

GR: Background evolution

GR : Linear perturbation theory

Non-Linear simulations

Outlines

- Part I: The Big Bang Theory
  - Homogeneous Universe
    - (Metric, Friedmann Eqs, Distances, Hubble law, SNIa)
  - Hot Big Bang Model
    - (thermal history, BBN)

- Part II: The Standard Model of the Cosmology
  - Cosmic Inflation
  - A pinch of phenomenology of the cosmological perturbations
  - observables (LSS, CMB, BAO)

- Part III: Recent results
  - Planck 2013
  - BICEP (2014-2015)
  - perspectives

The Big Bang Theory

Part I: The Big Bang Theory

Homogeneous Universe

(Metric, Friedmann Eqs, Distances, Hubble law, SNIa)

Hot Big Bang Model

(thermal history, BBN)

Theoretical Framework

General Relativity (GR) theory (Einstein 1915)

Gravitation is described as curvature of the 4D space-time

- the space-time geometry is defined by a *metric* :

$$ds^2 = \sum_{\mu=1}^{4} \sum_{\nu=1}^{4} g_{\mu\nu} dx^\mu dx^\nu$$

useful to tell us the shortest distance between 2 points (= *geodesics*)

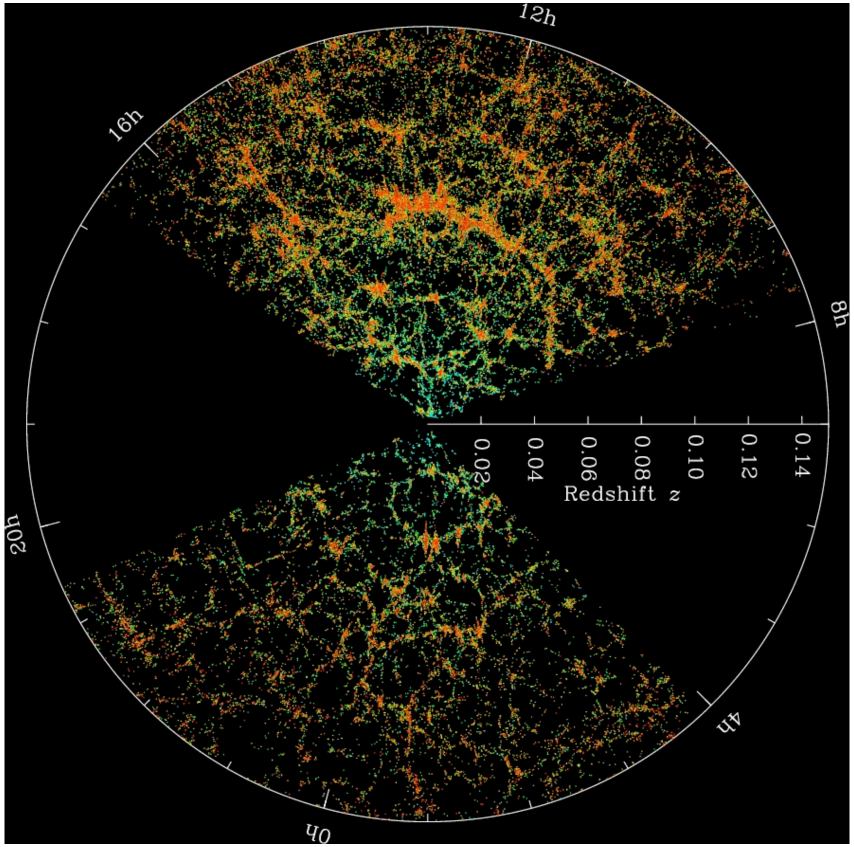
- Einstein field equations describe how the *space-time geometry* is curved by *matters*

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

«tricky» in general…
if symmetries are assumed, they can be applied to the whole Universe

The cosmological principle

The Universe is homogeneous and isotropic at large scales



Sloan Digital Sky Survey (SDSS) galaxies
(color shows local density)

Homogeneous if smoothing in a sphere > 100 Mpc

(a parsec is about 3×10^{16} meters)
(larger structures of the Universe = galaxy clusters are of the order of few 10 Mpc)

Friedmann-Lemaître-Robertson-Walker Metric

The Cosmological Principle imposes the form of the space-time metric

- Existence of a *covoming coordinate system* (that of an ensemble of free-falling observers *at rest* with respect to the homogeneous cosmologic fluid, who share the *same definition of the time*)
- At a given time, the curvature of the Universe should be the same everywhere

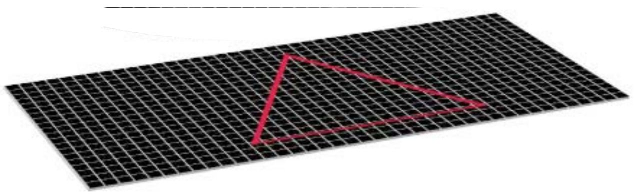
The most general form is :

$$ds^2 = -cdt + a(t)^2 \left[\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2 \right]$$

scale factor

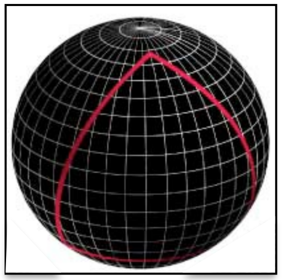
spatial curvature

k = 0



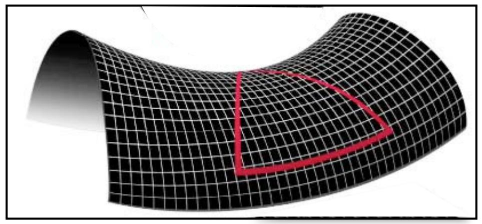
flat Euclidian

k >0



3-sphere
closed

k < 0



3-hyperboloid
open

Propagation of light-rays in the FLRW Universe

- Light propagation equation
  - Photons propagate along null geodesics: ds = 0
  - One stands at the origin of a comoving coordinate system and receives a photon at (t_r, 0, 0, 0), which has been emitted at (t_e, r_e, 0, 0). The photon trajectory is:

$$\int_{r_\mathrm{e}}^{0} -\frac{dr}{\sqrt{1-kr^2}} = \int_{t_\mathrm{e}}^{t_\mathrm{r}} \frac{cdt}{a(t)}$$

- A new definition of the redshift
  - Writing the previous equation for two consecutive wavecrests of a monochromatic light we receive from a galaxy, one obtains:

$$z = \frac{\Delta\lambda}{\lambda} = \frac{\lambda_r - \lambda_e}{\lambda_e} = \frac{a(t_r)}{a(t_e)} - 1$$

Propagation of light-rays in the FLRW Universe

- A new definition of the Hubble parameter
  - In the nearby Universe, Hubble Law gives : cz = v = Hr
  - Applying the previous redshift definition to a nearby source, emitting at t_0-dt :

$$z \simeq \frac{\dot{a}(t_0)}{a(t_0)} \frac{L}{c}$$

  - We recover Hubble law. Even comoving objects (at rest) undergo the Hubble flow, driven by the Universe expansion.
  - Hubble parameter :

$$H(t) = \frac{\dot{a}(t)}{a(t)}$$

  - H_0 is the current expansion rate, the parameter measured by Hubble.

$$H = 100\ h\ \mathrm{km\ s^{-1} Mpc^{-1}}$$

The Friedmann-Lemaître equations

- Ingesting the FLRW metric in the Einstein equation: $G_{\mu\nu} = 8\pi G T_{\mu\nu}$
  - both tensors should be diagonal and isotropic
  - energy-momentum tensor should be that of a smooth perfrect fluid

$$T_{\mu\nu} = (\rho + P)\, v_\mu v_\nu + P g_{\mu\nu}$$

- The Friedmann equations are:

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} + \frac{\Lambda}{3}$$

$$\frac{\ddot{a}}{a} = \frac{-4\pi G}{3}\,(\rho + 3P) + \frac{\Lambda}{3}$$

Multi-component fluid

- A fluid is described by a state equation $P = w\rho$
- several flavours:

$w = 0$	matter (pressureless)
$w = \frac{1}{3}$	radiation (radiation pression)
$w = -1$	Cosmological constant

- Local energy-momentum conservation

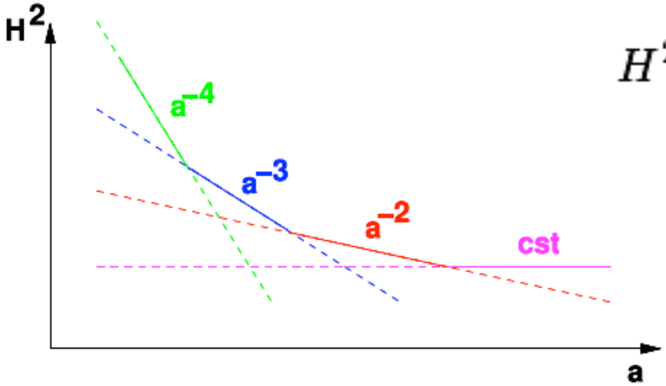
$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + P) = 0 \longrightarrow \rho \propto a^{-3(1+w)}$$

NR particles (matter)	$w = 0$	$\rho \propto a^{-3}$	dilution in volume
UR particles (radiation)	$w = \frac{1}{3}$	$\rho \propto a^{-4}$	extra-dilution due to the redshifting
Cosmological constant	$w = -1$	$\rho \quad \mathrm{constant}$	negative pressure and no dilution

Curvature $\rho_k \equiv \frac{-k}{a^2}\,, \quad \rho_k \propto a^{-2}\,, \quad w = -\frac{1}{3}$

Expansion history of the Universe

- Friedmann law:



$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi\mathcal{G}}{3}\rho_{\rm R} + \frac{8\pi\mathcal{G}}{3}\rho_{\rm M} - \frac{kc^2}{a^2} + \frac{\Lambda}{3}$$

scales as : a^{-4} a^{-3} a^{-2} 1

- Expansion dynamics
  - Radiation domination (RD)
    - decelerated expansion

$$\frac{\dot{a}^2}{a^2} \propto a^{-4}, \qquad a(t) \propto t^{1/2}, \qquad H(t) = \frac{1}{2t}$$

  - Matter domination (MD)
    - Slower decelerated expansion

$$\frac{\dot{a}^2}{a^2} \propto a^{-3}, \qquad a(t) \propto t^{2/3}, \qquad H(t) = \frac{2}{3t}$$

  - Λ domination (ΛD)
    - Exponentially accelerated expansion

$$\frac{\dot{a}^2}{a^2} \text{ constant} \qquad a(t) \propto \exp(\Lambda t/3), \qquad H = \sqrt{\Lambda/3}.$$

The cosmological parameters

- Multi-fluid first Friedmann equation

$$1 = \frac{8\pi G}{3H^2}\,\rho_\mathrm{r} + \frac{8\pi G}{3H^2}\,\rho_\mathrm{m} - \frac{k}{a^2H^2} + \frac{1}{3H^2}$$

- Critical density and density parameters

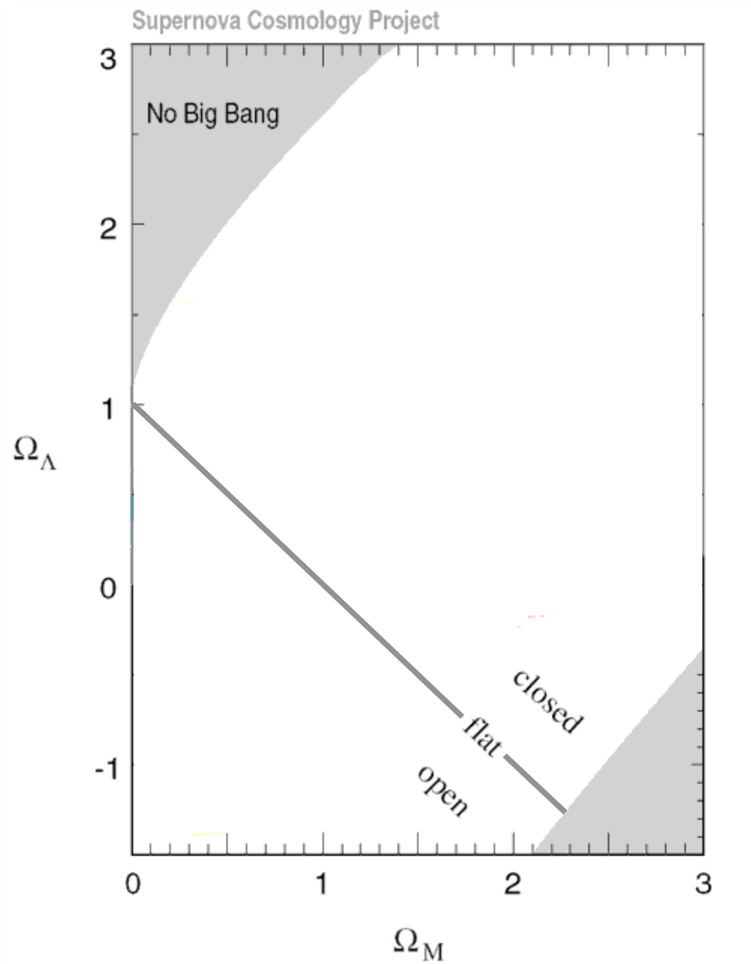
$$\rho_\mathrm{c} = \frac{3H^2}{8\pi G}$$

$$\Omega_\mathrm{X} = \frac{\rho_\mathrm{X}}{\rho_\mathrm{c}}\,, \quad \Omega_\mathrm{k} = \frac{-k}{a^2H^2}\,, \quad \Omega_\Lambda = \frac{1}{3H^2}$$

- Reduced first Friedmann equation

$$1 = \Omega_\mathrm{r} + \Omega_\mathrm{m} + \Omega_k + \Omega_\Lambda$$

$$\Omega_\mathrm{tot} = 1 - \Omega_k$$



The cosmological parameters

Fluid Density evolution: $$\frac{\Omega_{\rm X}}{\Omega_{\rm X}^{(0)}} = \left(\frac{a}{a_0}\right)^{-3(1+w_{\rm X})} \left(\frac{H_0}{H}\right)^2$$

The evolution of the expansion:

$$H(a)^2 = H_0^2 \left[\sum_{\rm X} \Omega_{\rm X}^{(0)} \left(\frac{a}{a_0}\right)^{-3(1+w_{\rm X})} + \Omega_k^{(0)} \left(\frac{a}{a_0}\right)^{-2} + \Omega_\Lambda\right]$$

The whole expansion history of the Universe is described by 5 parameters

$$\mathrm{H}_0, \quad \Omega_{\rm r}^{(0)}, \quad \Omega_{\rm m}^{(0)}, \quad \Omega_{\rm k}^{(0)}, \quad \Omega_\Lambda$$

Dynamical evolution

The second FL equation rewrites:

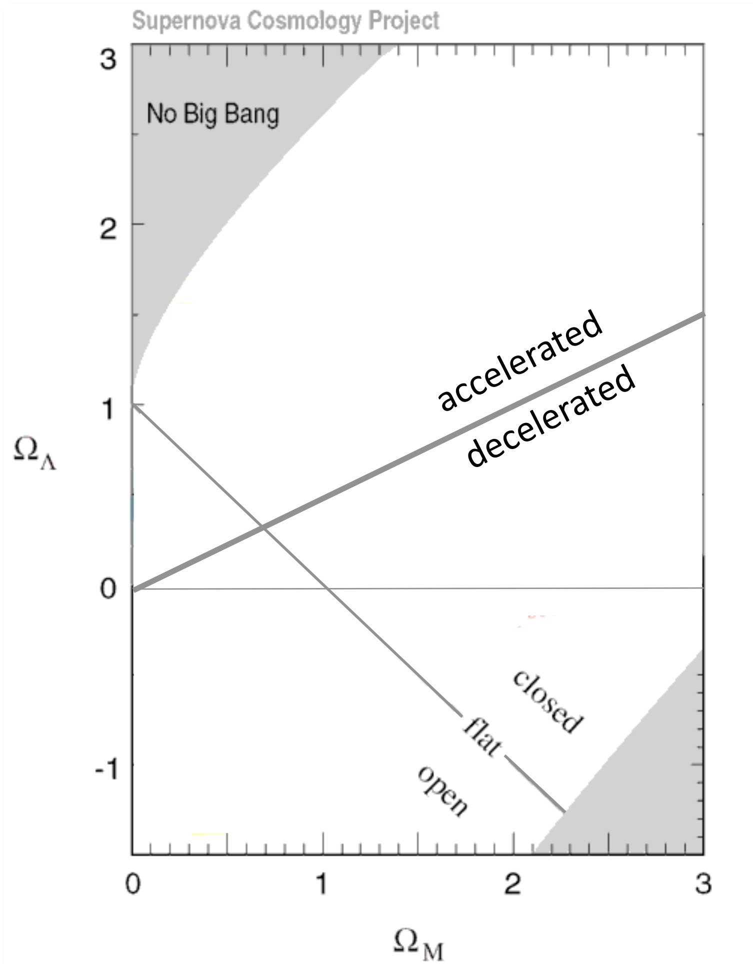
$$\frac{\ddot{a}}{aH^2} = -\frac{1}{2}\sum_{\mathrm{X}}(1+3w_{\mathrm{X}})\Omega_{\mathrm{X}} + \Omega_\Lambda$$

The deceleration parameter:

$$q_0 \equiv - \left.\frac{\ddot{a}}{aH^2}\right|_{t=t_0}$$

Measuring the expansion dynamics:

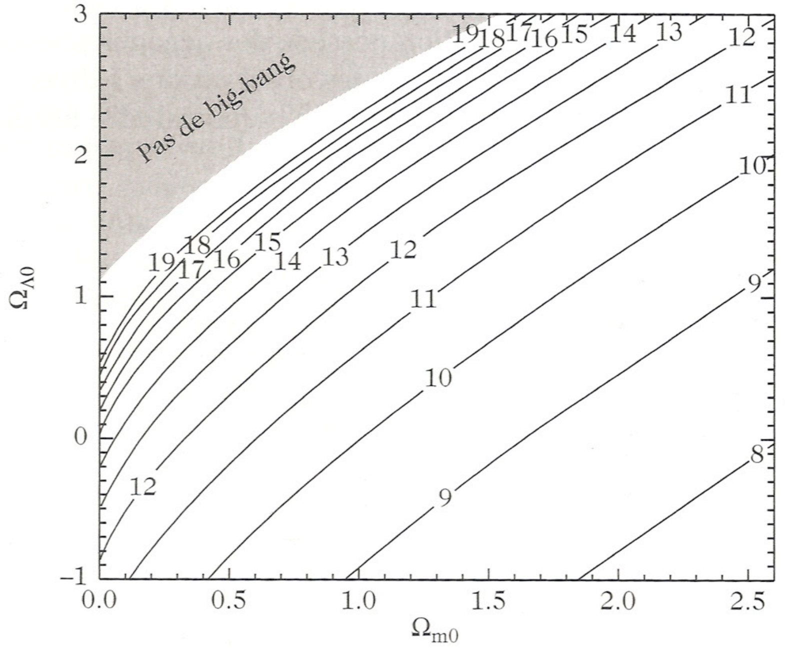
$$q_0 = \frac{1}{2}\Omega_{\mathrm{m}} - \Omega_\Lambda$$



Laurence Perotto

Age of the Universe

Peter&Uzan



Integrating over the whole history:

$$H = \frac{\dot{a}}{a} = \frac{da}{adt} \longrightarrow dt = \frac{da}{aH(a)}$$

RD or MD

$$t_0 = \int_0^1 \frac{da}{aH(a)} = \frac{2}{3(1+w)H_0}$$

Hubble time, Hubble radius:

$$R_{\rm H} = cH^{-1}$$

Useful rough estimate of the maximum scale on which physical phenomenon can happen

Comoving distance

- The comoving distance is

$$\chi(r_e) = \int_0^{r_e} \frac{dr}{\sqrt{(1-kr^2)}}$$

when multiplied by a_0, this is the distance to a comoving source emitting light at (t_e, r_e)

it has solutions depending on the curvature:

$$\chi(r) = \begin{cases} sin^{-1}(r) & \text{if k} = 1 \\ r & \text{if k} = 0 \\ sinh^{-1}(r) & \text{if k} = -1 \end{cases}$$

one defines

$$f_k(x) = \begin{cases} sin(x) \\ x \\ sinh(x) \end{cases} \text{if k} = \begin{cases} 1 \\ 0 \\ -1 \end{cases}$$

so that $f_k\left(\chi(r)\right) = r$

- Comoving distance to redshift relation:

$$\chi(r_e) = \int_{t_e}^{t_0} \frac{cdt}{a(t)} = \int_{a_e}^{a_0} \frac{cda}{a^2 H(a)} = \int_0^{z_e} \frac{cdz}{a_0 H(z)}$$

Luminosity distance

- The observed flux emitted by a distant source defines a distance:

$$\phi^{\mathrm{obs}} \equiv \frac{L^{\mathrm{source}}}{4\pi d_{\mathrm{L}}^2}$$

The observed flux is

$$\phi^{\mathrm{obs}} = \frac{L^{\mathrm{obs}}}{4\pi \left(a_0 f_k(\chi)\right)^2}$$

The observed total luminosity is redshifted by the expansion

$$L^{\mathrm{obs}} = \frac{\Delta E^{\mathrm{obs}}}{\Delta t^{\mathrm{obs}}} = \frac{\frac{a}{a_0}\Delta E^{\mathrm{source}}}{\frac{a_0}{a}\Delta t^{\mathrm{source}}} = (1+z)^2 L^{\mathrm{source}}$$

- Luminosity distance to redshift relation:

$$d_{\mathrm{L}}(z) = a_0\, f_k(\chi(r))\,(1+z)$$

It depends on the whole expansion history

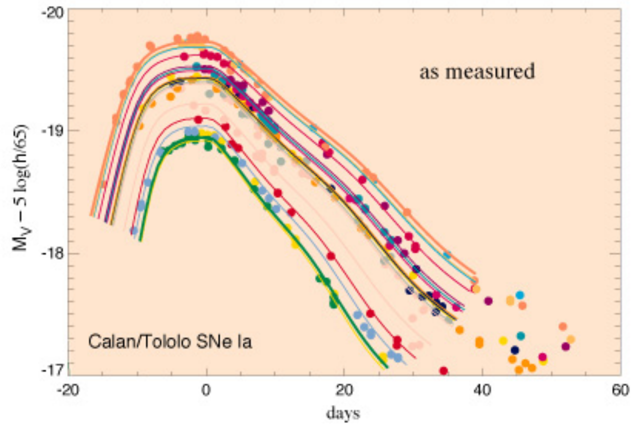
Luminosity distance measurements

• Type Ia Supernovae (SNIa) as « standard candels »

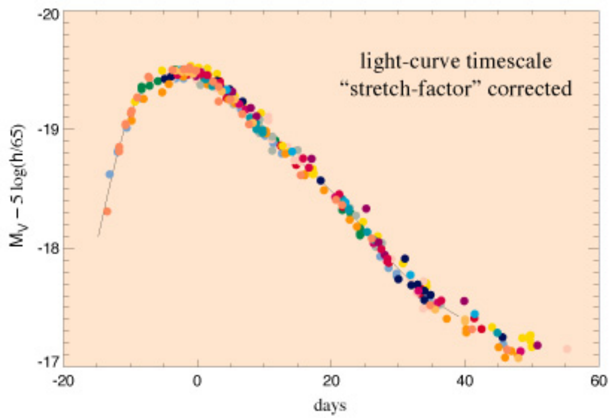
binary stellar systems: white dwarf accreting mass from the companion star until a critical mass, then exploding

• « Standardization »

empirical law relating the maximum and the width of the light curve



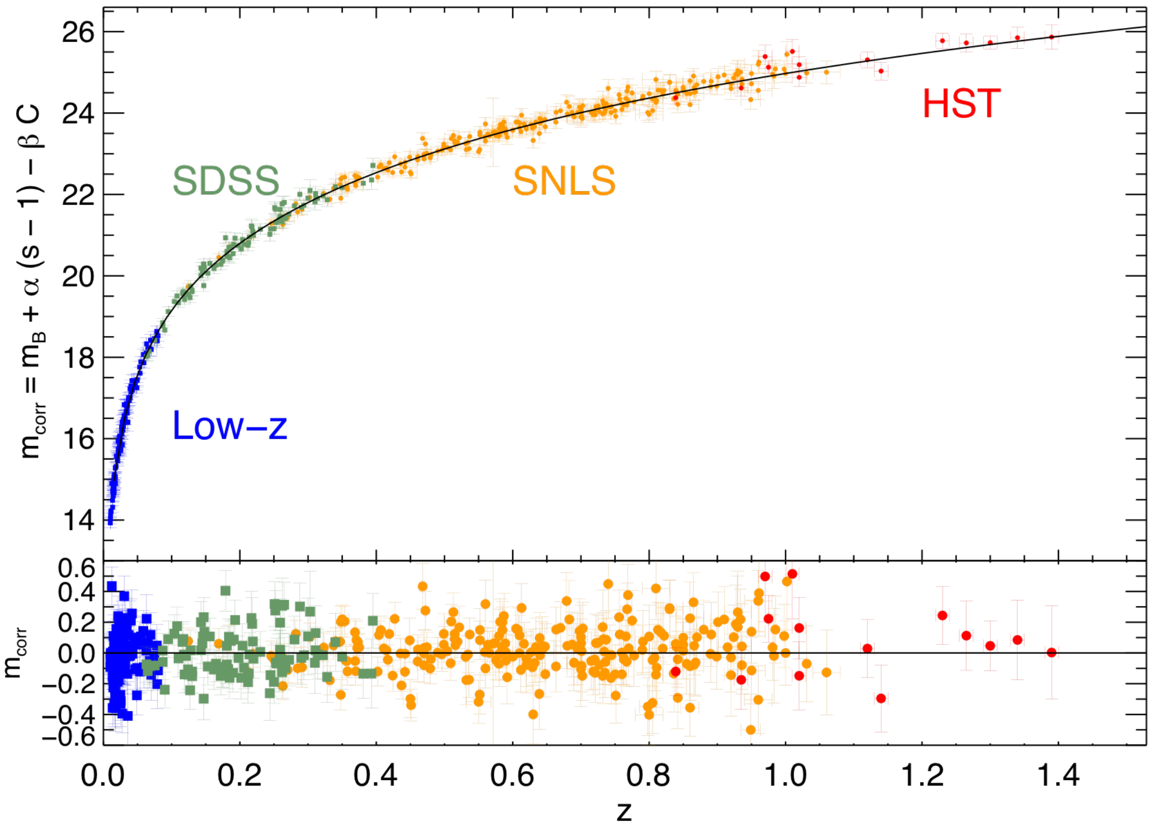
• Redshift can be measured using spectroscopy



• Calibration using nearby SNIa measurement:

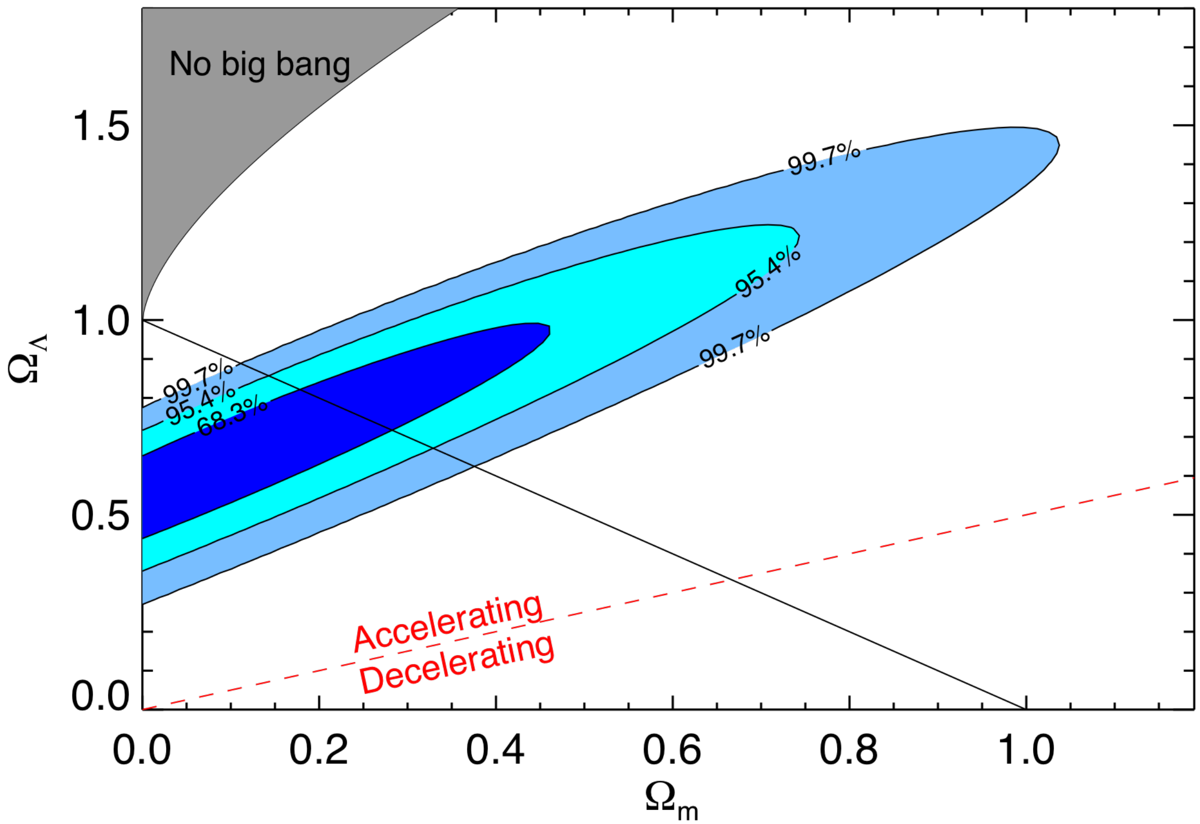
$$\frac{d_{\rm L}(z)}{d_{\rm L}(z_{\rm ref})} = \sqrt{\frac{\phi_{\rm ref}}{\phi}}$$

Modern Hubble diagram and implications



SuperNova Legacy Survey compilation(SNLS+low-z+SDSS+HST)
Conley et al. (2011) http://arxiv.org/1104.1443

Modern Hubble diagram and implications



- expansion is an etablished fact
- the expansion is accelerated due to a late phase of « Dark Energy » domination

Bibliography

References for this lecture :

- Introduction to Cosmology, by Julien Lesgourgues
  http://lesgourg.web.cern.ch/lesgourg/courses.html
- Primordial Cosmology, Patrick Peter, Jean-Philippe Uzan, Oxford Univ. Press (2009)
  Cosmologie Primordial, Collection Échelles (2005)
- Cosmological Physics, John, A., Peacock, Cambridge edition (1999)

History of cosmology: Amedeo Balbi, Rossano Piccioni
Cosmicomic, Gli uomini che scoprirono il big bang
Cosmicomic, Enquête(s) sur les traces du Big Bang

Other references :

- Gravitation and Cosmology : Steven Weinberg, Wiley edition (1972)
- The early Universe, Edward W. Kolb & Michael S. Turner, Frontiers in Physics (1990)
- Structure formation in the Universe, T. Padmanabhan, Cambridge Univ. Press (1993)