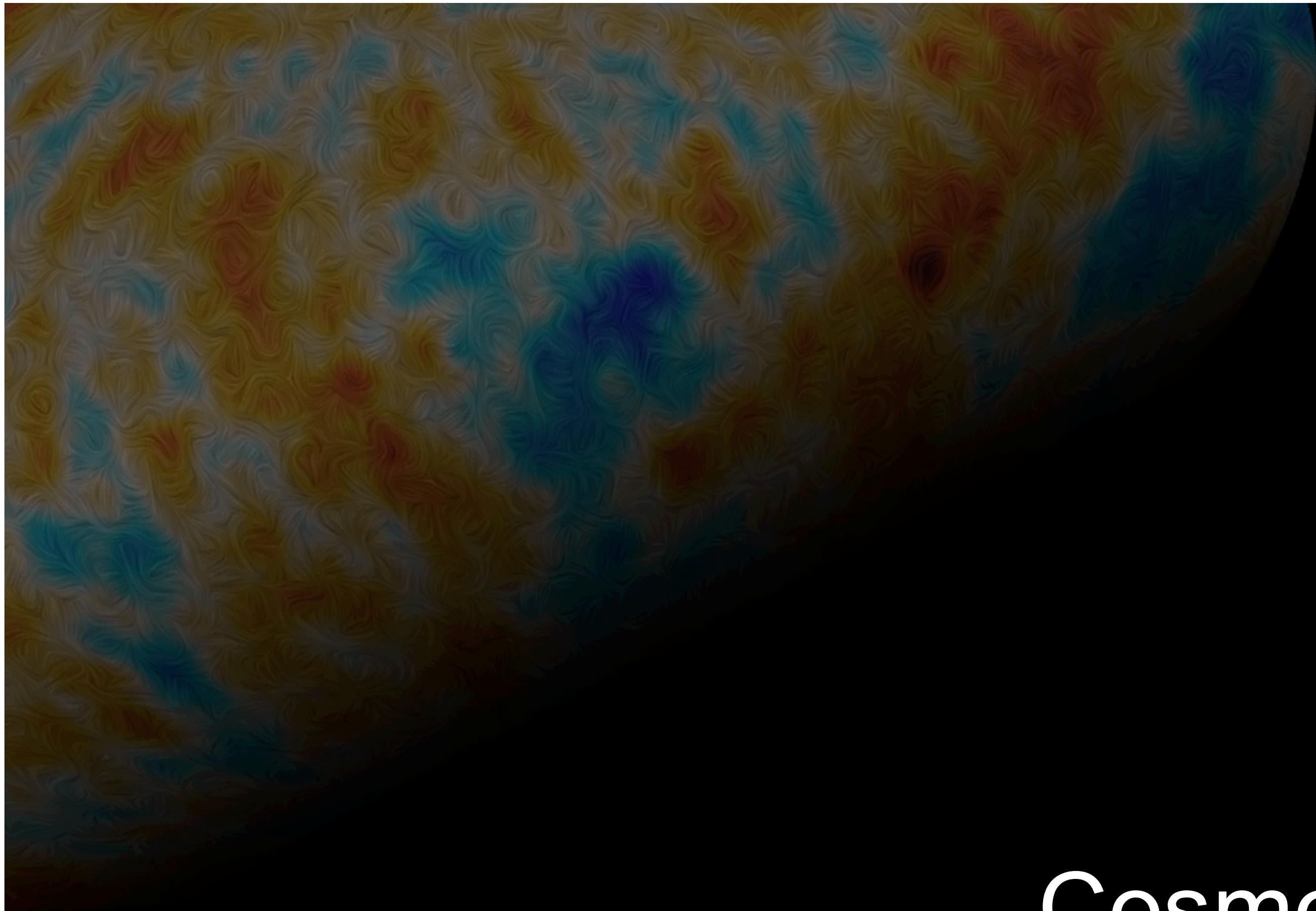
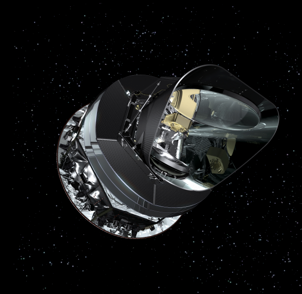




Cosmology
(2/3)

Laurence Perotto

Annecy-Le-Vieux, 2015, 16-22 July

Part I: The Big Bang Theory

Homogeneous Universe

(Metric, Friedmann Eqs, Distances, Hubble law, SNIa)

Hot Big Bang Model

(thermal history, BBN)

Thermal history of the Universe

- interaction rates compete with the expansion rate:

$\Gamma >> H$ equilibrium

$\Gamma \sim H$ decoupling

$\Gamma << H$ freeze-out

- Universe in thermal equilibrium

  Dense, hot radiation dominated (RD) early universe
  rapid rate of interactions faster than the expansion rate
  thermal equilibrium can be etablished

- decoupling

  due to expansion, any species will eventually decouple from the thermal plasma
  the interaction rate typically reads: $\Gamma = n < \sigma v >$, where *n* is the number density

- relics

  then they freeze-out, keeping their equilibrium distribution at a decreasing temperature as the universe expands

Thermal equilibrium

- particles in equilibrium are described as perfect Fermi-Dirac or Bose-Einstein gas

- Statistical thermodynamics tells us how to calculate the relevant quantities:

For a species i

$$n_i = \int g_i f(\vec{p}, T) \frac{d^3\vec{p}}{(2\pi)^3}$$
$$\rho_i = \int g_i E_i f(\vec{p}, T) \frac{d^3\vec{p}}{(2\pi)^3}$$
$$P_i = \int \frac{p^2}{3E} f(\vec{p}, T) \frac{d^3\vec{p}}{(2\pi)^3}$$

For FD(+) or BE(-) gas, the state occupation distribution is $$f(\vec{p}, T) = \frac{1}{\exp\left[\frac{E(\vec{p})}{T}\right] \pm 1}$$

NB 1: natural units

NB 2: negligible chemical potential $\mu \sim 0$

$$E^2 = m^2 + |\vec{p}|^2$$

- 2 limits

  UR: ultra-relativistic particles (T >> m)
  NR: non-relativistic particles (T << m)

Thermal equilibrium

UR: ultra-relativistic particles (T >> m)
NR: non-relativistic particles (T << m)

	Relativistic Bosons	Relativistic Fermions	Non-relativistic (Either)	
n_i	$\frac{\zeta(3)}{\pi^2} g_i T^3$	$\left(\frac{3}{4}\right) \frac{\zeta(3)}{\pi^2} g_i T^3$	$g_i \left(\frac{m_i T}{2\pi}\right)^{3/2} e^{-m_i/T}$	
ρ_i	$\frac{\pi^2}{30} g_i T^4$	$\left(\frac{7}{8}\right) \frac{\pi^2}{30} g_i T^4$	$m_i n_i$	rest-mass dom.
p_i	$\frac{1}{3}\rho_i$	$\frac{1}{3}\rho_i$	$n_i T \ll \rho_i$	pressureless

UR species dominate the universe in equilibrium

Adiabatic expansion

- Entropy conservation

  Vast amount of entropy relaxed during the matter-antimatter annihilation in the form of photon
  Any irreversible processes are overwhelmed
  The entropy in a covoming volume is concerved

$$\frac{n_\mathrm{b}}{n_\gamma} \sim 10^{-10}$$

$$dQ = 0$$

$$d(sa^3) = 0 \qquad s \propto a^{-3}$$

- Temperature of UR particles

$$s = \frac{\rho + P}{T}$$

$$s^{\mathrm{UR}} \propto T^3$$

$$T \propto a^{-1}$$

if no change of degrees of freedom

Radiation decoupling

- Distribution of a UR species at decoupling

$$f_{\mathrm{UR}}(\vec{p}, t_{\mathrm{d}}) = g_i \frac{1}{\exp\left[\frac{E(\vec{p})}{T(t_{\mathrm{d}})}\right] \pm 1}$$

- after decoupling, the species is frozen, experiencing only the redshifting due to the expansion

$$\vec{p}(t) = \vec{p}_{\mathrm{d}} \frac{a_{\mathrm{d}}}{a(t)} \qquad E(t) = E_{\mathrm{d}} \frac{a_{\mathrm{d}}}{a(t)}$$

$$T(t) = T_{\mathrm{d}} \frac{a_{\mathrm{d}}}{a(t)}$$

The equilibrium distribution is conserved at lower temperature

- NB: same reasoning holds for NR species, but the temperature decreases faster

$$E \sim m \left(1 + \frac{1}{2} \frac{p^2}{m^2}\right)$$

$$f_{\mathrm{NR}}(\vec{p}, t_{\mathrm{d}}) = g_i \exp\left[-\frac{m}{T_{\mathrm{d}}}\right] \exp\left[-\frac{p^2}{2mT(t_{\mathrm{d}})}\right] \qquad T(t) = T_{\mathrm{d}} \left(\frac{a_{\mathrm{d}}}{a(t)}\right)^2$$

Cosmic Microwave Background spectrum

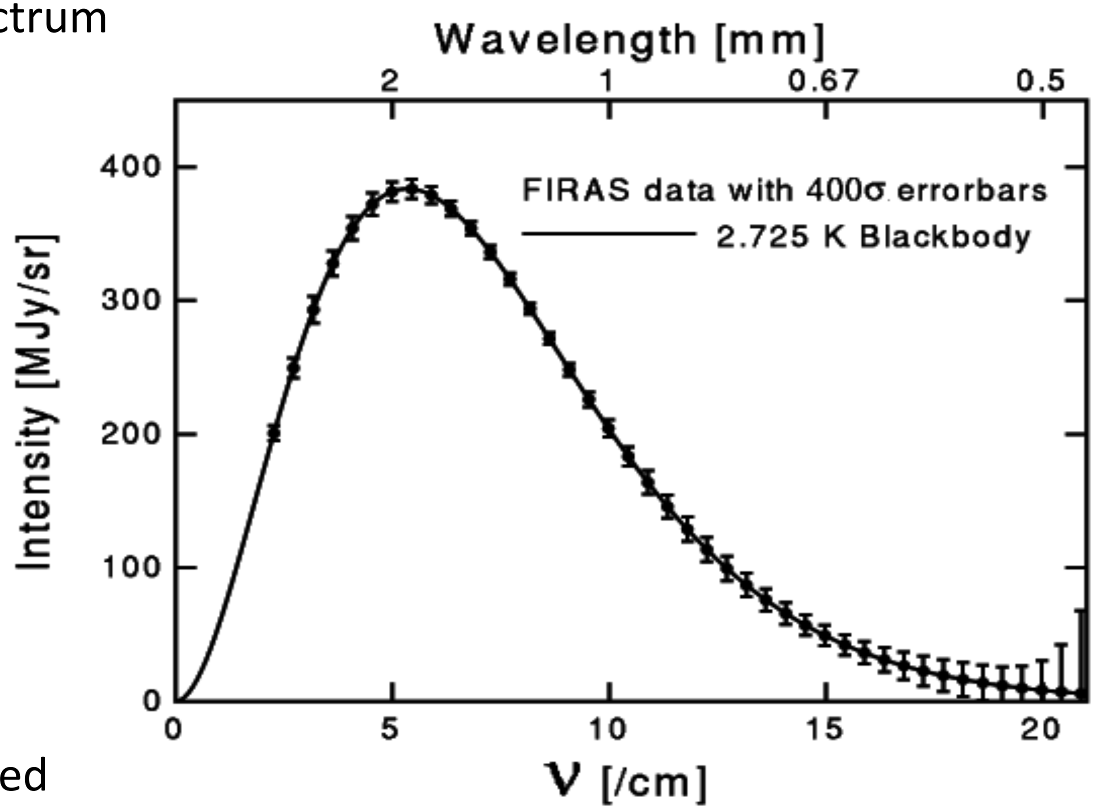
1990: Cosmic Background Explorer satellite, COBE

Measure of the CMB spectrum using the spectrometer: FIRAS

The CMB has a Planck's Black-Body spectrum
at a temperature of 2.725 K

$$I(\nu, T) \propto \frac{\nu^3}{\exp\left[\frac{E(\nu)}{T}\right] - 1}$$

$$\left[\mathrm{W\,sr^{-1}\,m^{-2}\,Hz^{-1}}\right]$$



J. C. Mather et al. (1990)

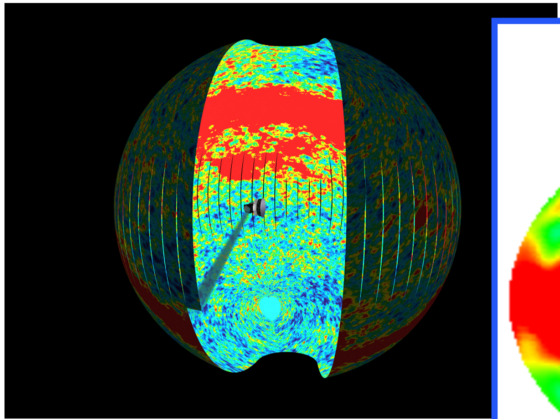
Alpher&Hermann prediction is validated

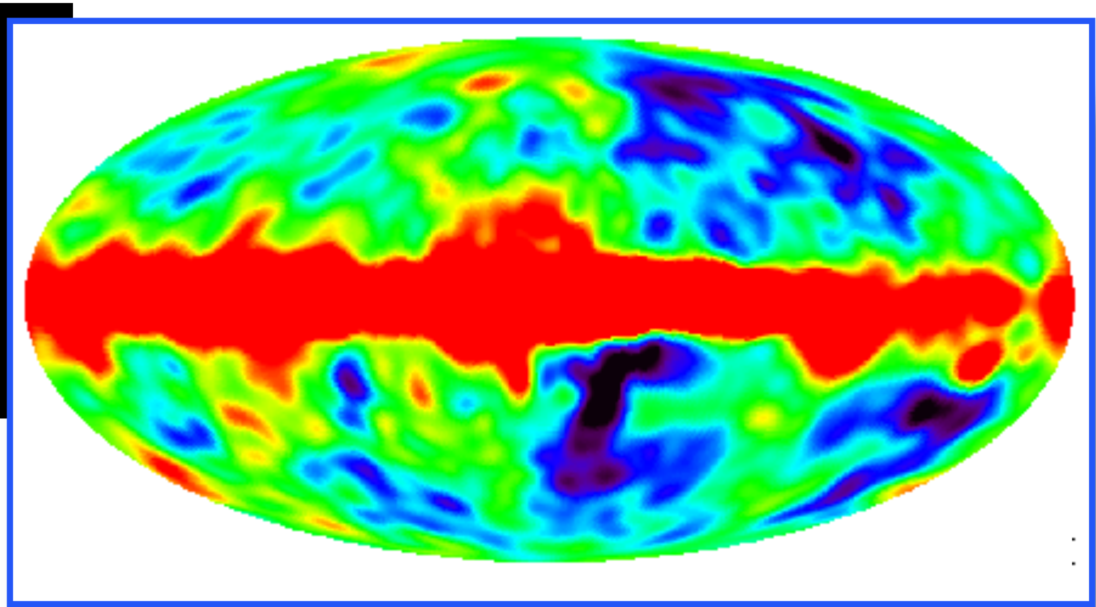
One of the most important observation in favour of the BB theory

Cosmic Microwave Background map

Measure of the CMB map using the imager DIRBE

G. Smoot et al (1992)





J. C. Mather & G.F. Smoot : prix Nobel 2006

Very isotropic radiation $\frac{\Delta T}{T_0} \sim 10^{-5}$

The CMB is a relic of the isotropic homogeneous primordial Universe in thermal equilibrium
The cosmological principle is validated.

The Universe cooling

Entropy of the Universe

summing over all species $$S = \frac{2\pi^2}{45}\left(\sum_{\text{bosons}} g_i \left(\frac{T_i}{T}\right)^3 + \frac{7}{8}\sum_{\text{fermions}} g_i \left(\frac{T_i}{T}\right)^3\right) T^3 a^3$$

g_*^s effective number of degree of freedom

- when the species i decoupled, its entropy S_i is also conserved but its temperature decreases
- the entropy of the thermal plasma ($S-S_i$) is conserved

$$S - S_i = \frac{2\pi^2}{45}\, g_\gamma^s\, T^3 a^3$$

g_γ^s effective number of degrees of freedom in equilibrium with the plasma

Temperature of the Universe

$$T \propto \left(g_\gamma^s\right)^{-1/3} a^{-1}$$

The Cosmic Neutrino Background

- T >> 1 MeV, plasma of $e^-,\, e^+,\, \gamma,\, \nu$ maintened in equilibrium by weak interactions

$$\nu + \bar{\nu} \longleftrightarrow e^+ + e^-$$
$$\nu + e \longleftrightarrow \nu + e$$
$$e^+ + e^- \longleftrightarrow 2\gamma$$

- T ~ 1 MeV, neutrino decoupling $T = T_\nu^{\mathrm{d}}$

for $T < T_\nu^{\mathrm{d}}$, $T_\nu \propto a^{-1}$ as photons

$$g_\gamma^s(T > m_e) = \frac{7}{8} g_{e^-} + \frac{7}{8} g_{e^+} + g_\gamma = \frac{11}{2}$$

- T ~ m_e , electron-positron annihilation occurs heating the Universe

$$g_\gamma^s(T < m_e) = 2$$
$$S(T > m_e) = S(T > m_e)$$
$$g_\gamma^s(T > m_e)\, T_\nu^3 a^3 = g_\gamma^s(T < m_e)\, T_\gamma^3 a^3$$
$$T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma$$

a Cosmic Neutrino Background exists at T about 1.9 K and number density $n_\nu = \dfrac{3}{11} n_\gamma$

Thermal relics

- UR when decoupling = hot relics (e.g. photon, neutrino)

After freeze-out, the number density is diluted by the expansion as $a(t)^{-3}$

The aboundance today $n_i \sim n_\gamma \sim 100\,\mathrm{cm}^{-3}$ $n_\gamma = 411\,\mathrm{cm}^{-3}$

with a factor between 1 and 10^{-2} depending on the number of dof at decoupling

- NR when decoupling = cold relics

number density at equilibrium $g_i \left(\frac{m_i T}{2\pi}\right)^{3/2} e^{-m_i/T}$

precise follow-up of the decoupling process is required
numerical integration of the Boltzmann equation

approximation $n_i(T < T^{\mathrm{d}}) \sim \frac{1}{<\sigma v> m_i M_{\mathrm{P}}} n_\gamma$ $M_{\mathrm{P}} = \sqrt{\frac{\hbar c}{G}} \sim 10^{19}\,\mathrm{GeV/c^2}$

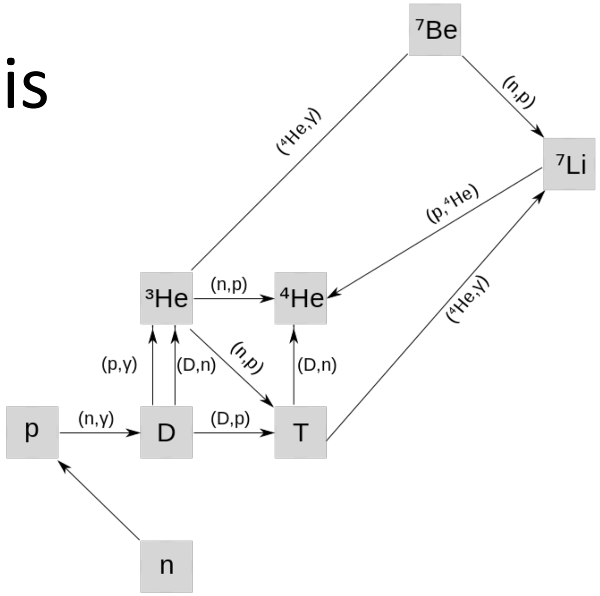
relic density

$$\Omega_i \equiv \frac{\rho_i}{\rho_c} = \frac{m_i n_i}{\rho_c} \sim \frac{1}{<\sigma v> M_{\mathrm{P}}^3 H_0^2}\, n_\gamma$$

For a WIMP (Weak interaction cross-section), one founds $\Omega_i \sim 1$

WIMP are natural candidate for the Cold Dark Matter (see Pierre Salati's lecture)

Primordial Nucleosynthesis



The most important evidence in favour of a Hot Big Bang

- Stellar nucleosynthesis fails at explaining the observed abundances of light elements in the old nebulea (Hydrogen, Deuteron, Helium, Lithium)
- In the primordial Universe, T reaches that of stellar centers and nuclear reactions occurs

Pre-nucleosynthesis:

T >> 1 MeV, plasma of e^-, e^+, γ, ν in equilibrium,

n and p (NR) also in equilibrium through Weak Interactions

$$\frac{n_n}{n_p} = \left(\frac{m_n}{m_p}\right)^{3/2} \exp\left[-\frac{m_n - m_p}{T}\right]$$

$$e^+ + e^- \longleftrightarrow 2\gamma$$
$$n + \nu_e \longleftrightarrow p + e$$
$$n \longleftrightarrow p + e + \bar{\nu}_e$$

T ~ 1 MeV, weak interaction rate below the expansion rate, $\frac{n_n}{n_p}$ freeze-out

$$\frac{n_n}{n_p} \sim \frac{1}{6}$$

then, neutron number drops slowly

$$n \longrightarrow p + e + \bar{\nu}_e$$

Helium abundance

At T < 1MeV, nucleus formation begins

Deuteron formation $p + n \longrightarrow D + \gamma$

happening at T=T_{nuc}, when the photon density has dropped enough : $\frac{n_{\rm D}}{n_\gamma} \sim 1$

$$T_{\rm nuc} \sim 0.1\,{\rm MeV}$$

Helium formation

$$\begin{aligned} {\rm D} + {\rm D} &\longrightarrow {}^3{\rm H_e} + {\rm n} \\ {}^3{\rm H_e} + {\rm D} &\longrightarrow {}^4{\rm H_e} + {\rm p} \end{aligned}$$

if all the neutron in Helium 4 :

$$\rm Y_{H_e} \equiv \frac{4n_{H_e}}{n_b} = \frac{2n_n}{n_n + n_p} \sim 0.25$$

$$\frac{n_n}{n_p}(T_{\rm nuc}) \sim \frac{1}{7} \longrightarrow \frac{n_{\rm H_e}}{n_{\rm b}} \sim \frac{1}{16}$$

precise numerical integration forecasts : $\rm Y_{H_e} \sim 0.245$

Nuclear reactions in the Early Universe inefficient to produce significant abundance of elements heavier than Helium 4

Implication on neutrino family number

one more neutrino family would increase the radiation density, $t \sim H^{-1} \propto \rho_{\rm r}^{-1/2} \searrow$
triggering earlier nucleosynthesis, reducing the time left for the neutron to decrease

BBN favours 3 families

Baryon density constraints

The relative abundance of light elements depends almost only of the baryon-to-photon ratio

$$\eta = \frac{n_p + n_n}{n_\gamma}$$

Baryon density today

$$\Omega_\mathrm{b} = \frac{\rho_\mathrm{b}}{\rho_\mathrm{c} H_0^2} \sim \frac{m_p n_\mathrm{b}}{\rho_\mathrm{c} H_0^2} \quad \longrightarrow \quad \eta \sim 2.74 \times 10^{-8} (\Omega_\mathrm{b} h^2)$$

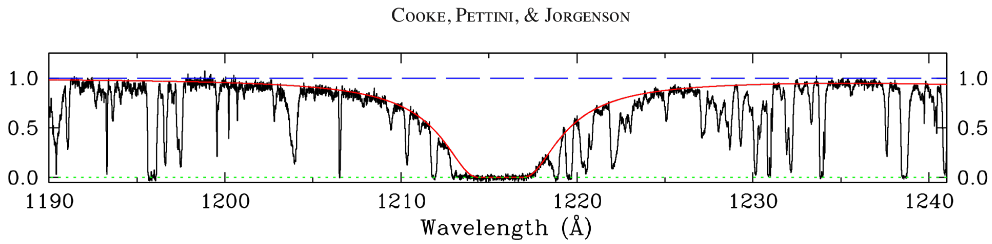
$$\Omega_\mathrm{b} = \frac{n_\gamma (m_p \eta)}{\rho_\mathrm{c} H_0^2} \quad \longrightarrow \omega_\mathrm{b} \equiv \Omega_\mathrm{b} h^2 = 3.66 \times 10^7 \eta$$ (Particle Data Group)

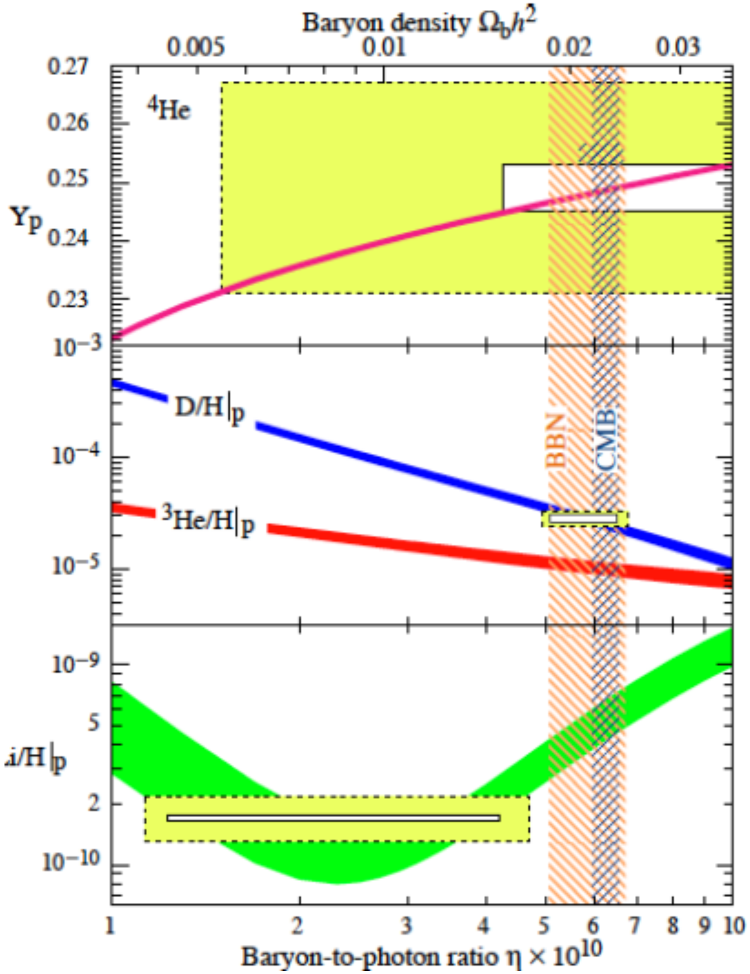
BBN gives the first evidence for a low baryon density

Baryon density constraints

Spectroscopic measurements in metal-poor objects

- low star formation rate (dwarf galaxies)
- high redshift (distant quasars)





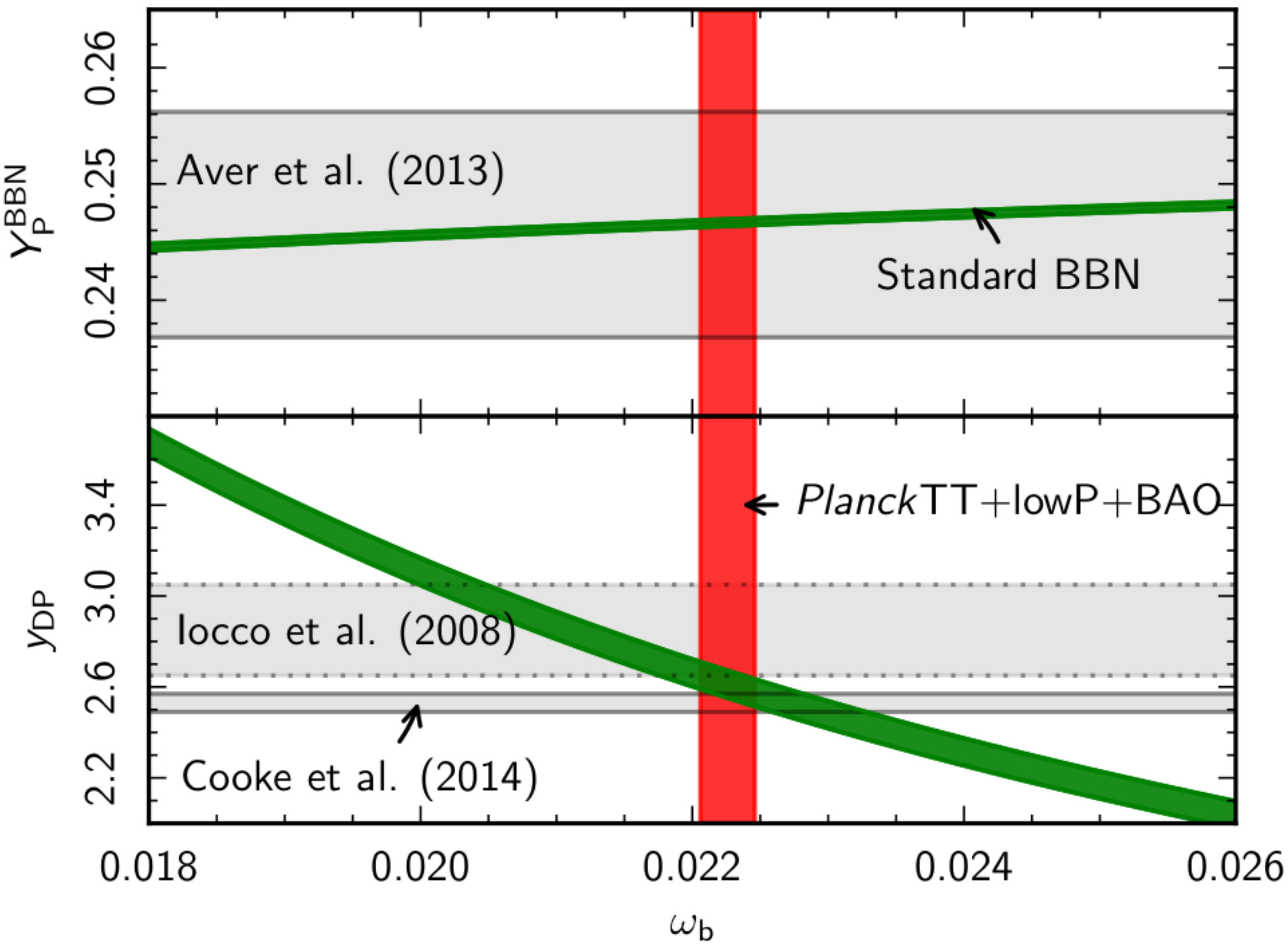
neutral hydrogen and metal absorption lines

For Helium, extrapolation to primordial abundances

Constraints on the baryon-photon ratio by comparing measured and predicted abundances

Recent results

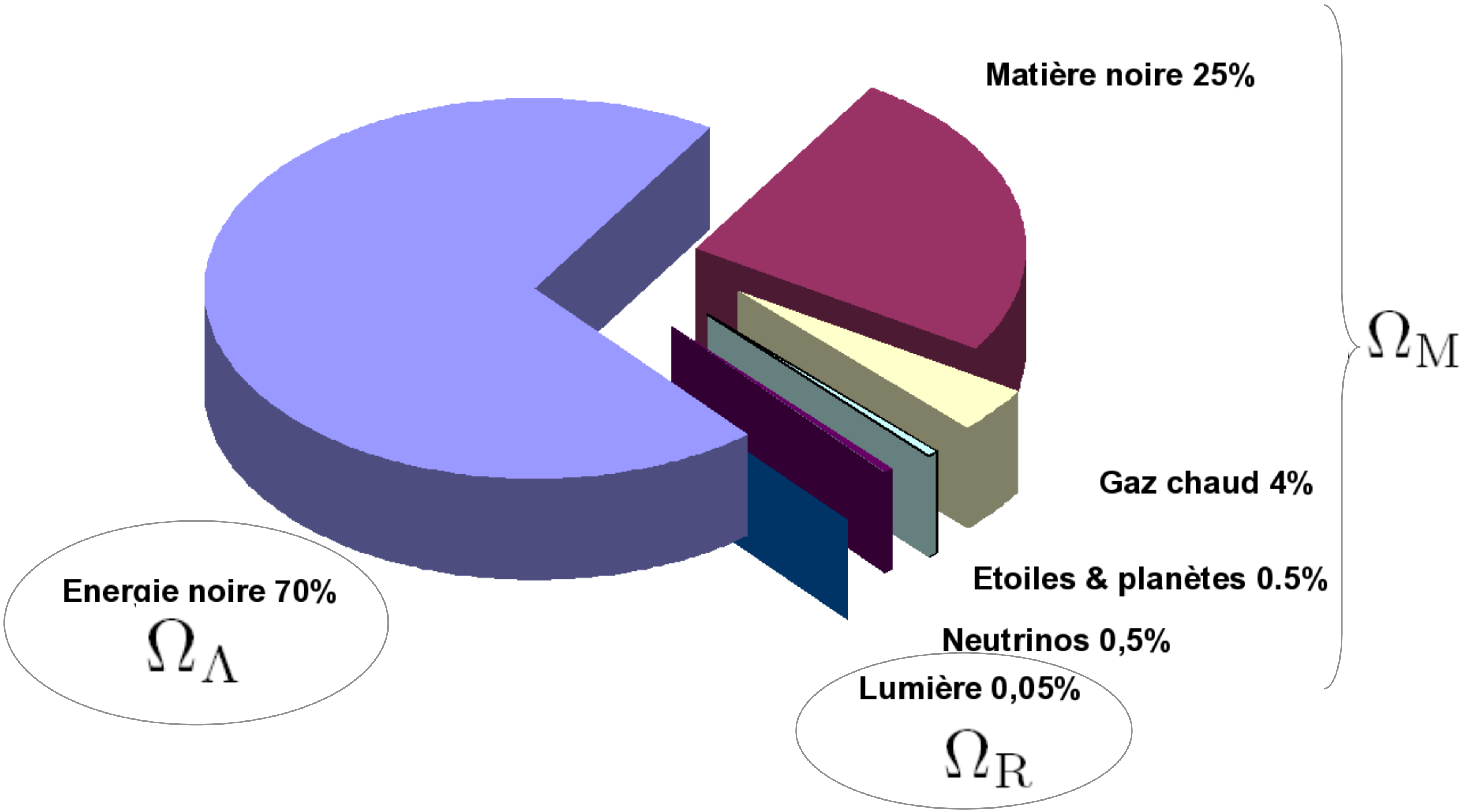
Planck Collaboration (2015)



NB: theoretical dispersion from nuclear reaction rates and neutron lifetime

$$\Omega_b \sim 0.05$$

The standard model of the Cosmology



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BACKUP

Thermal history of the Universe: 1 second

• Quantum gravity	10^{18} GeV	10^{-36} s	?
• Inflation	10^{16} GeV	10^{-32} s	ϕ
• Reheating			↓
			$(q, \bar{q})$, $(l, \bar{l})$, (H^+, H^0), gauge bosons, WIMP
• Beginning of Rad. Dom.			↓
• EW breaking	100 GeV	10^{-6} s	$(q, \bar{q})$, $(l, \bar{l})$, H, $W^{+/-}$, Z^0, γ
• baryon annihilation	0.1 GeV	$\frac{n_B}{n_\gamma} \sim 5 \times 10^{-10}$	↓
• Hadronisation	100 MeV	10^{-4} s	baryons, mesons, $(l, \bar{l})$, γ
• Pre-Nucleosynthesis	1 MeV	1 s	p, n, e^+, e^-, ν, $\bar{\nu}$, γ