Phenomenology of the Accelerating Universe

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In progress also with: L. Hui, L. Perenon, V. Salvatelli





Nobel Prize in Physics 2011

The Universe is accelerating!





• Two main classes of observables:

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However: beware of theoretical prejudice!

MEASUREMENTS¹ OF THE COSMOLOGICAL PARAMETERS Ω AND Λ FROM THE FIRST SEVEN SUPERNOVAE AT $z \ge 0.35$

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ABSTRACT

We have developed a technique to systematically discover and study high-redshift supernovae that can be used to measure the cosmological parameters. We report here results based on the initial seven of more than 28 supernovae discovered to date in the high-redshift supernova search of the Supernova Cosmology Project. We find an observational dispersion in peak magnitudes of $\sigma_{M_B} = 0.27$; this dispersion narrows to $\sigma_{M_{B,corr}} = 0.19$ after "correcting" the magnitudes using the light-curve "widthluminosity" relation found for nearby ($z \le 0.1$) Type Ia supernovae from the Calán/Tololo survey (Hamuy et al.). Comparing light-curve width-corrected magnitudes as a function of redshift of our distant (z = 0.35 - 0.46) supernovae to those of nearby Type Ia supernovae yields a global measurement of the mass density, $\Omega_{\rm M} = 0.88^{+0.69}_{-0.60}$ for a $\Lambda = 0$ cosmology. For a spatially flat universe (i.e., $\Omega_{\rm M} + \Omega_{\Lambda} = 1$), we find $\Omega_{\rm M} = 0.94^{+0.34}_{-0.28}$ or, equivalently, a measurement of the cosmological constant, $\Omega_{\Lambda} = 0.06^{+0.28}_{-0.34}$ (<0.51 at the 95% confidence level). For the more general Friedmann-Lemaître cosmologies with independent $\Omega_{\rm M}$ and Ω_{Λ} , the results are presented as a confidence region on the $\Omega_{\rm M}$ - Ω_{Λ} plane. This region does not correspond to a unique value of the deceleration parameter q_0 . We present analyses and checks for statistical and systematic errors and also show that our results do not depend on the specifics of the width-luminosity correction. The results for Ω_{Λ} -versus- Ω_{M} are inconsistent with Λ -dominated, lowdensity, flat cosmologies that have been proposed to reconcile the ages of globular cluster stars with higher Hubble constant values.

- Two main classes of observables:
 - Background: compatible with w=-1 (LCDM)
 - Perturbations: some interesting tension...



Macaulay et al. '13

Two main classes of observables



- Two main classes of observables
- EUCLID, DESI etc... are specifically designed to target perturbation sector
- `No shortage' of dark energy models (>5000 papers on Spires) Need for a Phenomenology

Ideally...

 A limited number of effective operators, each one responsible for an observable dynamical feature (e.g. flavor-changing neutral currents in physics beyond Standard Model)

The effective field theory (EFT) of dark energy

- Most general description of 1 scalar degree of freedom added to GR
- Cosmological perturbations as the relevant objects of the theory
- Background (0th order) and perturbation (linear and +) sectors
- Unifying framework for DE observables, stability
- Good parameter space to constrain with data (see Planck 2015)

Unitary gauge in Cosmology (technical detour)

The Effective Field Theory of Inflation (Creminelli et al. `06, Cheung et al. `07)

Main idea: scalar degrees of freedom are `eaten' by the metric. Ex:

$$\phi(t,\vec{x}) \to \phi_0(t) \quad (\delta\phi=0) \qquad -\frac{1}{2}\partial\phi^2 \to -\frac{1}{2}\dot{\phi}_0^2(t) \ g^{00}$$



Unitary gauge in Cosmology (technical detour)

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Effective Field Theory of Dark Energy: (Gubitosi, F.P., Vernizzi 2012)

I) Assume WEP (universally coupled metric $S_m[g_{\mu\nu}, \Psi_i]$)

2) Write the most generic action for $g_{\mu\nu}$ compatible with the residual un-broken symmetries (3-diff).

The Action

$$S = \int d^4x \sqrt{-g} \frac{M^2(t)}{2} \left[R - 2\lambda(t) - 2\mathcal{C}(t)g^{00} + \mu_2^2(t)(\delta g^{00})^2 - \mu_3(t) \,\delta K \delta g^{00} + \epsilon_4(t) \left(\delta K^{\mu}_{\ \nu} \,\delta K^{\nu}_{\ \mu} - \delta K^2 + \frac{R^{(3)} \,\delta g^{00}}{2} \right) + \dots \right]$$

The Action

Background (expansion history)

$$S = \int d^4x \sqrt{-g} \underbrace{\frac{M^2(t)}{2}}_{2} \left[R - 2\lambda(t) - 2\mathcal{C}(t)g^{00} + \underbrace{\mu_2^2(t)}_{2} (\delta g^{00})^2 - \underbrace{\mu_3(t)}_{3} \delta K \delta g^{00} + \underbrace{\epsilon_4(t)}_{4} \left(\delta K^{\mu}_{\ \nu} \, \delta K^{\nu}_{\ \mu} - \delta K^2 + \frac{R^{(3)} \, \delta g^{00}}{2} \right) + \dots \right]$$

only affect perturbations

Time-dependent couplings

$$S = \int d^4x \sqrt{-g} \, \frac{M^2(t)}{2} \left[R - 2\lambda(t) - 2\mathcal{C}(t)g^{00} + \mu_2^2(t)(\delta g^{00})^2 - \mu_3(t)\,\delta K \delta g^{00} + \epsilon_4(t) \left(\delta K^{\mu}_{\ \nu}\,\delta K^{\nu}_{\ \mu} - \delta K^2 + \frac{R^{(3)}\,\delta g^{00}}{2} \right) + \dots \right]$$

Examples

$$S = \int d^4x \sqrt{-g} \frac{M^2(t)}{2} \left[R - 2\lambda(t) - 2\mathcal{C}(t)g^{00} + \mu_2^2(t)(\delta g^{00})^2 - \mu_3(t) \,\delta K \delta g^{00} + \epsilon_4(t) \left(\delta K^{\mu}_{\ \nu} \,\delta K^{\nu}_{\ \mu} - \delta K^2 + \frac{R^{(3)} \,\delta g^{00}}{2} \right) + \dots \right]$$

Quintessence

$$S = \int d^4x \sqrt{-g} \underbrace{\frac{M^2(t)}{2}}_{2} \left[R - 2\lambda(t) - 2\mathcal{C}(t)g^{00} + \mu_2^2(t)(\delta g^{00})^2 - \mu_3(t) \,\delta K \delta g^{00} + \epsilon_4(t) \left(\delta K^{\mu}_{\ \nu} \,\delta K^{\nu}_{\ \mu} - \delta K^2 + \frac{R^{(3)} \,\delta g^{00}}{2} \right) + \dots \right]$$

Non-minimally coupled scalar field (Brans-Dicke, f(R) etc.)

Examples

$$S = \int d^4x \sqrt{-g} \frac{M^2(t)}{2} \left[R - 2\lambda(t) - 2C(t)g^{00} + \mu_2^2(t) (\delta g^{00})^2 - \mu_3(t) \,\delta K \delta g^{00} + \epsilon_4(t) \left(\delta K^{\mu}_{\ \nu} \,\delta K^{\nu}_{\ \mu} - \delta K^2 + \frac{R^{(3)} \,\delta g^{00}}{2} \right) + \dots \right]$$

K-essence (Amendariz-Picon et al., 2000)

$$S = \int d^4x \sqrt{-g} P(\phi, X) \qquad X \equiv g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

$$S = \int d^4x \sqrt{-g} \frac{M^2(t)}{2} \left[R - 2\lambda(t) - 2\mathcal{C}(t)g^{00} + \mu_2^2(t) (\delta g^{00})^2 - \mu_3(t) \delta K \delta g^{00} + \epsilon_4(t) \left(\delta K^{\mu}_{\ \nu} \, \delta K^{\nu}_{\ \mu} - \delta K^2 + \frac{R^{(3)} \, \delta g^{00}}{2} \right) + \dots \right]$$

"Galilean Cosmology" (Chow and Khoury, 2009)

$$S = \int d^4x \sqrt{-g} \left[\frac{M^2}{2} e^{-2\phi/M} R - \frac{r_c^2}{M} (\partial\phi)^2 \Box\phi \right]$$

$$S = \int d^4x \sqrt{-g} \frac{M^2(t)}{2} \left[R - 2\lambda(t) - 2\mathcal{C}(t)g^{00} + \mu_2^2(t) (\delta g^{00})^2 - \mu_3(t) \delta K \delta g^{00} + \epsilon_4(t) \left(\delta K^{\mu}_{\ \nu} \, \delta K^{\nu}_{\ \mu} - \delta K^2 + \frac{R^{(3)} \, \delta g^{00}}{2} \right) + \dots \right]$$

"Generalized Galileons" (\equiv Horndeski)

(Deffayet et al., 2011)

$$\begin{aligned} \mathcal{L}_2 &= A(\phi, X) ,\\ \mathcal{L}_3 &= B(\phi, X) \Box \phi ,\\ \mathcal{L}_4 &= C(\phi, X) R - 2C_{,X}(\phi, X) \left[(\Box \phi)^2 - (\nabla_\mu \nabla_\nu \phi)^2 \right] ,\\ \mathcal{L}_5 &= D(\phi, X) G^{\mu\nu} \nabla_\mu \nabla_\nu \phi + \frac{1}{3} D_{,X}(\phi, X) \left[(\Box \phi)^3 - 3(\Box \phi) (\nabla_\mu \nabla_\nu \phi)^2 + 2(\nabla_\mu \nabla_\nu \phi)^3 \right] ,\end{aligned}$$

(see David's talk before this)

The most general (linear) theory without higher derivatives on the propagating degree of freedom

Complete background separation/redundancies

$$S = \int d^4x \sqrt{g^{(M^2(t))}} R - 2\lambda(t) - 2C(t)g^{00} + \mu_2^2(t)(\delta g^{00})^2 - \mu_3(t) \,\delta K \delta g^{00} + \epsilon_4(t) \left(\delta K^{\mu}_{\ \nu} \,\delta K^{\nu}_{\ \mu} - \delta K^2 + \frac{R^{(3)} \,\delta g^{00}}{2}\right) + \dots \right]$$

also participates in perturbations

Complete background separation/redundancies

$$S = \int d^4x \sqrt{-g} \, \frac{M^2(t)}{2} \left[R - 2\lambda(t) - 2\mathcal{C}(t)g^{00} + \mu_2^2(t)(\delta g^{00})^2 - \mu_3(t) \,\delta K \delta g^{00} + \epsilon_4(t) \left(\delta K^{\mu}_{\ \nu} \,\delta K^{\nu}_{\ \mu} - \delta K^2 + \frac{R^{(3)} \,\delta g^{00}}{2} \right) + \dots \right]$$

$$\lambda(t), \ \mathcal{C}(t), \ \mu(t) \equiv \frac{d \ln M^2(t)}{dt} \begin{cases} \overline{w}(t) & \text{Expansion History} \\ \mu(t) \\ \mu_3(t) \\ \epsilon_4(t) \\ \mu_2^2(t) \end{cases}$$
Perturbation sector

Complete background separation/redundancies

$$S = \int d^4x \sqrt{-g} \, \frac{M^2(t)}{2} \left[R - 2\lambda(t) - 2\mathcal{C}(t)g^{00} + \mu_2^2(t)(\delta g^{00})^2 - \mu_3(t)\,\delta K \delta g^{00} + \epsilon_4(t) \left(\delta K^{\mu}_{\ \nu}\,\delta K^{\nu}_{\ \mu} - \delta K^2 + \frac{R^{(3)}\,\delta g^{00}}{2} \right) + \dots \right]$$



Stability

$$S_{\pi} = \int a^{3}(t)M^{2}(t) \left[A\left(\mu,\mu_{2}^{2},\mu_{3},\epsilon_{4}\right) \dot{\pi}^{2} + B\left(\mu,\mu_{3},\epsilon_{4}\right) \frac{(\vec{\nabla}\pi)^{2}}{a^{2}} \right] + \text{lower order in derivatives.}$$
No ghost: A>0 No gradient instabilities: B>0
$$\mu_{2}^{2} = 0$$

Effective Newton constant



If modified gravity is responsible for acceleration

 $\succ G_{\rm eff}(z,k)$

See also Baker et. al. 1409.8284

Effective Newton constant

If modified gravity is responsible for acceleration \blacksquare $G_{\rm eff}(z, k)$

$$4\pi G_{\text{eff}} = \frac{1}{2M^2} \frac{2\mathcal{C} + 2(\mu + \mathring{\epsilon}_4)^2 + \mathring{\mu}_3 - 2\dot{H}\epsilon_4 + 2H\mathring{\epsilon}_4 + 3(a/k)^2\mathcal{A}}{(1 + \epsilon_4)^2 [2\mathcal{C} + \mathring{\mu}_3 - 2\dot{H}\epsilon_4 + 2H\mathring{\epsilon}_4] + 2(1 + \epsilon_4)(\mu + \mathring{\epsilon}_4)(\mu - \mu_3) - (\mu - \mu_3)^2/2 + 3(a/k)^2\mathcal{A}'}$$

More than just parameterize $G_{\text{eff}}(z)$!

F. P., C. Marinoni, H. Steigerwald 1312.6111



 γ_0

Growth rate (preliminary) Modified gravity: less growth than LCDM?





Other observables (preliminary)



Conclusions

- Unifying framework for dark energy/modified gravity
- Perturbations "know" about the background (stability)
- G_{eff} : more information than just a "blind fit"
- Observational constraints and forecasts: much work in progress

Perturbations "know" about the background

stability conditions

$$S_{\pi} = \int a^{3} M^{2} \left\{ \left[(\mathcal{C} + 2\mu_{2}^{2})(1 + \epsilon_{4}) + \frac{3}{4}(\mu - \mu_{3})^{2} \right] \dot{\pi}^{2} - \left[(\mathcal{C} + \frac{\ddot{\mu}_{3}}{2} - \dot{H}\epsilon_{4} + H\dot{\epsilon}_{4})(1 + \epsilon_{4}) - (\mu - \mu_{3})\left(\frac{\mu - \mu_{3}}{4(1 + \epsilon_{4})} - \mu - \dot{\epsilon}_{4}\right) \right] \frac{(\vec{\nabla}\pi)^{2}}{a^{2}} \right\}$$

$$\mathring{\mu}_3 \equiv \dot{\mu}_3 + \mu \mu_3 + H \mu_3, \qquad \mathring{\epsilon}_4 \equiv \dot{\epsilon}_4 + \mu \epsilon_4 + H \epsilon_4$$

Stability

$$S_{\pi} = \int a^{3}(t)M^{2}(t) \begin{bmatrix} A(\mu, \mu_{2}^{2}, \mu_{3}, \epsilon_{4}) \dot{\pi}^{2} + B(\mu, \mu_{3}, \epsilon_{4}) \frac{(\vec{\nabla}\pi)^{2}}{a^{2}} \end{bmatrix} + \text{lower order in derivatives.}$$

$$\uparrow$$
No ghost: A>0 No gradient instabilities: B>0
$$\mu_{2}^{2} \gg H^{2}$$



$$S_{\pi} = \int a^{3} M^{2} \left\{ \left[(\mathcal{C} + 2\mu_{2}^{2})(1 + \epsilon_{4}) + \frac{3}{4}(\mu - \mu_{3})^{2} \right] \dot{\pi}^{2} - \left[(\mathcal{C} + \frac{\dot{\mu}_{3}}{2} - \dot{H}\epsilon_{4} + H\dot{\epsilon}_{4})(1 + \epsilon_{4}) - (\mu - \mu_{3})\left(\frac{\mu - \mu_{3}}{4(1 + \epsilon_{4})} - \mu - \dot{\epsilon}_{4}\right) \right] \frac{(\vec{\nabla}\pi)^{2}}{a^{2}} \right\}$$

$$4\pi G_{\text{eff}} = \frac{1}{2M^2} \frac{2\mathcal{C} + 2(\mu + \mathring{\epsilon}_4)^2 + \mathring{\mu}_3 - 2\dot{H}\epsilon_4 + 2H\mathring{\epsilon}_4 + 3(a/k)^2\mathcal{A}}{(1 + \epsilon_4)^2 [2\mathcal{C} + \mathring{\mu}_3 - 2\dot{H}\epsilon_4 + 2H\mathring{\epsilon}_4] + 2(1 + \epsilon_4)(\mu + \mathring{\epsilon}_4)(\mu - \mu_3) - (\mu - \mu_3)^2/2 + 3(a/k)^2\mathcal{A}'}$$

$$\mathring{\mu}_3 \equiv \dot{\mu}_3 + \mu \mu_3 + H \mu_3, \qquad \qquad \mathring{\epsilon}_4 \equiv \dot{\epsilon}_4 + \mu \epsilon_4 + H \epsilon_4$$

 $\mathcal{A} \equiv 2\dot{H}\mathcal{C} - \dot{H}\mathring{\mu}_{3} + \ddot{H}(\mu - \mu_{3}) - 2H\dot{H}\mu_{3} - 2H^{2}(\mu^{2} + \dot{\mu}), \qquad \mathcal{A}' \equiv (1 + \epsilon_{4})^{2}\mathcal{A}$

The Action

$$S = \int d^4x \sqrt{-g} \left[\frac{M^2}{2} f(t)R - \Lambda(t) - c(t)g^{00} \right] + S_{DE}^{(2)}$$

Enough for background equations:

$$c = \frac{1}{2}(-\ddot{f} + H\dot{f})M^{2} + \frac{1}{2}(\rho_{D} + p_{D})$$

$$\Lambda = \frac{1}{2}(\ddot{f} + 5H\dot{f})M^{2} + \frac{1}{2}(\rho_{D} - p_{D})$$

$$H^{2} = \frac{1}{3fM^{2}}(\rho_{m} + \rho_{D})$$

$$H^{2} = \frac{1}{3fM^{2}}(\rho_{m} + \rho_{D} + p_{m} + p_{D})$$

$$\dot{H} = -\frac{1}{2fM^{2}}(\rho_{m} + \rho_{D} + p_{m} + p_{D})$$

"Bare" Planck Mass Defined by the modified Friedman equations

Mixing with gravity 1: Brans-Dicke

$$S = \int \sqrt{-g} \left(\frac{M^2}{2} fR - \Lambda \left(cg^{00} + \frac{M_2^4}{2} (\delta g^{00})^2 - \frac{\bar{m}_1^3}{2} \delta g^{00} \delta K + \frac{1}{2} T^{\mu\nu} \delta g_{\mu\nu} \right)$$

Apply Stueckelberg and go to
Newtonian Gauge

$$ds^{2} = -(1+2\Phi)dt^{2} + a^{2}(1-2\Psi)\delta_{ij}dx^{i}dx^{j}$$

$$S \stackrel{\text{kinetic}}{=} \int M^{2}f \left[-3\dot{\Psi}^{2} - 2\vec{\nabla}\Phi\vec{\nabla}\Psi + (\vec{\nabla}\Psi)^{2} + c\,\dot{\pi}^{2} - c(\vec{\nabla}\pi)^{2} + 3(\dot{f}/f)\dot{\Psi}\dot{\pi} + (\dot{f}/f)\vec{\nabla}\pi(\vec{\nabla}\Phi - 2\vec{\nabla}\Psi) \right]$$
De-mixing = conformal transformation

$$\Phi_{E} = \Phi + \frac{1}{2}(\dot{f}/f)\pi$$

$$\Psi_{E} = \Psi - \frac{1}{2}(\dot{f}/f)\pi$$
Mixing

Mixing with gravity 1: Brans-Dicke

$$S = \int \sqrt{-g} \left(\frac{M^2}{2} fR - \Lambda \left(cg^{00} + \frac{M_2^4}{2} (\delta g^{00})^2 - \frac{\bar{m}_1^3}{2} \delta g^{00} \delta K + \frac{1}{2} T^{\mu\nu} \delta g_{\mu\nu} \right)$$

Apply Stueckelberg and go to
Newtonian Gauge

$$ds^{2} = -(1+2\Phi)dt^{2} + a^{2}(1-2\Psi)\delta_{ij}dx^{i}dx^{j}$$

$$S^{\text{kinetic}} \int M^{2}f \left[-3\dot{\Psi}^{2} - 2\vec{\nabla}\Phi\vec{\nabla}\Psi + (\vec{\nabla}\Psi)^{2} + c\,\dot{\pi}^{2} - c(\vec{\nabla}\pi)^{2} + 3(\dot{f}/f)\dot{\Psi}\dot{\pi} + (\dot{f}/f)\vec{\nabla}\pi(\vec{\nabla}\Phi - 2\vec{\nabla}\Psi) \right]$$

$$1 - \gamma \equiv \frac{\Phi - \Psi}{\Phi} = \frac{M^{2}\dot{f}^{2}/f}{2(c + M^{2}\dot{f}^{2}/f)} \quad \text{anisotropic stress}$$
Newtonian
limit

$$G_{\text{eff}} = \frac{1}{8\pi M^{2}f} \frac{c + M^{2}\dot{f}^{2}/f}{c + \frac{3}{4}M^{2}\dot{f}^{2}/f} \quad \text{dressed Newton constant}$$

Mixing with gravity 2:

(Cf. braiding: Deffayet et al., 2010)

$$f(t) = 1$$

$$S = \int \sqrt{-g} \left(\frac{M^2}{2} fR - \Lambda + cg^{00} + \frac{M_2^4}{2} (\delta g^{00})^2 - \frac{\bar{m}_1^3}{2} \delta g^{00} \delta K + \frac{1}{2} T^{\mu\nu} \delta g_{\mu\nu} \right)$$

Apply Stueckelberg and go to
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$$ds^{2} = -(1+2\Phi)dt^{2} + a^{2}(1-2\Psi)\delta_{ij}dx^{i}dx^{j}$$

$$S^{\text{kinetic}} \int M^{2} \left[-3\dot{\Psi}^{2} - 2\vec{\nabla}\Phi\vec{\nabla}\Psi + (\vec{\nabla}\Psi)^{2} \right] + c\dot{\pi}^{2} - c(\vec{\nabla}\pi)^{2} + 3\bar{m}_{1}^{3}\dot{\Psi}\dot{\pi} + \bar{m}_{1}^{3}\vec{\nabla}\Phi\vec{\nabla}\pi$$
De-mixing \neq conformal transformation

$$\Phi_{E} = \Phi + \frac{\bar{m}_{1}^{3}}{2M^{2}}\pi$$

$$\Psi_{E} = \Psi + \frac{\bar{m}_{1}^{3}}{2M^{2}}\pi$$

Mixing with gravity 2:

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Apply Stueckelberg and go to
Newtonian Gauge

$$ds^{2} = -(1+2\Phi)dt^{2} + a^{2}(1-2\Psi)\delta_{ij}dx^{i}dx^{j}$$

$$S \stackrel{\text{kinetic}}{=} \int M^{2} \left[-3\dot{\Psi}^{2} - 2\vec{\nabla}\Phi\vec{\nabla}\Psi + (\vec{\nabla}\Psi)^{2} \right] + c\dot{\pi}^{2} - c(\vec{\nabla}\pi)^{2} + 3\bar{m}_{1}^{3}\dot{\Psi}\dot{\pi} + \bar{m}_{1}^{3}\vec{\nabla}\Phi\vec{\nabla}\pi$$

Speed of Sound of DE

Mixing

$$c_s^2 = \frac{c + \frac{1}{2}(H\bar{m}_1^3 + \dot{\bar{m}}_1^3) - \frac{1}{4}\bar{m}_1^6/M^2}{c + \frac{3}{4}\bar{m}_1^6/M^2}$$

Mixing with gravity 2:

(Cf. braiding: Deffayet et al., 2010)

$$f(t) = 1$$

$$S = \int \sqrt{-g} \left(\frac{M^2}{2} fR - \Lambda + cg^{00} + \frac{M_2^4}{2} (\delta g^{00})^2 - \frac{\bar{m}_1^3}{2} \delta g^{00} \delta K + \frac{1}{2} T^{\mu\nu} \delta g_{\mu\nu} \right)$$

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$$1 - \gamma = \frac{\Phi - \Psi}{\Phi} = 0$$
Newtonian
limit

$$G_{\text{eff}} = \frac{1}{8\pi M^{2}f} \left(1 - \frac{\bar{m}_{1}^{3}}{4cM^{2}} \right)^{-1} \text{dressed Newton constant}$$