Towards NNLO Event Generators for the LHC

Emanuele Re

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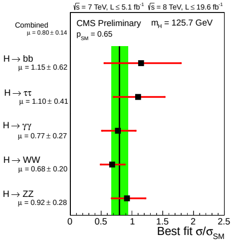
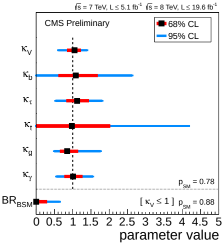
University of Oxford

LAPTh, Annecy, 5 February 2015

Status after LHC "run I"

- Scalar at 125 GeV found, study of properties begun





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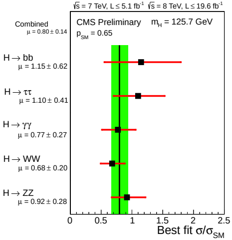
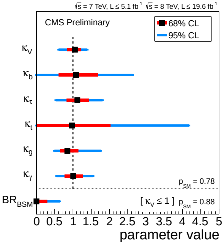
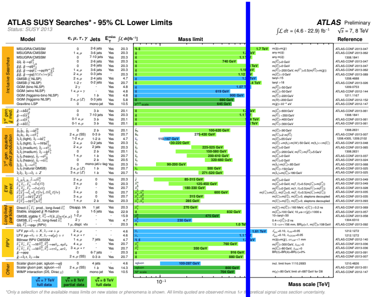
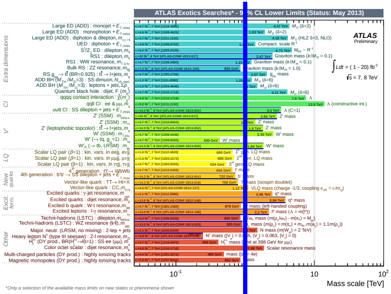




- In general no smoking-gun signal of new-physics





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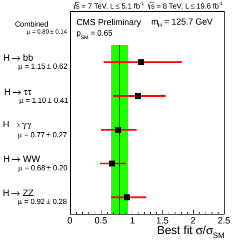

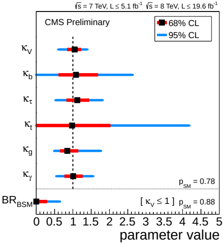

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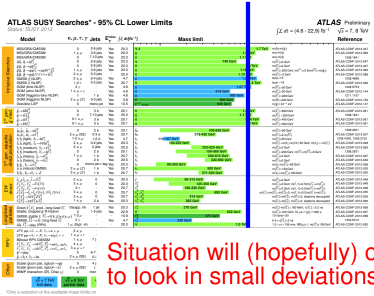
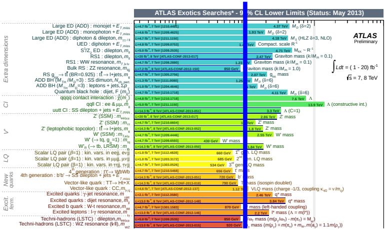
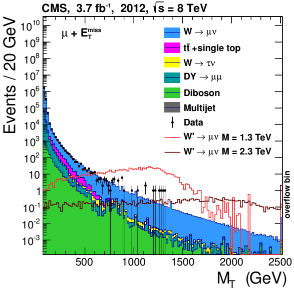




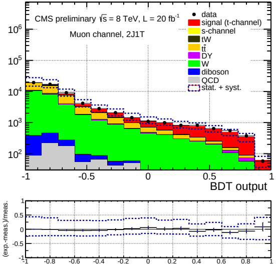
Situation will (hopefully) change at 13-14 TeV. If not, then we have to look in small deviations wrt SM: “precision physics”.

Where are QCD precision and MC important?

1 : s-channel resonance



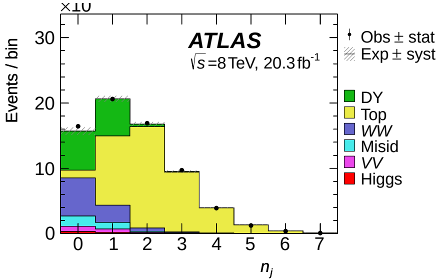
2 : BDT output



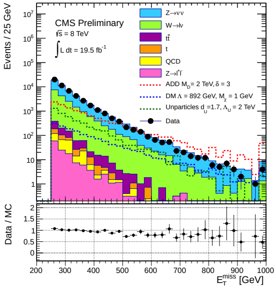
- s-channel resonance "easy" to discover; Higgs discovery in $\gamma\gamma$ and ZZ belongs to 1
- Some analysis techniques (e.g. 2) heavily relies on using MC event generators to separate signal and backgrounds
- MC very often needed also in more standard analysis...

Where are QCD precision and MC important?

3 : jet-binned x-section



4 : high-pt excess



- For 3 and 4, need to control as much as possible QCD effects (i.e. rates and shapes, and also uncertainties!).
- Similar issues when extracting a SM parameters very precisely (e.g. the W mass).

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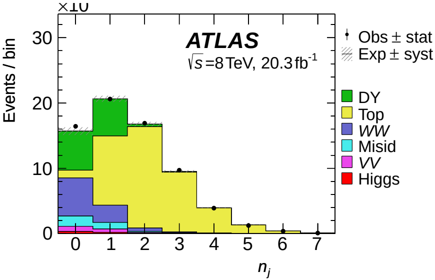

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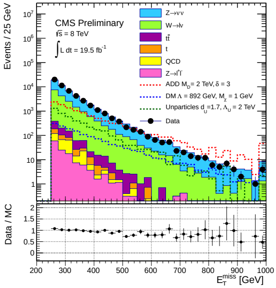


- at some level, MC event generators enter in almost all experimental analyses

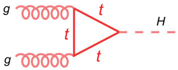
precise tools ⇒ smaller uncertainties on measured quantities

⇓

"small" deviations from SM accessible

Event generators: what they are?

ideal world: high-energy collision and detection of elementary particles

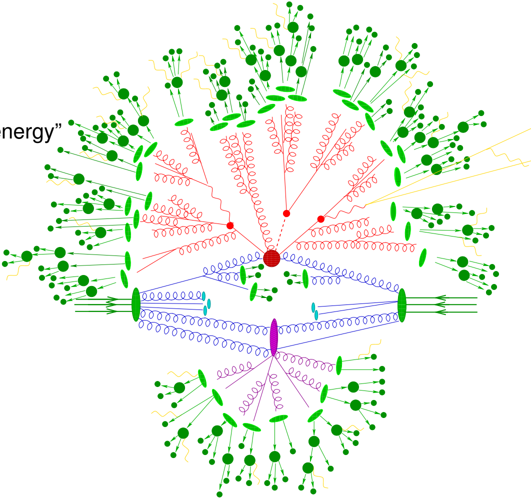


Event generators: what they are?

ideal world: high-energy collision and detection of elementary particles
real world:

- collide non-elementary particles
- we detect e, μ, γ,hadrons, “missing energy”

- we want to predict final state
  - realistically
  - precisely
  - from first principles



[sherpa’s artistic view]

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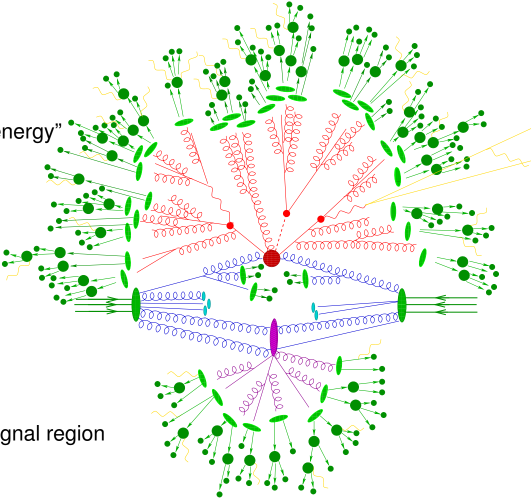

$\Rightarrow$ full event simulation needed to:
- compare theory and data
- estimate how backgrounds affect signal region
- test/build analysis techniques
- ...

[sherpa’s artistic view]

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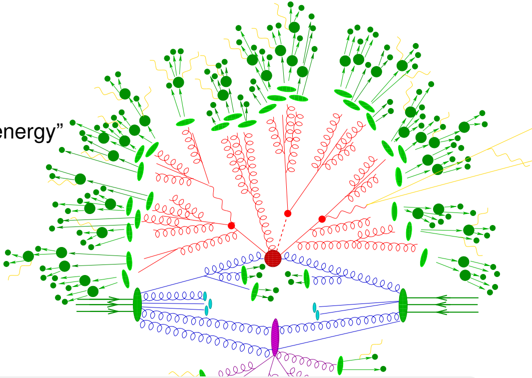


- hard scattering: QCD, EW, BSM (fixed order) $\mu \approx Q \gg \Lambda_{\text{QCD}}$
- multiple soft and collinear emissions $\Lambda_{\text{QCD}} < \mu < Q$
  - $\hookrightarrow$ pQCD (parton shower approximation)
- large distance: hadronisation $\mu \approx \Lambda_{\text{QCD}}$
  - $\hookrightarrow$ non-perturbative QCD $\rightarrow$ phenomenological models, tuned on data.

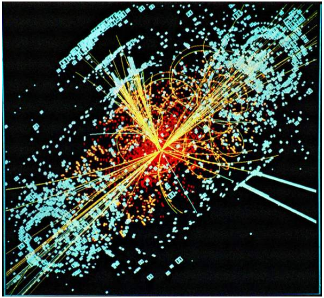
Event generators: what's the output?

- in practice: momenta of all outgoing leptons and hadrons:

IHEP	ID	IDPDG	IST	MO1	MO2	DA1	DA2	P-X	P-Y	P-Z	ENERGY
31	NU_E	12	1	29	22	0	0	60.53	37.24	-1185.0	1187.1
32	E+	-11	1	30	22	0	0	-22.80	2.59	-232.4	233.6
148	K+	321	1	109	9	0	0	-1.66	1.26	1.3	2.5
151	PI0	111	1	111	9	0	0	-0.01	0.05	11.4	11.4
152	PI+	211	1	111	9	0	0	-0.19	-0.13	2.0	2.0
153	PI-	-211	1	112	9	0	0	0.84	-1.07	1626.0	1626.0
154	K+	321	1	112	9	0	0	0.48	-0.63	945.7	945.7
155	PI0	111	1	113	9	0	0	-0.37	-1.16	64.8	64.8
156	PI-	-211	1	113	9	0	0	-0.20	-0.02	3.1	3.1
158	PI0	111	1	114	9	0	0	-0.17	-0.11	0.2	0.3
159	PI0	111	1	115	18	0	0	0.18	-0.74	-267.8	267.8
160	PI-	-211	1	115	18	0	0	-0.21	-0.13	-259.4	259.4
161	N	2112	1	116	23	0	0	-8.45	-27.55	-394.6	395.7
162	NBAR	-2112	1	116	23	0	0	-2.49	-11.05	-154.0	154.4
163	PI0	111	1	117	23	0	0	-0.45	-2.04	-26.6	26.6
164	PI0	111	1	117	23	0	0	0.00	-3.70	-56.0	56.1
167	K+	321	1	119	23	0	0	-0.40	-0.19	-8.1	8.1
186	PBAR	-2212	1	130	9	0	0	0.10	0.17	-0.3	1.0

Plan of the talk

1. brief review how these tools work
2. discuss how their accuracy can be improved
3. explain how to build an event generator that is NNLO accurate (NNLOPS)



parton showers and fixed order

Parton showers I

- connect the hard scattering ($\mu \approx Q$) with the final state hadrons ($\mu \approx \Lambda_{\mathrm{QCD}}$)
- need to simulate production of many quarks and gluons

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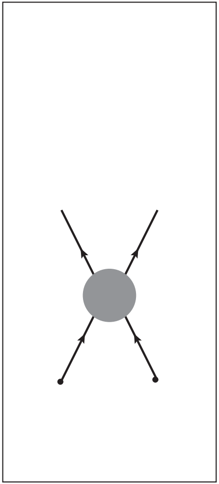

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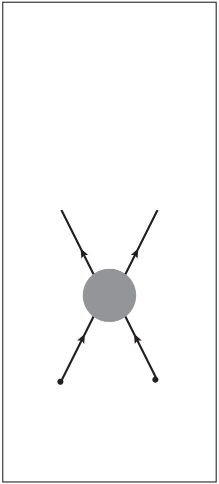
2. quarks and gluons are color-charged $\Rightarrow$ they radiate



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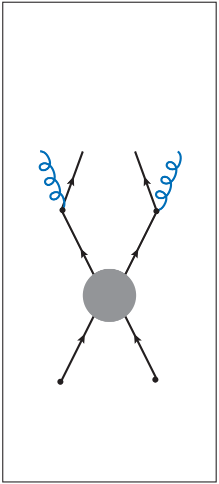

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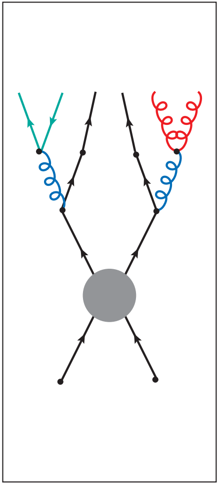

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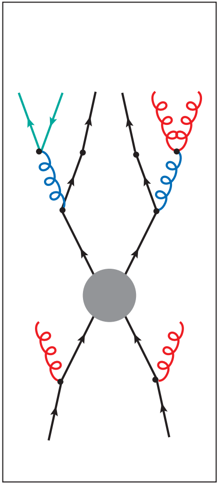

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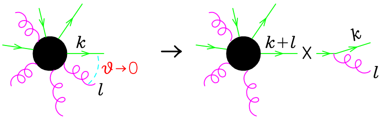
3. soft-collinear emissions are ennhanced:

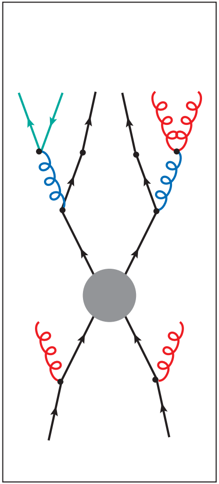
$$\frac{1}{(p_1+p_2)^2} = \frac{1}{2E_1E_2(1-\cos\theta)}$$

4. in soft-collinear limit, factorization properties of QCD amplitudes



$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \to |\mathcal{M}_n|^2 d\Phi_n \; \frac{\alpha_S}{2\pi} \frac{dt}{t} P_{q,qg}(z) dz \frac{d\varphi}{2\pi}$$

$z = k^0/(k^0+l^0)$	quark energy fraction
$t = \left\{(k+l)^2, l_T^2, E^2\theta^2\right\}$	splitting hardness
$P_{q,qg}(z) = C_F \frac{1+z^2}{1-z}$	AP splitting function



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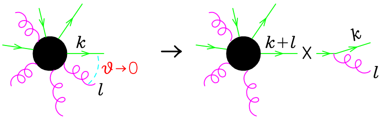

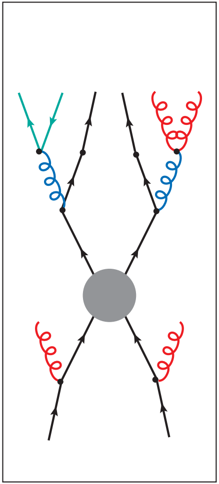

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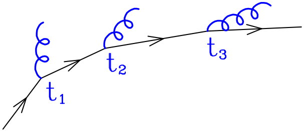
$P_{q,qg}(z) = C_F \dfrac{1+z^2}{1-z}$ AP splitting function

probabilistic interpretation!

Parton showers II

5. dominant contributions for multiparticle production due to strongly ordered emissions

$$t_1 > t_2 > t_3 \ldots$$



6. at any given order, we also have virtual corrections: for consistency we should include them with the same approximation

- LL virtual contributions included by assigning to each internal line a Sudakov form factor:

$$\Delta_a(t_i, t_{i+1}) = \exp\left[-\sum_{(bc)} \int_{t_{i+1}}^{t_i} \frac{dt'}{t'} \int \frac{\alpha_s(t')}{2\pi} P_{a,bc}(z)\, dz\right]$$

- Δ_a corresponds to the probability of having no resolved emission between t_i and t_{i+1} off a line of flavour a

☞ resummation of collinear logarithms

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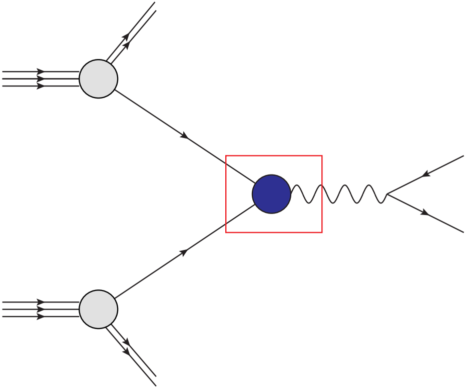
7. At scales $\mu \approx \Lambda_{\text{QCD}}$, $\alpha_S \gtrsim 1$ and hadrons form: non-perturbative effect, simulated with models fitted to data

Parton showers: summary

$$d\sigma_{\rm SMC} = \underbrace{|\mathcal{M}_B|^2 d\Phi_B}_{d\sigma_B} \Big\{ \qquad \Big\}$$



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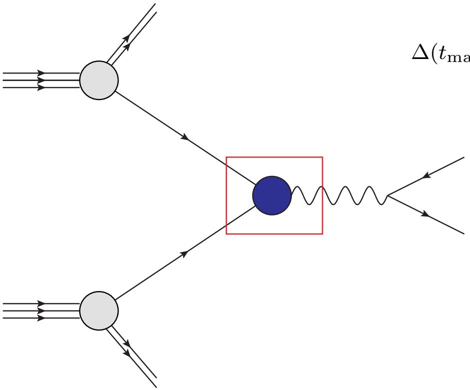

$$\Delta(t_{\max}, t) = \exp\left\{ - \int_t^{t_{\max}} d\Phi'_r \, \frac{\alpha_s}{2\pi} \frac{1}{t'} P(z') \right\}$$

Parton showers: summary

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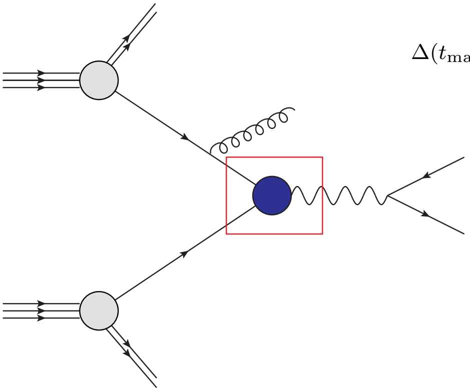

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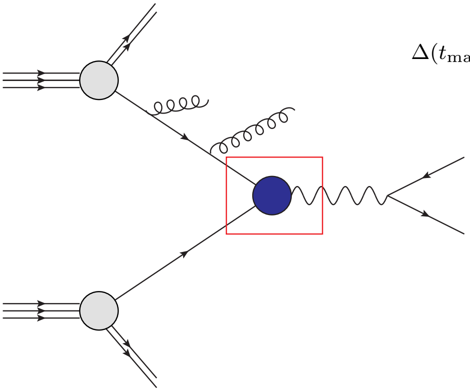

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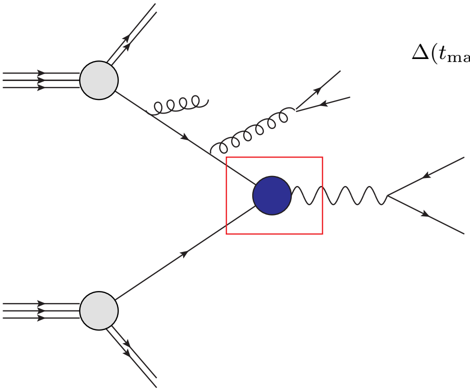

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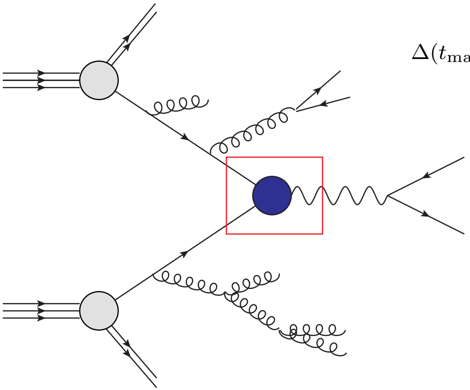

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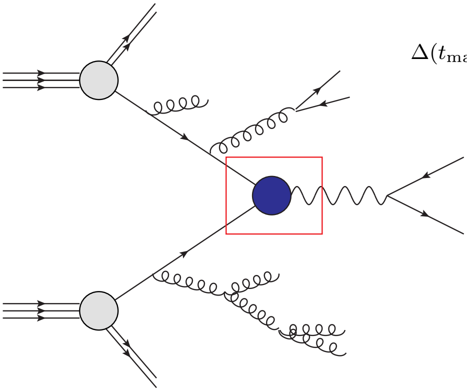

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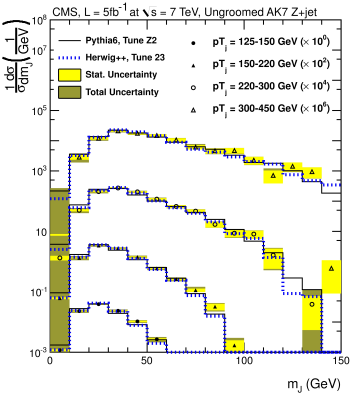
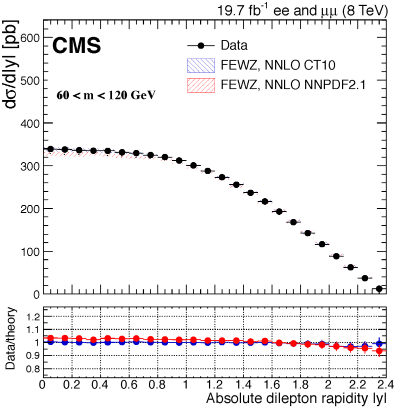
This is “LOPS”

- A parton shower changes shapes, not the overall normalization, which stays LO (*unitarity*)

Do they work?





- ok when observables dominated by soft-collinear radiation [✓]
- not surprisingly, they fail when looking for hard multijet kinematics [✗]
- they are only LO+LL accurate (whereas we want (N)NLO QCD corrections) [✗]

⇒ Not enough if interested in precision (10% or less), or in multijet regions

Next-to-Leading Order

$\alpha_S \sim 0.1 \Rightarrow$ to improve the accuracy, use exact perturbative expansion

$$d\sigma = d\sigma_{\text{LO}} + \left(\frac{\alpha_S}{2\pi}\right) d\sigma_{\text{NLO}} + \left(\frac{\alpha_S}{2\pi}\right)^2 d\sigma_{\text{NNLO}} + \ldots$$

LO: *Leading Order*
NLO: *Next-to-Leading Order*
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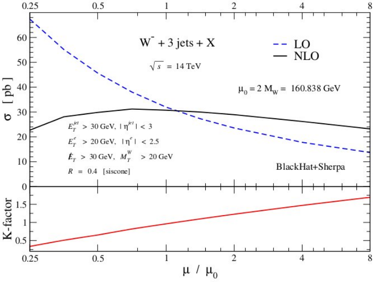
☞ Why NLO is important?

- first order where rates are reliable
- shapes are, in general, better described
- possible to attach sensible theoretical uncertainties



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...

☞ Why NLO is important?

- first order where rates are reliable
- shapes are, in general, better described
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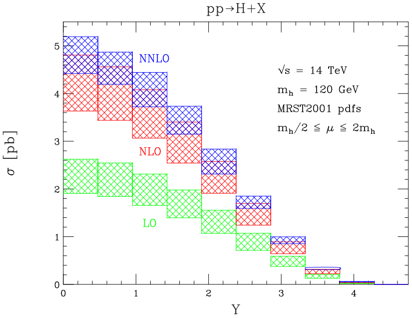
☞ When NNLO is needed?

- NLO corrections large
- very high-precision needed

$\Rightarrow$ Drell-Yan, Higgs, $t\bar{t}$ production



plot from [Anastasiou et al., '03]

PS vs. NLO

NLO

- ✓ precision
- ✓ nowadays this is the standard
- ✗ limited multiplicity
- ✗ (fail when resummation needed)

parton showers

- ✓ realistic + flexible tools
- ✓ widely used by experimental coll's
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☞ can we merge them and build an NLOPS generator?

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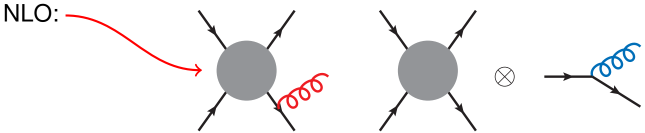

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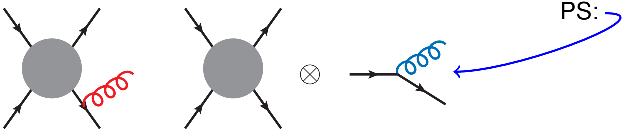

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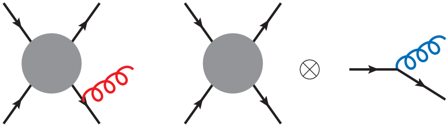


✓ many proposals, 2 well-established methods available to solve this problem: MC@NLO and POWHEG [Frixione-Webber '03, Nason '04]

matching NLO and PS

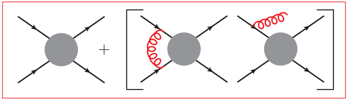
- POWHEG (POsitive Weight Hardest Emission Generator)

NLOPS: POWHEG I

$$d\sigma_{\mathrm{POW}} = d\Phi_n \quad \bar{B}(\Phi_n) \quad \left\{ \Delta(\Phi_n; k_{\mathrm{T}}^{\mathrm{min}}) + \Delta(\Phi_n; k_{\mathrm{T}}) \frac{\alpha_s}{2\pi} \frac{R(\Phi_n, \Phi_r)}{B(\Phi_n)} \, d\Phi_r \right\}$$

NLOPS: POWHEG I

$$B(\Phi_n) \Rightarrow \bar{B}(\Phi_n) = B(\Phi_n) + \frac{\alpha_s}{2\pi}\left[V(\Phi_n) + \int R(\Phi_{n+1})\, d\Phi_r\right]$$



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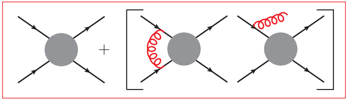

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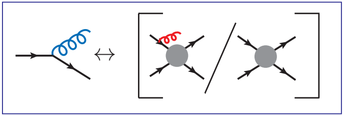

$$\Delta(t_{\rm m}, t) \Rightarrow \Delta(\Phi_n; k_{\rm T}) = \exp\left\{-\frac{\alpha_s}{2\pi} \int \frac{R(\Phi_n, \Phi'_r)}{B(\Phi_n)} \theta(k'_{\rm T} - k_{\rm T})\, d\Phi'_r\right\}$$

NLOPS: POWHEG II

$$d\sigma_{\mathrm{POW}} = d\Phi_n\ \bar{B}(\Phi_n) \left\{ \Delta(\Phi_n; k_{\mathrm{T}}^{\mathrm{min}}) + \Delta(\Phi_n; k_{\mathrm{T}}) \frac{\alpha_s}{2\pi} \frac{R(\Phi_n, \Phi_r)}{B(\Phi_n)}\ d\Phi_r \right\}$$

[+ p_{T}-*vetoing subsequent emissions*, to avoid double-counting]

- inclusive observables: @NLO
- first hard emission: full tree level ME
- (N)LL resummation of collinear/soft logs
- extra jets in the shower approximation

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This is "NLOPS"

POWHEG BOX

[Alioli,Nason,Oleari,ER '10]

- large library of SM processes, (largely) automated
- widely used by LHC collaborations and other theorists, also thanks to standardised interfaces (BLHA)
- not really a closed chapter; some important issues are still to be addressed...

NNLO+PS: why and where?

NLO(+PS) not always enough: NNLO needed when

1. large NLO/LO "K-factor"
   [as in Higgs Physics]

2. very high precision needed
   [e.g. Drell-Yan, top pairs]

- last couple of years:
  huge progress in NNLO

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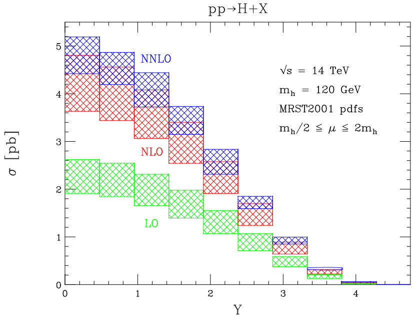

[Anastasiou et al., ’03]

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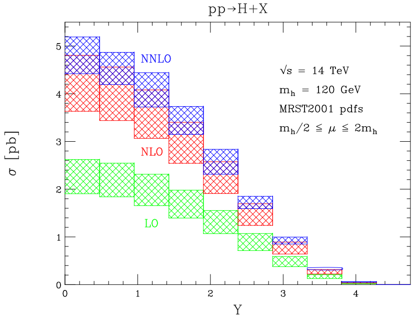

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Q: can we merge NNLO and PS?

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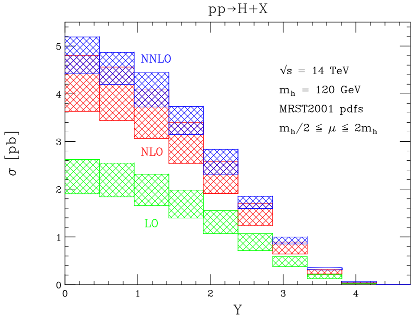

[Anastasiou et al., ’03]

Q: can we merge NNLO and PS?

☞ realistic event generation with state-of-the-art perturbative accuracy !
☞ important for precision studies for several processes

- method presented here: based on POWHEG+MiNLO, used so far for
  - Higgs production [Hamilton,Nason,ER,Zanderighi, 1309.0017]
  - neutral & charged Drell-Yan [Karlberg,ER,Zanderighi, 1407.2940]

towards NNLO+PS

- what do we need and what do we already have?

	H (inclusive)	H+j (inclusive)	H+2j (inclusive)
H @ NLOPS	NLO	LO	shower
HJ @ NLOPS	/	NLO	LO
H @ NNLOPS	NNLO	NLO	LO

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H-HJ @ NLOPS	NLO	NLO	LO
H @ NNLOPS	NNLO	NLO	LO

☞ a merged H-HJ generator is almost OK

towards NNLO+PS

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H-HJ @ NLOPS	NLO	NLO	LO
H @ NNLOPS	NNLO	NLO	LO

☞ a merged H-HJ generator is almost OK

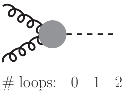
- many of the multijet NLO+PS merging approaches work by combining 2 (or more) NLO+PS generators, introducing a merging scale

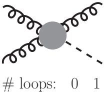
- `POWHEG` + `MiNLO`: no need of merging scale: it extends the validity of an NLO computation with jets in the final state to phase-space regions where jets become unresolved

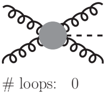
rest of the talk: explain how to do this...

rest of the talk:

Higgs at NNLO:







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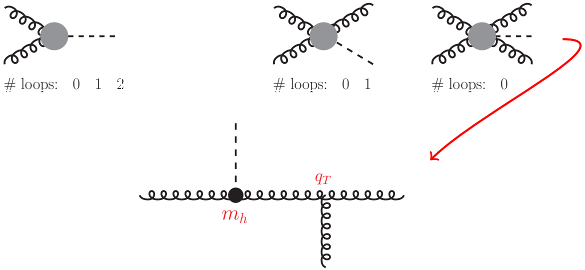

(a) 1 and 2 jets: POWHEG H+1j

rest of the talk:

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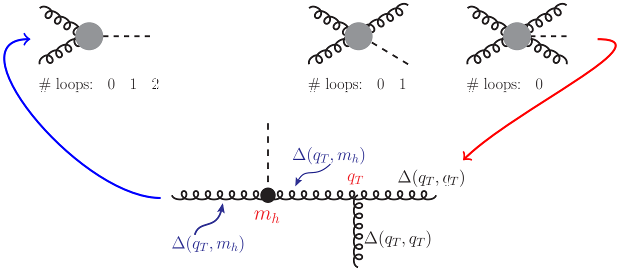

(b) - integrate down to $q_T = 0$ with MiNLO
- "Improved MiNLO" allows to build a H-HJ @ NLOPS generator

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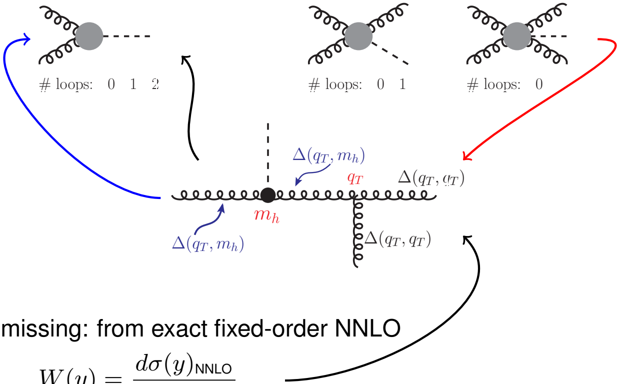

(c) 2 loops missing: from exact fixed-order NNLO

$$W(y) = \frac{d\sigma(y)_{\text{NNLO}}}{d\sigma(y)_{\text{MiNLO}}}$$

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- "Improved MiNLO" allows to build a H-HJ @ NLOPS generator

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NLOPS merging

- MiNLO (Multiscale Improved NLO)

MiNLO

Multiscale Improved NLO

[Hamilton,Nason,Zanderighi, 1206.3572]

- original goal: method to a-priori choose scales in multijet NLO computation
- non-trivial task, since phase space is by construction probed also in presence of widely separated energy scales

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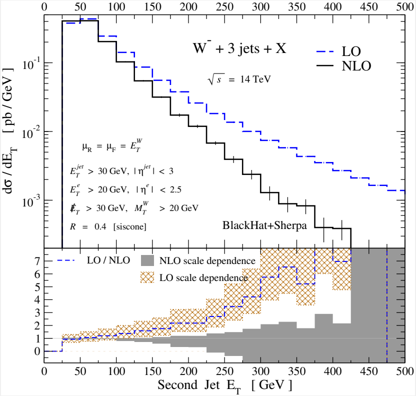
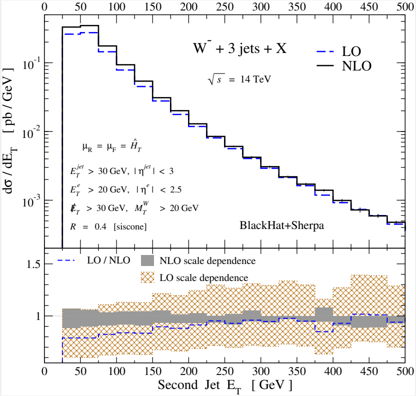


$$\mu = E_{T,W}$$



$$\mu = H_T$$

plot from [Berger et al., '09]

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- how: correct weights of different NLO terms with CKKW-inspired approach (without spoiling formal NLO accuracy)
  - for each point sampled, build the "more-likely" shower history that would have produced that kinematics (can be done by clustering kinematics with k_T-algo, then, by undoing the clustering, build "skeleton")
  - "CKKW-correct" original NLO: $\alpha_{\rm S}$ evaluated at nodal scales and Sudakov FFs

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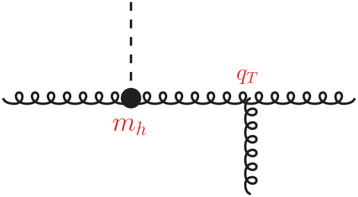

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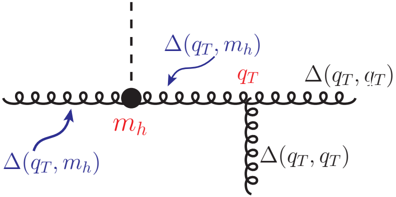
$$\bar{B}_{\mathrm{MiNLO}} = \alpha_{\mathrm{S}}^2(m_h)\alpha_{\mathrm{S}}(q_T)\Delta_g^2(q_T, m_h)\Big[B\left(1 - 2\Delta_g^{(1)}(q_T, m_h)\right) + \alpha_{\mathrm{S}}\, V(\bar{\mu}_R) + \alpha_{\mathrm{S}} \int d\Phi_{\mathrm{r}} R\Big]$$



- $\bar{\mu}_R = (m_h^2 q_T)^{1/3}$
- $\log \Delta_{\mathrm{f}}(q_T, m_h) = -\int_{q_T^2}^{m_h^2} \frac{dq^2}{q^2} \frac{\alpha_{\mathrm{S}}(q^2)}{2\pi}\left[A_f \log\frac{m_h^2}{q^2} + B_f\right]$
- $\Delta_{\mathrm{f}}^{(1)}(q_T, m_h) = -\frac{\alpha_{\mathrm{S}}}{2\pi}\left[\frac{1}{2}A_{1,\mathrm{f}} \log^2\frac{m_h^2}{q_T^2} + B_{1,\mathrm{f}} \log\frac{m_h^2}{q_T^2}\right]$
- $\mu_F = q_T$

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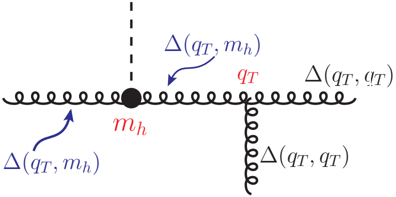

☞ Sudakov FF included on $H+j$ Born kinematics

- MiNLO-improved HJ yields finite results also when 1st jet is unresolved ($q_T \to 0$)
- $\bar{B}_{\text{MiNLO}}$ ideal to extend validity of HJ−POWHEG [called "HJ−MiNLO" hereafter]

“Improved” MiNLO & NLOPS merging

- formal accuracy of `HJ-MiNLO` for inclusive observables carefully investigated

  [Hamilton et al., 1212.4504]

- `HJ-MiNLO` describes inclusive observables at order α_{S}
- to reach genuine NLO when fully inclusive (NLO$^{(0)}$), “spurious” terms must be of relative order α_{S}^2, *i.e.*

$$O_{\mathrm{HJ-MiNLO}} = O_{\mathrm{H@NLO}} + \mathcal{O}(\alpha_{\mathrm{S}}^{2+2}) \qquad \text{if } O \text{ is inclusive}$$

- “Original `MiNLO`” contains ambiguous “$\mathcal{O}(\alpha_{\mathrm{S}}^{2+1.5})$” terms

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- Possible to improve `HJ-MiNLO` such that inclusive NLO is recovered (NLO$^{(0)}$), without spoiling NLO accuracy of $H+j$ (NLO$^{(1)}$).
- accurate control of subleading small-p_T logarithms is needed
  (scaling in low-p_T region is $\alpha_S L^2 \sim 1$, *i.e.* $L \sim 1/\sqrt{\alpha_S}$!)

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Effectively as if we merged NLO$^{(0)}$ and NLO$^{(1)}$ samples, without merging different samples (no merging scale used: there is just one sample).

“Improved” MiNLO & NLOPS merging

- Resummation formula

$$\frac{d\sigma}{dq_T^2 dy} = \sigma_0 \frac{d}{dq_T^2}\Big\{[C_{ga} \otimes f_a](x_A, q_T) \times [C_{gb} \otimes f_b](x_B, q_T) \times \exp S(q_T, Q)\Big\} + R_f$$

$$S(q_T, Q) = -2\int_{q_T^2}^{Q^2} \frac{dq^2}{q^2} \frac{\alpha_S(q^2)}{2\pi}\Big[A_f \log\frac{Q^2}{q^2} + B_f\Big]$$

- If $C_{ij}^{(1)}$ included and R_f is LO$^{(1)}$, then upon integration we get NLO$^{(0)}$
- Take derivative, then compare with `MiNLO` :

$$\sim \sigma_0 \frac{1}{q_T^2}[\alpha_S, \boxed{\alpha_S^2}, \alpha_S^3, \alpha_S^4, \alpha_S L, \alpha_S^2 L, \alpha_S^3 L, \alpha_S^4 L] \exp S(q_T, Q) + R_f \qquad L = \log(Q^2/q_T^2)$$

- highlighted terms are needed to reach NLO$^{(0)}$:

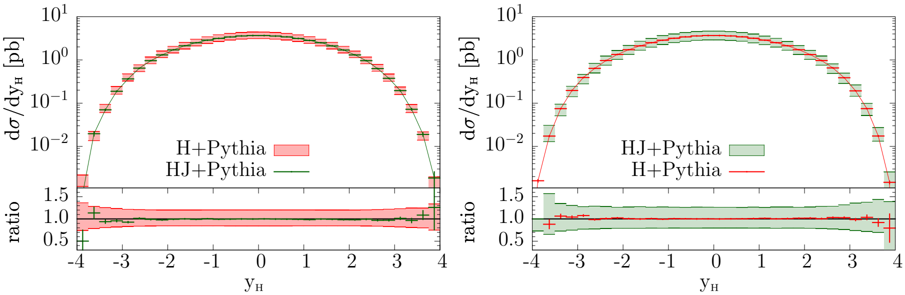
$$\int^{Q^2} \frac{dq_T^2}{q_T^2} L^m \alpha_S{}^n(q_T) \exp S \sim \left(\alpha_S(Q^2)\right)^{n-(m+1)/2}$$

(scaling in low-p_T region is $\alpha_S L^2 \sim 1$!)

- if I don’t include B_2 in `MiNLO` Δ_g, I miss a term $(1/q_T^2)\ \boxed{\alpha_S^2}\ B_2 \exp S$
- upon integration, violate NLO$^{(0)}$ by a term of <u>relative</u> $\mathcal{O}(\alpha_S^{3/2})$

MiNLO merging: results

[Hamilton et al., 1212.4504]



- "H+Pythia": standalone POWHEG ($gg \to H$) + PYTHIA (PS level) [7pts band, $\mu = m_H$]
- "HJ+Pythia": HJ–MiNLO* + PYTHIA (PS level) [7pts band, μ from MiNLO]

- very good agreement (both value and band) [✓]

☞ Notice: band is $\sim 20 - 30\%$

matching NNLO with PS

- Higgs and Drell-Yan production at NNLOPS

Higgs at NNLO+PS I

- `HJ−MiNLO+POWHEG` generator gives H-HJ @ NLOPS

	H (inclusive)	H+j (inclusive)	H+2j (inclusive)
✓ H-HJ @ NLOPS	NLO	NLO	LO
H @ NNLOPS	NNLO	NLO	LO

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- reweighting (differential on Φ_B) of "MiNLO-generated" events:

$$W(\Phi_B) = \frac{\left(\frac{d\sigma}{d\Phi_B}\right)_{\rm NNLO}}{\left(\frac{d\sigma}{d\Phi_B}\right)_{\rm HJ-MiNLO*}}$$

  - by construction NNLO accuracy on fully inclusive observables $(\sigma_{\rm tot}, y_H; m_{\ell\ell}, ...)$ [✓]
  - to reach NNLOPS accuracy, need to be sure that the reweighting doesn't spoil the NLO accuracy of HJ−MiNLO in 1-jet region [ ]

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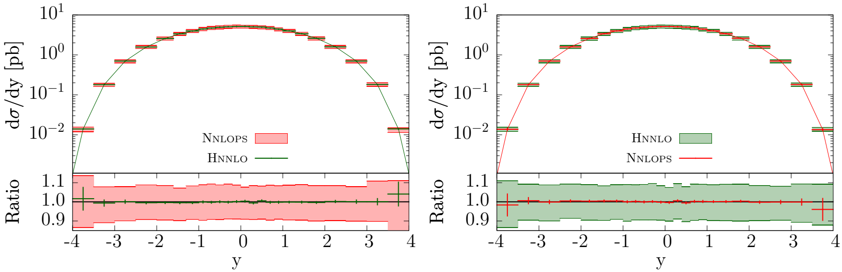
  - to reach NNLOPS accuracy, need to be sure that the reweighting doesn't spoil the NLO accuracy of HJ−MiNLO in 1-jet region [✓]
  - notice: formally works because no spurious $\mathcal{O}(\alpha_S^{2+1.5})$ terms in H-HJ @ NLOPS

H@NNLOPS (fully incl.)

To reweight, use y_H

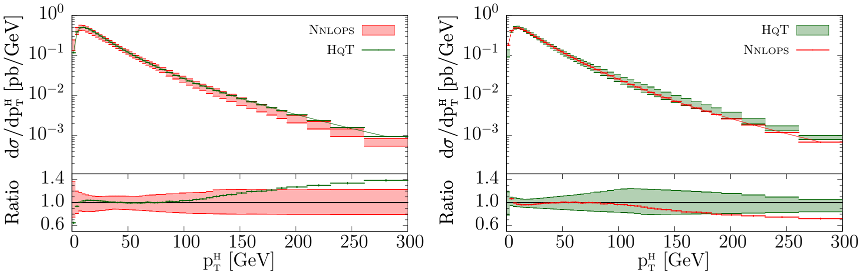
- NNLO with $\mu = m_H/2$, HJ−MiNLO "core scale" m_H [NNLO from HNNLO, Catani,Grazzini]
- $(7_{\rm Mi} \times 3_{\rm NN})$ pts scale var. in NNLOPS, 7pts in NNLO



☞ Notice: band is 10% (at NLO would be $\sim$ 20-30%) [✓]

[Until and including $\mathcal{O}(\alpha_S^4)$, PS effects don't affect y_H (first 2 emissions controlled properly at $\mathcal{O}(\alpha_S^4)$ by MiNLO+POWHEG)]

H@NNLOPS (p_T^H)



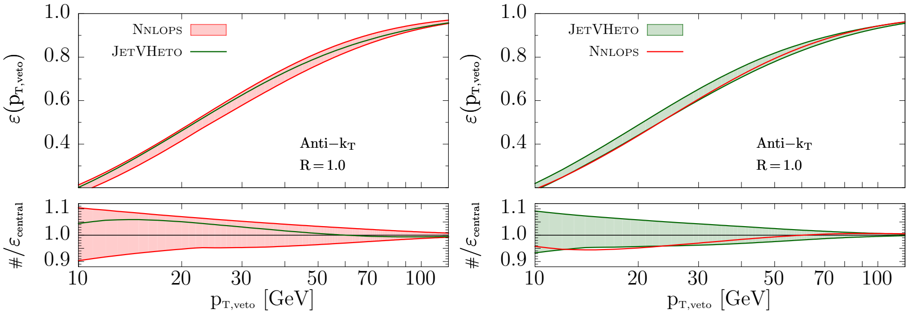
- HqT: NNLL+NNLO, $\mu_R = \mu_F = m_H/2$ [7pts], $Q_{\rm res} \equiv m_H/2$ [HqT, Bozzi et al.]

✓ uncertainty bands of HqT contain NNLOPS at low-/moderate p_T

- HqT tail harder than NNLOPS tail ($\mu_{\rm HqT} <$ "$\mu_{\rm MiNLO}$")
  HJ @ NNLO will allow to say more for large $p_{T,H}$

- very good agreement with HqT resummation
  ["$\sim$ expected", since $Q_{\rm res} \equiv m_H/2$, and $\beta = 1/2$]

H@NNLOPS ($p_T^{j_1}$)



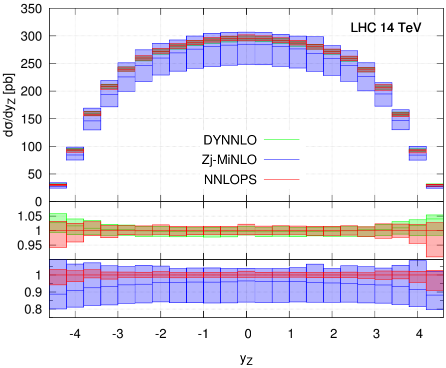
$$\varepsilon\left(p_{\mathrm{T,veto}}\right) = \frac{\Sigma(p_{\mathrm{T,veto}})}{\sigma^{\mathrm{tot}}} = \frac{1}{\sigma^{\mathrm{tot}}} \int d\sigma\, \theta\left(p_{\mathrm{T,veto}} - p_{\mathrm{T}}^{\mathrm{j_1}}\right)$$

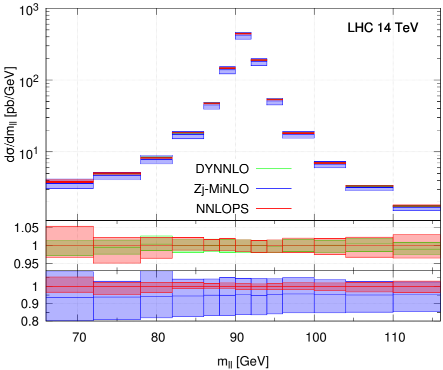
- `JetVHeto`: NNLL resum, $\mu_R = \mu_F = m_H/2$ [7pts], $Q_{\mathrm{res}} \equiv m_H/2$, (a)-scheme only
  [`JetVHeto`, Banfi et al.]
- nice agreement, differences never more than 5-6 %

☞ Separation of $H \to WW$ from $t\bar{t}$ bkg: x-sec binned in N_{jet}
0-jet bin ⇔ jet-veto accurate predictions needed !

Z@NNLOPS, PS level

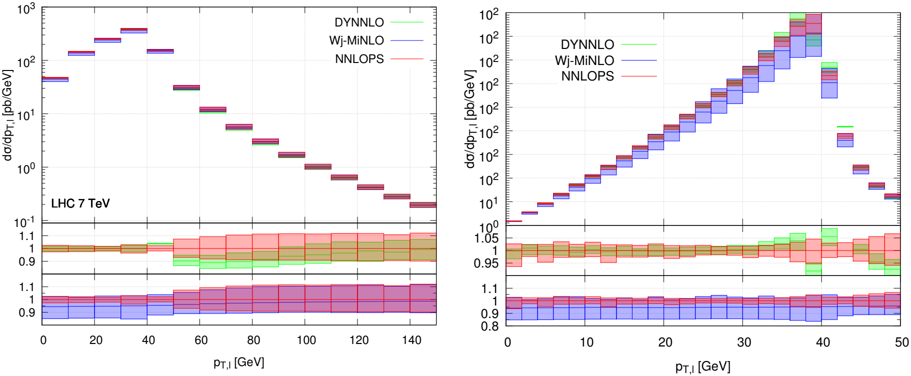
To reweight, use $(y_{\ell\ell}, m_{\ell\ell}, \cos\theta_\ell)$





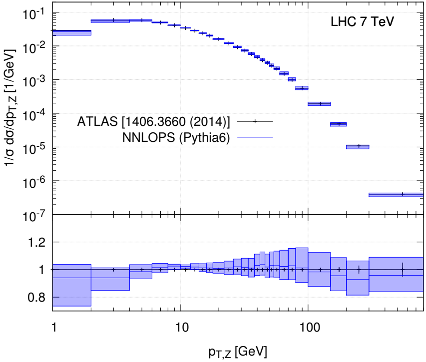
- $(7_{\mathrm{Mi}} \times 3_{\mathrm{NN}})$ pts scale var. in NNLOPS, 7pts in NNLO
- agreement with DYNNLO
- scale uncertainty reduction wrt ZJ-MiNLO

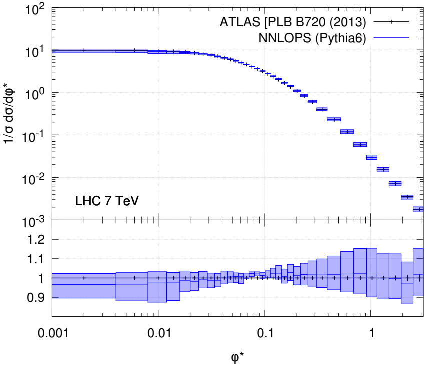
W@NNLOPS, PS level



- not the observables we are using to do the NNLO reweighting
  - observe exactly what we expect:
    $p_{T,\ell}$ has NNLO uncertainty if $p_T < M_W/2$, NLO if $p_T > M_W/2$
  - smooth behaviour when close to Jacobian peak (also with small bins) (due to resummation of logs at small $p_{T,V}$)
- just above peak, DYNNLO uses $\mu = M_W$, WJ-MiNLO uses $\mu = p_{T,W}$
  - here $0 \lesssim p_{T,W} \lesssim M_W$ (so resummation region does contribute)

Vector boson: comparison with data ($p_{T,Z}$, ϕ^*)





- good agreement with data (PS+hadronisation+MPI)
- ϕ^* is an alternative probe to measure low-$p_{T,V}$ domain

$$\phi^* = \tan\left(\frac{\pi - \Delta\phi}{2}\right)\sin\theta^*$$

- θ^*: angle between electron and beam axis, in Z boson rest frame
- ATLAS uses slightly different definition: $\cos\theta^* = \tanh((y_{l^-} - y_{l^+})/2)$

Conclusions and Outlook

- Especially in absence of very clear singals of new-physics, accurate tools are needed for LHC phenomenology
- In the last decade, impressive amount of progress: new ideas, and automated tools

⇒ briefly reviewed how Event Generators work, and how they can be upgraded to NLO

⇒ shown results of merging NLOPS for different jet-multiplicities *without* merging scale

⇒ shown first working examples of NNLOPS

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Thank you for your attention!