# Cosmology V: CMB Polarization and other themes

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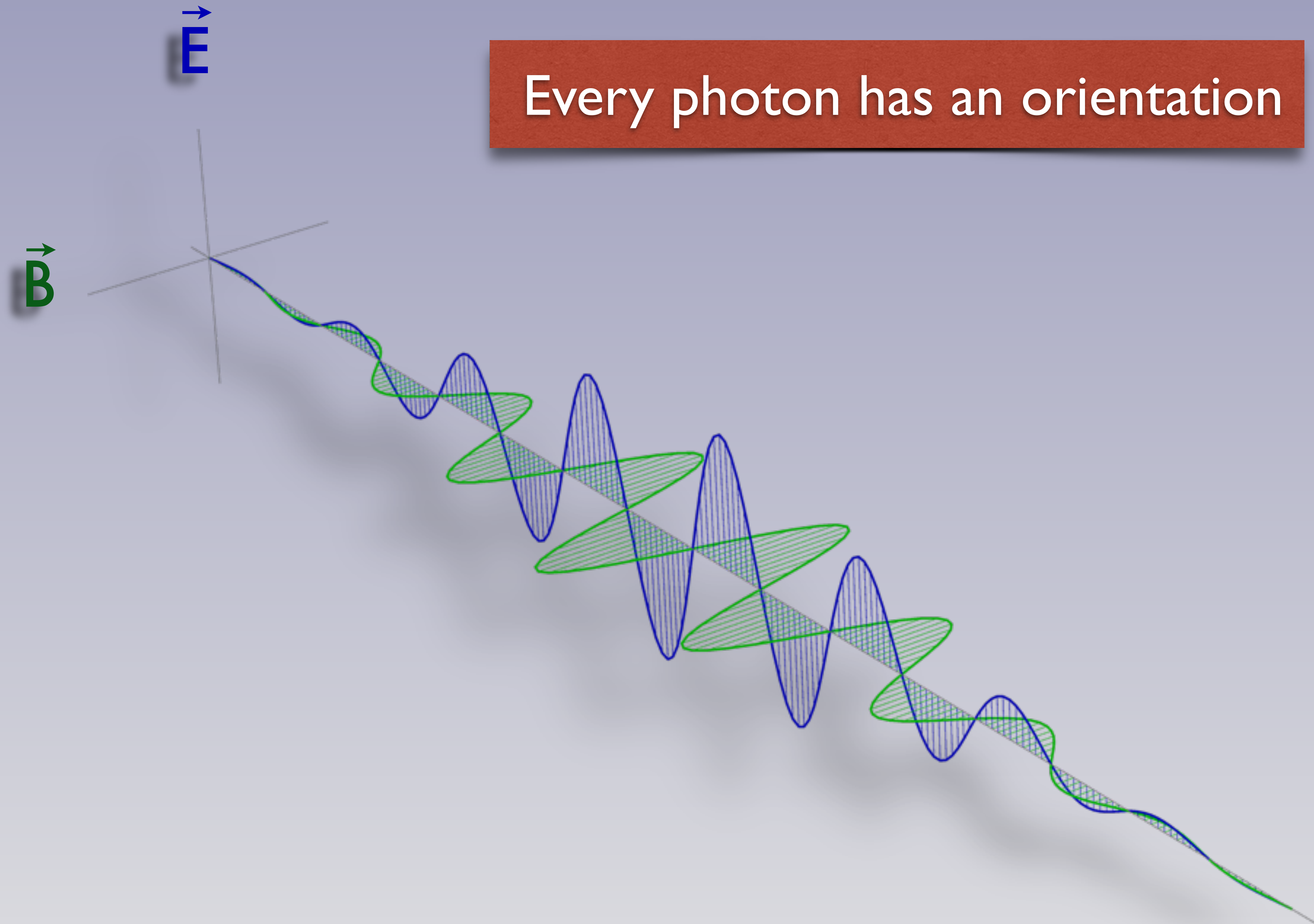
# Outline

# Every photon has an orientation



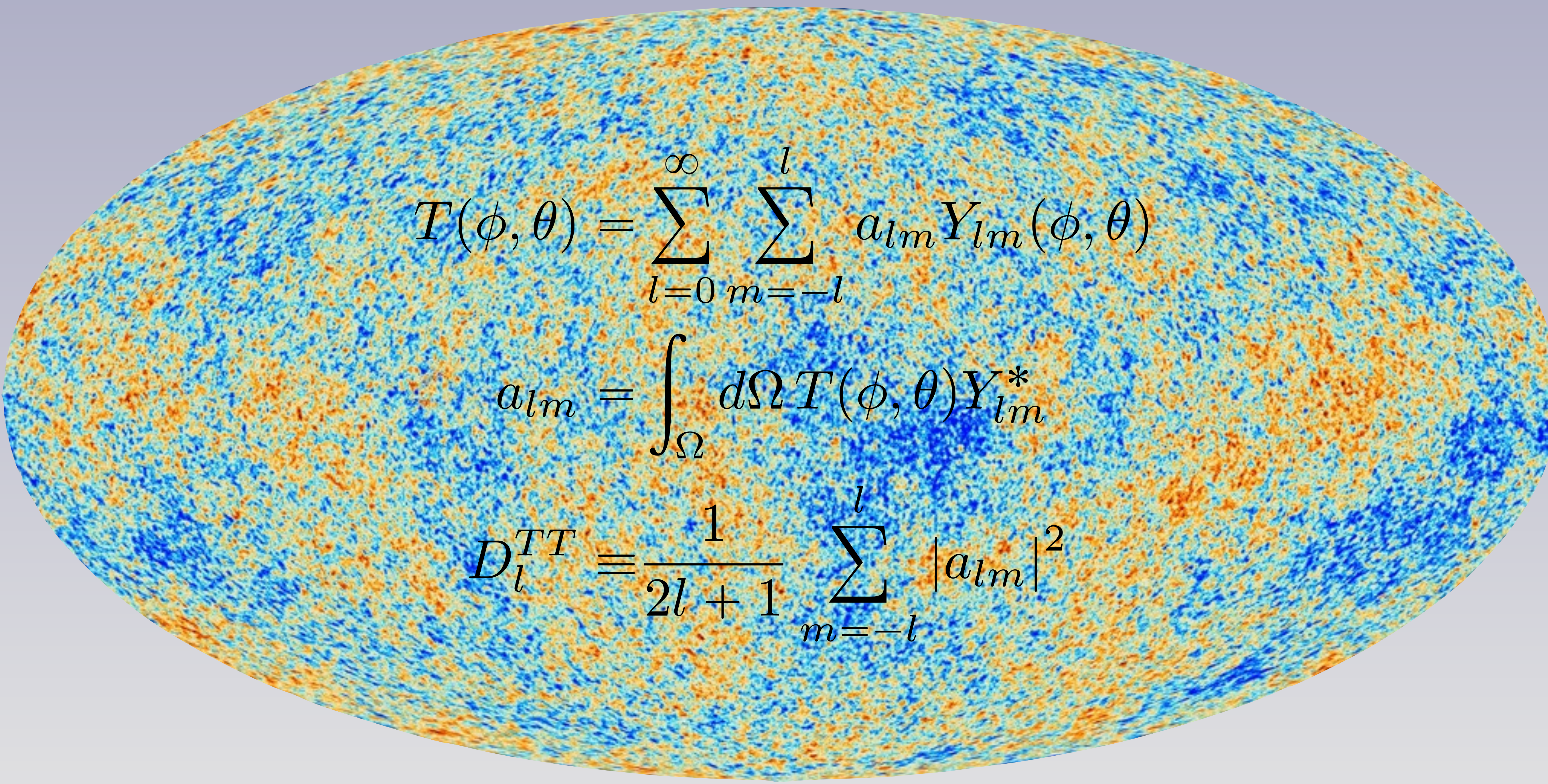
# Spherical harmonics

$$T(\phi, \theta) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} a_{lm} Y_{lm}(\phi, \theta)$$

$$a_{lm} = \int_{\Omega} d\Omega \, T(\phi, \theta) Y_{lm}^{*}$$

$$D_l^{TT} \equiv \frac{1}{2l+1} \sum_{m=-l}^{l} |a_{lm}|^2$$

# Spherical harmonics



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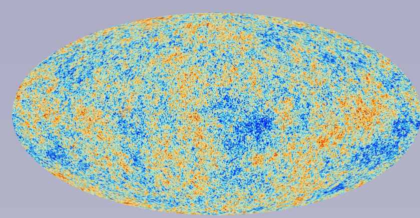

+

$$T(\phi,\theta) = \sum_{l=0}^{\infty}\sum_{m=-l}^{l} a_{lm} Y_{lm}(\phi,\theta)$$

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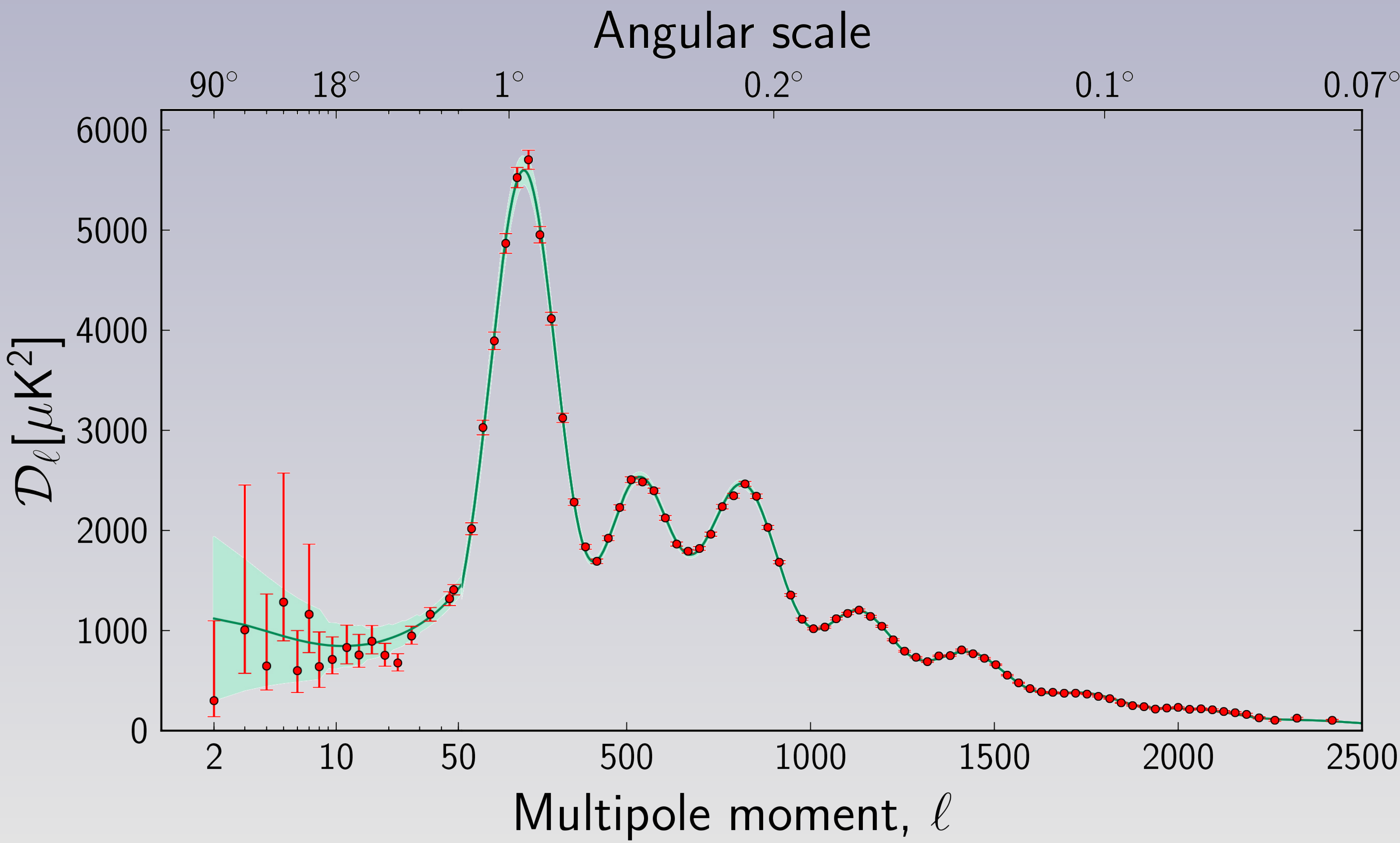
=



# Correlations

$$\langle T(\phi_1,\theta_1)T^*(\phi_2,\theta_2)\rangle = \sum_{l=0}^{\infty} C_l \frac{2l+1}{4\pi} P_l(\hat{n}_1 \cdot \hat{n}_2)$$

For a gaussian random field:

$$\langle a_{lm} a^*_{l'm'} \rangle \equiv C_l \delta_{ll'} \delta_{mm'}$$

independent of $m$!

So, $D_l^{TT} \equiv \frac{1}{2l+1} \sum_{m=-l}^{l} |a_{lm}|^2$ measures $2l+1$ independent random variables, drawn from the same gaussian distribution with variance $C_l$.

# Perturbation Theory

$$\frac{\Delta T(\mathbf{n})}{T} = \left[\frac{1}{4}D_g^{(r)} + V_j^{(b)} n^j + \Psi - \Phi\right](\eta_{dec}, \mathbf{x}_{dec}) + \int_{\eta_{dec}}^{\eta_0} (\dot{\Psi} - \dot{\Phi})(\eta, \mathbf{x}(\eta)) d\eta$$

see [Durrer, astro-ph/0109522] or
[text book “The Cosmic Microwave Background” by Durrer]

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Photon density

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Photon density

Velocity (Doppler)
*negligible*

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Photon density

Gravitational potential

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Photon density

Velocity (Doppler)
*negligible*

Gravitational potential

Time change of gravitational potential along the photon path

see [Durrer, astro-ph/0109522] or
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# For adiabatic perturbations:

$$D_g^{(r)}(k,\eta) = -20/3\Psi(k,\eta)$$

$$\left(\frac{\Delta T(\mathbf{n})}{T}\right)^{(OSW)}_{\text{adiabatic}} = \frac{1}{3}\Psi(\eta_{dec}, \mathbf{x}_{dec})$$

see [Durrer, astro-ph/0109522] or
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Ordinary Sachs-Wolfe effect

Integrated Sachs-Wolfe effect

$$\frac{\Delta T(\mathbf{n})}{T} = \left[\frac{1}{4}D_g^{(r)} + V_j^{(b)}n^j + \Psi - \Phi\right](\eta_{dec}, \mathbf{x}_{dec}) + \int_{\eta_{dec}}^{\eta_0} (\dot{\Psi} - \dot{\Phi})(\eta, \mathbf{x}(\eta))d\eta$$

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Gravity wins!

Hot = underdense

Cold=overdense

$$\left(\frac{\Delta T(\mathbf{n})}{T}\right)_{\text{adiabatic}}^{(OSW)} = \frac{1}{3}\Psi(\eta_{dec}, \mathbf{x}_{dec})$$

see [Durrer, astro-ph/0109522] or
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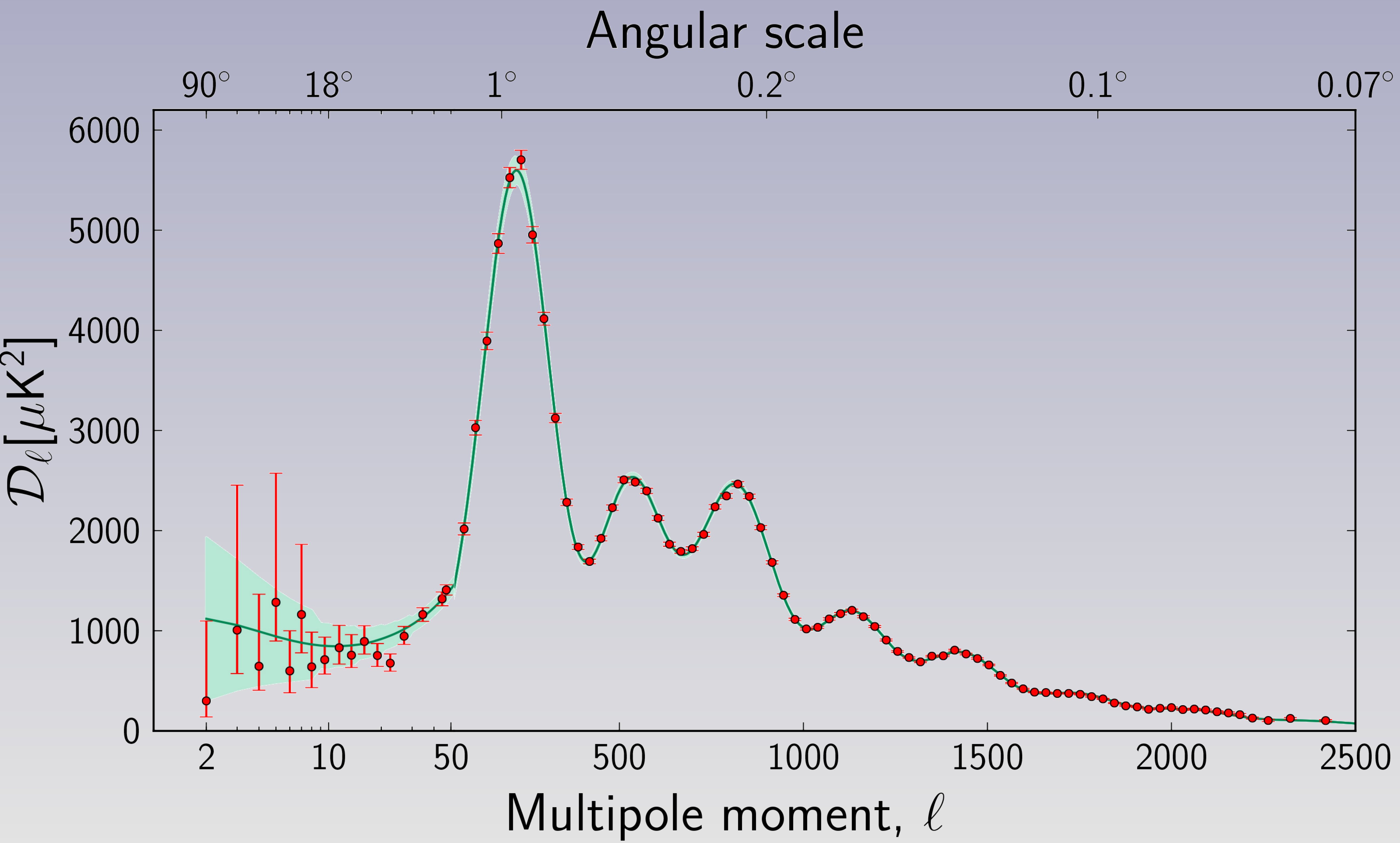
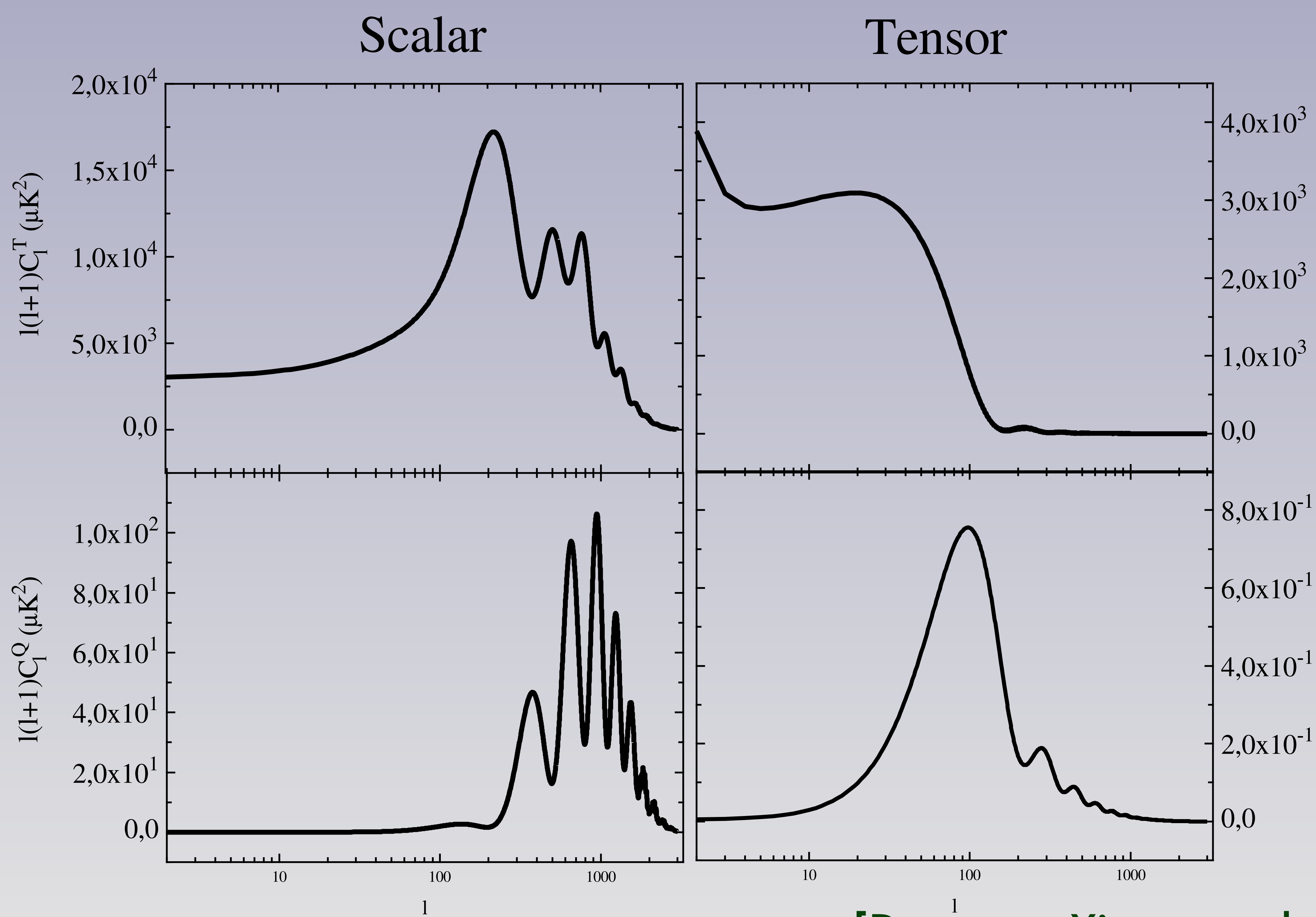
# Shortcut

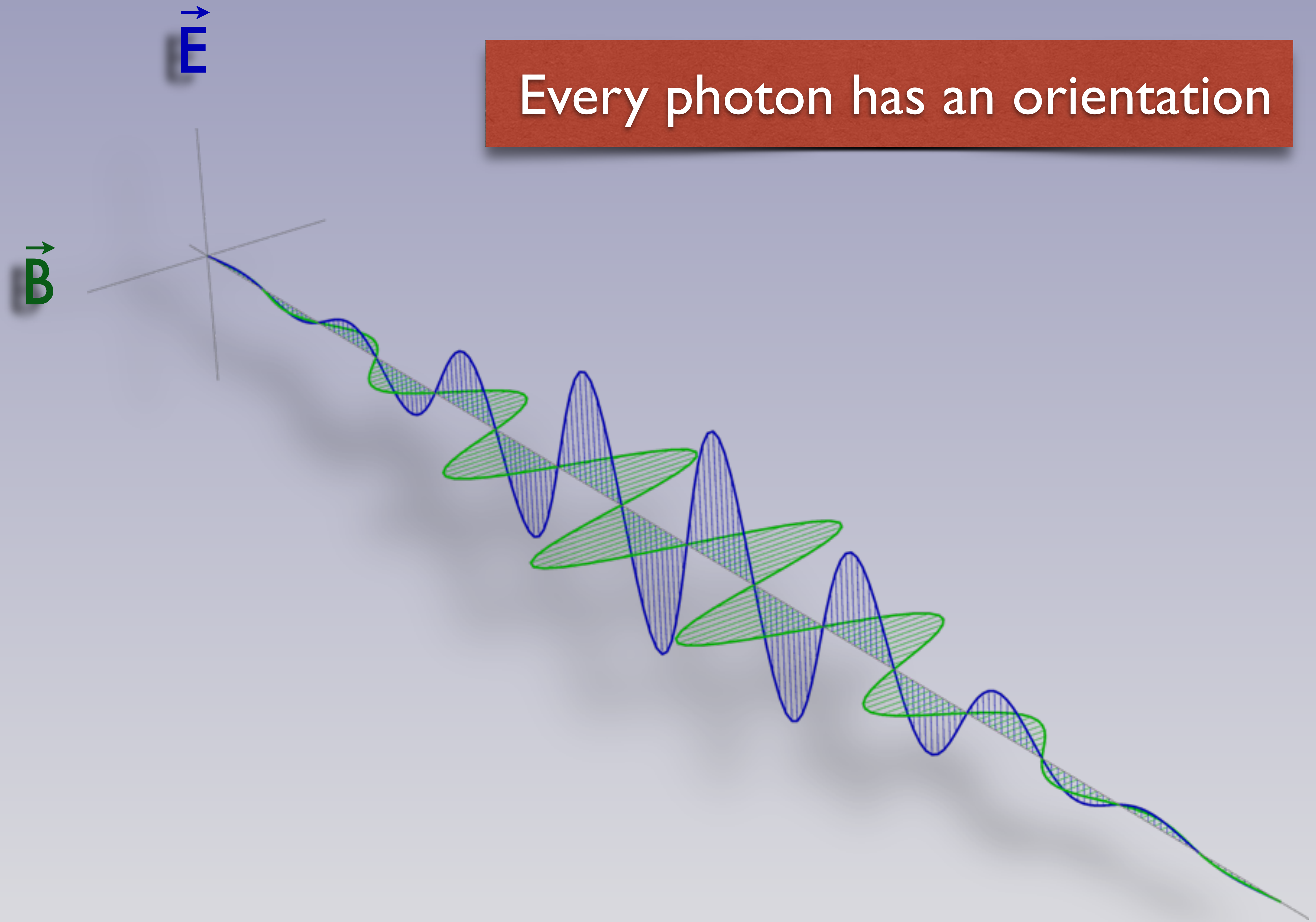

Angular scale
90°
18°
1°
0.2°
0.1°
0.07°
6000
5000
4000
3000
2000
1000
0
$\mathcal{D}_\ell [\mu K^2]$
2
10
50
500
1000
1500
2000
2500
Multipole moment, $\ell$

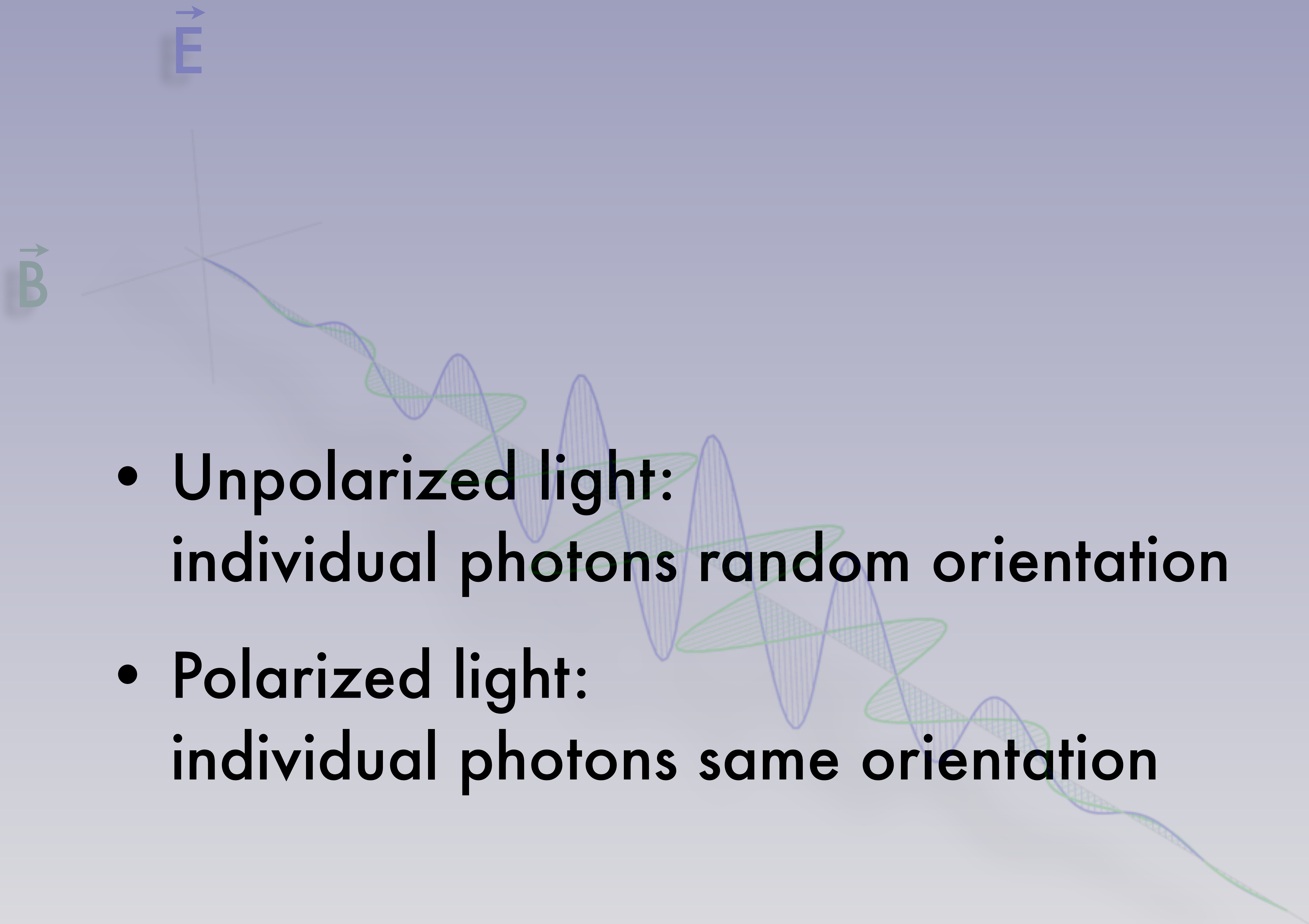
# CMB




[Durrer, arXiv:astro-ph/0109522]

# Every photon has an orientation






- Unpolarized light:
  individual photons random orientation
- Polarized light:
  individual photons same orientation

# Polarization by reflection

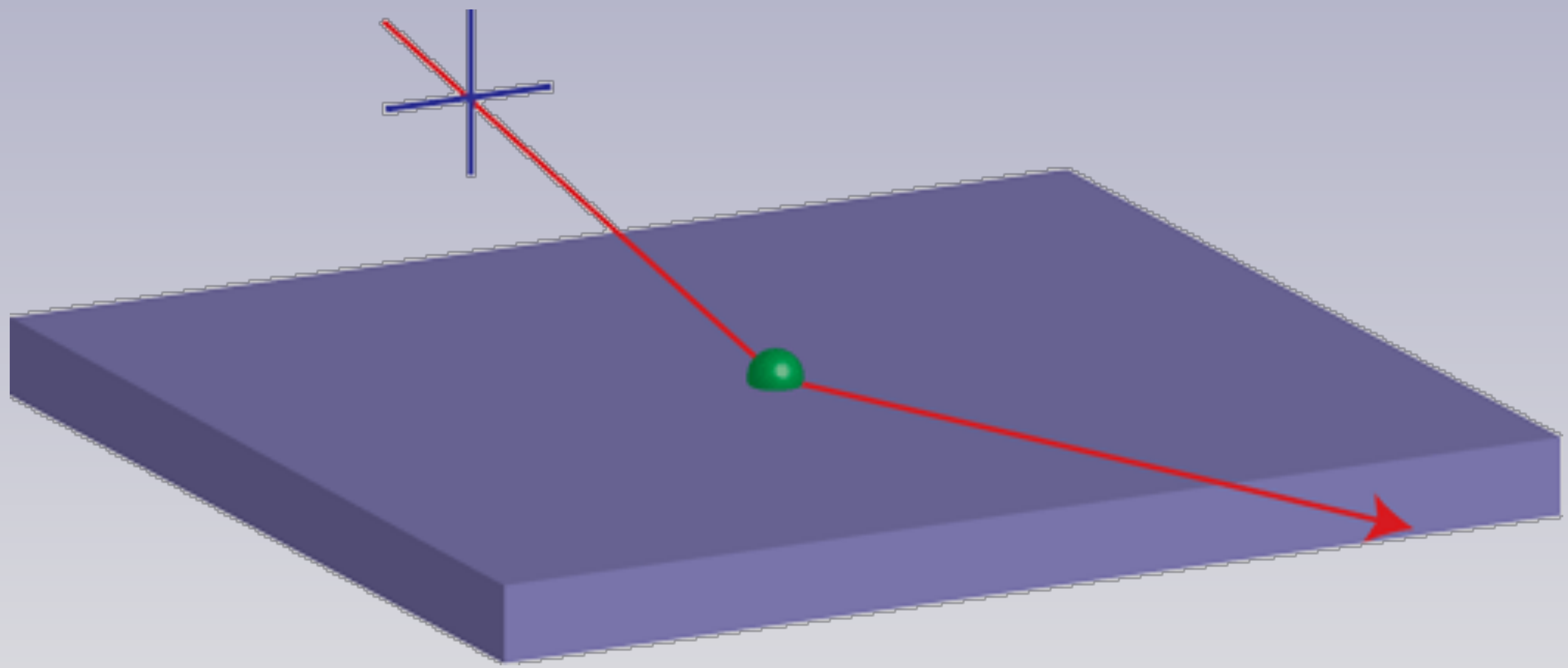
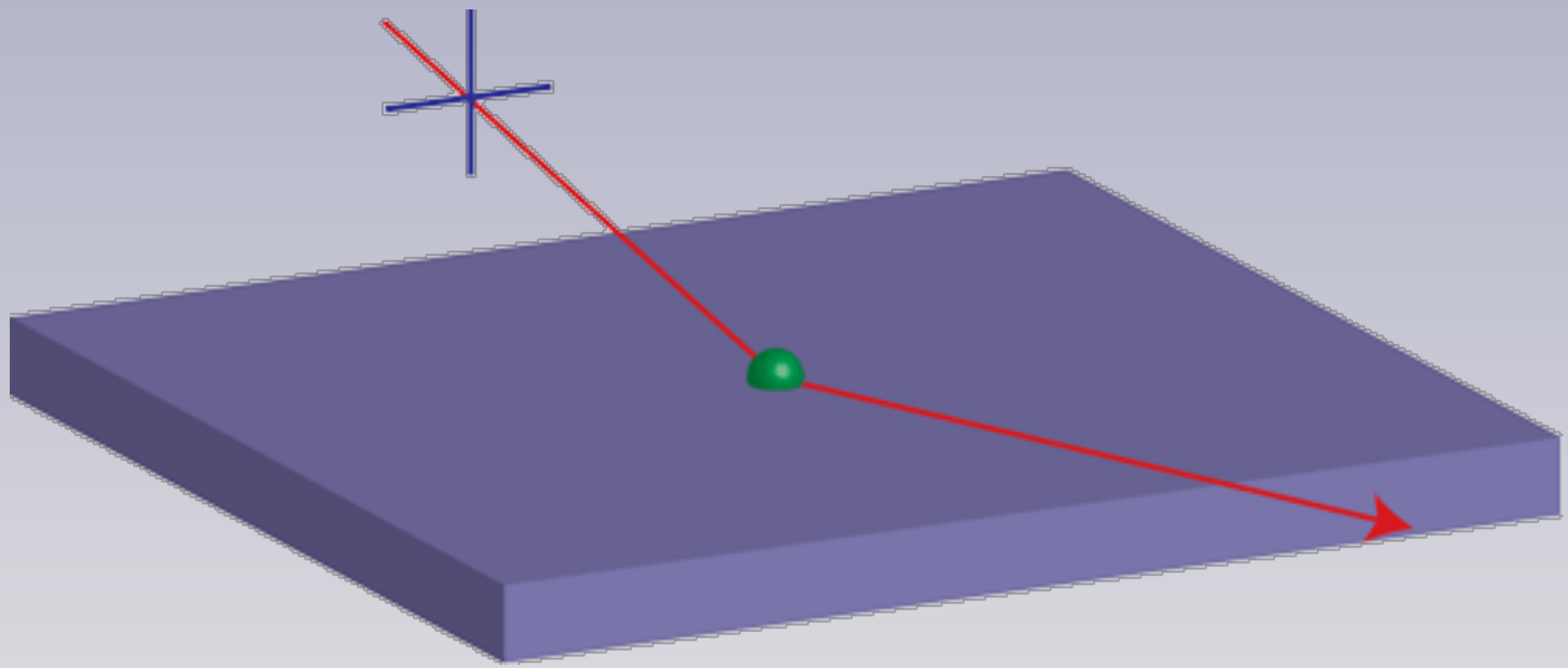


Image by Waynu Hu [http://background.uchicago.edu/~whu/]

# Polarization by reflection



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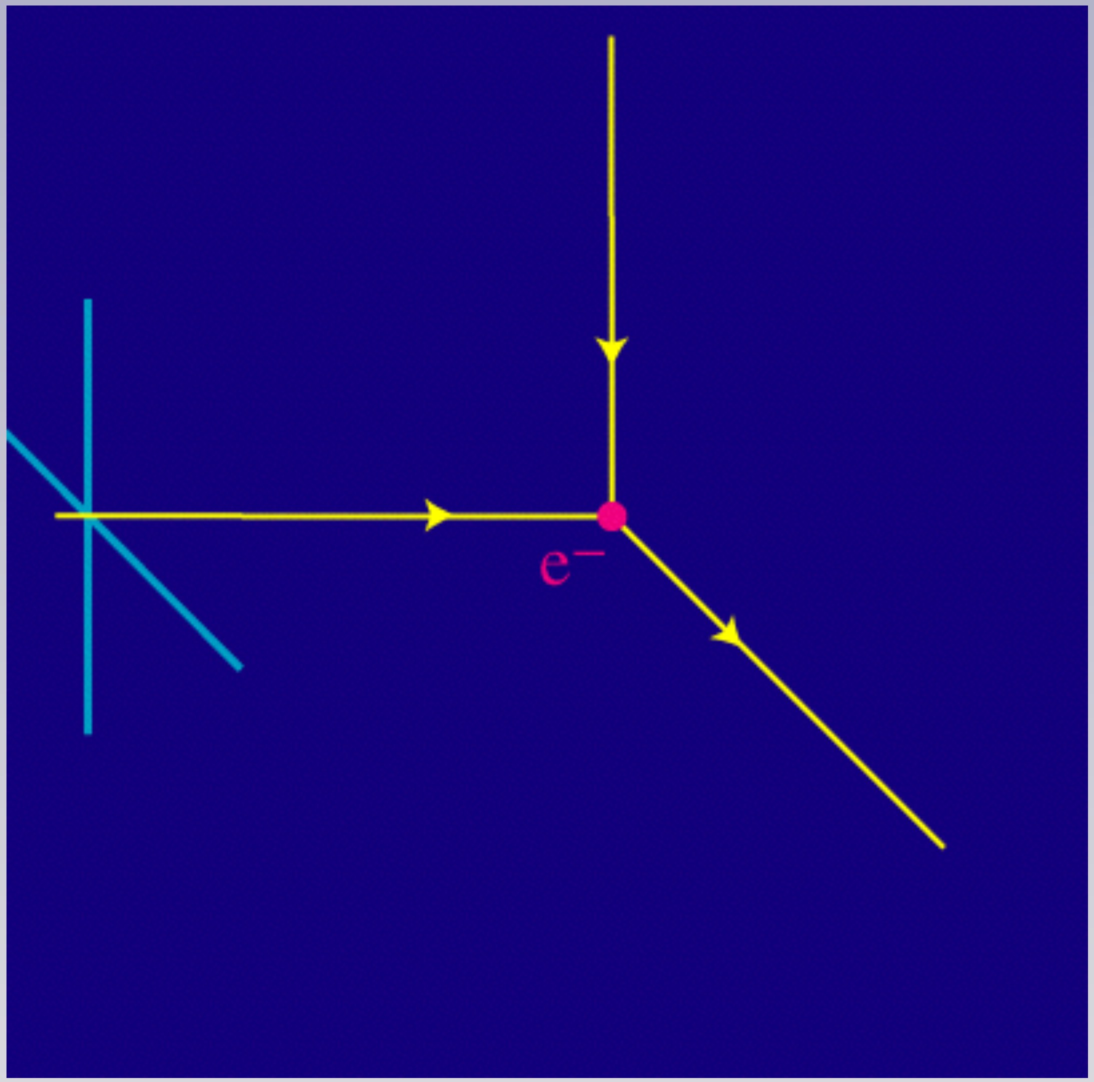
# Polarization by Thomson scattering

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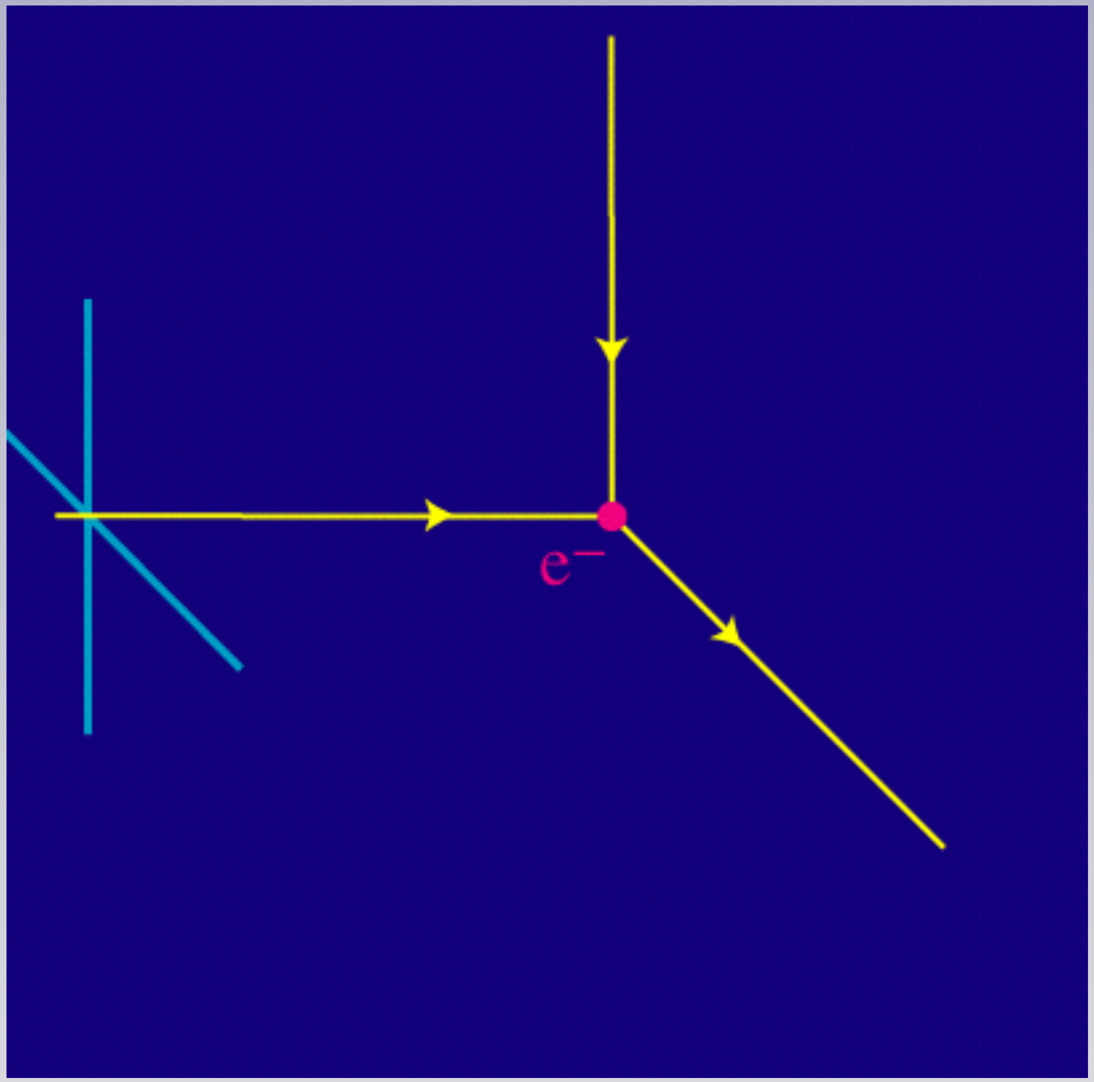
# Polarization by Thomson scattering

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# No polarization by Thomson scattering for isotropic influx

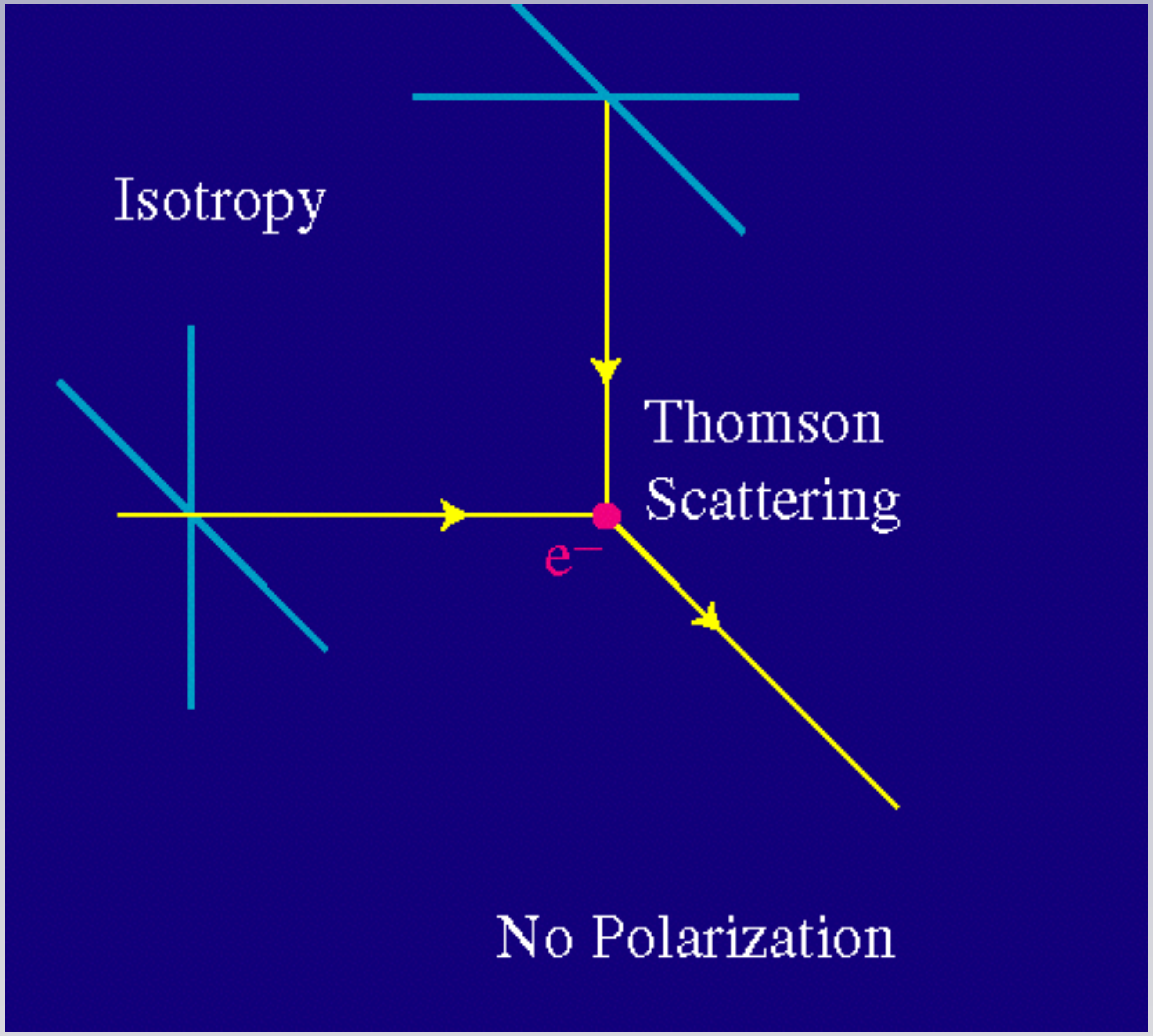


Image by Waynu Hu [http://background.uchicago.edu/~whu/]

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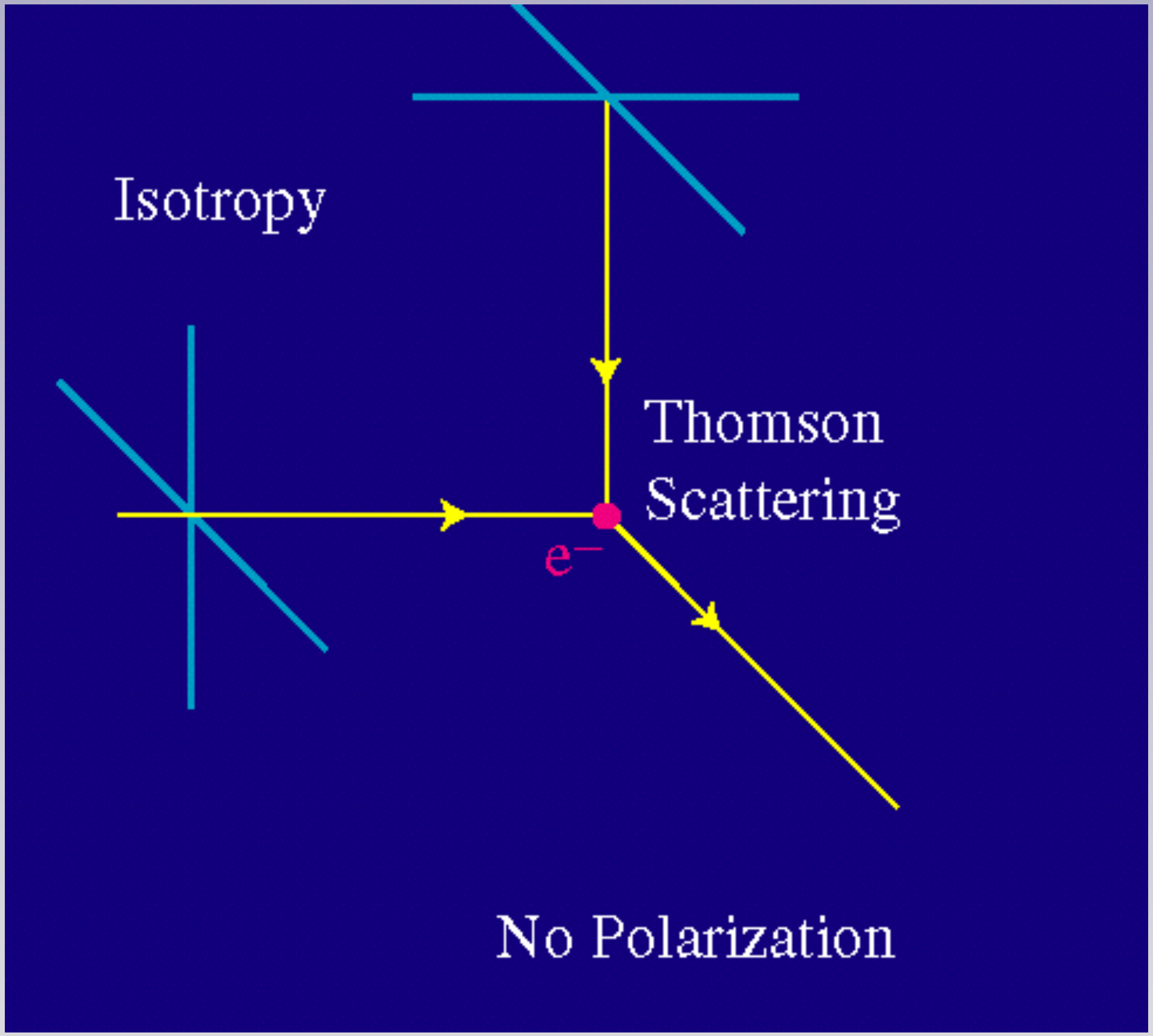


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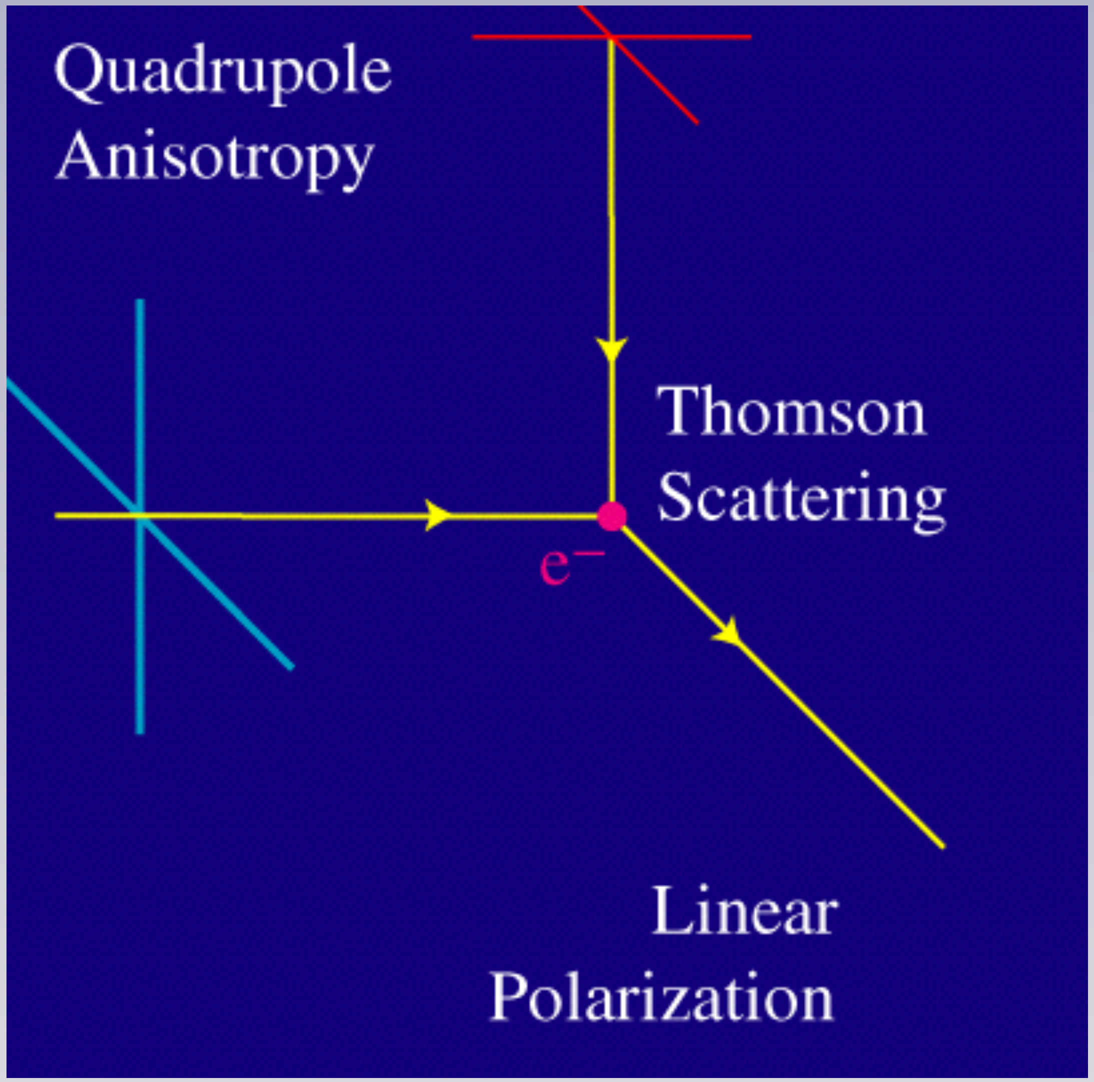
# Polarization by Thomson scattering of influx with quadrupolar anisotropy

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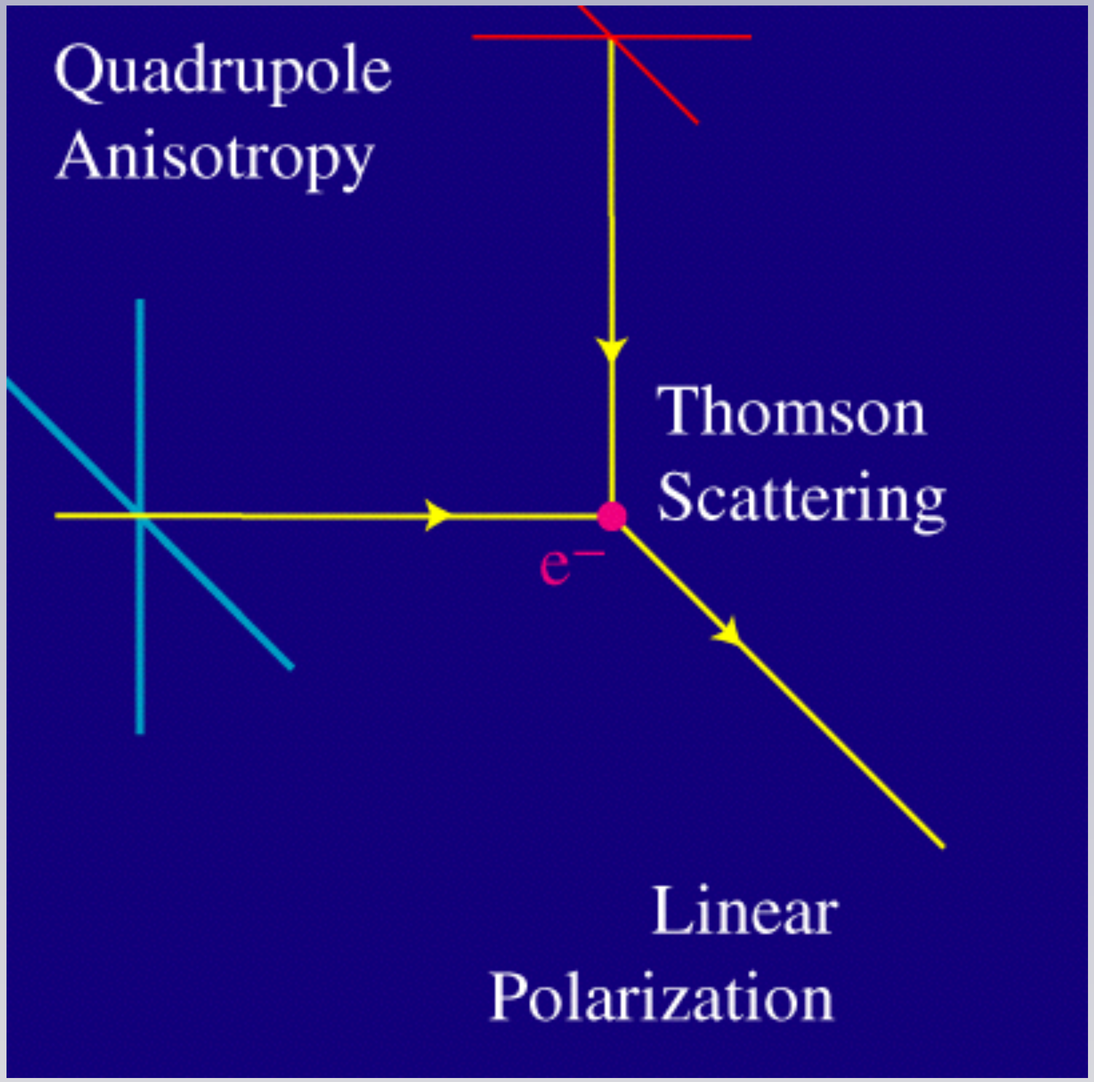


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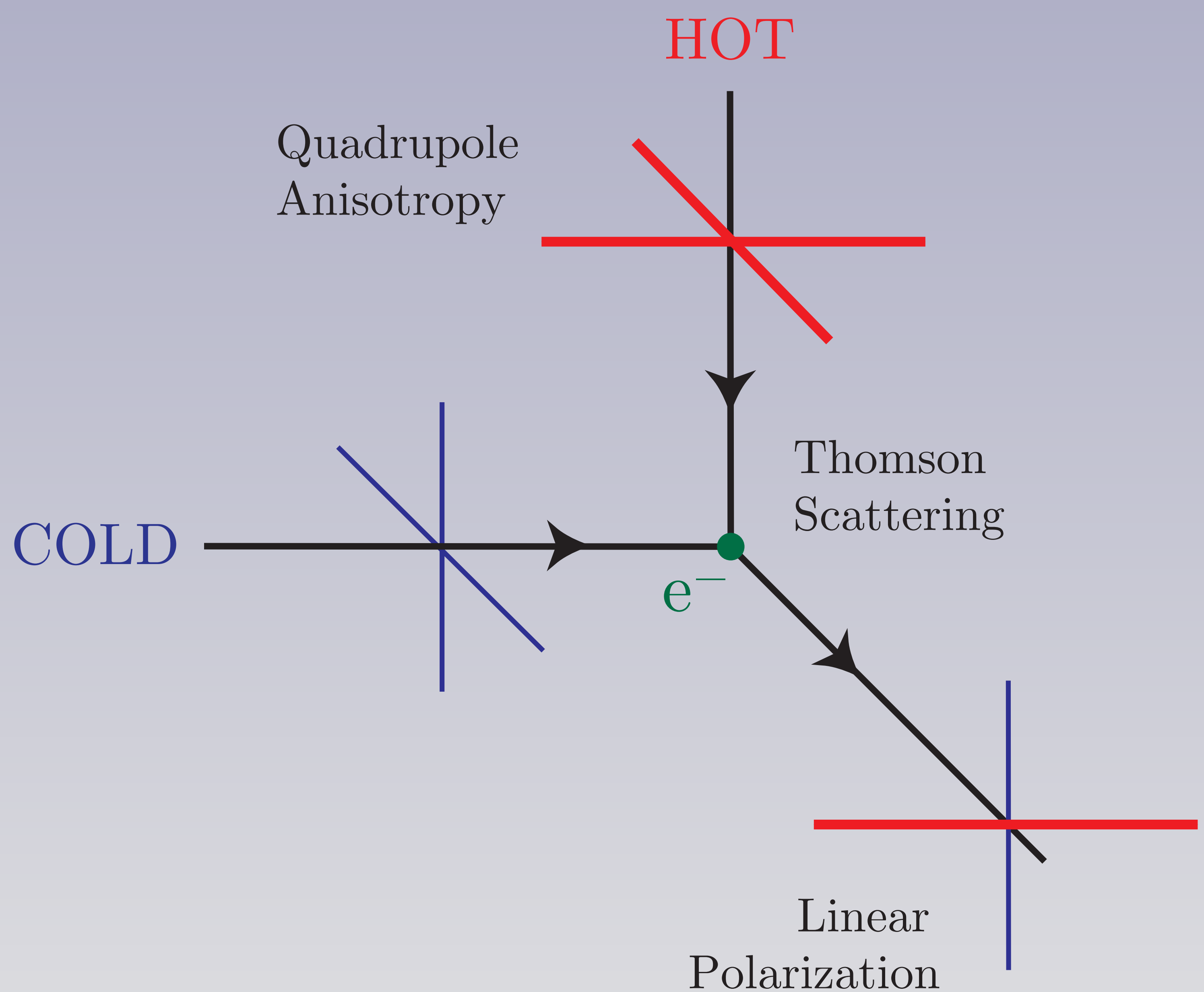
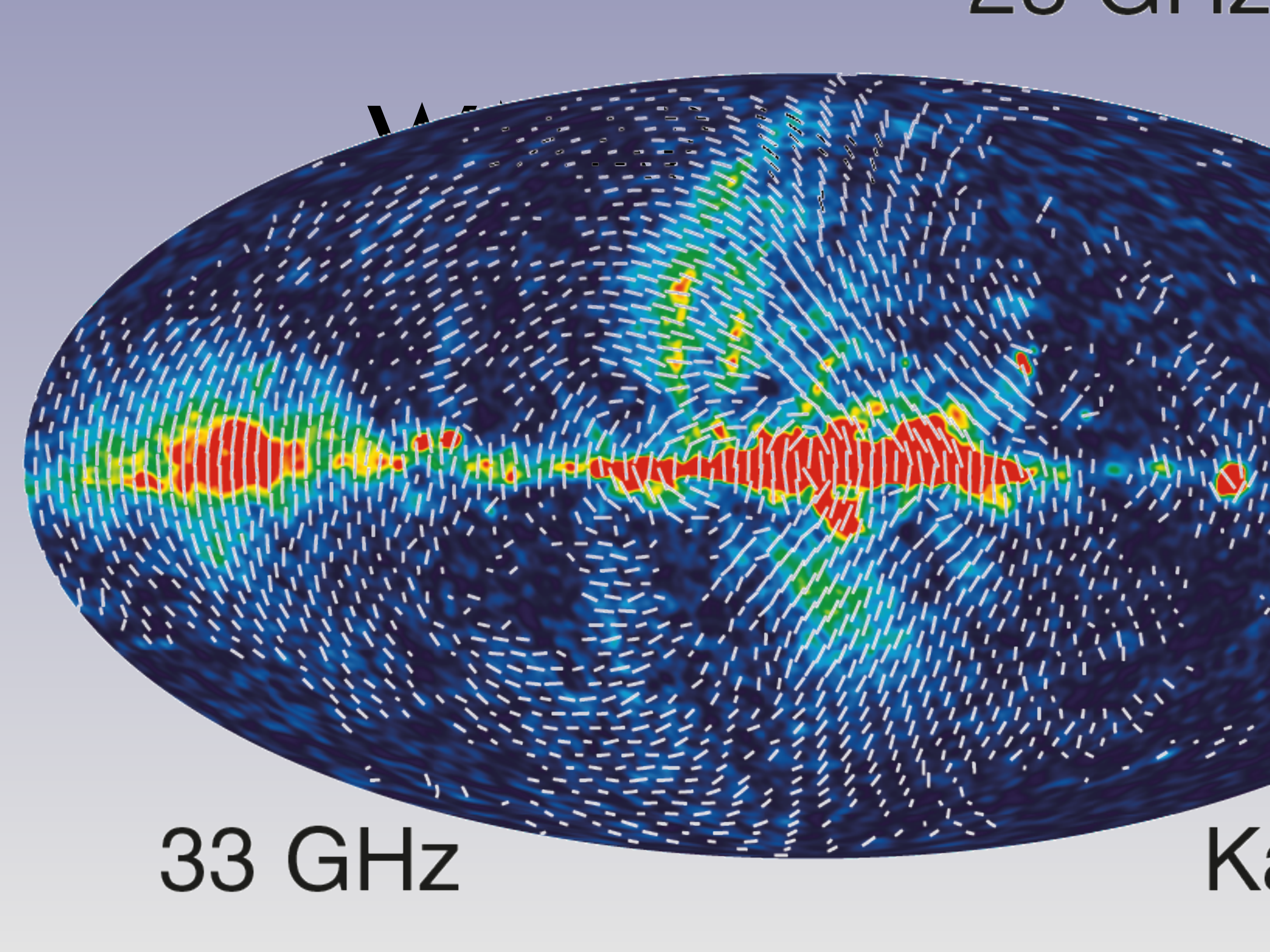
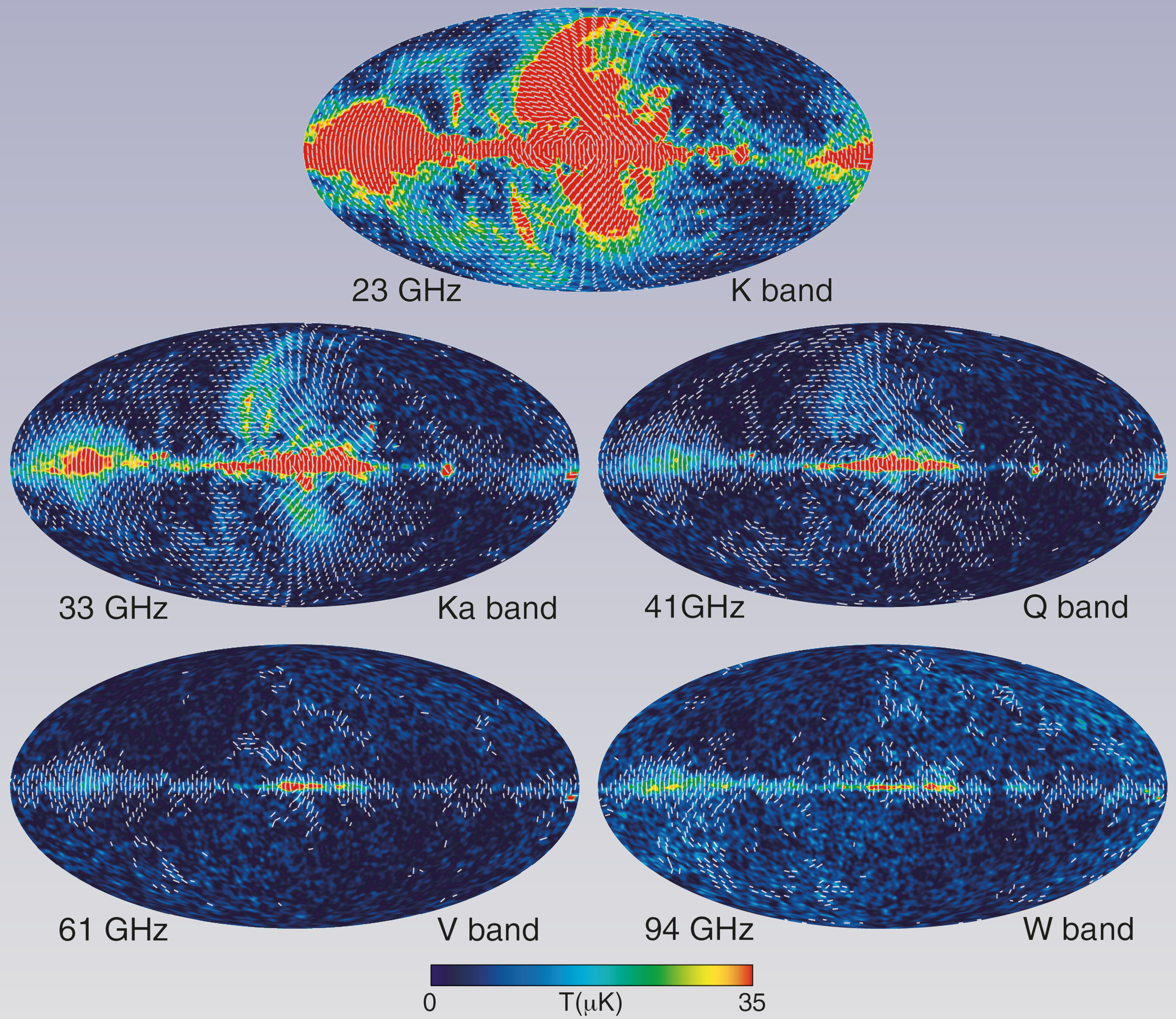



[from Baumann et al. 2008]

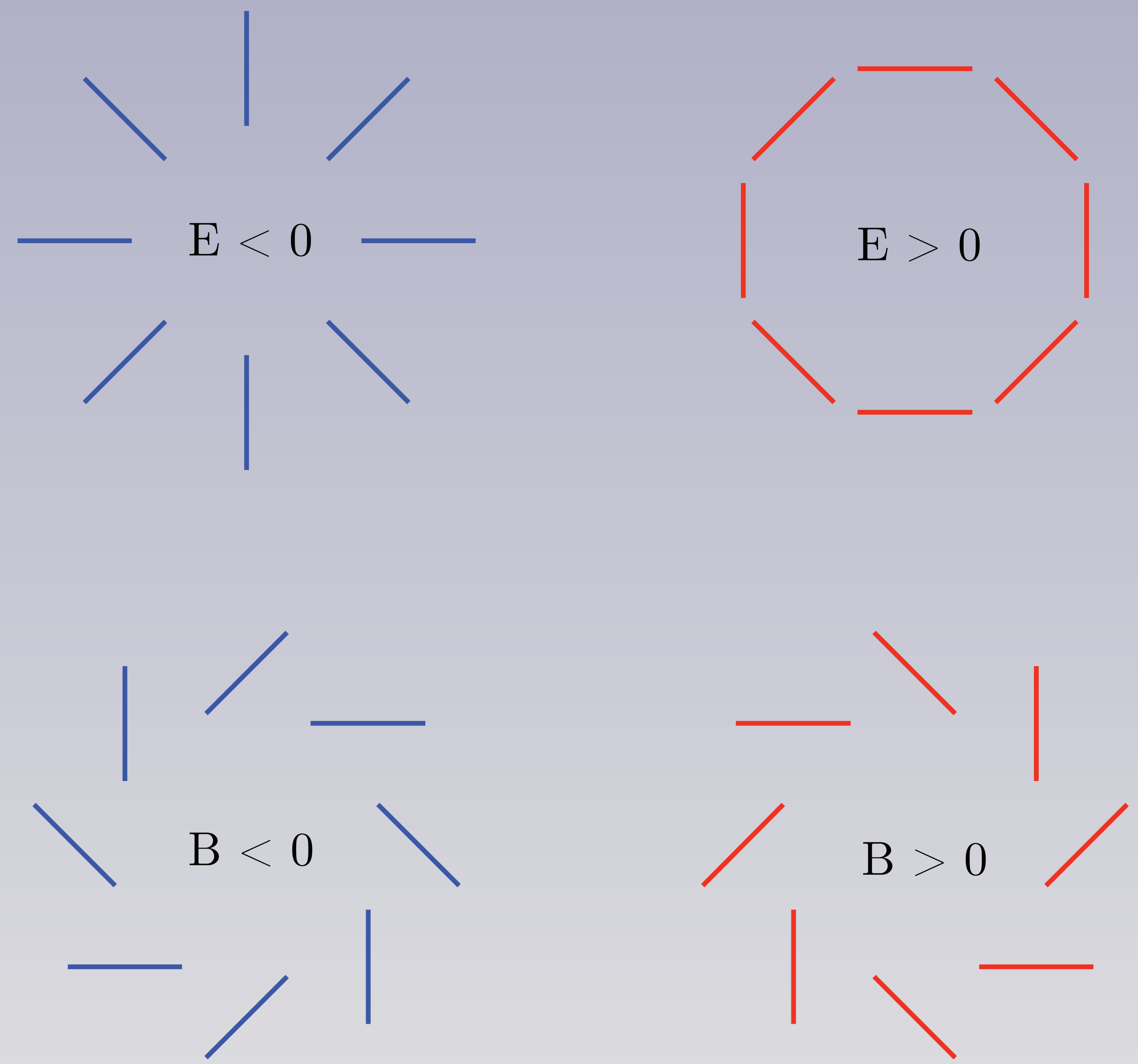

33 GHz

# WMAP 9yr



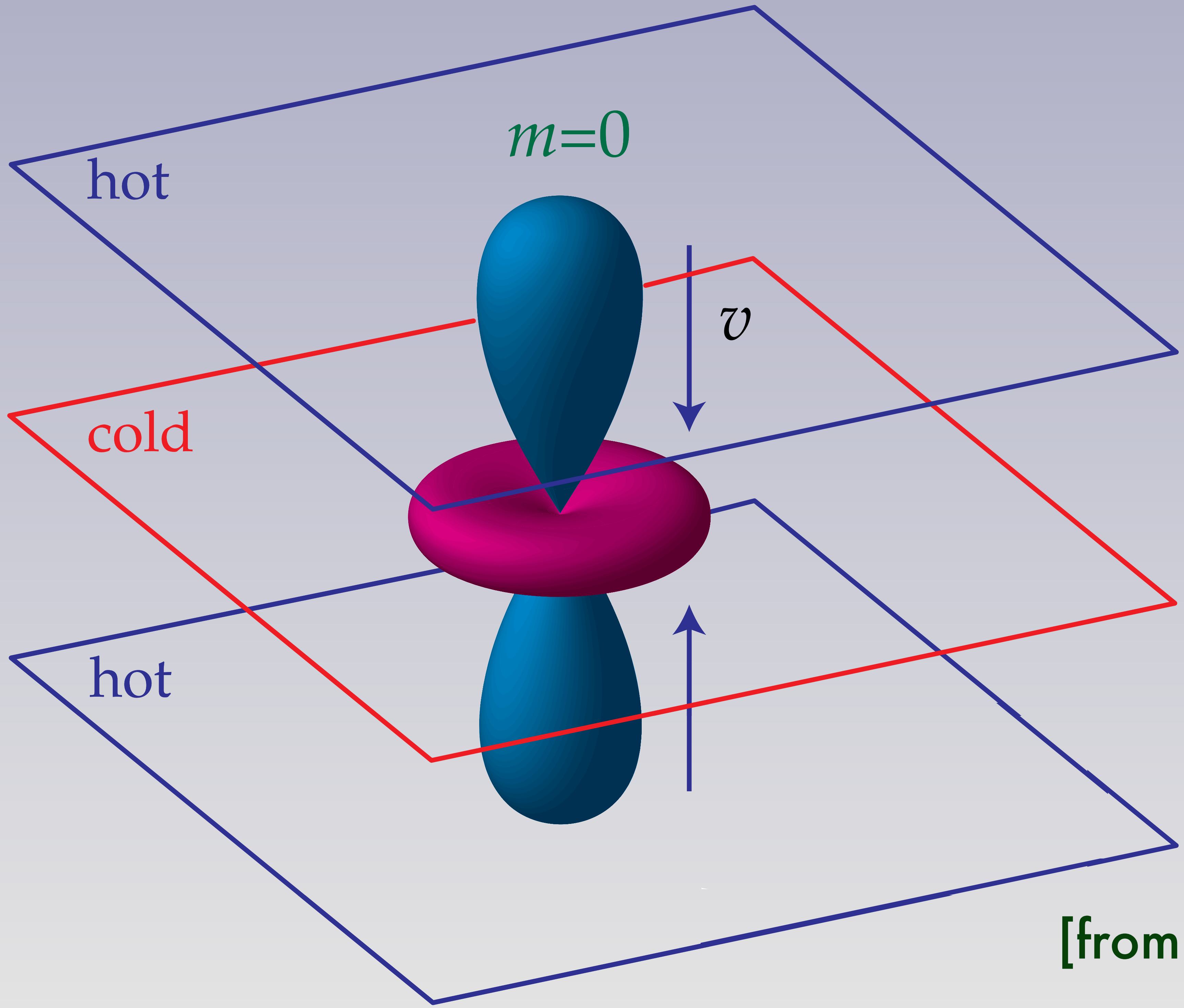
23 GHz
K band
33 GHz
Ka band
41GHz
Q band
61 GHz
V band
94 GHz
W band
0
T(μK)
35

# Decompose the vector field



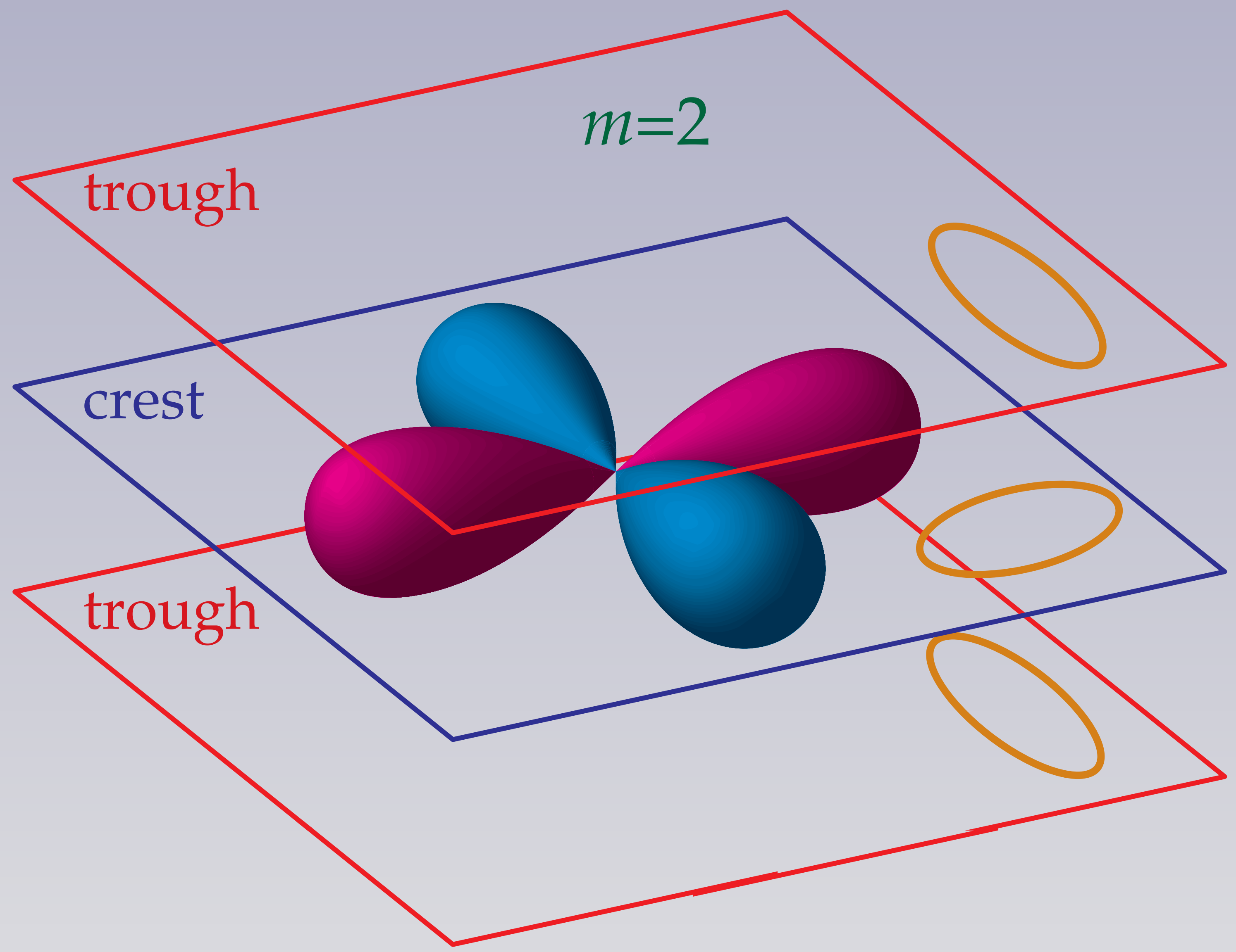

[from Baumann et al. 2008]

# Local quadrupole leads to polarized radiation

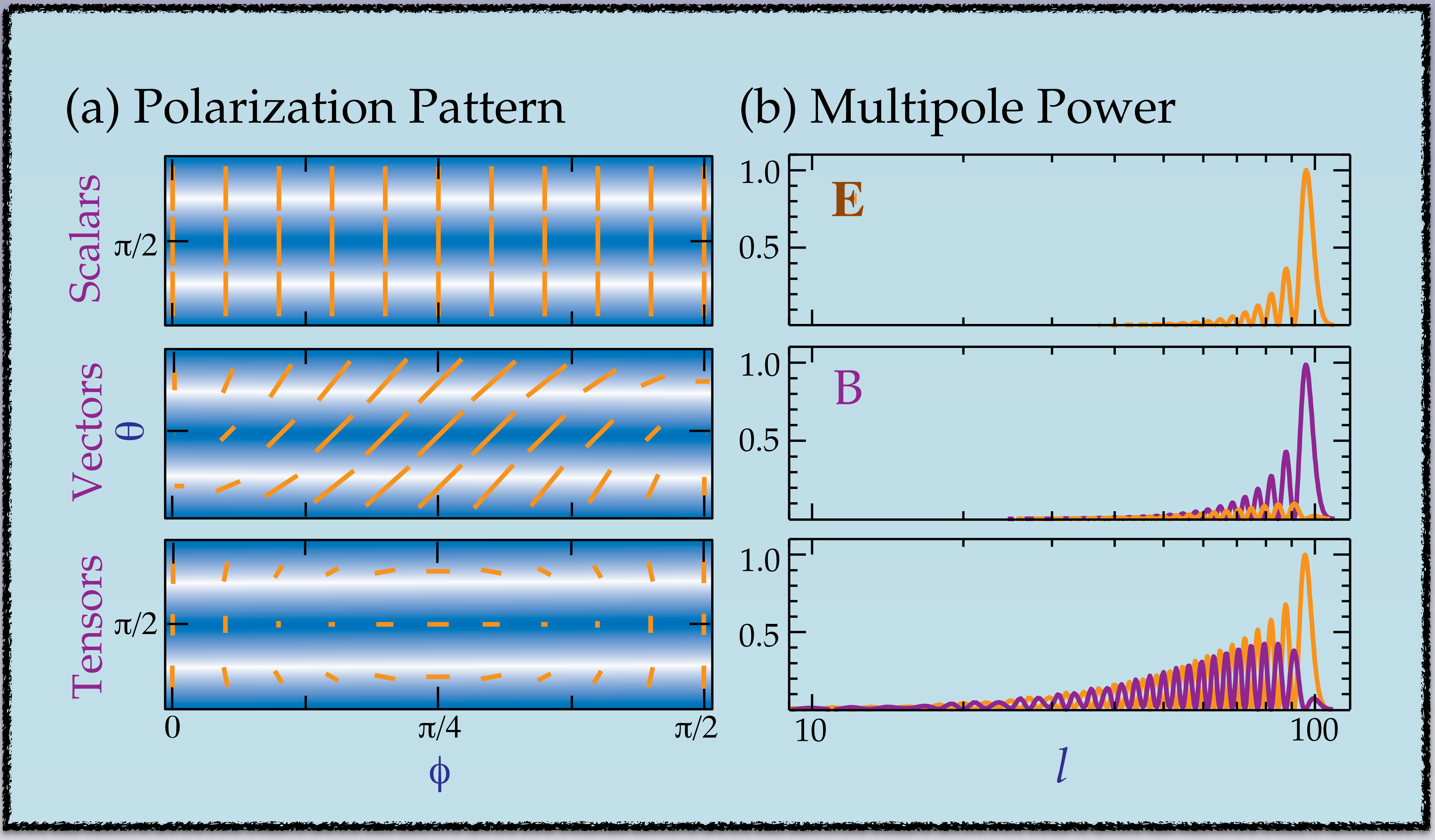



[from Hu & White 1997]

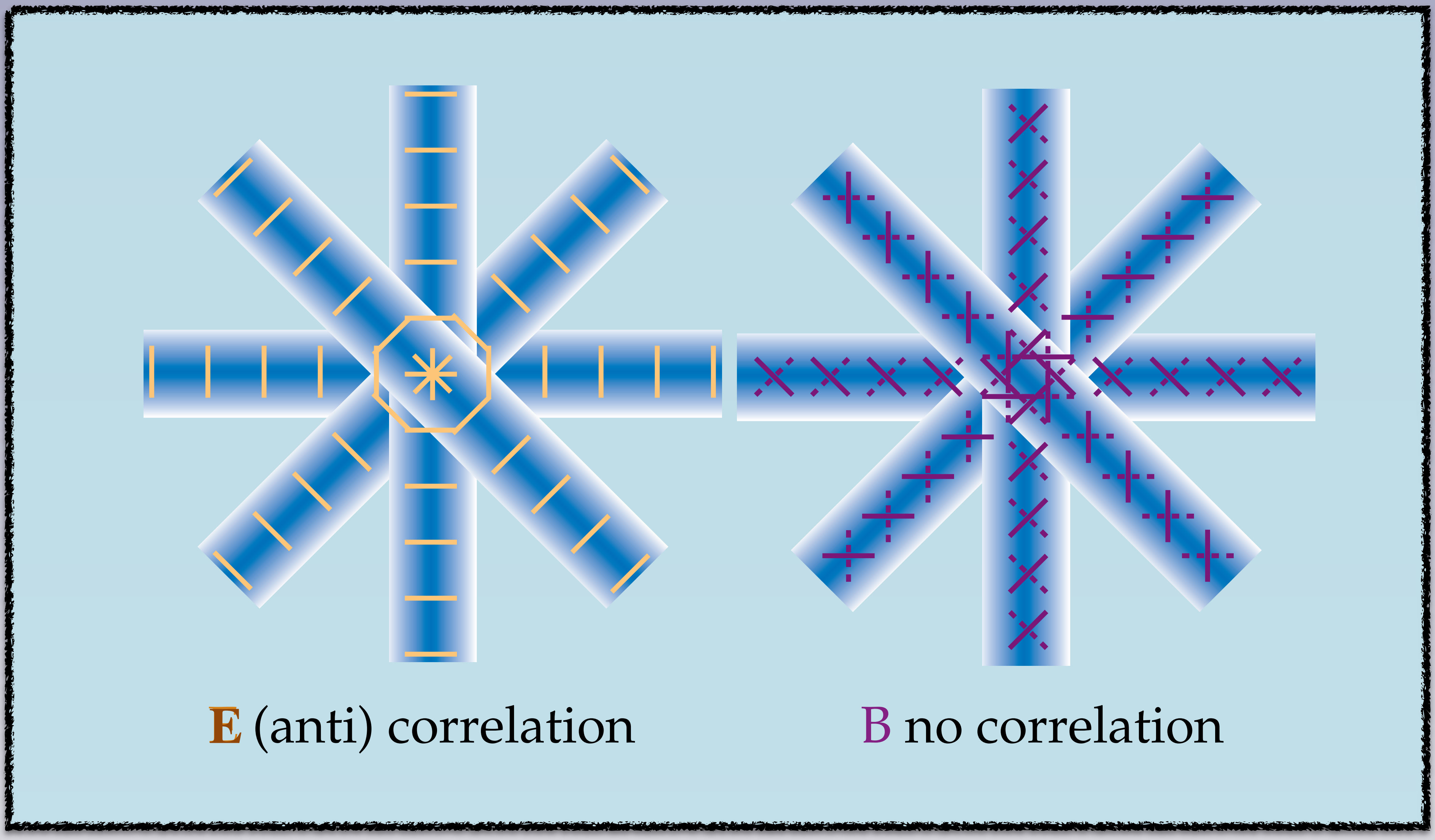
# Local quadrupole with gravity wave sources E&B modes




[from Hu & White 1997]




[from Hu & White 1997]




[from Hu & White 1997]

# Equations of motion

$$\dot{\mathcal{M}}_\ell^{(m)} - k\left[\frac{{}_0\kappa_\ell^m}{2\ell-1}\mathcal{M}_{\ell-1}^{(m)} - \frac{{}_0\kappa_{\ell+1}^m}{2\ell+3}\mathcal{M}_{\ell+1}^{(m)}\right] = -n_e\sigma_T a\mathcal{M}_\ell^{(m)} + S_\ell^{(m)} \quad (\ell \geq m)$$

$$\dot{E}_\ell^{(m)} - k\left[\frac{{}_2\kappa_\ell^m}{2\ell-1}E_{\ell-1}^{(m)} - \frac{2m}{\ell(\ell+1)}B_\ell^{(m)} - \frac{{}_2\kappa_{\ell+1}^m}{2\ell+3}E_{\ell+1}^{(m)}\right] = -n_e\sigma_T a[E_\ell^{(m)} + \sqrt{6}C^{(m)}\delta_{\ell,2}$$

$$\dot{B}_\ell^{(m)} - k\left[\frac{{}_2\kappa_\ell^m}{2\ell-1}B_{\ell-1}^{(m)} + \frac{2m}{\ell(\ell+1)}E_\ell^{(m)} - \frac{{}_2\kappa_{\ell+1}^m}{2\ell+3}B_{\ell+1}^{(m)}\right] = -n_e\sigma_T aB_\ell^{(m)}$$

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$$S_0^{(0)} = n_e\sigma_T a\mathcal{M}_0^{(0)}, \qquad S_1^{(0)} = n_e\sigma_T a4V_b + 4k(\Psi - \Phi),$$

$$S_2^{(0)} = n_e\sigma_T aC^{(0)}, \qquad S_1^{(1)} = n_e\sigma_T a4\omega_b,$$

$$S_2^{(1)} = n_e\sigma_T aC^{(1)} + 4k\Sigma, \qquad S_2^{(2)} = n_e\sigma_T aC^{(2)} + 4\dot{H}$$

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$$C^{(m)} = \tfrac{1}{10}[\mathcal{M}_2^{(m)} - \sqrt{6}E_2^{(m)}]$$

$$S_0^{(0)} = n_e\sigma_T a\mathcal{M}_0^{(0)}, \qquad S_1^{(0)} = n_e\sigma_T a4V_b + 4k(\Psi - \Phi),$$
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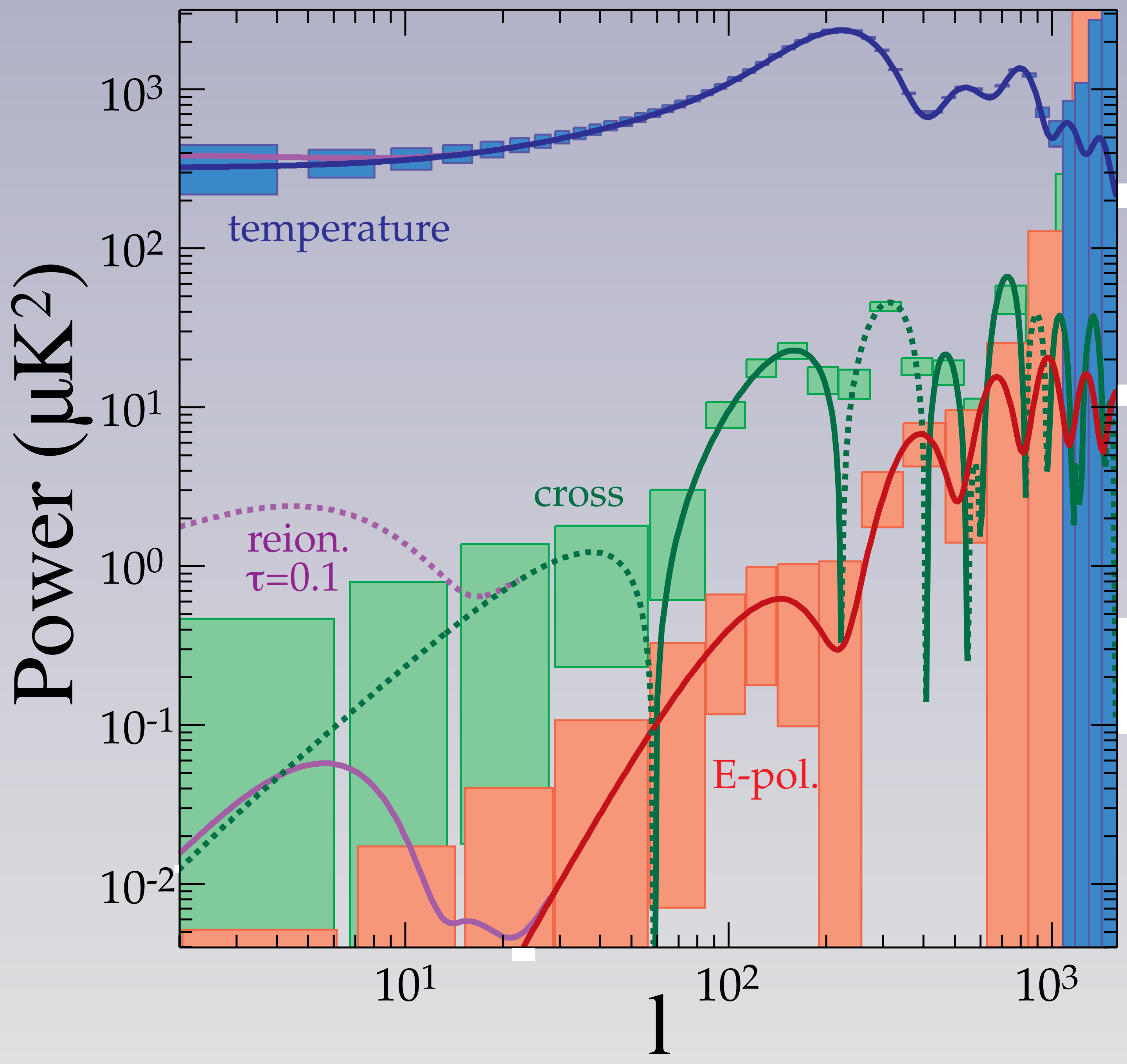
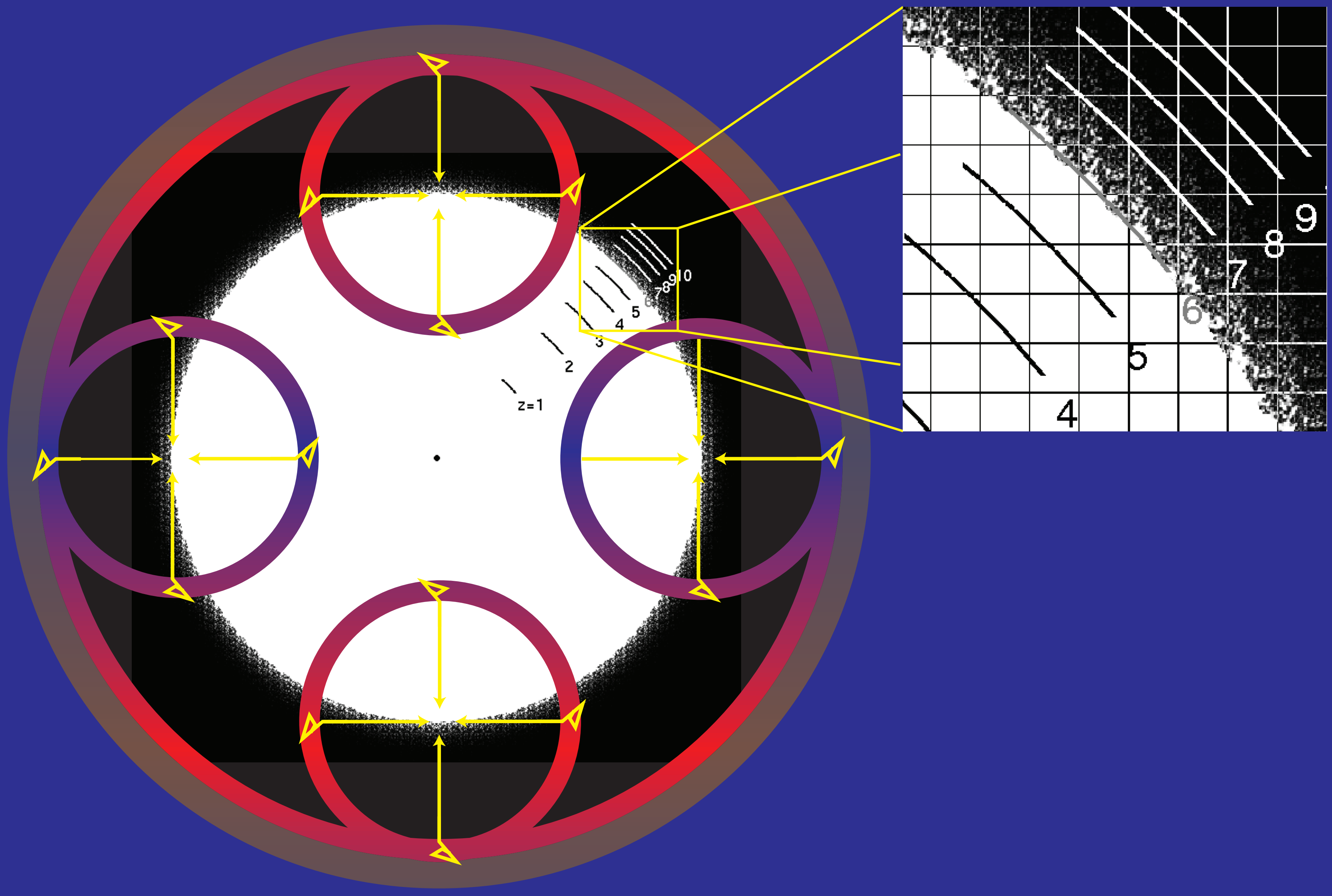
# Spectra

$$(2\ell+1)^2 C_\ell^{XY(m)} = \frac{n_m}{8\pi}\int k^2 dk X_\ell^{(m)} Y_\ell^{(m)*}$$



# Inhomogeneous Ionization

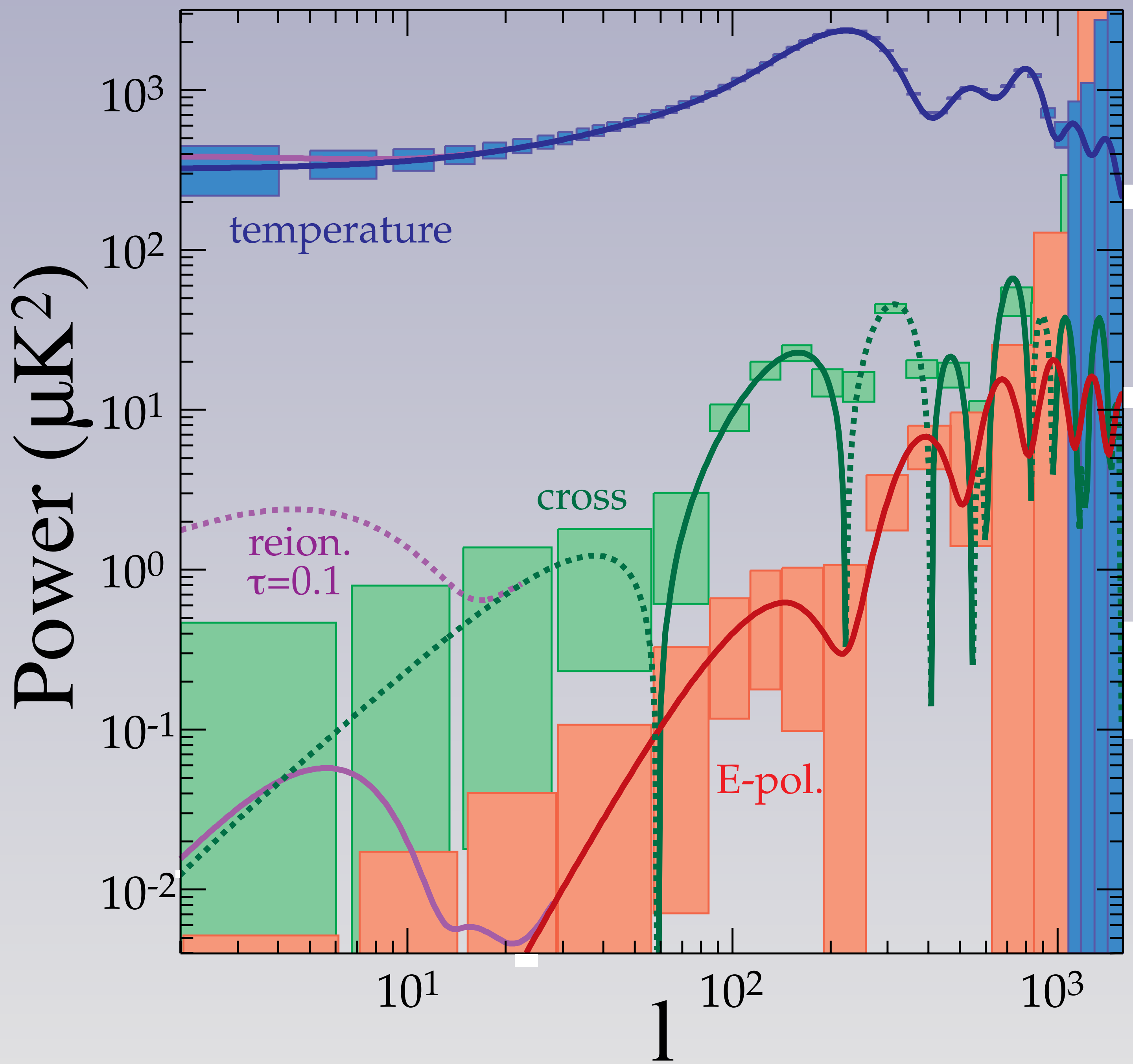
- As reionization completes, ionization regions grow and fill the space




Zahn et al. (2006) [Mortonson et al (2009)]

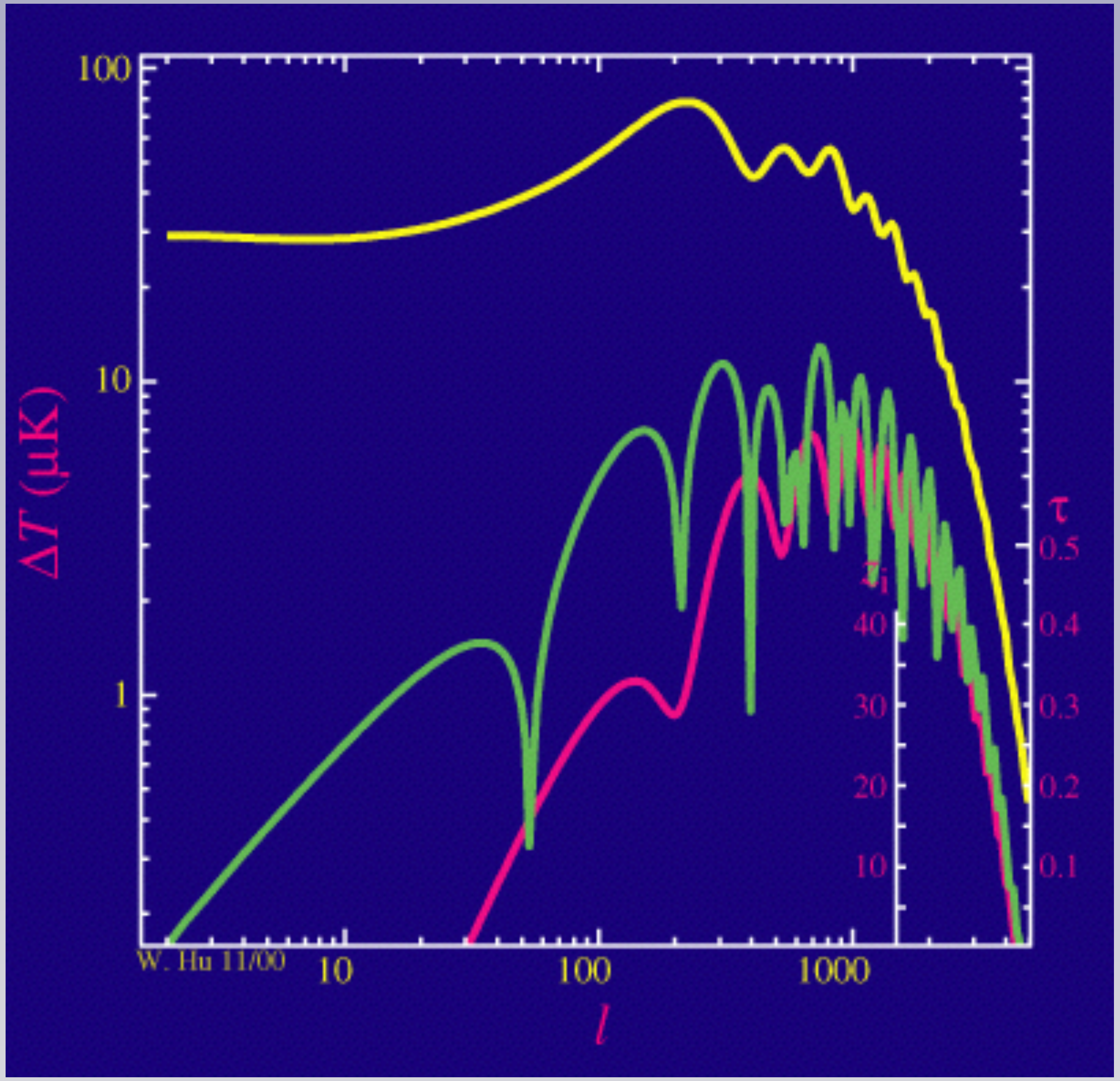
# Spectra

$$(2\ell+1)^2 C_\ell^{XY(m)} = \frac{n_m}{8\pi}\int k^2 dk X_\ell^{(m)} Y_\ell^{(m)*}$$



Secondary anisotropy

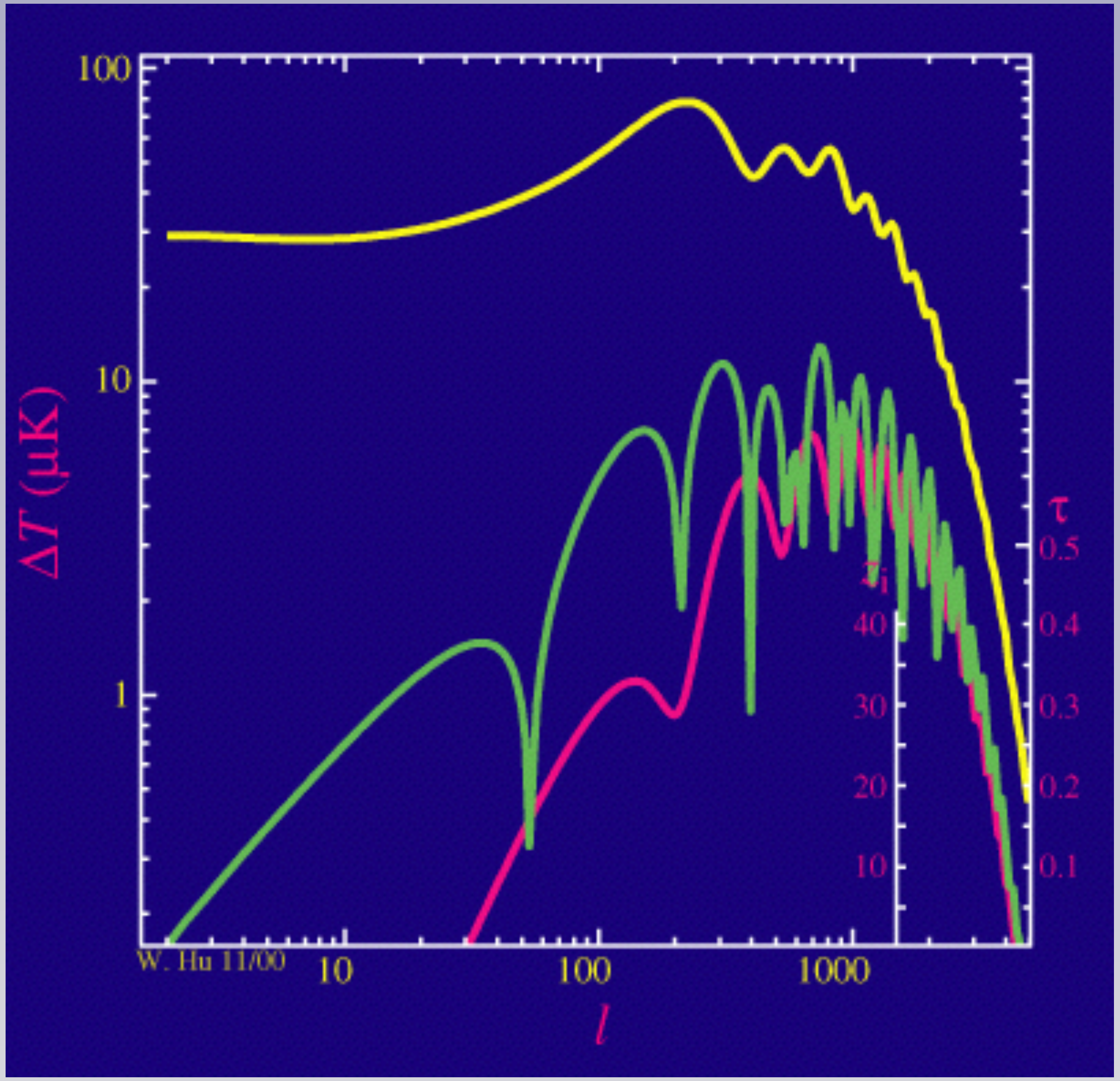
# Reionization




from Wayne Hu [http://background.uchicago.edu/~whu/animbut/anim4.html]

Secondary anisotropy

# Reionization

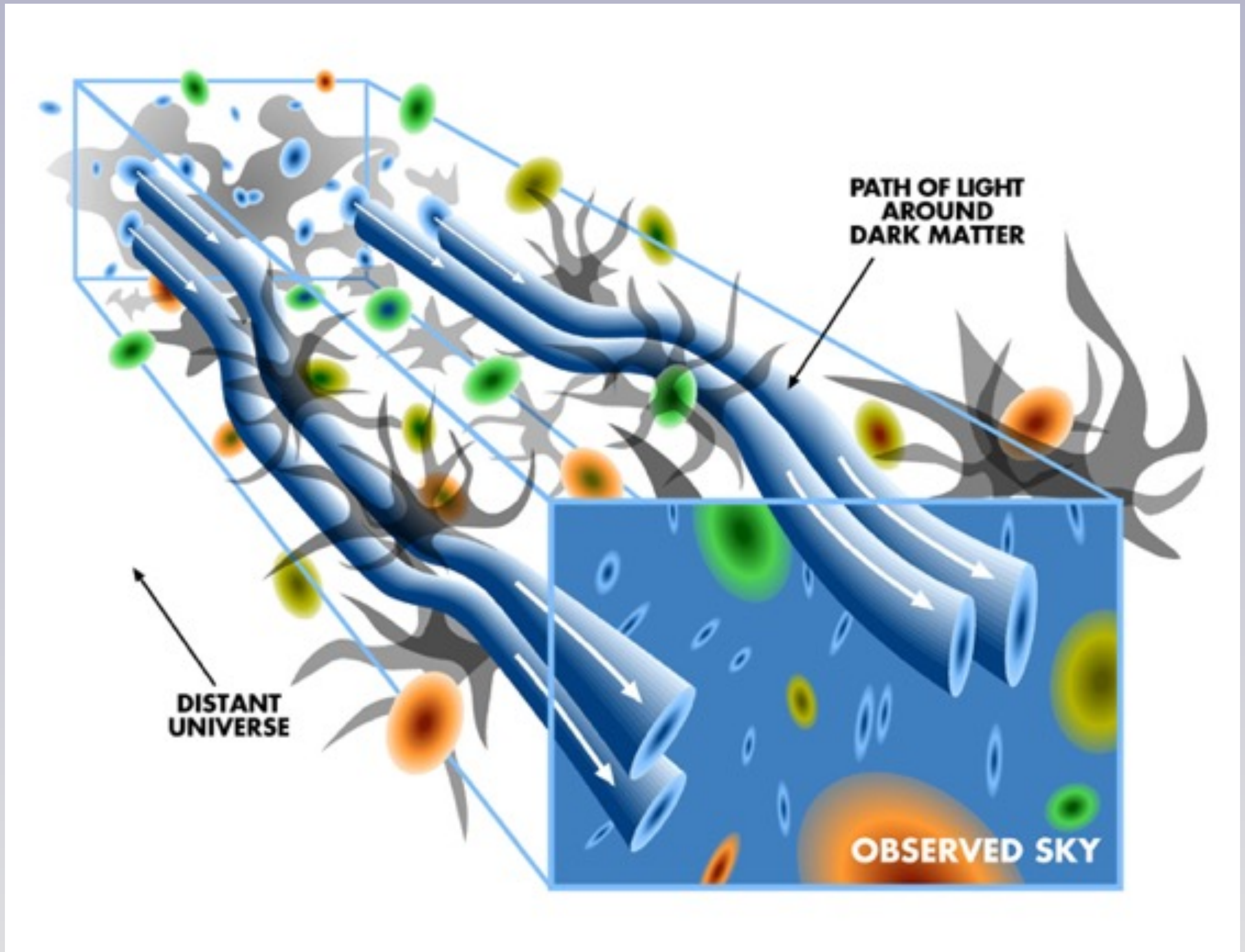



from Wayne Hu [http://background.uchicago.edu/~whu/animbut/anim4.html]

Reminder from lecture III

# Weak lensing

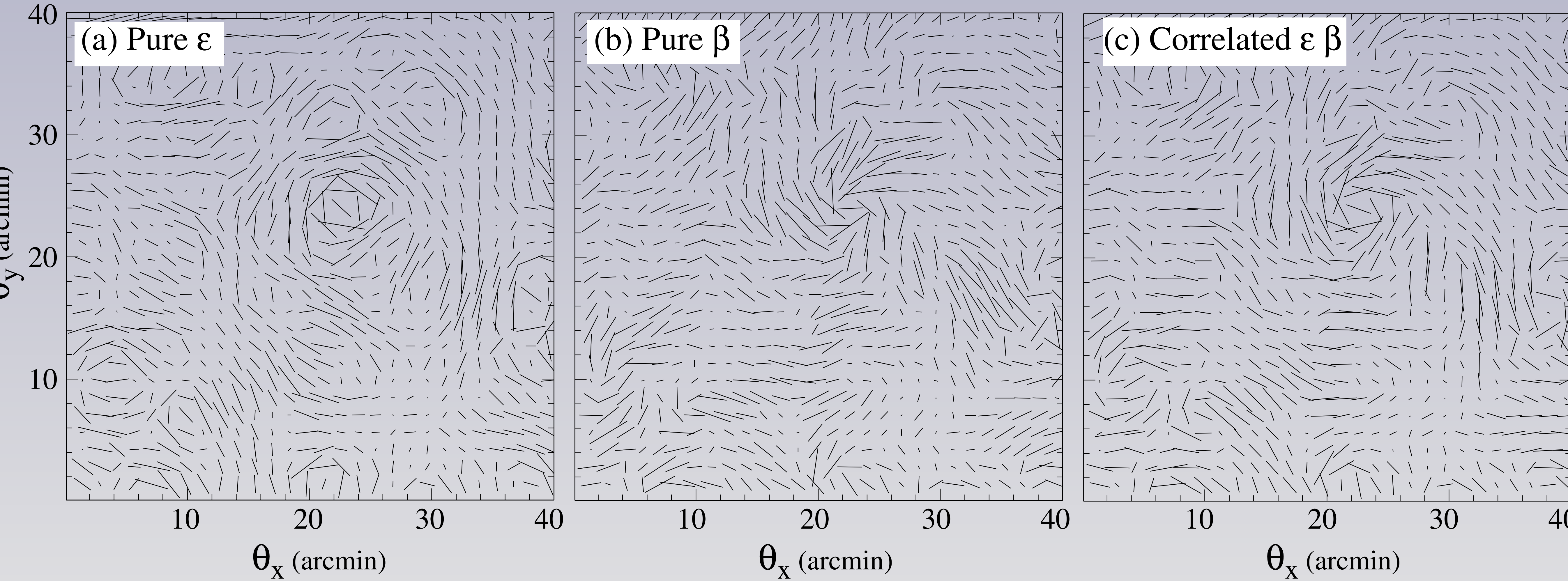
- Probe the potential, not the galaxies
- LSST, Euclid, ...




[http://lsst.org/lsst/science/scientist_cosmic_shear]

Reminder from lecture III

# Weak lensing

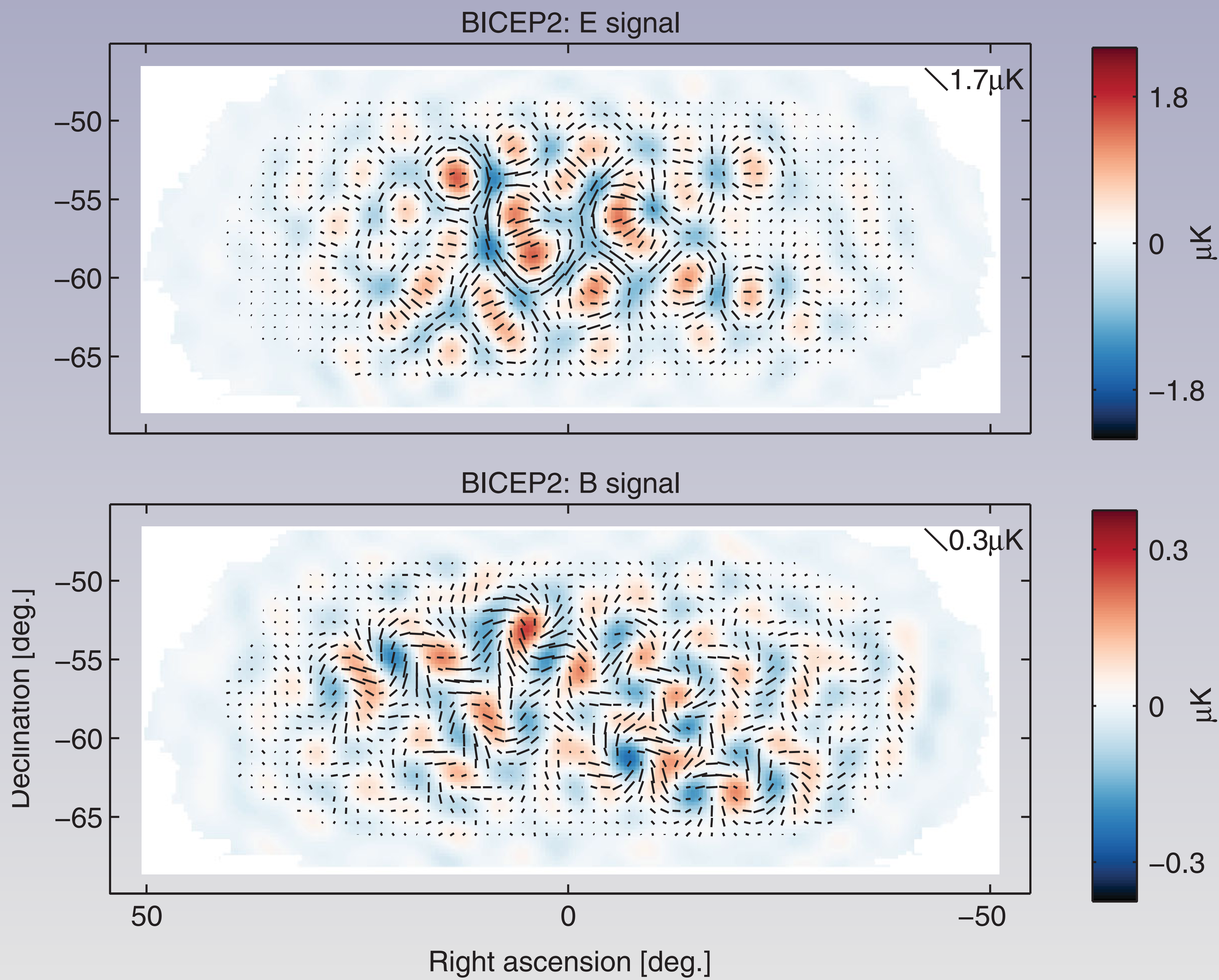



[Hu & White, 2001]

# CMB lensing

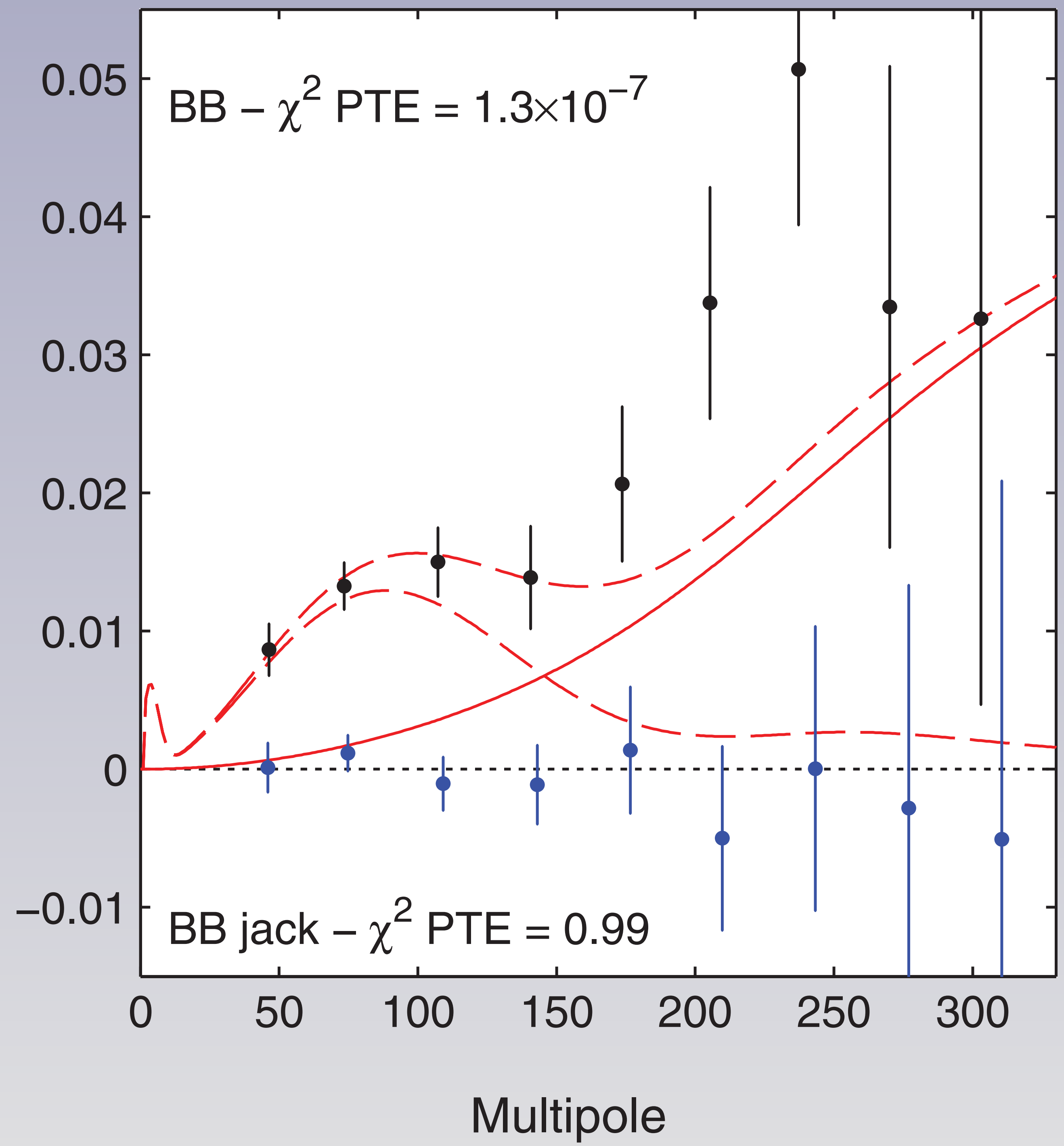
- Background of E-mode polarized light (from mostly scalar perturbations)
- Travelling through the gravitational potential (a scalar).
- Scalars only?
- Scalar + scalar = nonlinear
  - B-modes generated!

# CMB from BICEP2




[BICEP2, March 2014]

# CMB from BICEP2

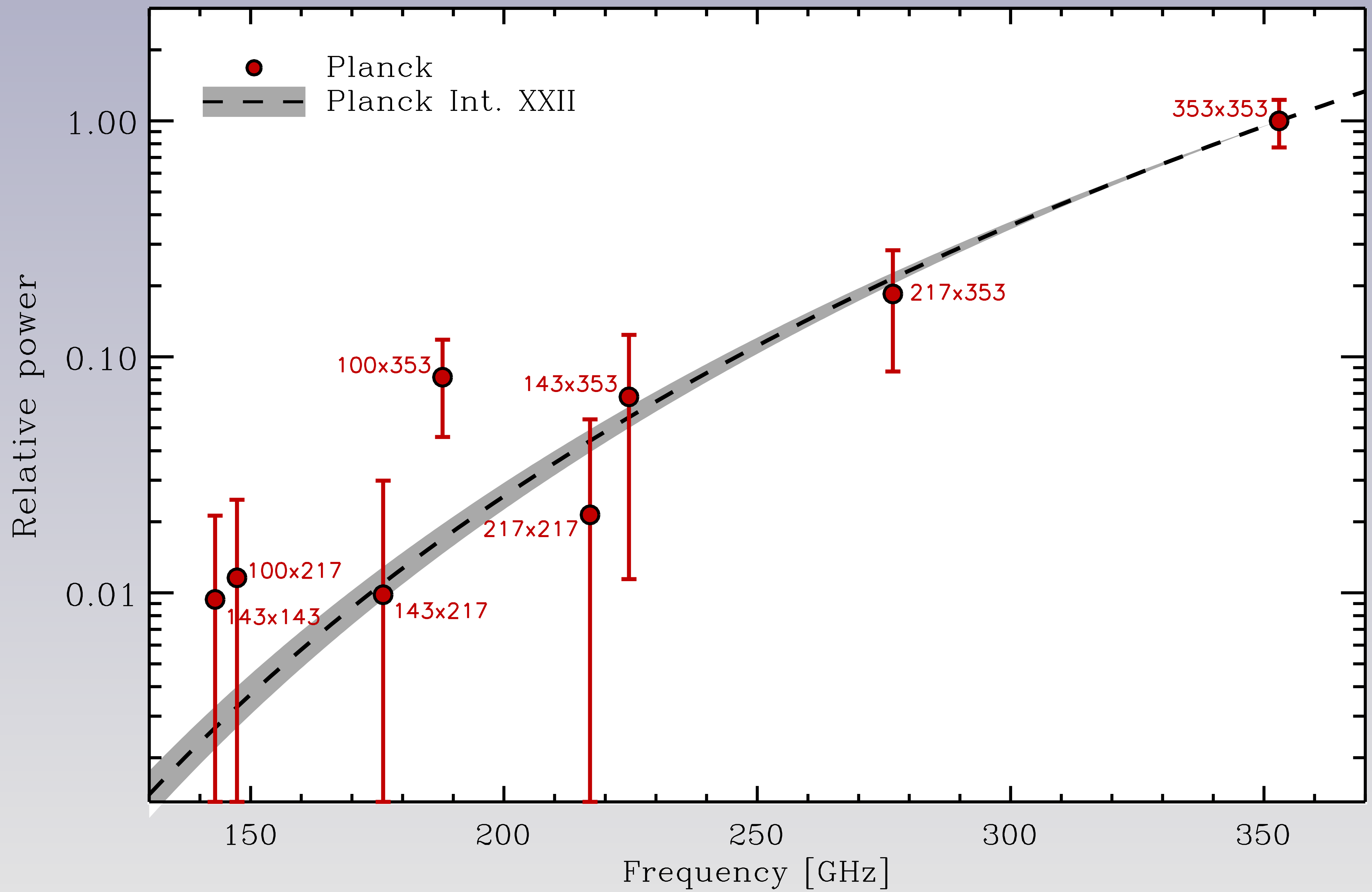



[BICEP2, March 2014]

# Spinning dust

- Spinning dust generates a magnetic field
- Will align with any present large scale magnetic field (e.g. galaxy)
- Hence an orientation (a vector field) enters the dynamics
- E- and B-modes sourced
- No “standard model of dust emission”

# Planck mesaurements of dust, paper XXX



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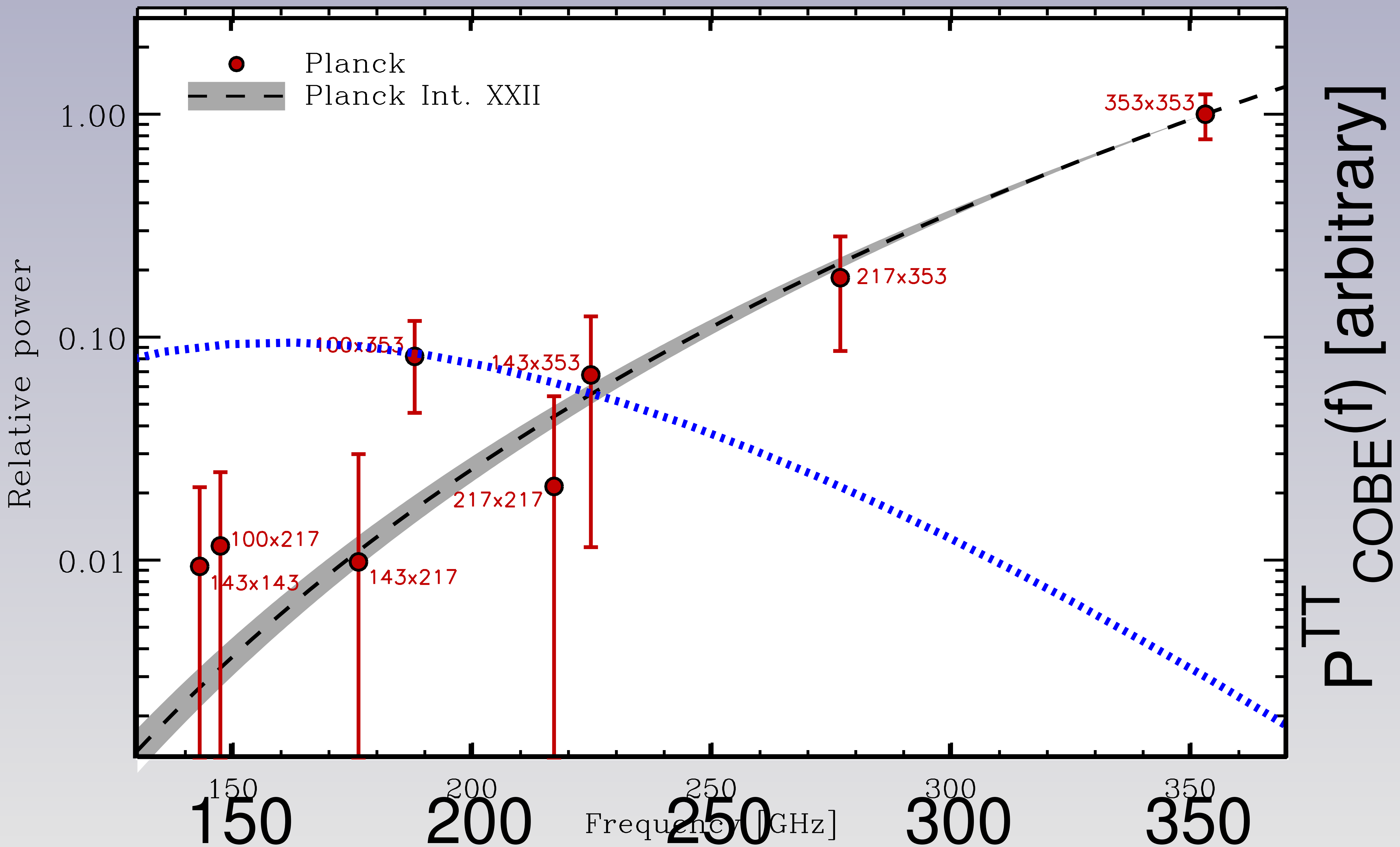
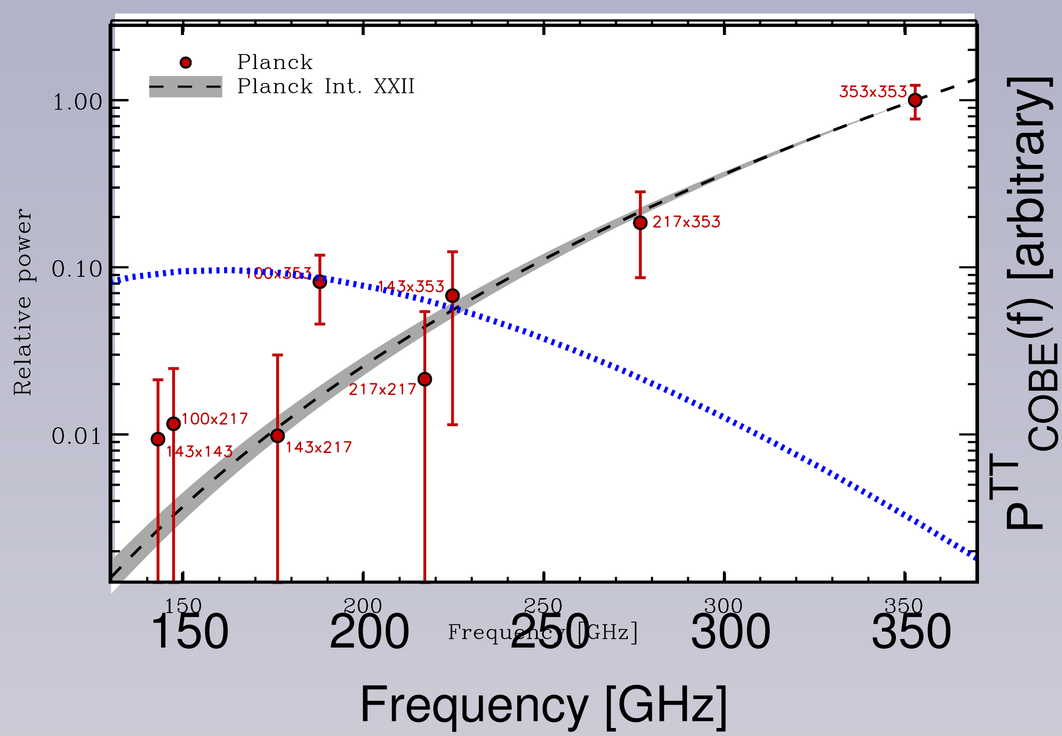


# Planck mesaurements of dust, paper XXX




$$C_\ell \propto \ell^\alpha$$

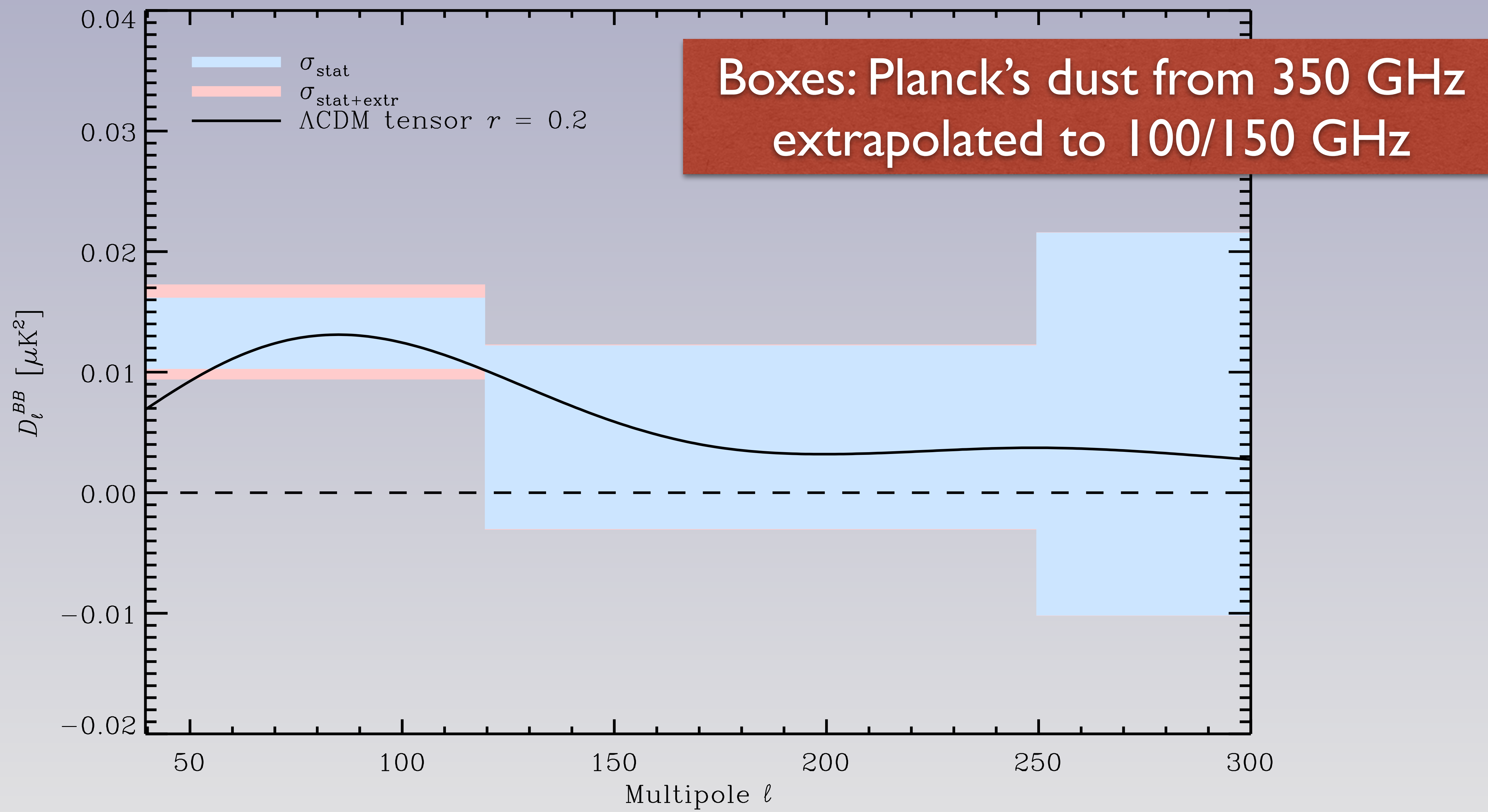
$$I_{\rm d}(\nu) \propto \nu^{\beta_{\rm d}} B_\nu(T_{\rm d}),$$

$$\beta_{\rm d} = 1.59$$

$$T_{\rm d} = 19.6\,{\rm K},$$

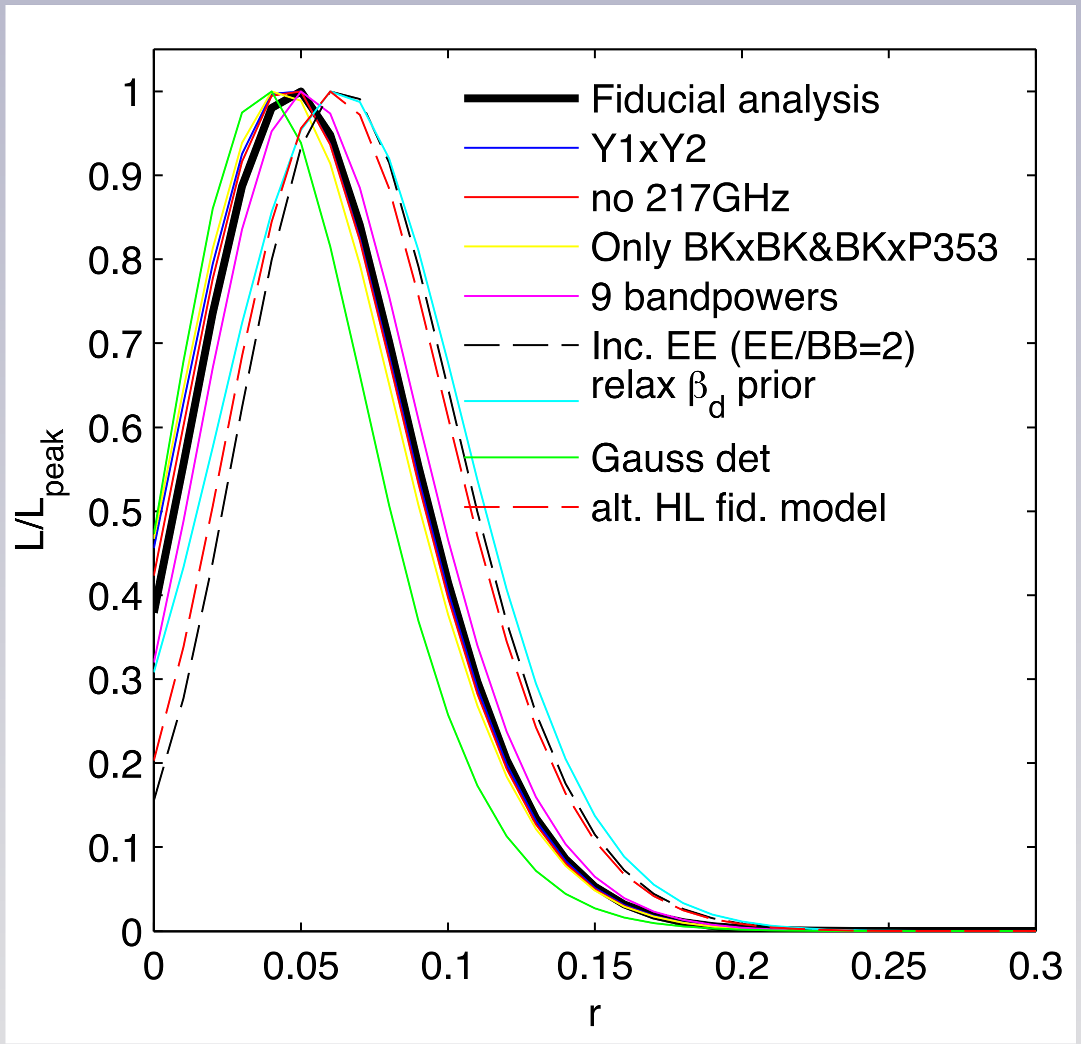
$B_\nu$ is the Planck function.

# Planck measurements of dust, paper XXX



# Last Friday's BICEP2+Keck +Planck paper

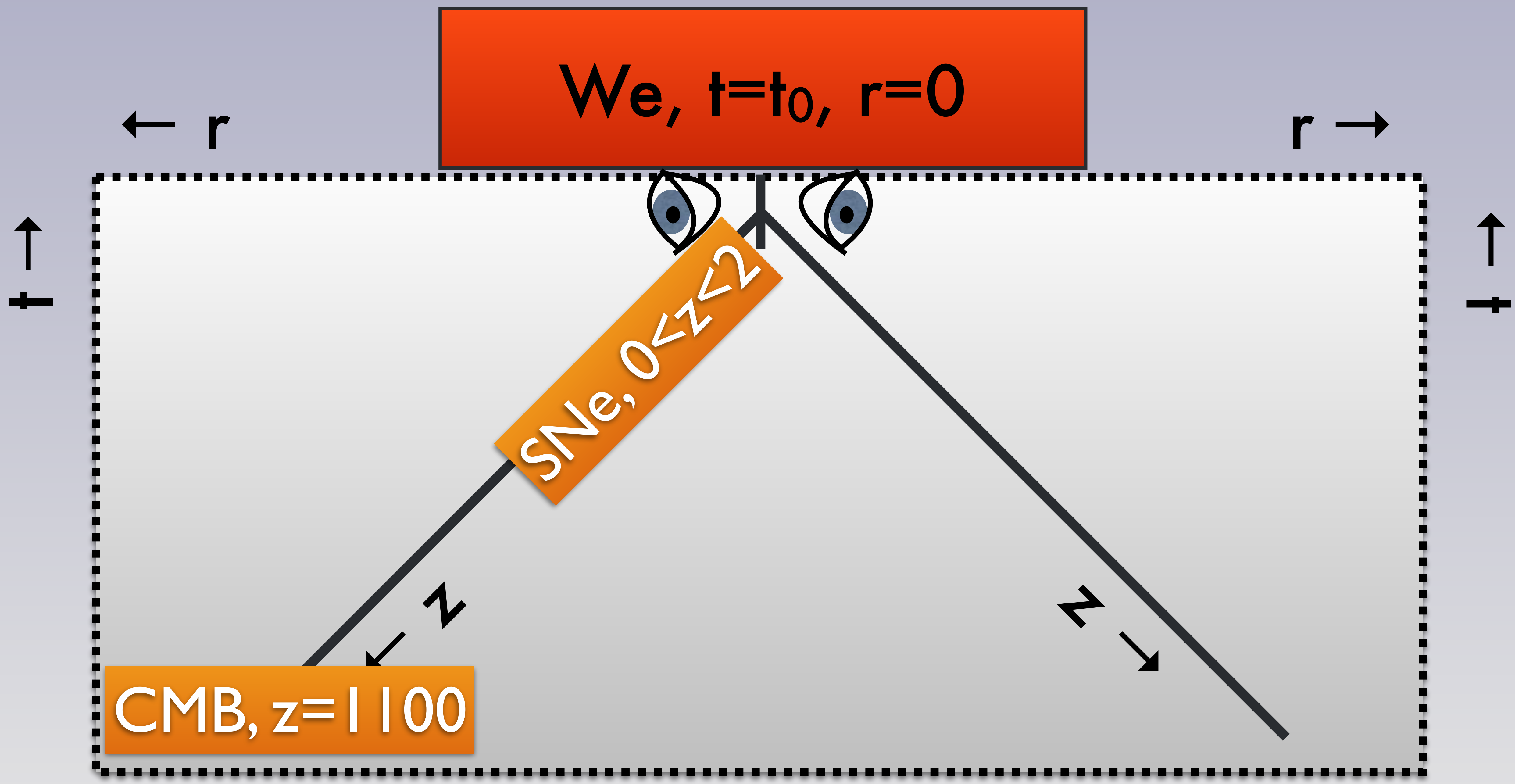
- Conclusion about dust foreground seems robust



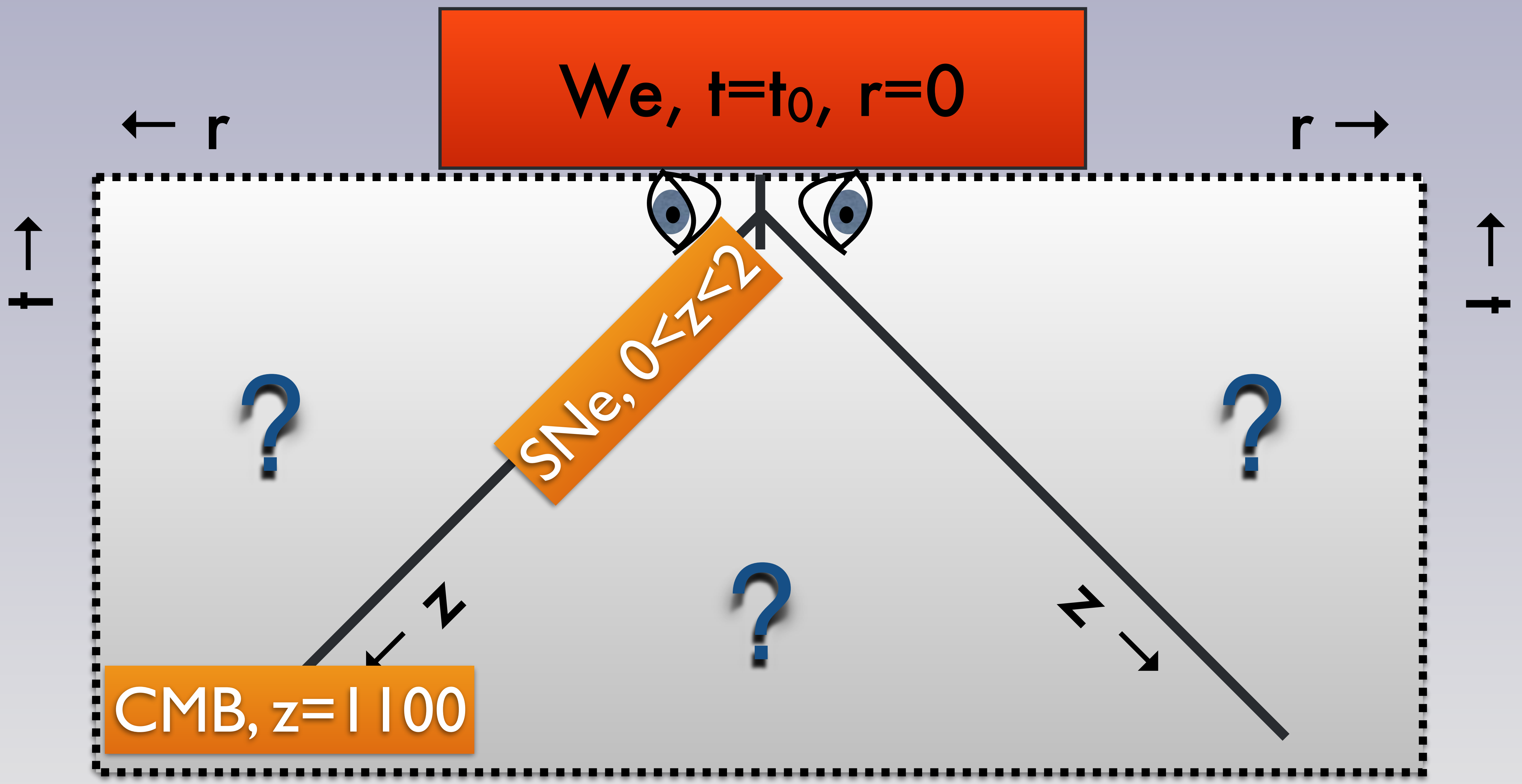

upper limit $r < 0.12$ at 95% confidence

# Summary of Cosmology

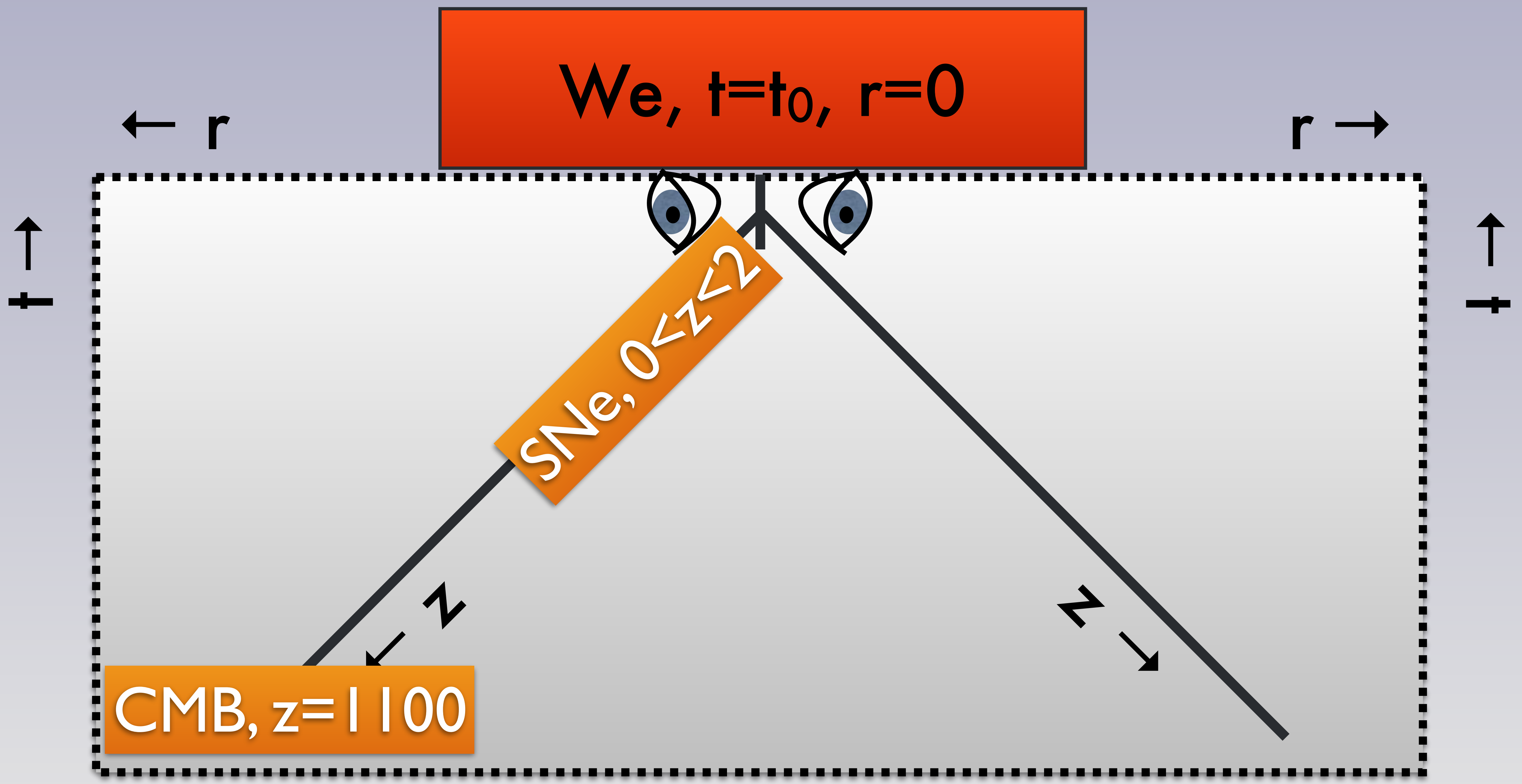
# Our light cone

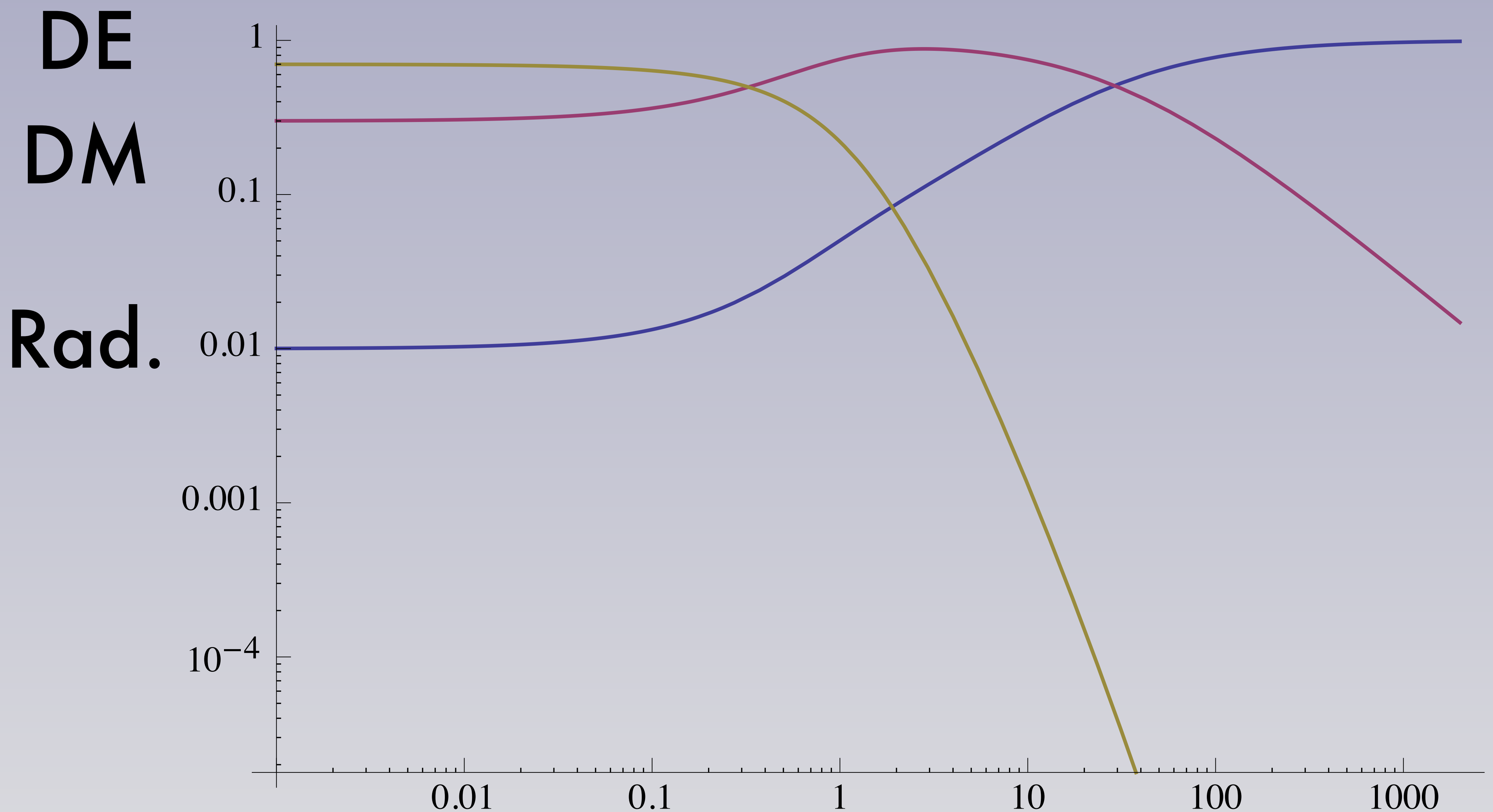


# Our light cone



# Our light cone




DE
DM
Rad.
1
0.1
0.01
0.001
$10^{-4}$
0.01
0.1
1
10
100
1000

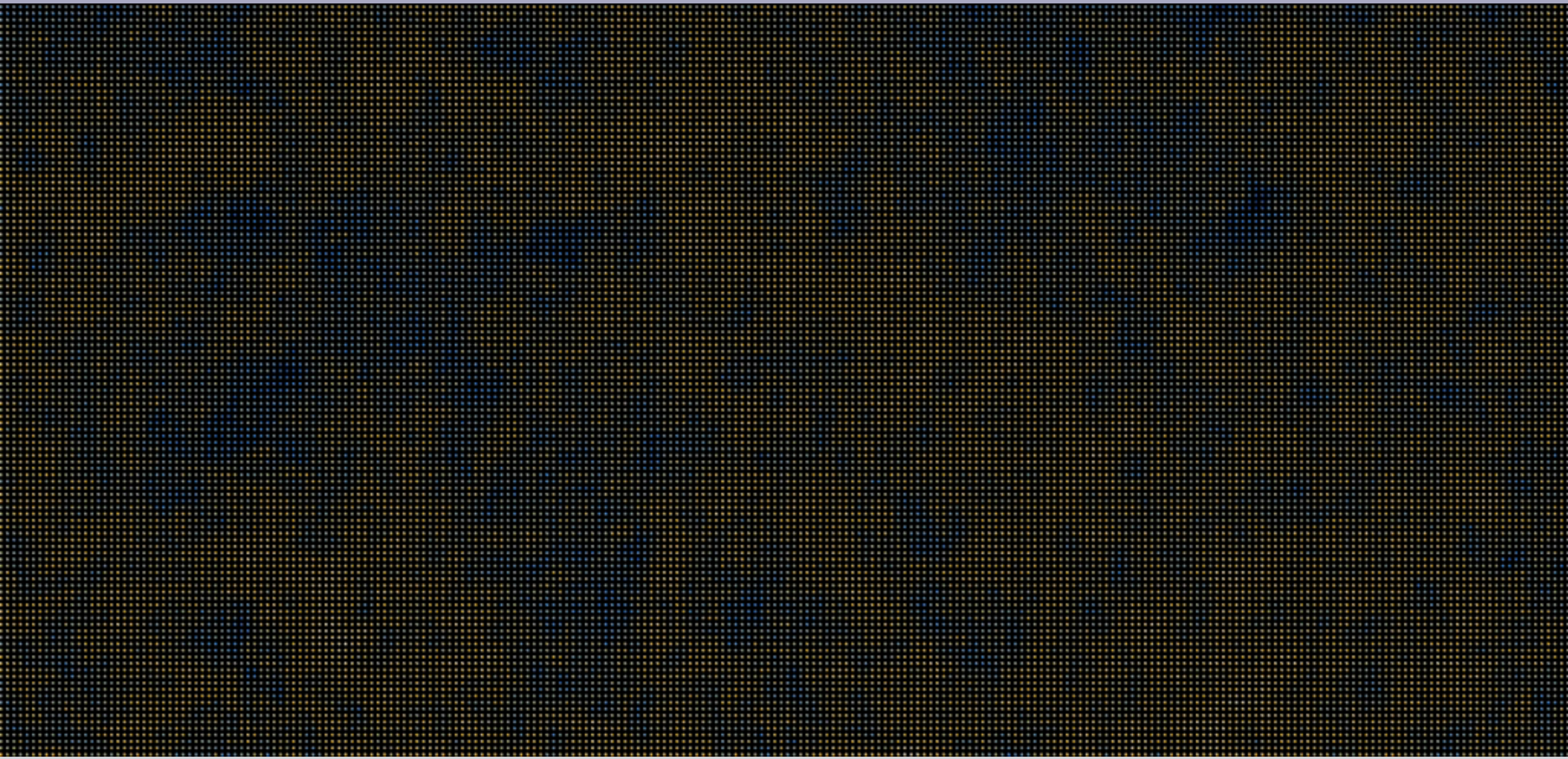
# Friedmann-Lemaître metric

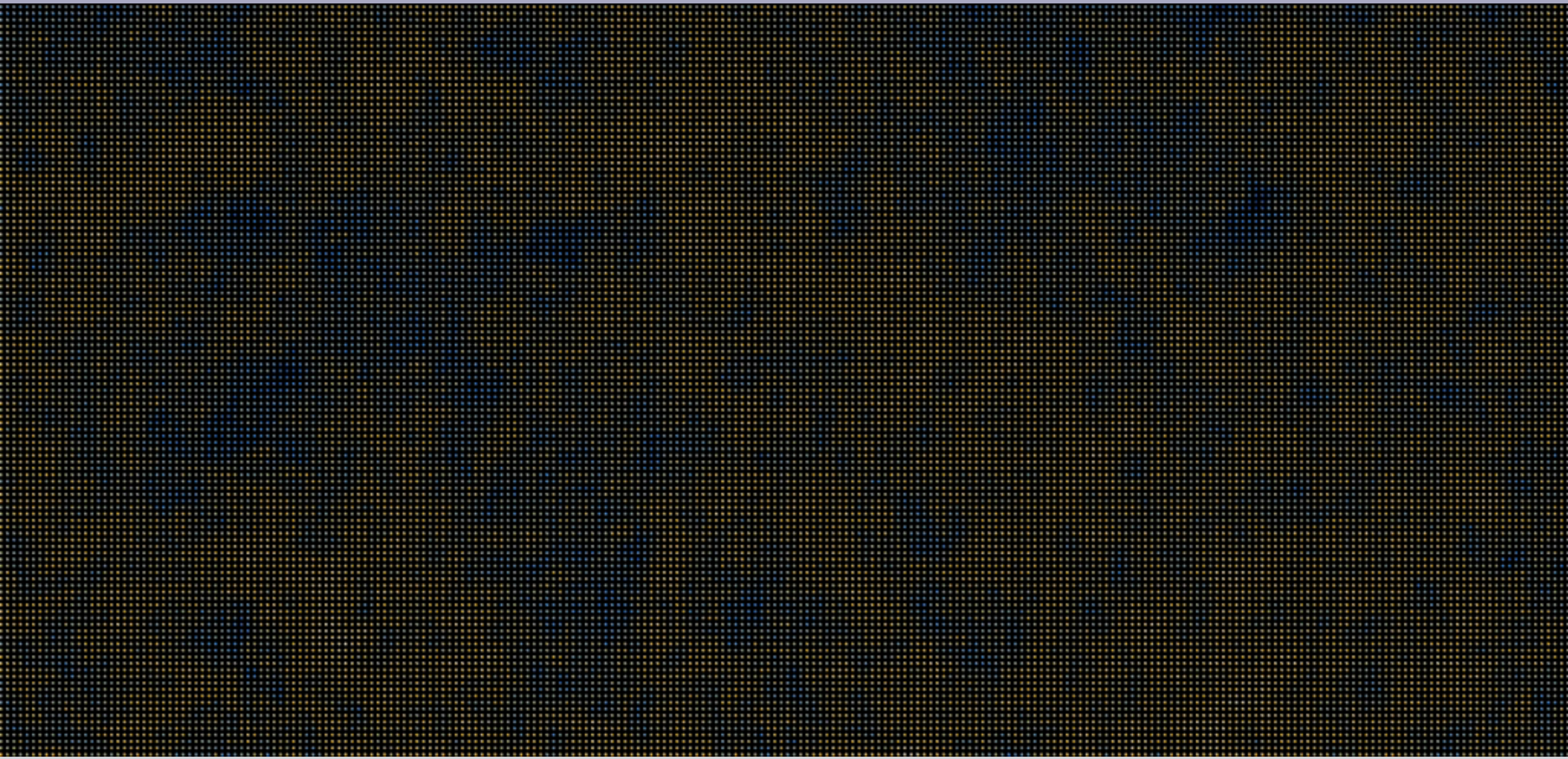
- If we are not in a special position
- Universe must be everywhere similar to here*
- First approximation: no space dependence, only time.
  - Invariant under rotations and translations

$$\left(\frac{\dot{a}(t)}{a(t)}\right)^2 = \frac{8\pi G_{\mathrm{N}}}{3}\rho + \frac{\Lambda}{3} - \frac{k^2}{a(t)^2}$$

**Big Bang model**

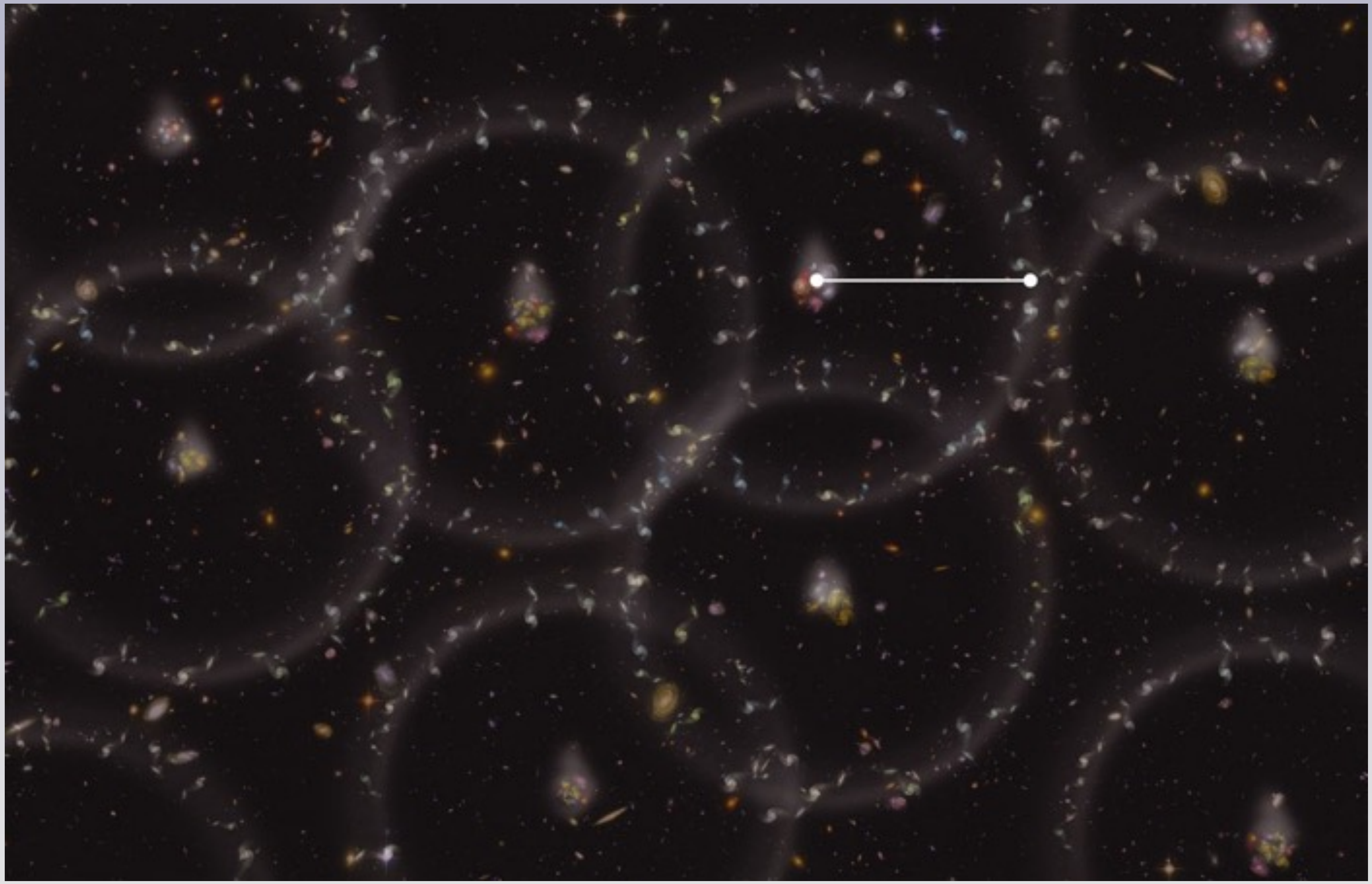
* realize how revolutionary that idea was (still is)



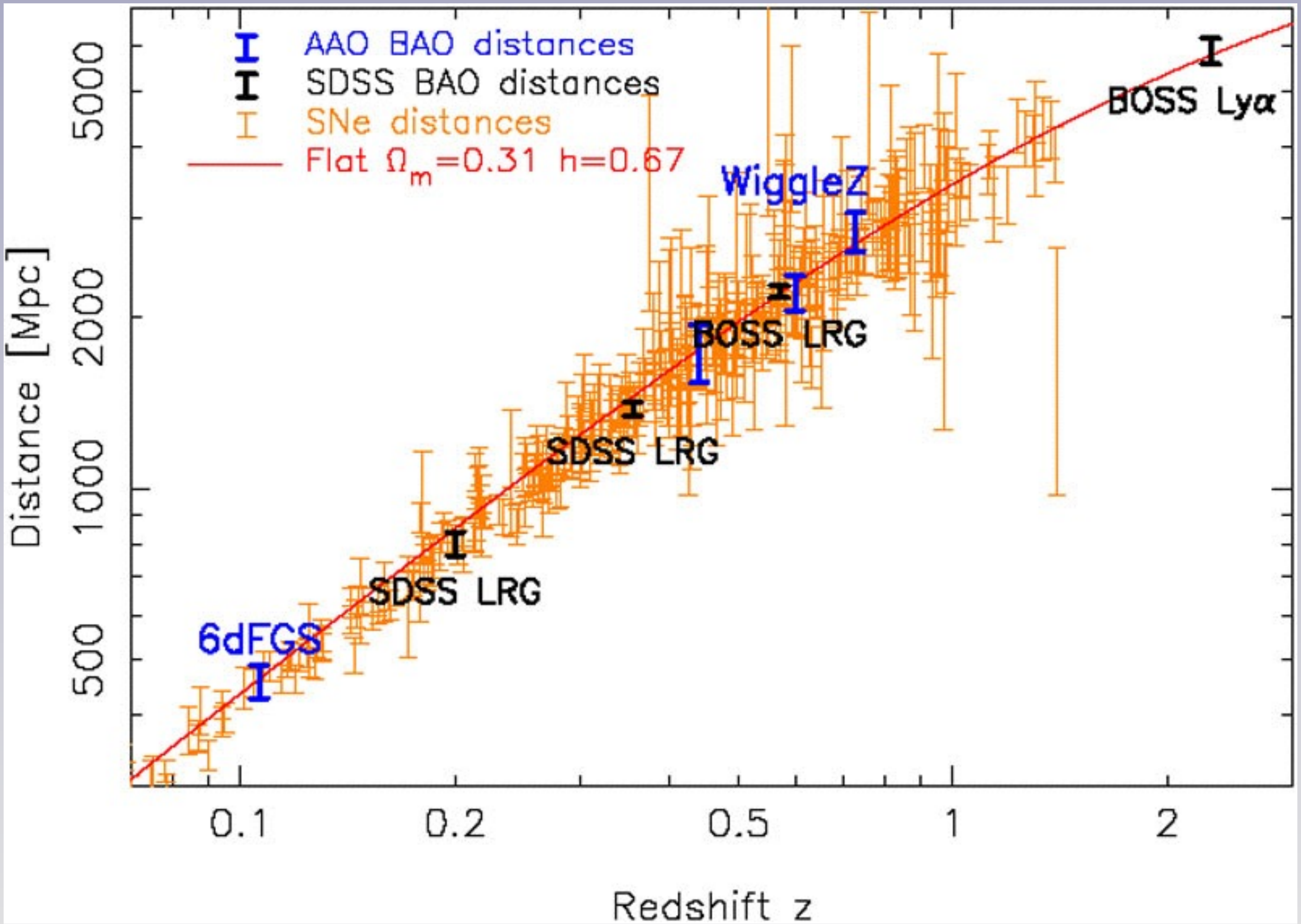


# Baryon Acoustic Oscillations

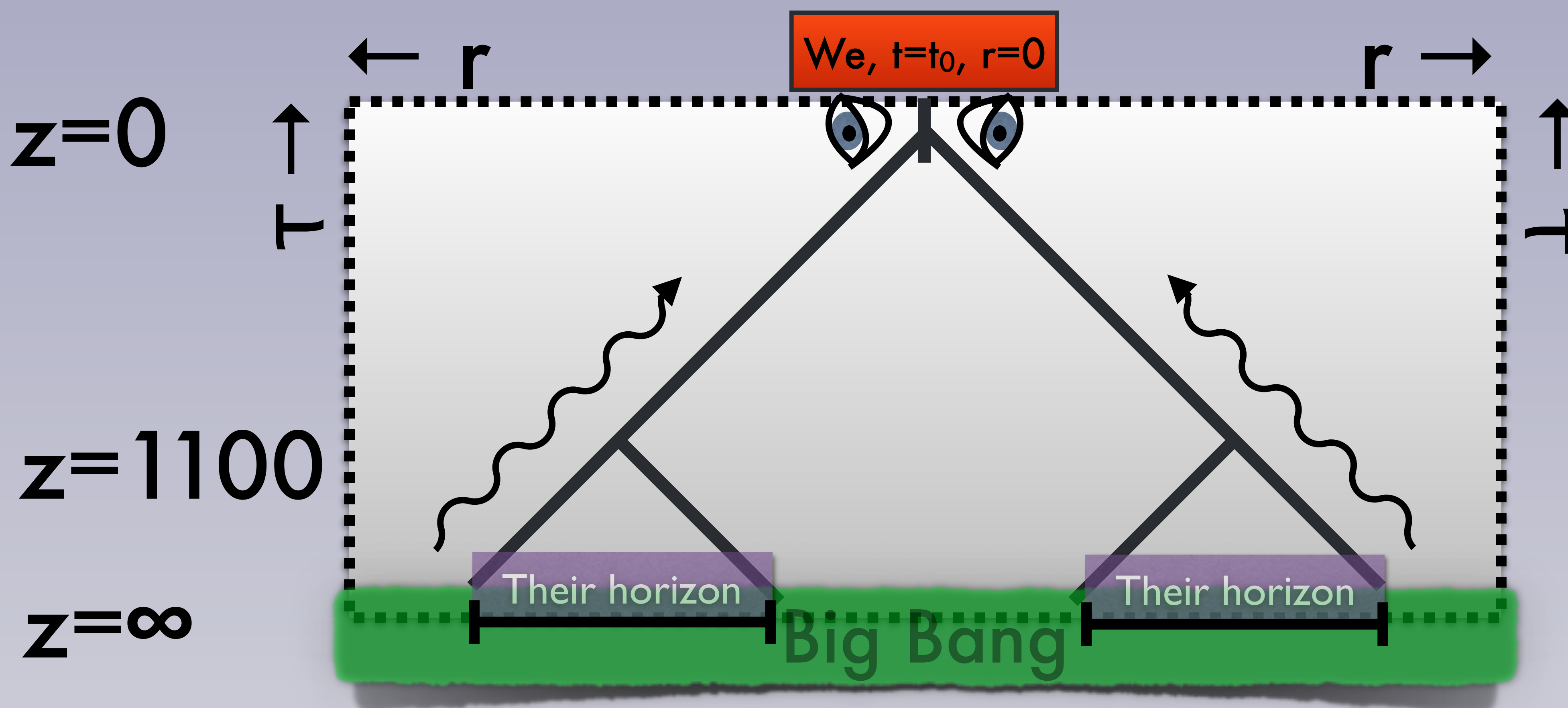
- Same peaks as in CMB



# BAO as a distance measure

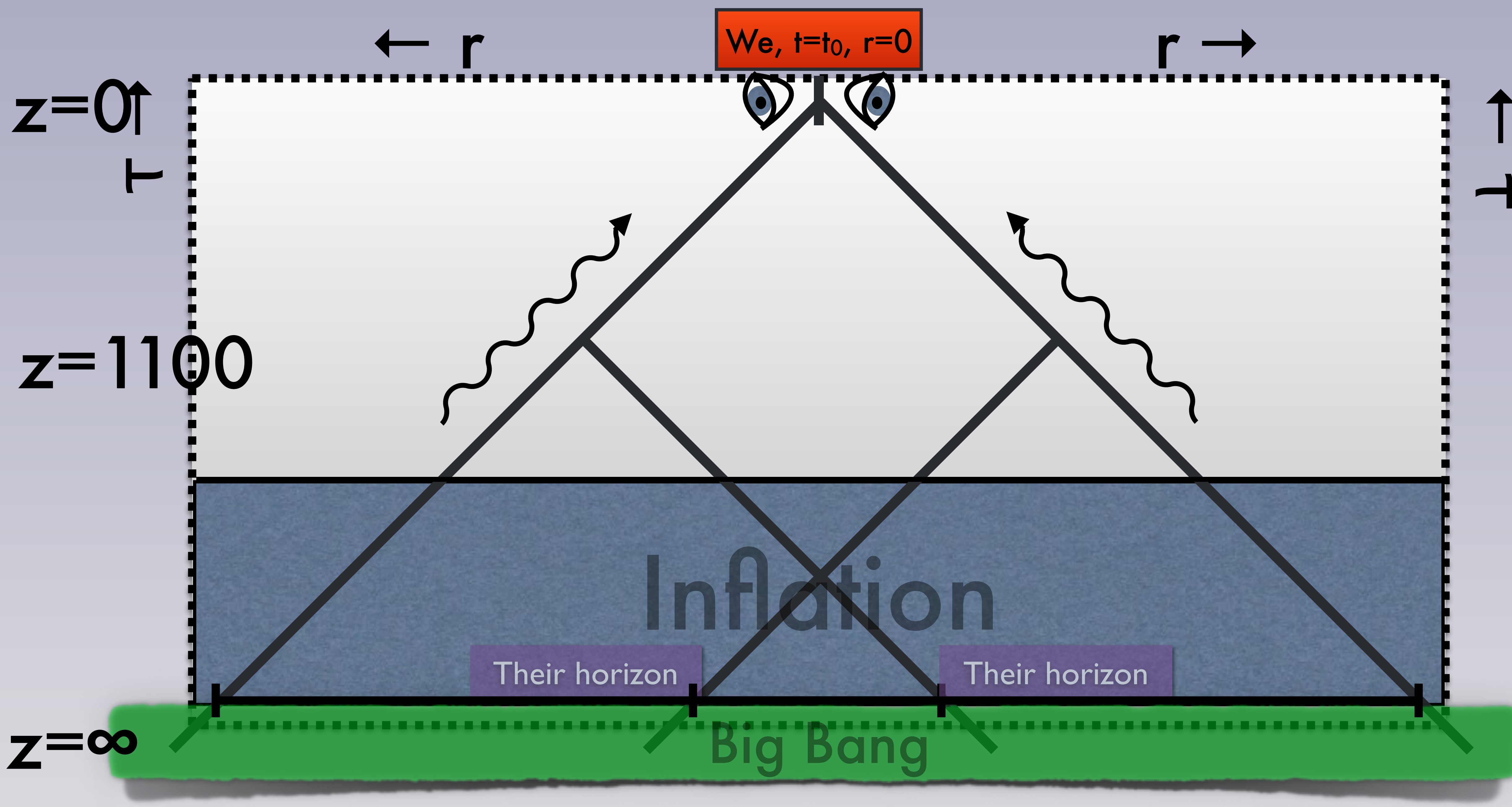


# Horizon problem solved



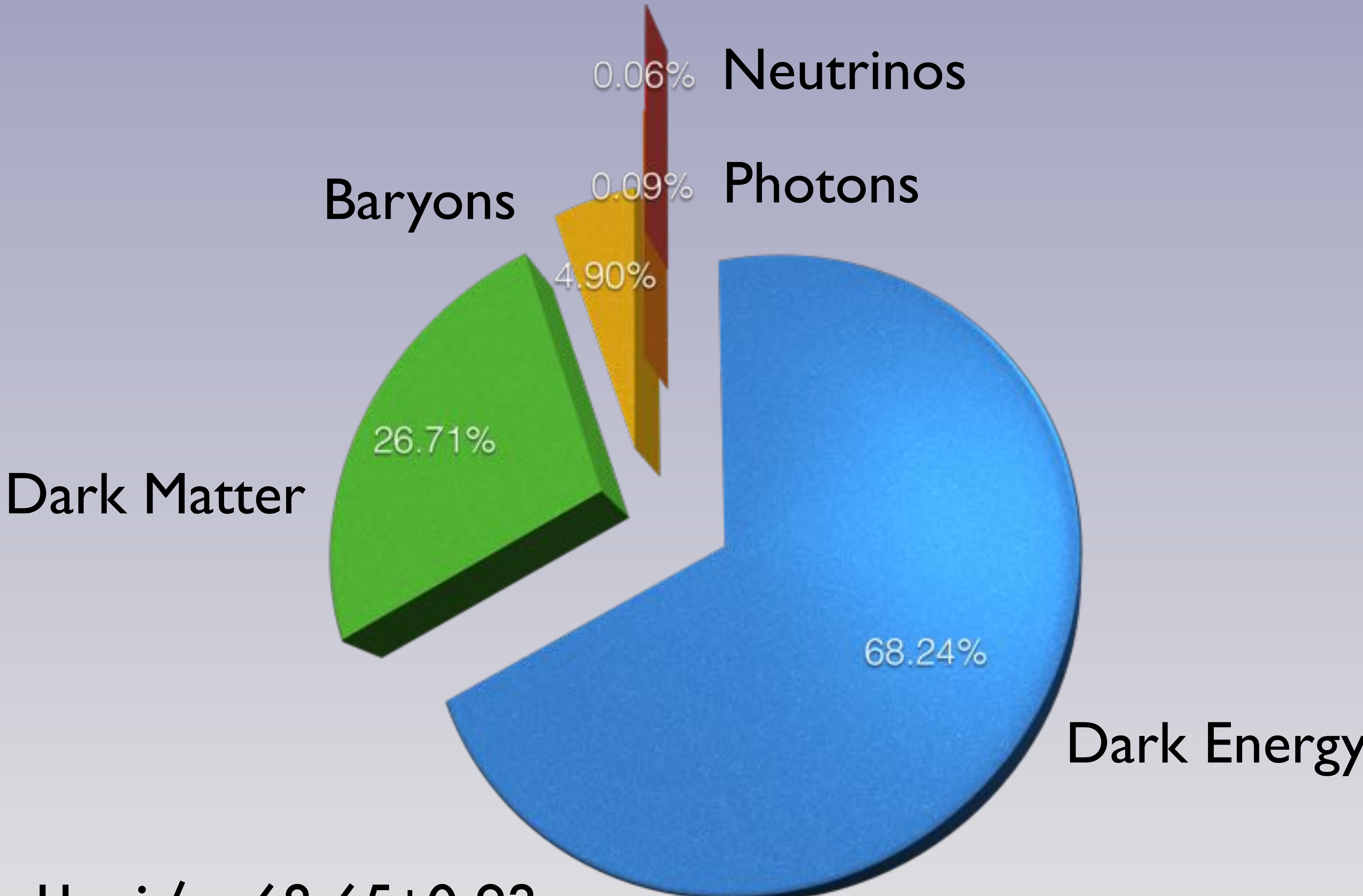

[following Will Kinney's lecture notes arXiv:0902.1529]

# Horizon problem solved




[following Will Kinney's lecture notes arXiv:0902.1529]

# Parameter constraints from Planck




$H_0 = \dot{a}/a = 68.65 \pm 0.93$