Cosmology IV: Cosmic Inflation

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Outline

- Connection to previous lectures
- Inflation
  - Solving problems
  - Generating perturbatinos
- CMB polarization
  - qualitative
  - E/B-modes and inflation and BICEP-II

Correlations

- Structure in the universe has a power spectrum.
  - CMB: C_l *or* D_l, at z=1100
  - LSS: $P(k,z)$
  - Not discussed the initial conditions for differential equation of $\Phi(k,z)$.

Correlations

- Spectrum if universe was made of white noise, 2-point correlation vanishes:

$$
\begin{aligned}
\left\langle \delta(\vec{x})\delta^*(\vec{x}')\right\rangle =& \delta^{(3)}(\vec{x}-\vec{x}')\sigma^2 \\
=& \int \frac{d^3k}{(2\pi)^3}\frac{d^3k'}{(2\pi)^3}\left\langle \delta_{\vec{k}}\delta^*_{\vec{k}'}\right\rangle e^{-i\vec{k}\cdot\vec{x}+i\vec{k}'\cdot\vec{x}'} \\
& \downarrow \\
\left\langle \delta_{\vec{k}}\delta^*_{\vec{k}'}\right\rangle =& (2\pi)^3\sigma^2\delta^{(3)}(\vec{k}-\vec{k}') \\
& \downarrow \\
P_{\rm ini}(k) =& \text{const.}
\end{aligned}
$$

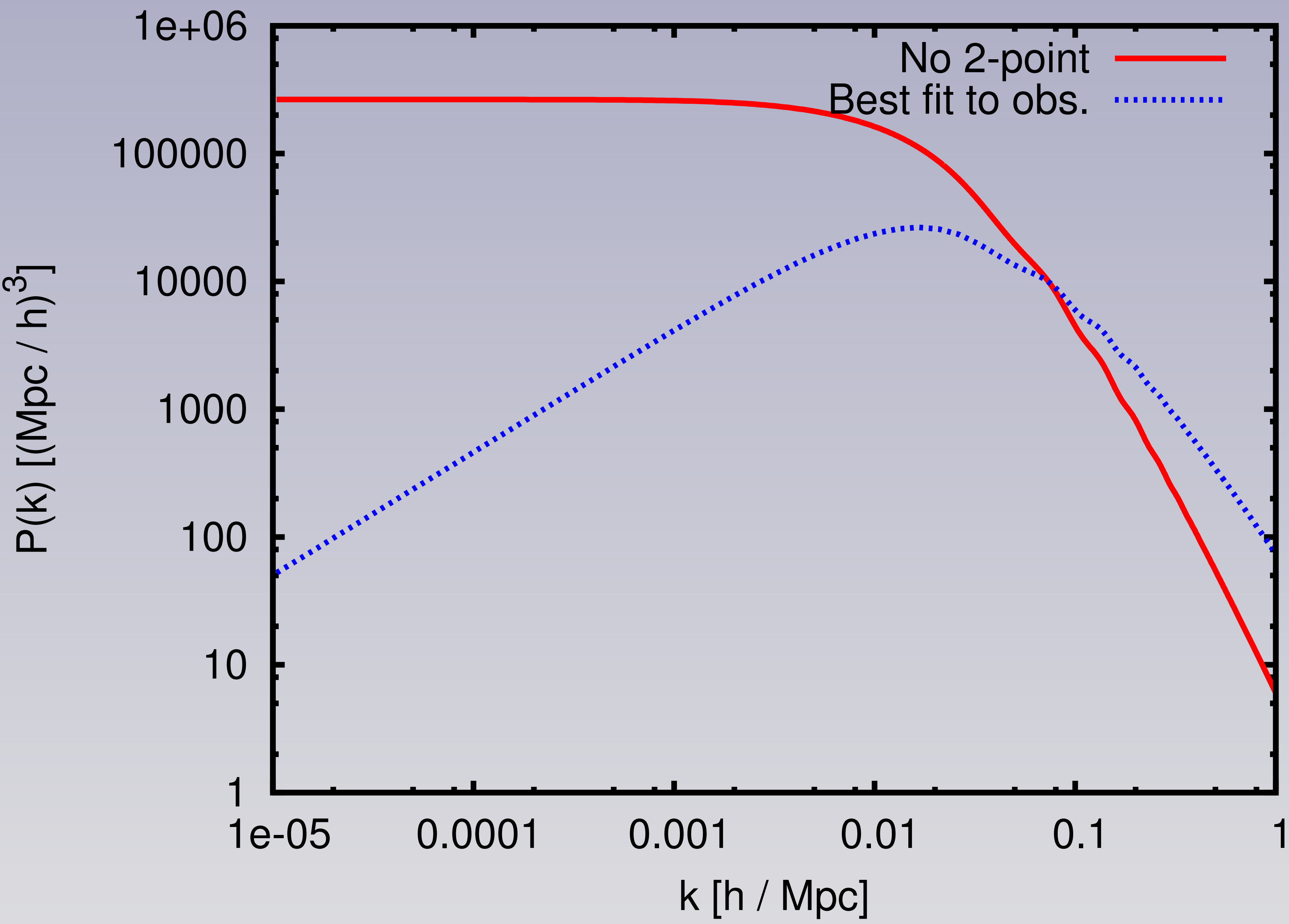
Correlations

- But white noise in what? Look at e.g. GR Poisson equation:

$$\nabla^2\Phi = 4\pi G_{\mathrm{N}} a^2(t)\bar{\rho}_{\mathrm{m}}\delta_m$$
$$P_\Phi(k) \propto k^4 P_\rho(k)$$

- If $P(k) \propto k^n$, origin may still be uncorrelated system

Correlations



Harrison-Zel’dovich

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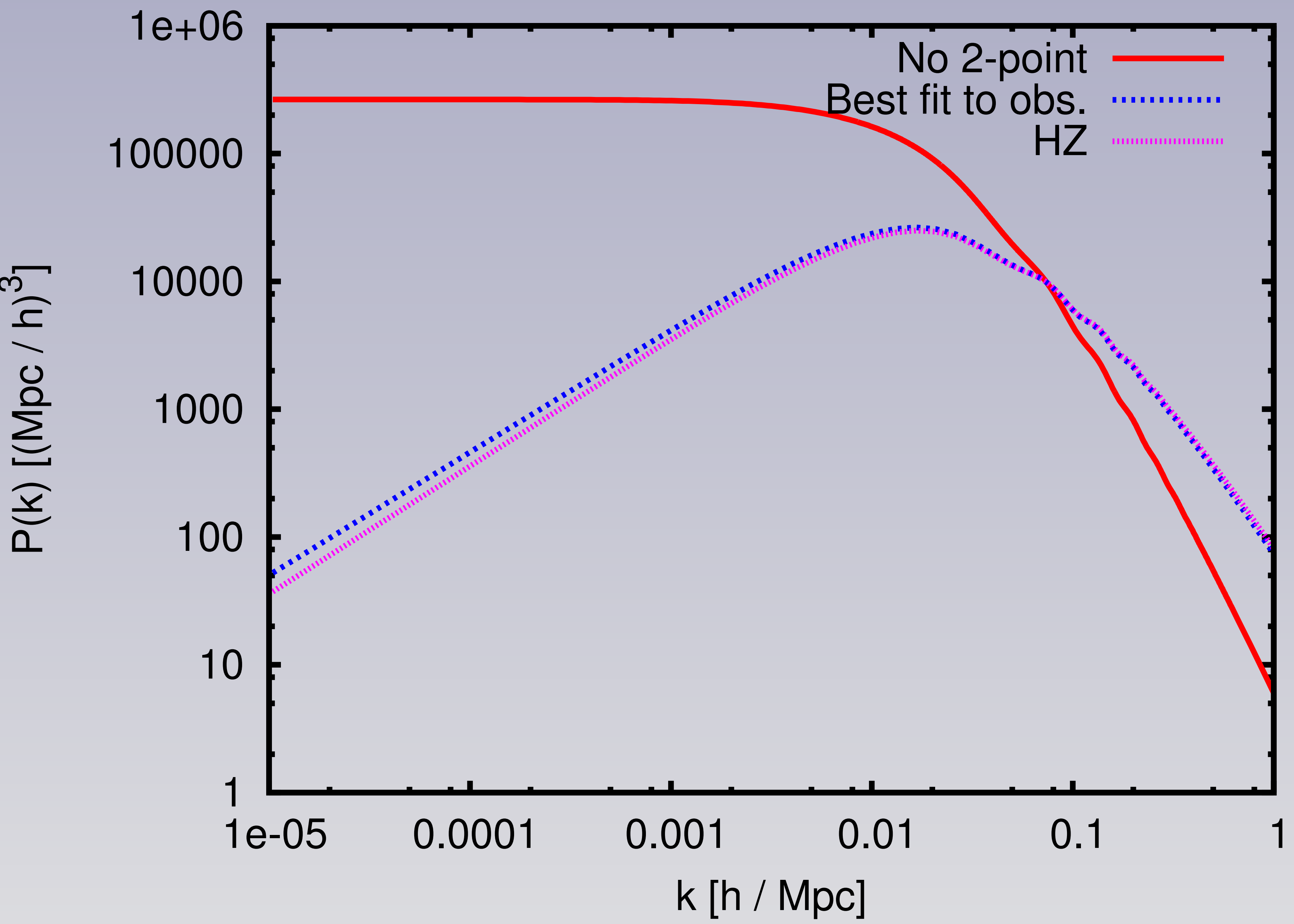
- Translates to $P(k) \propto k$

Harrison-Zel'dovich

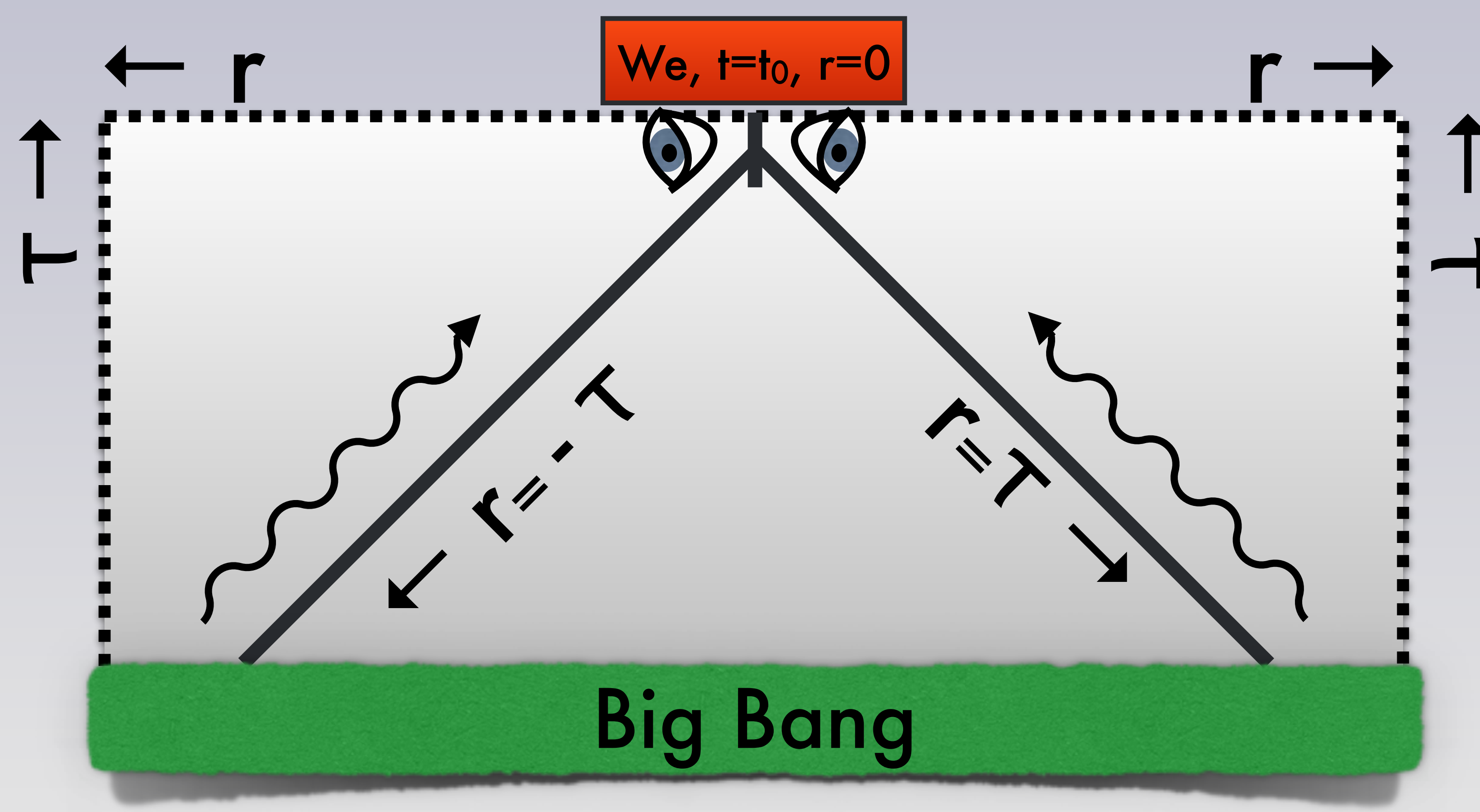


Horizon problem

- Conformal time:

$$ds^2 = -dt^2 + a(t)^2\left[dr^2 + r^2 d\theta^2 + r^2\sin^2\theta d\phi^2\right]$$
$$ds^2 = a(t)^2\left[-d\tau^2 + dr^2 + r^2 d\theta^2 + r^2\sin^2\theta d\phi^2\right]$$

- Photons travel on $ds^2=0$ or $d\tau = \pm dr$.



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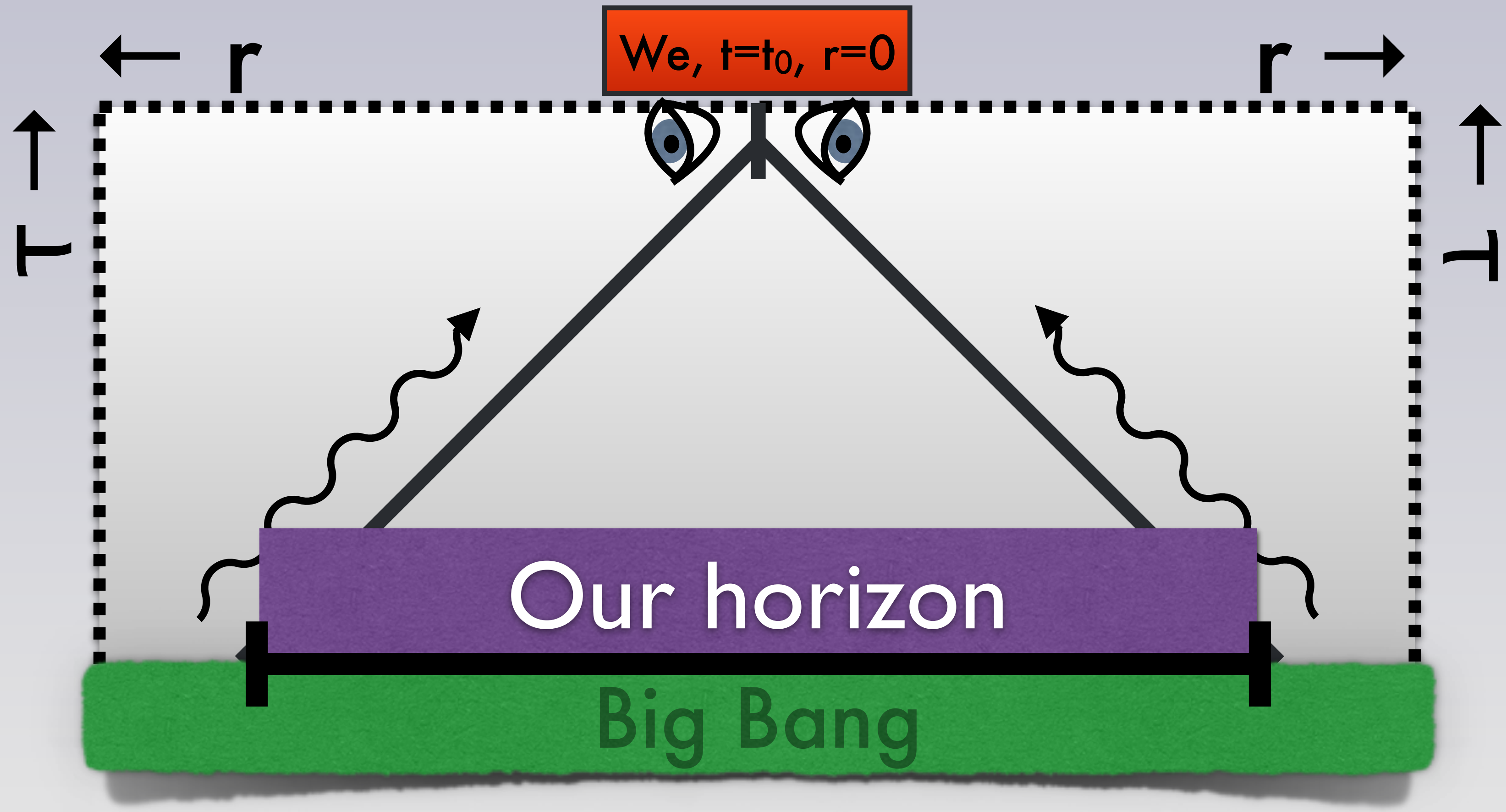

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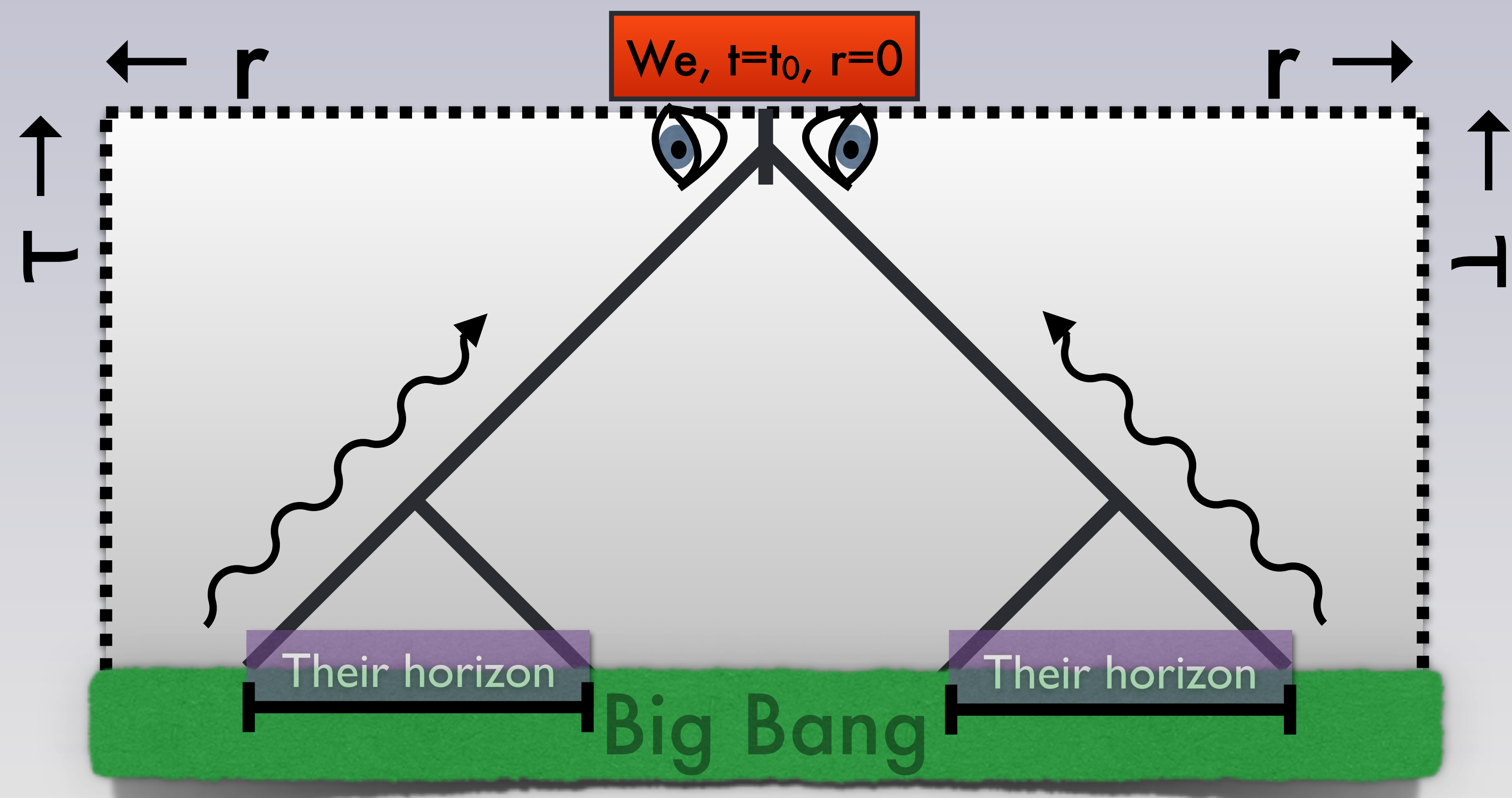

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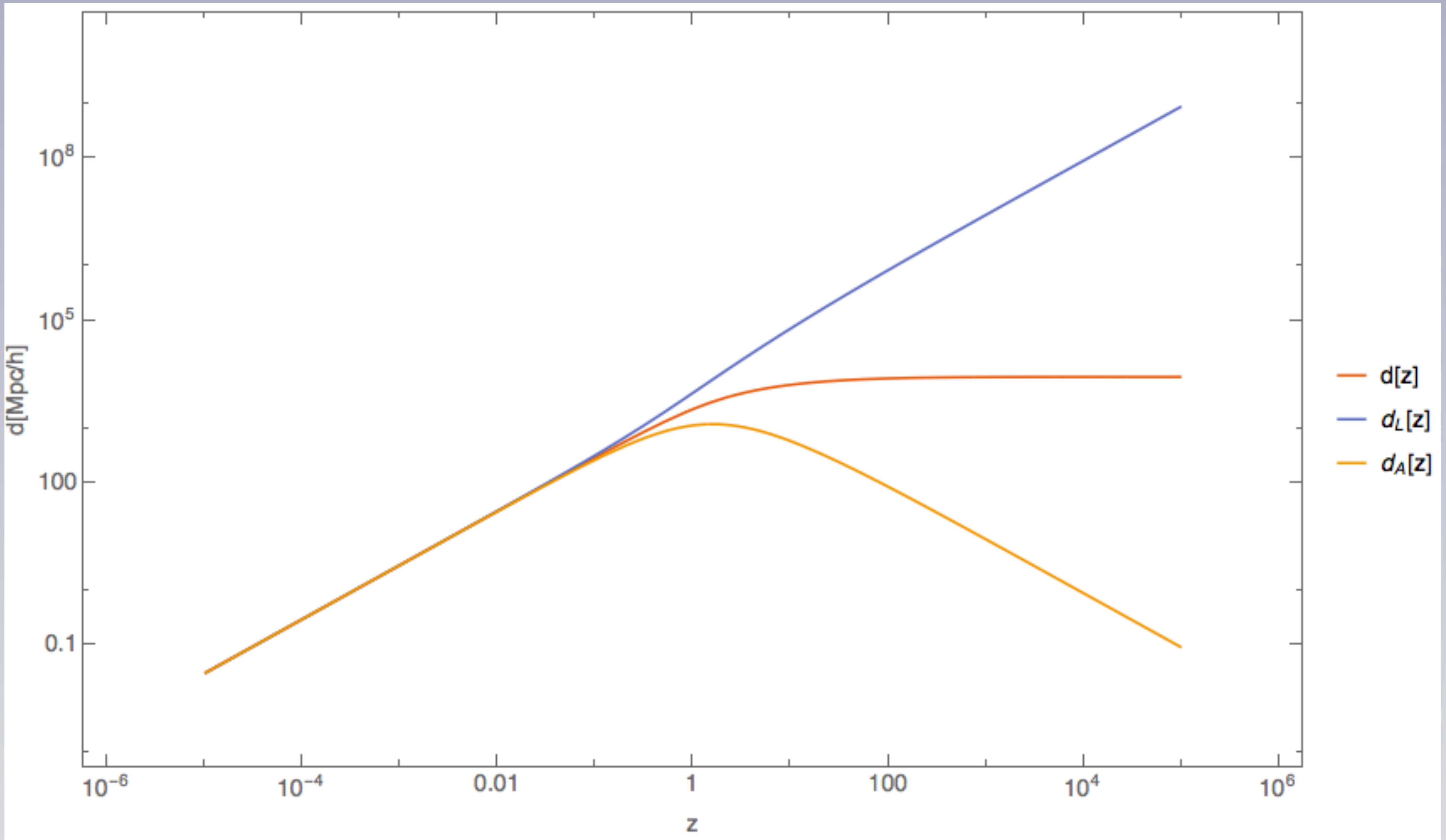

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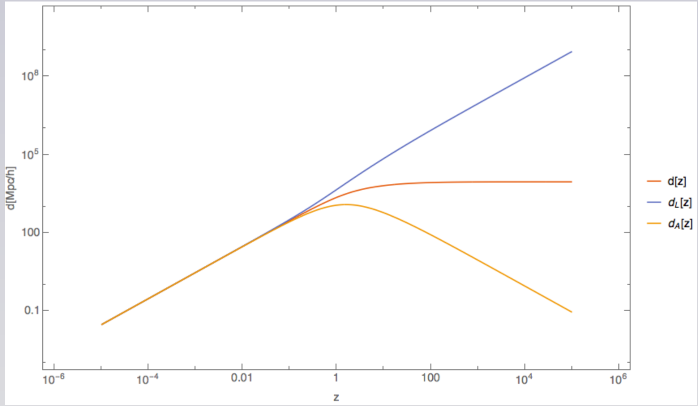

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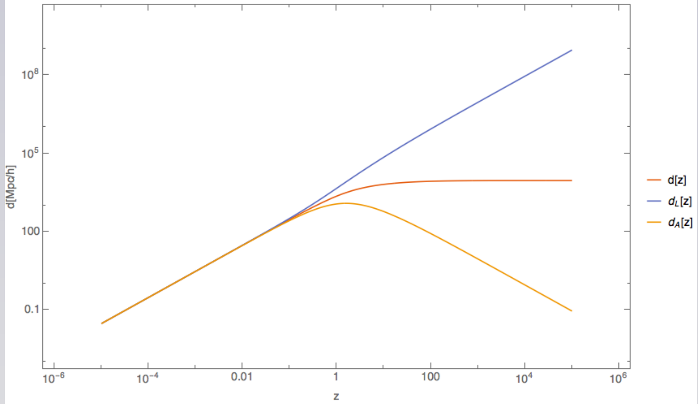
  - at z = 1100
  - Angular scale: 0.3°



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  - d_H = 50 kpc
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  - Angular scale: < 1°
  - Why is CMB so uniform?

Flatness problem

Flatness problem

- Spatial curvature:

$$ds^2 = a(t)^2 \left[-d\tau^2 + \frac{dr^2}{1-\kappa r^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \right]$$

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- If Ω(t)=1, universe is spatially flat.

$$H(t)^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_{\mathrm{N}}}{3}\rho + \frac{\Lambda}{3} - \frac{\kappa}{a^2}$$

$$\Omega(t) = H(t)^{-2}\left(\frac{8\pi G_{\mathrm{N}}}{3}\rho + \frac{\Lambda}{3}\right) = 1 - \frac{\kappa}{(aH(t))^2}$$

$$= 1 - \frac{\kappa}{(\dot{a})^2}$$

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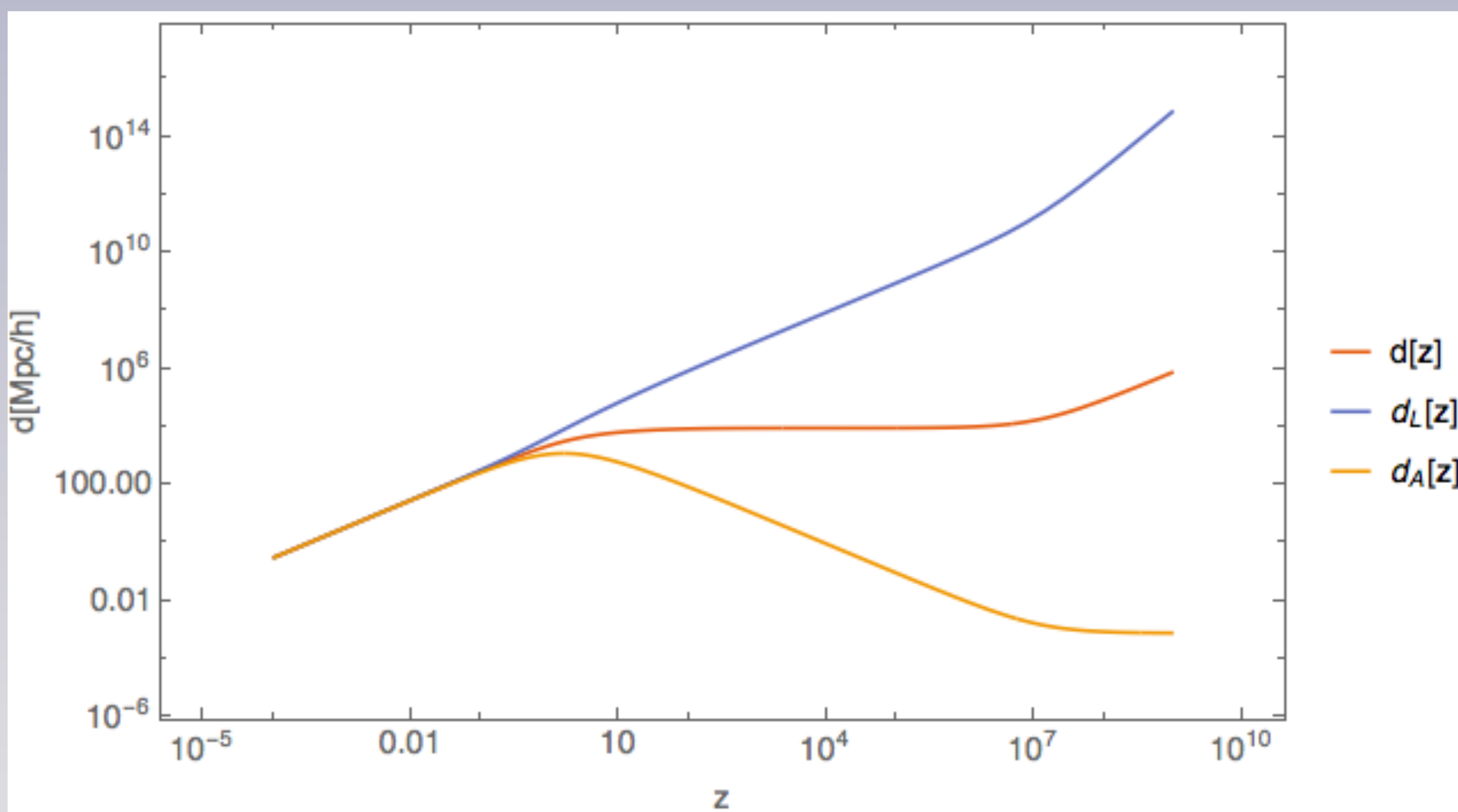
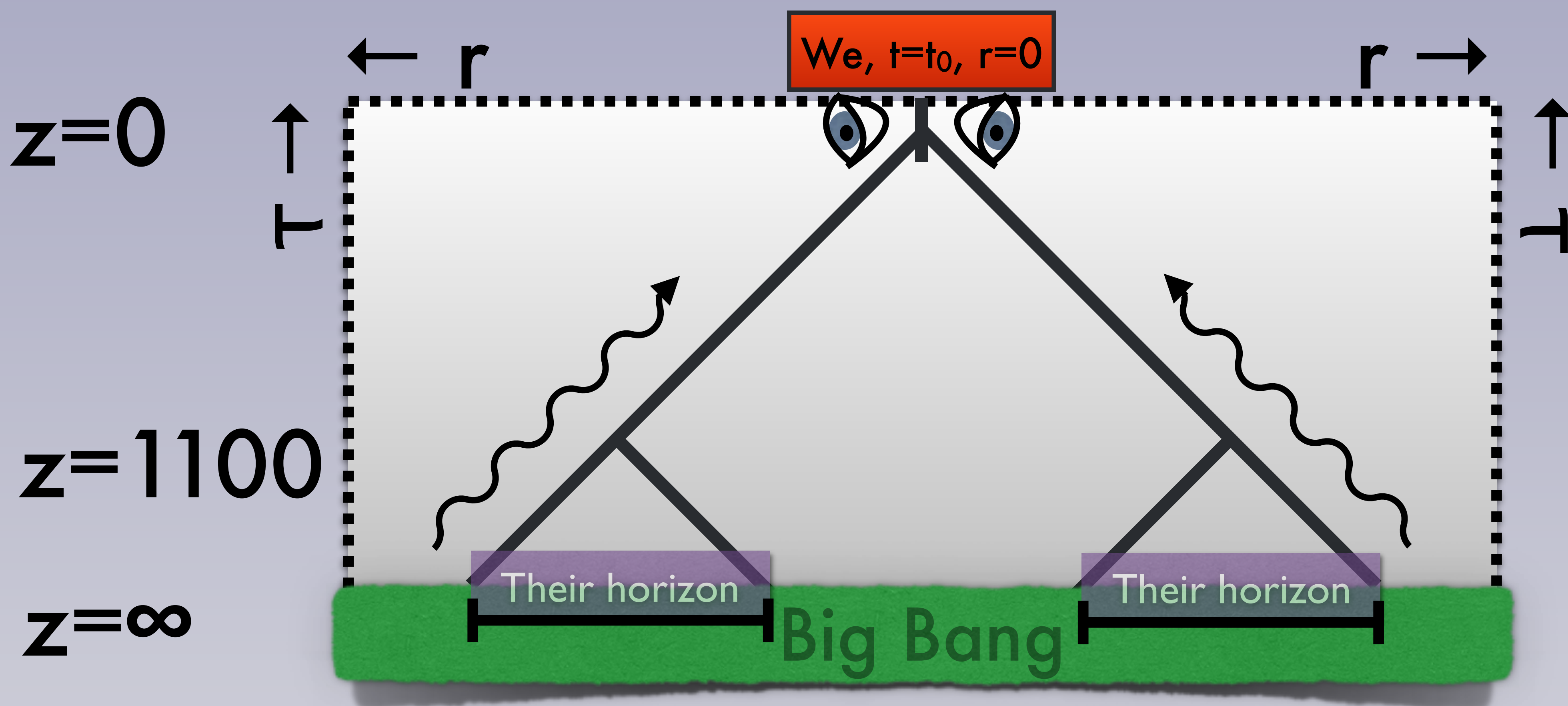
Inflation

$$H(t)^2 = \left(\frac{\dot{a}}{a}\right)^2 = \begin{cases} \frac{8\pi G_{\mathrm{N}}}{3}\rho + \frac{\Lambda}{3} - \frac{\kappa}{a^2} & \text{for } z < z_{\mathrm{reh}} \\ H_{\mathrm{Inf}} & \text{for } z \geqslant z_{\mathrm{reh}} \end{cases}$$



Horizon problem solved



[following Will Kinney's lecture notes arXiv:0902.1529]

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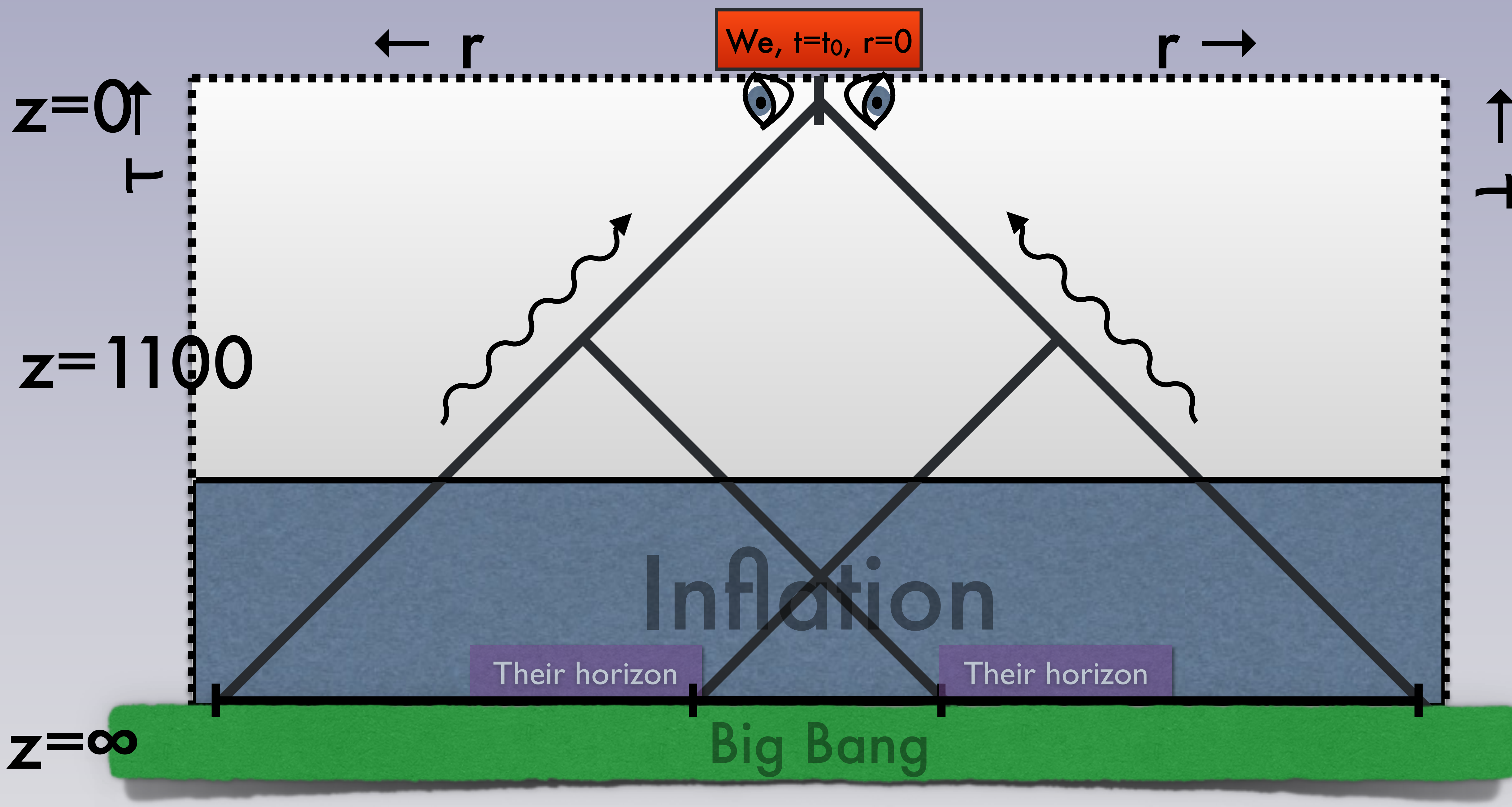

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Horizon problem solved

- Big bang is still at $z=\infty$
- More space time pushed into our light cone
- One way of phrasing it: our Universe stems from a much smaller patch of initial space time, blown up to big proportions by inflation, hence it is causally connected.

Scalar inflation

$$S_{\rm tot} = \int d^4x\sqrt{-g}\left[\frac{m_{\rm Pl}^2}{16\pi}R + \mathcal{L}_\phi\right]$$

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$$\begin{aligned} \eta &= -\frac{\ddot{\phi}}{H\dot{\phi}} \\ &\simeq \frac{m_{\rm Pl}^2}{8\pi}\left[\frac{V''\left(\phi\right)}{V\left(\phi\right)} - \frac{1}{2}\left(\frac{V'\left(\phi\right)}{V\left(\phi\right)}\right)^2\right] \end{aligned}$$

[following Will Kinney's lecture notes arXiv:0902.1529]

Perturbations in GR

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\begin{aligned}
g_{00} &= -a^2(\tau)\ \{1 + 2\psi(\vec{x}, \tau)\}\ , \\
g_{0i} &= a^2(\tau)\, w_i(\vec{x}, \tau)\, , \\
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scalar

scalar + vector

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scalar

Scalar + vector + tensor

Perturbations in the scalar field and in $g_{\mu\nu}$

$$\varphi\left(\tau,\mathbf{x}\right)=\int\frac{d^3k}{\left(2\pi\right)^{3/2}}\left[\varphi_{\mathbf{k}}\left(\tau\right)b_{\mathbf{k}}e^{i\mathbf{k}\cdot\mathbf{x}}+\varphi^*_{\mathbf{k}}\left(\tau\right)b^*_{\mathbf{k}}e^{-i\mathbf{k}\cdot\mathbf{x}}\right].$$

[following Will Kinney's lecture notes arXiv:0902.1529]

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$$u_k\equiv a\left(\tau\right)\varphi_{\mathbf{k}}\left(\tau\right)$$

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$$u''_k+\left[k^2-\frac{a''}{a}\right]u_k=0$$

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$$u_k''+\left[k^2-\frac{a''}{a}\right]u_k=0$$

$k \gg a''/a$ $\qquad u_k''+k^2u_k=0,$ $\qquad u_k\left(\tau\right)=\frac{1}{\sqrt{2k}}\left(A_ke^{-ik\tau}+B_ke^{ik\tau}\right)$

[following Will Kinney's lecture notes arXiv:0902.1529]

Perturbations in the scalar field and in $g_{\mu\nu}$

$$\varphi\left(\tau,\mathbf{x}\right) = \int \frac{d^3k}{\left(2\pi\right)^{3/2}}\left[\varphi_{\mathbf{k}}\left(\tau\right) b_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}} + \varphi^*_{\mathbf{k}}\left(\tau\right) b^*_{\mathbf{k}} e^{-i\mathbf{k}\cdot\mathbf{x}}\right].$$

$$u_k \equiv a\left(\tau\right)\varphi_{\mathbf{k}}\left(\tau\right)$$

$$u''_k + \left[k^2 - \frac{a''}{a}\right] u_k = 0$$

$k \gg a''/a$ $\qquad u''_k + k^2 u_k = 0,$ $\qquad u_k\left(\tau\right) = \frac{1}{\sqrt{2k}}\left(A_k e^{-ik\tau} + B_k e^{ik\tau}\right)$

$k \ll a''/a.$ $\qquad a'' u_k = a u''_k$ $\qquad u_k \propto a \;\Rightarrow \varphi_k = \text{const.}$

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$$\varphi(\tau, \mathbf{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} \left[\varphi_{\mathbf{k}}(\tau)\, b_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}} + \varphi^*_{\mathbf{k}}(\tau)\, b^*_{\mathbf{k}} e^{-i\mathbf{k}\cdot\mathbf{x}}\right].$$

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$$b_{\mathbf{k}} \rightarrow \hat{b}_{\mathbf{k}}, \quad b^*_{\mathbf{k}} \rightarrow \hat{b}^\dagger_{\mathbf{k}},$$

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$$b_{\mathbf{k}} \rightarrow \hat{b}_{\mathbf{k}}, \quad b_{\mathbf{k}}^{*} \rightarrow \hat{b}_{\mathbf{k}}^{\dagger},$$

$$\left[\hat{b}_{\mathbf{k}}, \hat{b}_{\mathbf{k}'}^{\dagger}\right] \equiv \delta^3\left(\mathbf{k} - \mathbf{k}'\right)$$

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$$\Pi\left(\tau,\mathbf{x}\right)\equiv\frac{\delta\mathcal{L}}{\delta\left(\partial_0\varphi\right)}=a^2\left(\tau\right)\frac{\partial\varphi}{\partial\tau}.$$

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$$\left|A_k\right|^2-\left|B_k\right|^2=1.$$

[following Will Kinney's lecture notes arXiv:0902.1529]

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$$u_k \frac{\partial u^*_k}{\partial \tau} - u^*_k \frac{\partial u_k}{\partial \tau} = i, \qquad |A_k|^2 - |B_k|^2 = 1.$$

[following Will Kinney's lecture notes arXiv:0902.1529]

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$$u_k\frac{\partial u^*_k}{\partial\tau}-u^*_k\frac{\partial u_k}{\partial\tau}=i,\qquad\left|A_k\right|^2-\left|B_k\right|^2=1.$$

$$u_k\left(\tau\right)\propto e^{-ik\tau}\ \Rightarrow A_k=1,\ B_k=0.$$

[following Will Kinney's lecture notes arXiv:0902.1529]

Perturbations in the scalar field and in $g_{\mu\nu}$

[following Will Kinney's lecture notes arXiv:0902.1529]

Perturbations in the scalar field and in $g_{\mu\nu}$

$$P_{\mathcal{R}}\left(k\right) = \left(\frac{\delta N}{\delta\phi}\delta\phi\right)^2 = \left.\frac{H^2}{\pi m_{\mathrm{Pl}}^2\epsilon}\right|_{k=aH} \propto k^{n_S-1},$$

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Perturbations in the scalar field and in $g_{\mu\nu}$

$$P_{\mathcal{R}}(k) = \left(\frac{\delta N}{\delta \phi}\delta\phi\right)^2 = \left.\frac{H^2}{\pi m_{\mathrm{Pl}}^2 \epsilon}\right|_{k=aH} \propto k^{n_S - 1};$$

$$n_S - 1 = \frac{\epsilon}{H^2(\epsilon - 1)}\frac{d}{dN}\left(\frac{H^2}{\epsilon}\right) \simeq -4\epsilon + 2\eta$$

[following Will Kinney's lecture notes arXiv:0902.1529]

Perturbations in the scalar field and in $g_{\mu\nu}$

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$$P_T = \left\langle \delta g_{ij}^2 \right\rangle = 2\times\frac{32}{m_{\rm Pl}^2}\left\langle \varphi^2\right\rangle = \frac{16H^2}{\pi m_{\rm Pl}^2} \propto k^{n_T},$$

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Perturbations in the scalar field and in $g_{\mu\nu}$

$$P_{\mathcal{R}}\left(k\right) = \left(\frac{\delta N}{\delta \phi}\delta\phi\right)^2 = \frac{H^2}{\pi m_{\mathrm{Pl}}^2 \epsilon}\bigg|_{k=aH} \propto k^{n_S - 1};$$

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[following Will Kinney's lecture notes arXiv:0902.1529]

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$$P_{\mathcal{R}}(k) = \left(\frac{\delta N}{\delta\phi}\delta\phi\right)^2 = \left.\frac{H^2}{\pi m_{\rm Pl}^2 \epsilon}\right|_{k=aH} \propto k^{n_S-1},$$

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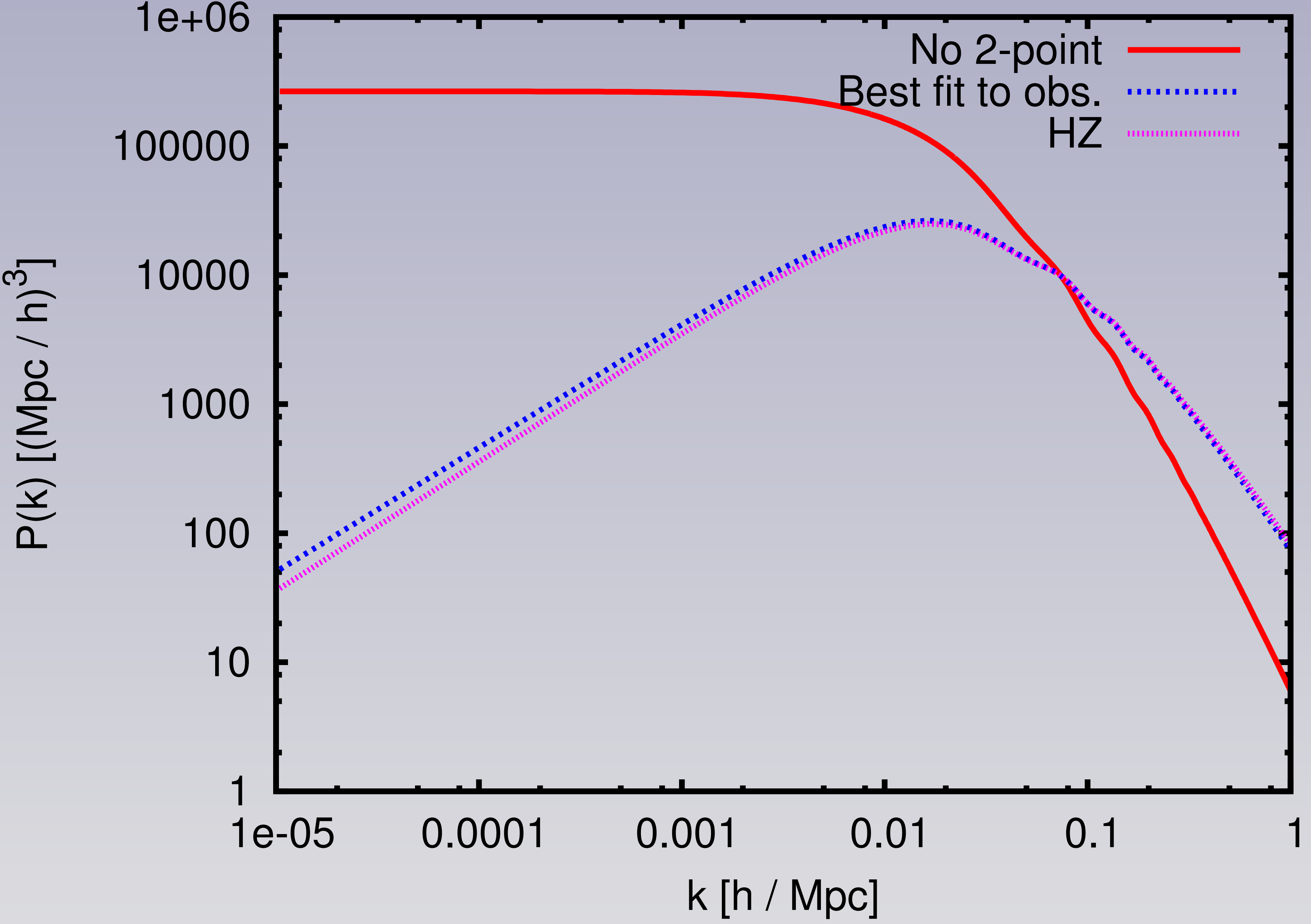
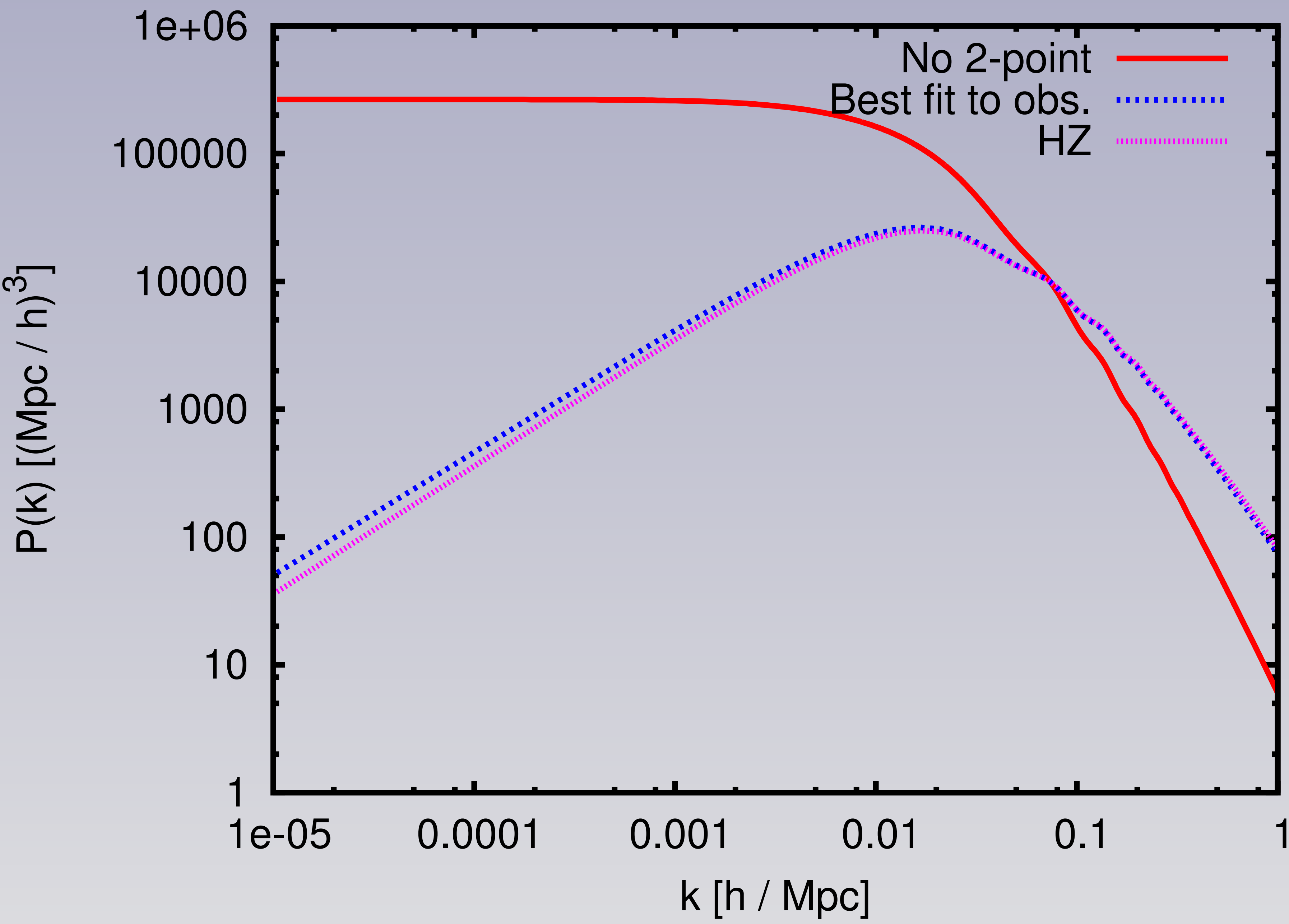
$$\text{note: } n_T < 0$$

[following Will Kinney's lecture notes arXiv:0902.1529]

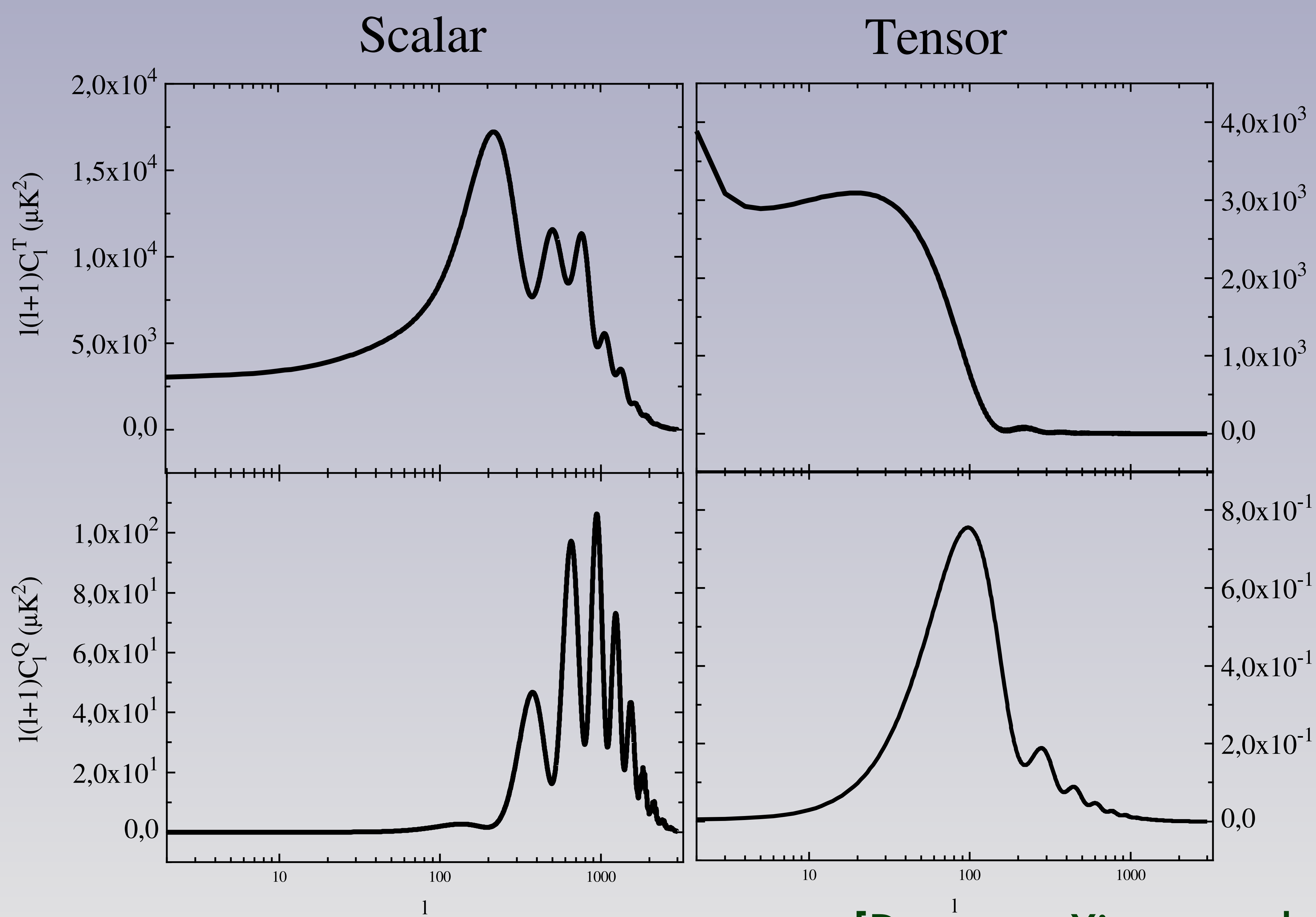
Harrison-Zel'dovich



Inflation


No 2-point
Best fit to obs.
HZ
1e+06
100000
10000
1000
100
10
1
1e-05
0.0001
0.001
0.01
0.1
1
k [h / Mpc]
P(k) [(Mpc / h)^3]

CMB



[Durrer, arXiv:astro-ph/0109522]

Knowns and unknowns

- Scalar perturbations give $V'(\varphi)$, but not $V(\varphi)$ and hence H_{Inf}.
- Need tensors for H_{Inf}.
- tensor spectrum is strictly *red*.
- *Blue* tensor spectrum would falsify inflation.

Not talked about

- Multi-field inflation:
  - turns in trajectories
- Gaussian or not Gaussian