# Cosmology II: Cosmic Microwave Background

Wessel Valkenburg
Instituut-Lorentz, Universiteit Leiden, Pays-Bas

# Today

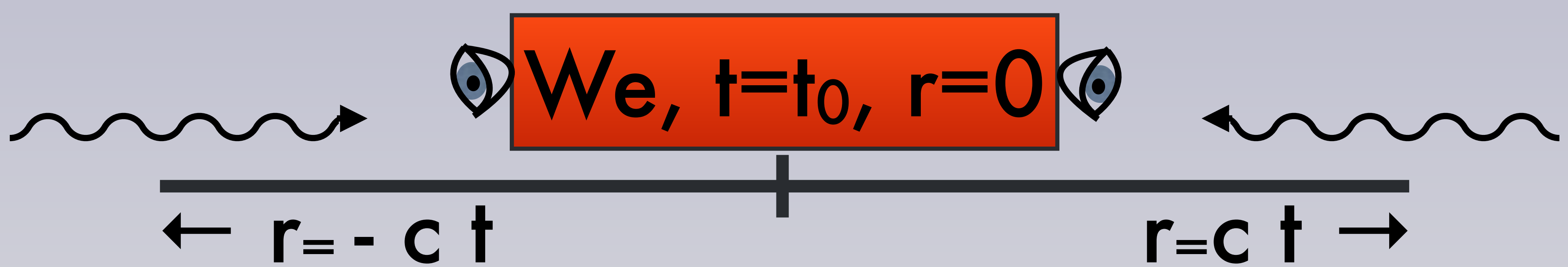
- Summary of previous.
- Correlation functions
- Perturbation theory of CMB
- CMB lensing
- Cosmological parameters in CMB
- Experimental constraints

# Summary of previous lecture

- Cosmologists observe light:
  - Intensity
  - Wavelength
  - Direction: 2 angles
- That is:
  - 2 dims of the 4 of our space time
  - intensity and wavelength help to pin down a third dimension

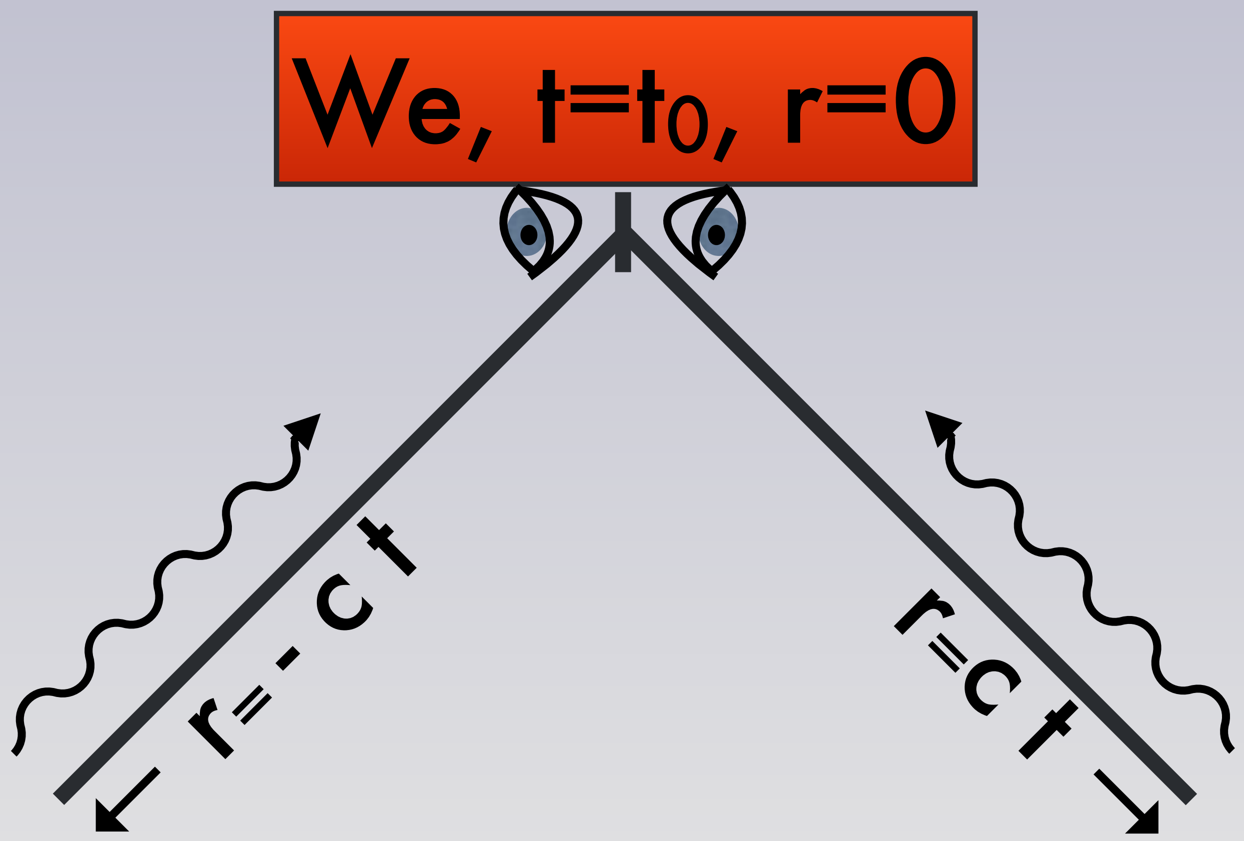
# Summary of previous lecture

- Intensity, wavelength, direction: 2 angles
- Light in vacuum has fixed velocity: $c = 1$
- (3+1)D space time $[t, r, \varphi, \vartheta]$, t&r constrained by $r = c\,t$.



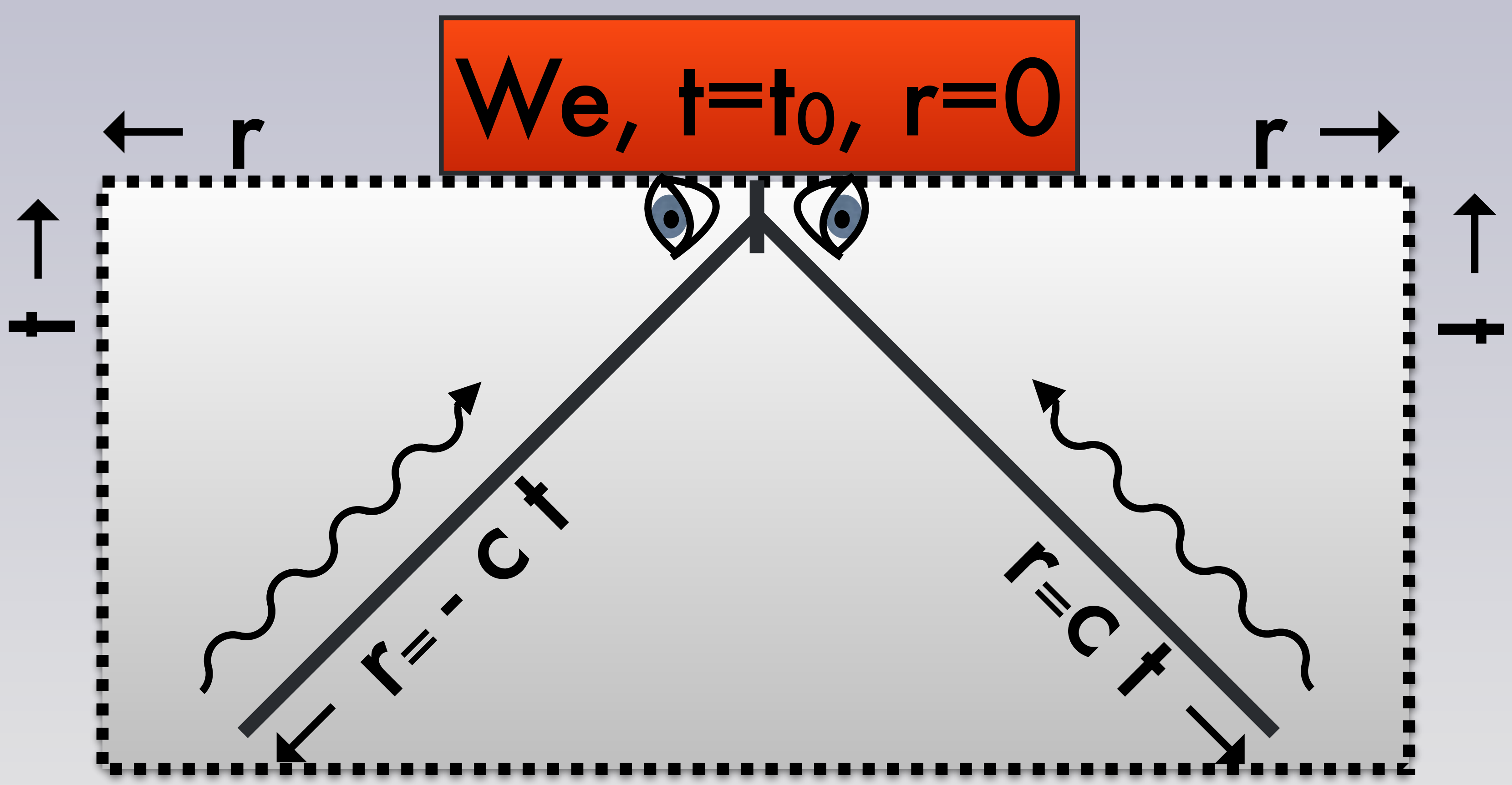
# Summary of previous lecture

- Intensity, wavelength, direction: 2 angles
- Light in vacuum has fixed velocity: $c = 1$
- (3+1)D space time $[t, r, \varphi, \vartheta]$, t&r constrained by $r = c\ t$.



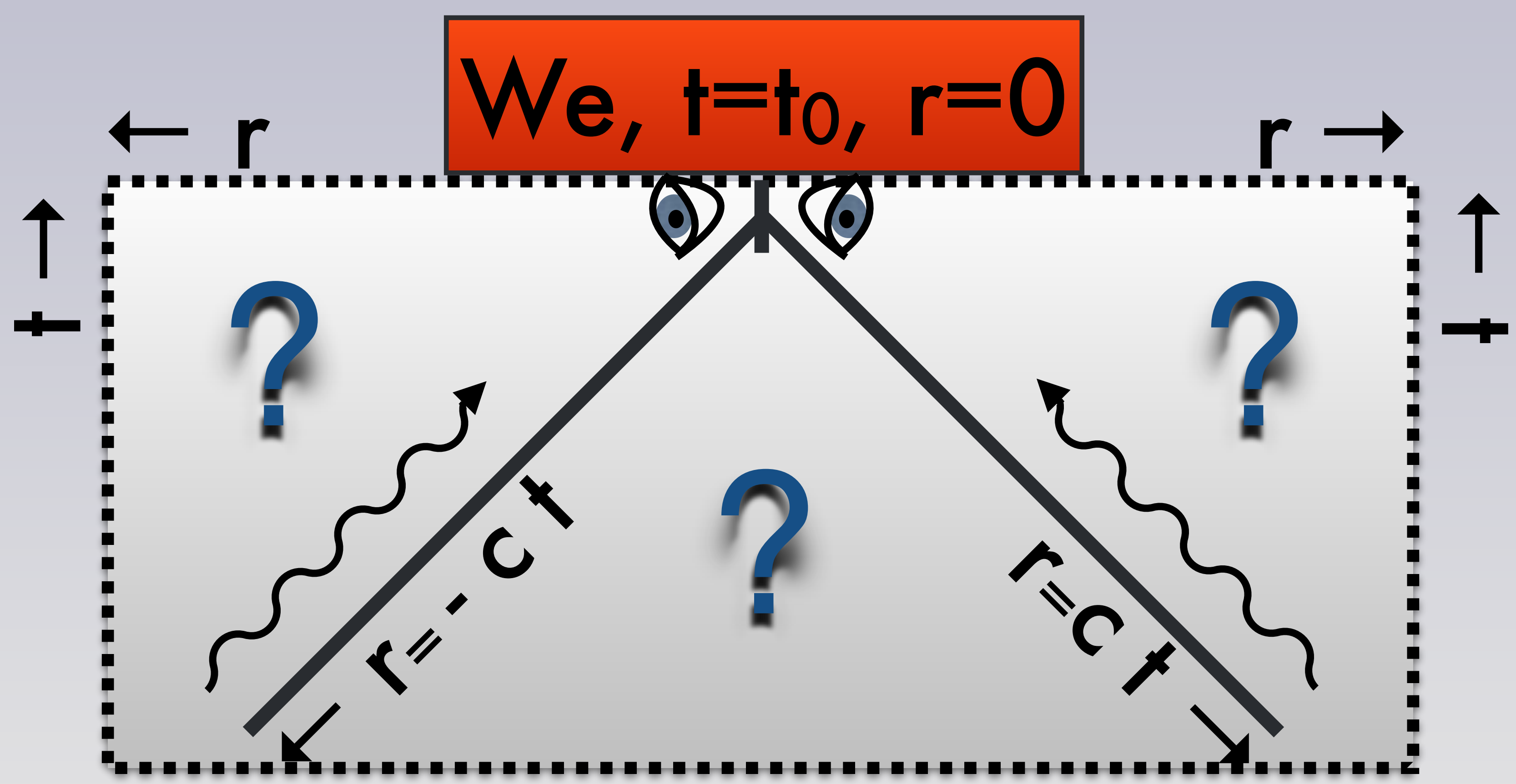
# Summary of previous lecture

- Intensity, wavelength, direction: 2 angles
- Light in vacuum has fixed velocity: $c = 1$
- (3+1)D space time $[t, r, \varphi, \vartheta]$, t&r constrained by $r = c\ t$.



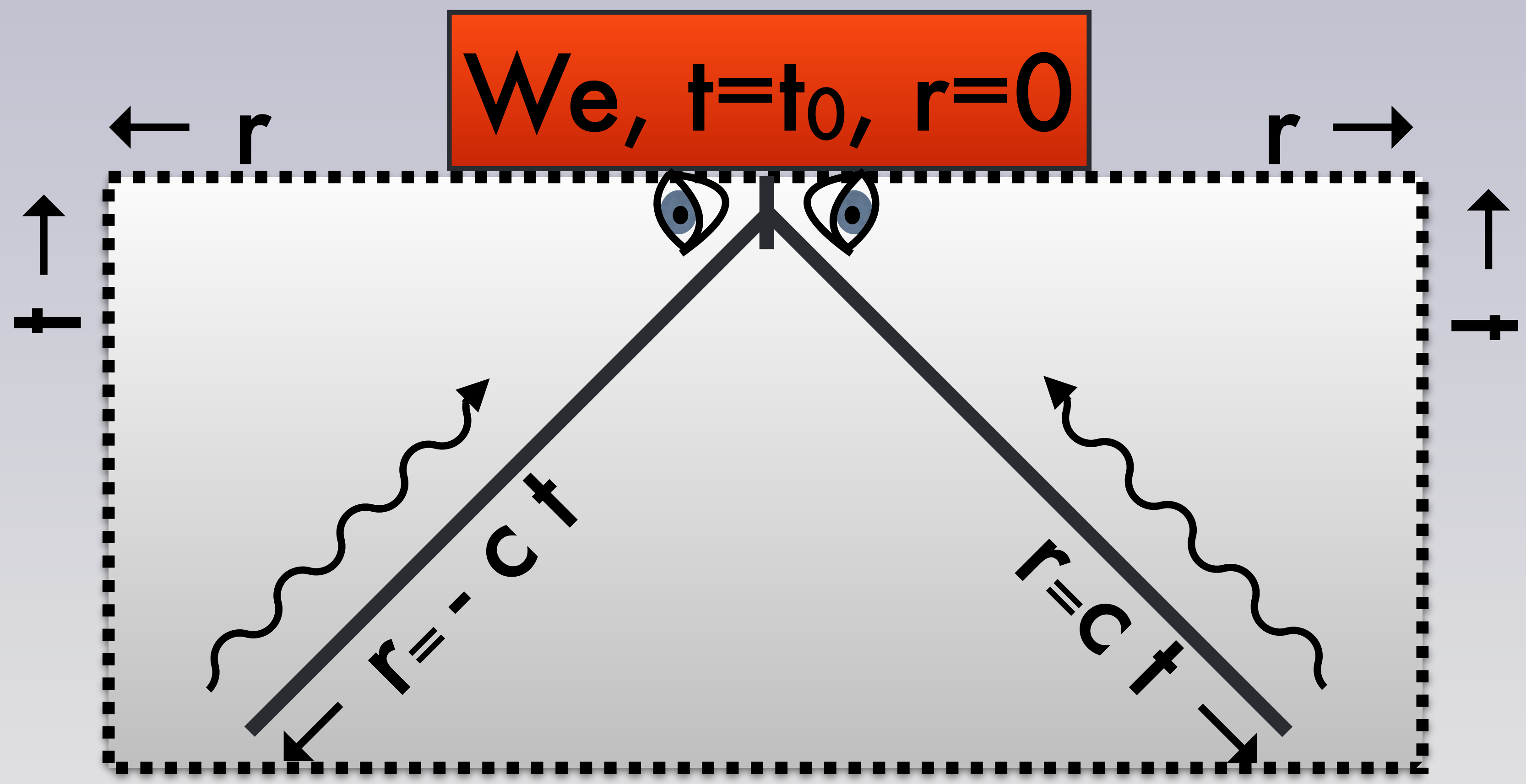
# Summary of previous lecture

- Intensity, wavelength, direction: 2 angles
- Light in vacuum has fixed velocity: c =1
- (3+1)D space time [t,r,φ,ϑ], t&r constrained by r = c t.



# Summary of previous lecture

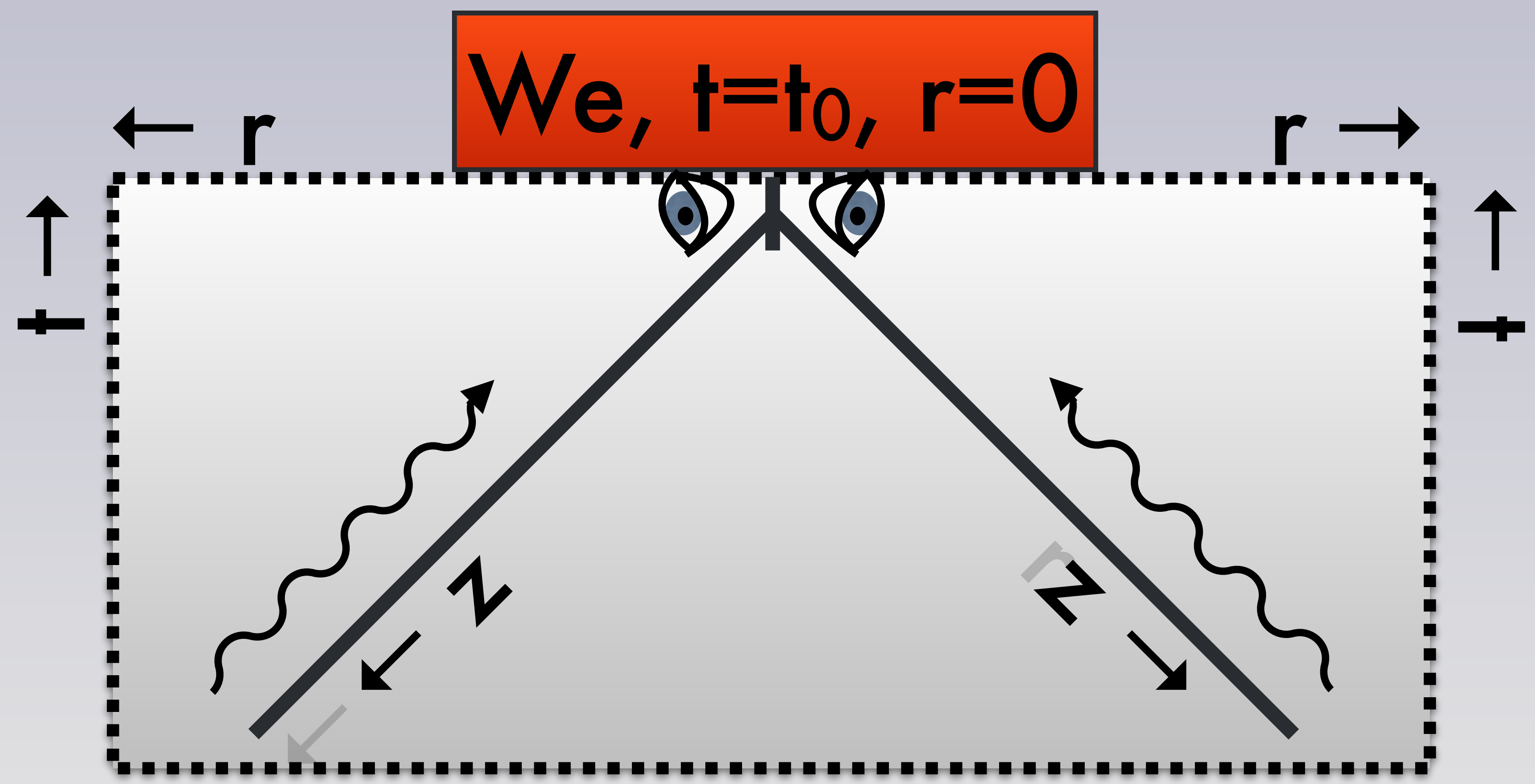
- Intensity, wavelength, direction: 2 angles
- Light in vacuum has fixed velocity: $c = 1$
- (3+1)D space time $[t, r, \varphi, \vartheta]$, t&r constrained by $r = c\ t$.



# Summary of previous lecture

- Intensity, wavelength, direction: 2 angles
- Light in vacuum has fixed velocity: c =1
- (3+1)

$$z \equiv \frac{\lambda_{\rm obs}}{\lambda_{\rm em}} - 1 \simeq \frac{\Delta v}{c} \simeq \frac{a(t_{\rm obs})}{a(t_{\rm em})} - 1$$



# $d_L$ - Luminosity distance

$$
\begin{aligned}
d_L &= \frac{d_A}{a(t)^2} = (1+z)^2 d_A \\
d_A(z) &= a(t) r \\
ds^2 &= 0 \;\rightarrow\; dt = a(t) dr \\
\int dr &= \int dt/a(t) \\
r &= \int dt/a(t) = \int da/(a\dot{a}) \\
&= \int da/(a^2 H(z)) = \int dz/H(z) \\
H_0 d_A(z) &= \frac{1}{1+z} \int \frac{dz}{\sqrt{\Omega_m (1+z)^3 + \Omega_r (1+z)^4 + \Omega_k (1+z)^2 + \Omega_\Lambda}}
\end{aligned}
$$

## $d_A$ - Angular-diameter distance

$$
\begin{aligned}
d_L &= \frac{d_A}{a(t)^2} = (1+z)^2 d_A \\
d_A(z) &= a(t) r \\
ds^2 &= 0 \;\rightarrow\; dt = a(t)dr \\
\int dr &= \int dt/a(t) \\
r &= \int dt/a(t) = \int da/(a\dot{a}) \\
&= \int da/(a^2 H(z)) = \int dz/H(z) \\
H_0 d_A(z) &= \frac{1}{1+z} \int \frac{dz}{\sqrt{\Omega_m(1+z)^3 + \Omega_r(1+z)^4 + \Omega_k(1+z)^2 + \Omega_\Lambda}}
\end{aligned}
$$

$$
\begin{aligned}
d_L &= \frac{d_A}{a(t)^2} = (1+z)^2 d_A \\
d_A(z) &= a(t) r \\
ds^2 &= 0 \;\rightarrow\; dt = a(t)dr \\
\int dr &= \int dt/a(t) \\
r &= \int dt/a(t) = \int da/(a\dot{a}) \\
&= \int da/(a^2 H(z)) = \int dz/H(z) \\
H_0 d_A(z) &= \frac{1}{1+z} \int \frac{dz}{\sqrt{\Omega_m(1+z)^3 + \Omega_r(1+z)^4 + \Omega_k(1+z)^2 + \Omega_\Lambda}}
\end{aligned}
$$

a = 1/(1+z)
in a truly
homogeneous universe

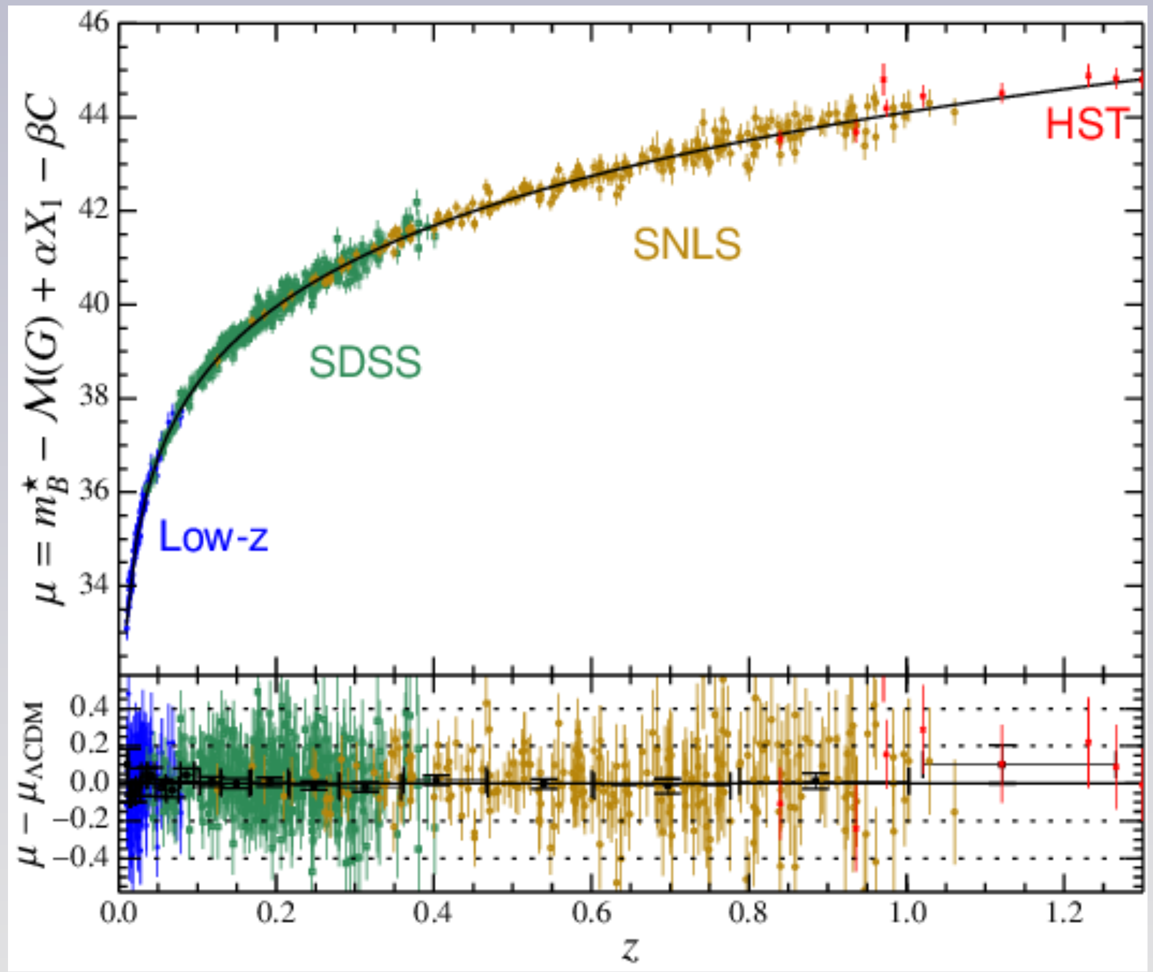
z - Redshift

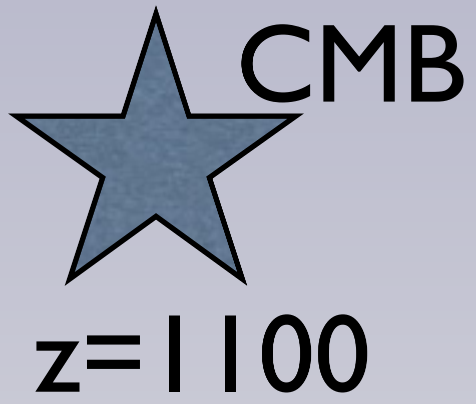
$$
\begin{aligned}
d_L &= \frac{d_A}{a(t)^2} = (1+z)^2 d_A \\
d_A(z) &= a(t) r \\
ds^2 &= 0 \;\rightarrow\; dt = a(t) dr \\
\int dr &= \int dt/a(t) \\
r &= \int dt/a(t) = \int da/(a\dot{a}) \\
&= \int da/(a^2 H(z)) = \int dz/H(z) \\
H_0 d_A(z) &= \frac{1}{1+z} \int \frac{dz}{\sqrt{\Omega_m (1+z)^3 + \Omega_r (1+z)^4 + \Omega_k (1+z)^2 + \Omega_\Lambda}}
\end{aligned}
$$

H(t)=H(a(t))=H(z(a(t)))=ȧ/a
Hubble rate

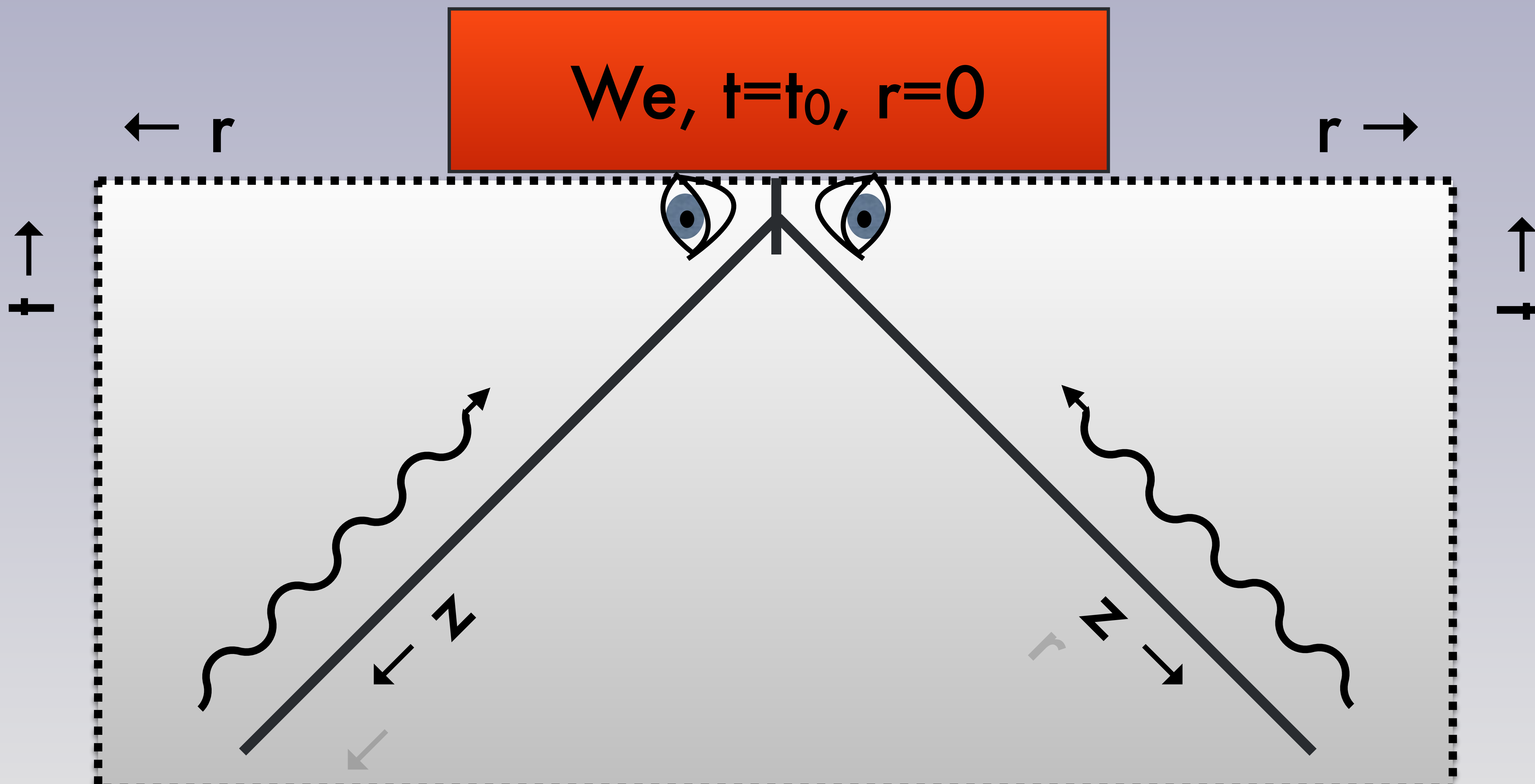
# CMB as a distance measure

- assume for the moment that you know the physical distance of this characteristic scale 1°

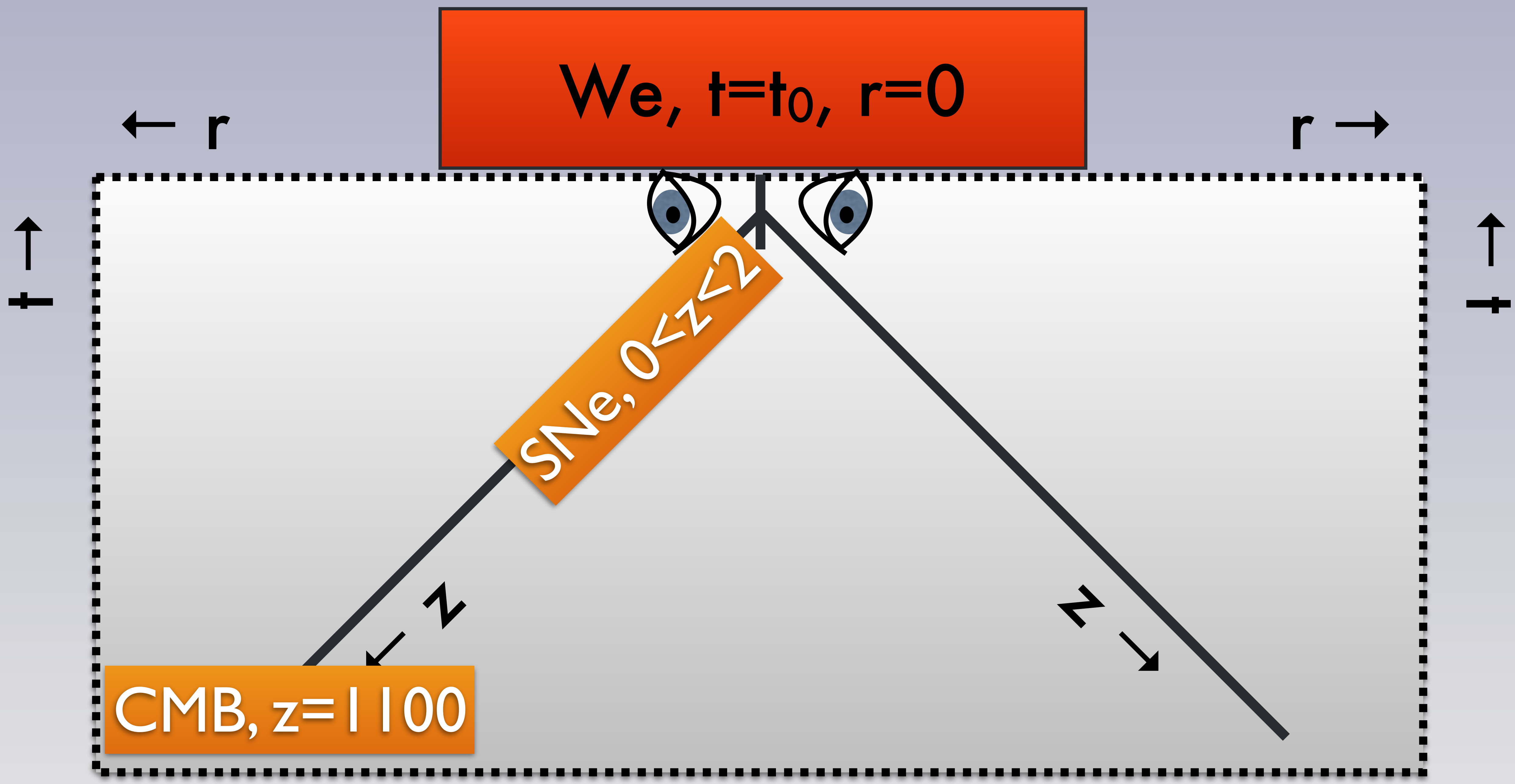


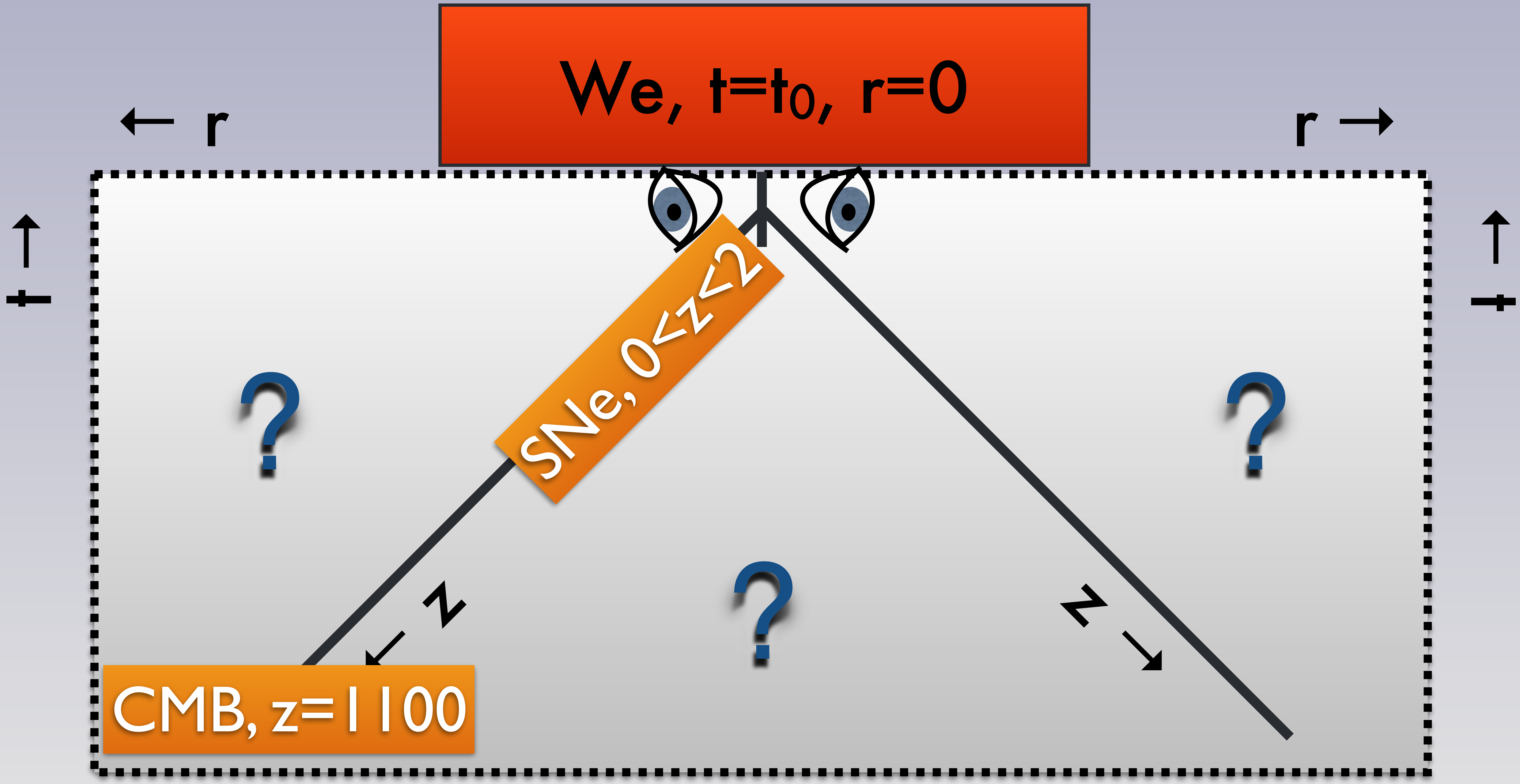
# Our light cone



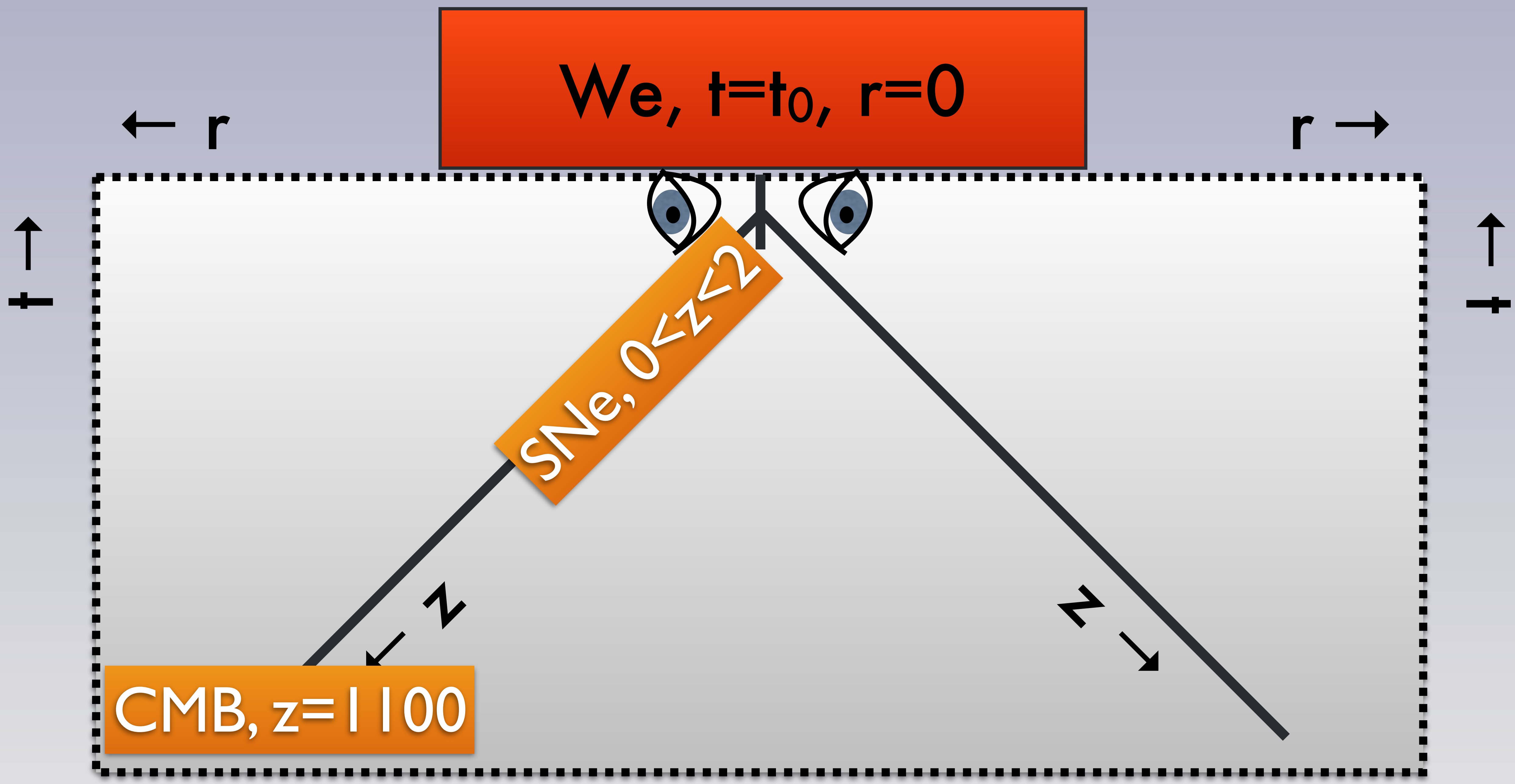
# Our light cone

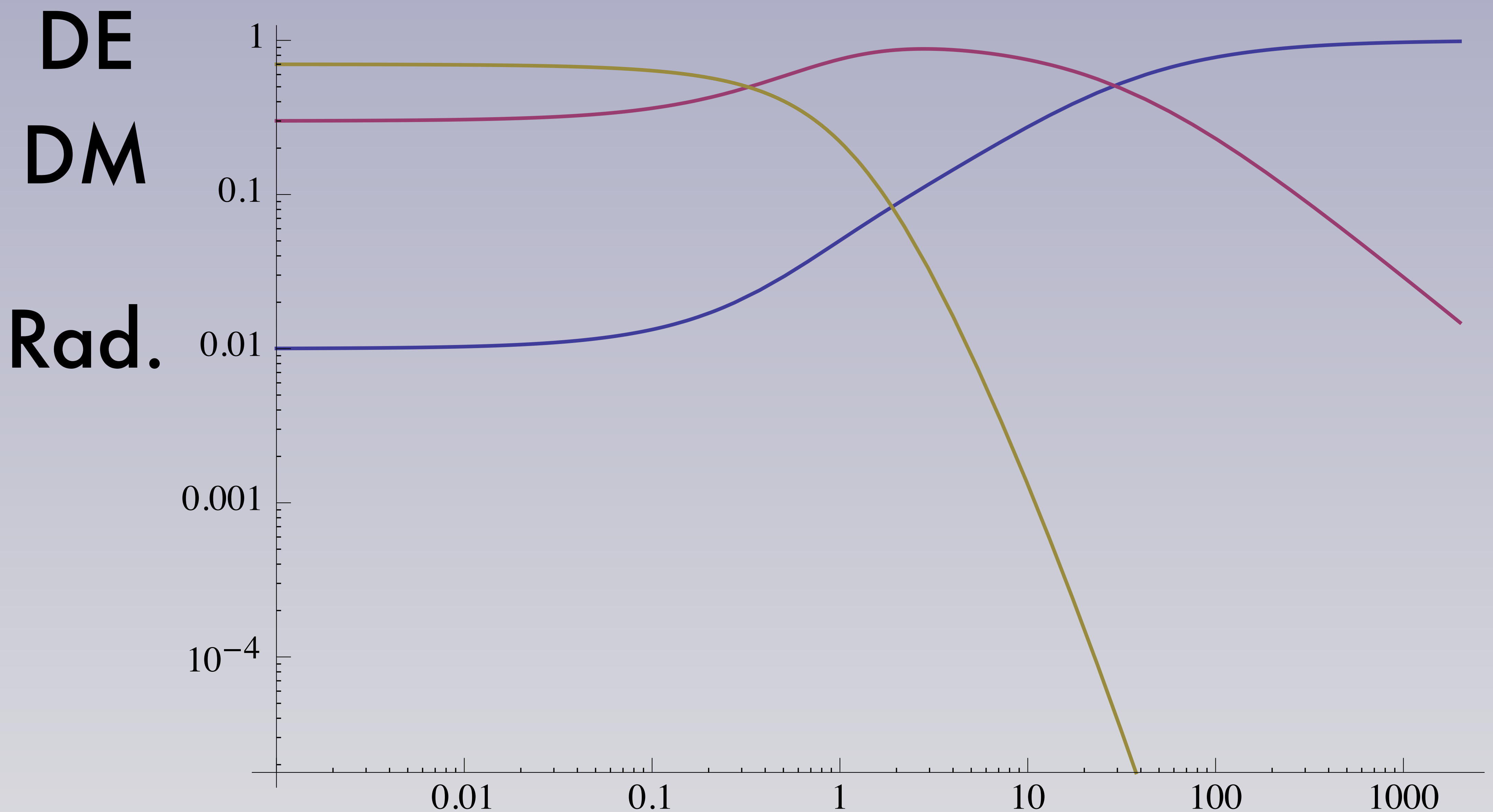


# Our light cone



# Our light cone





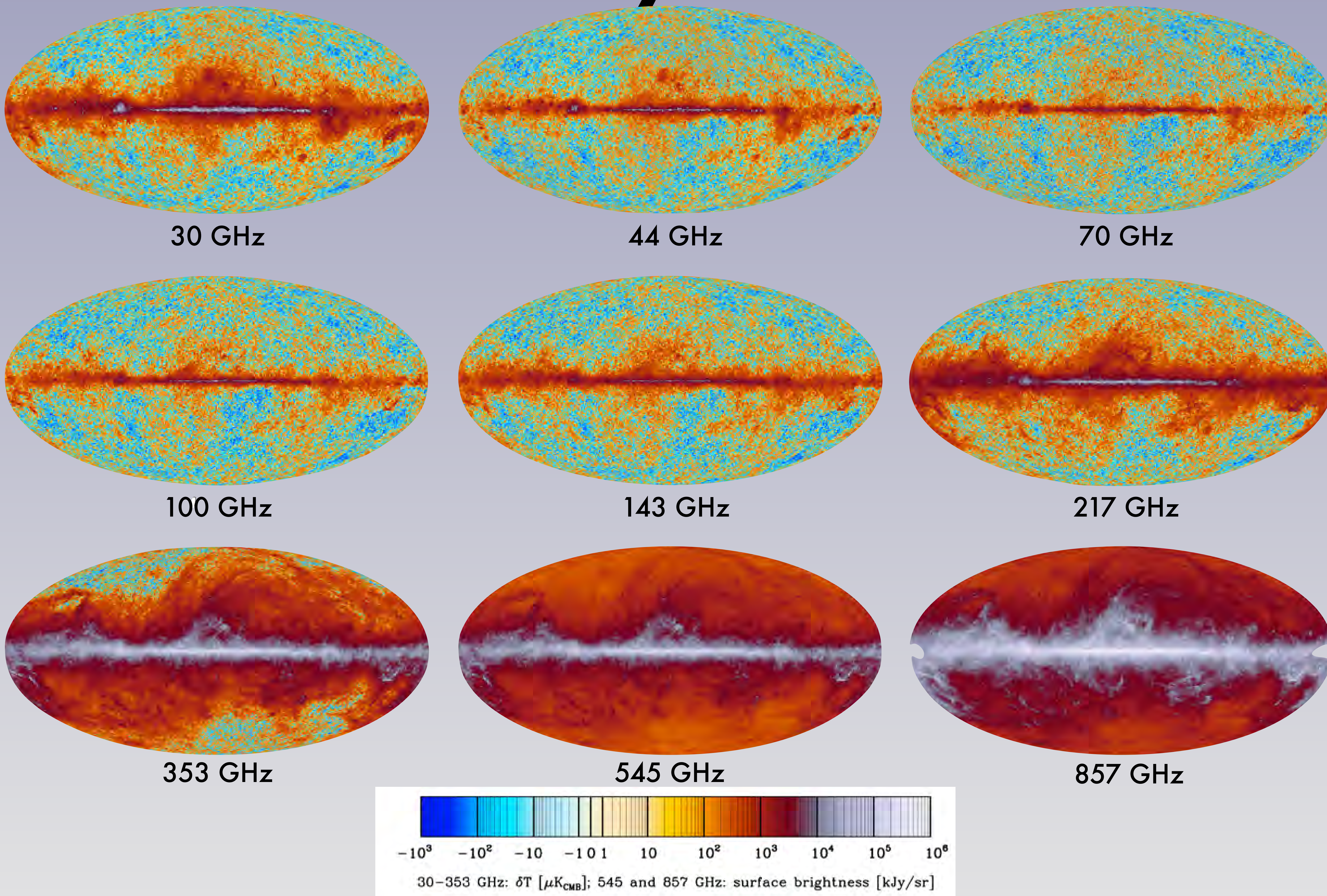
DE
DM
Rad.
1
0.1
0.01
0.001
$10^{-4}$
0.01
0.1
1
10
100
1000

# Today

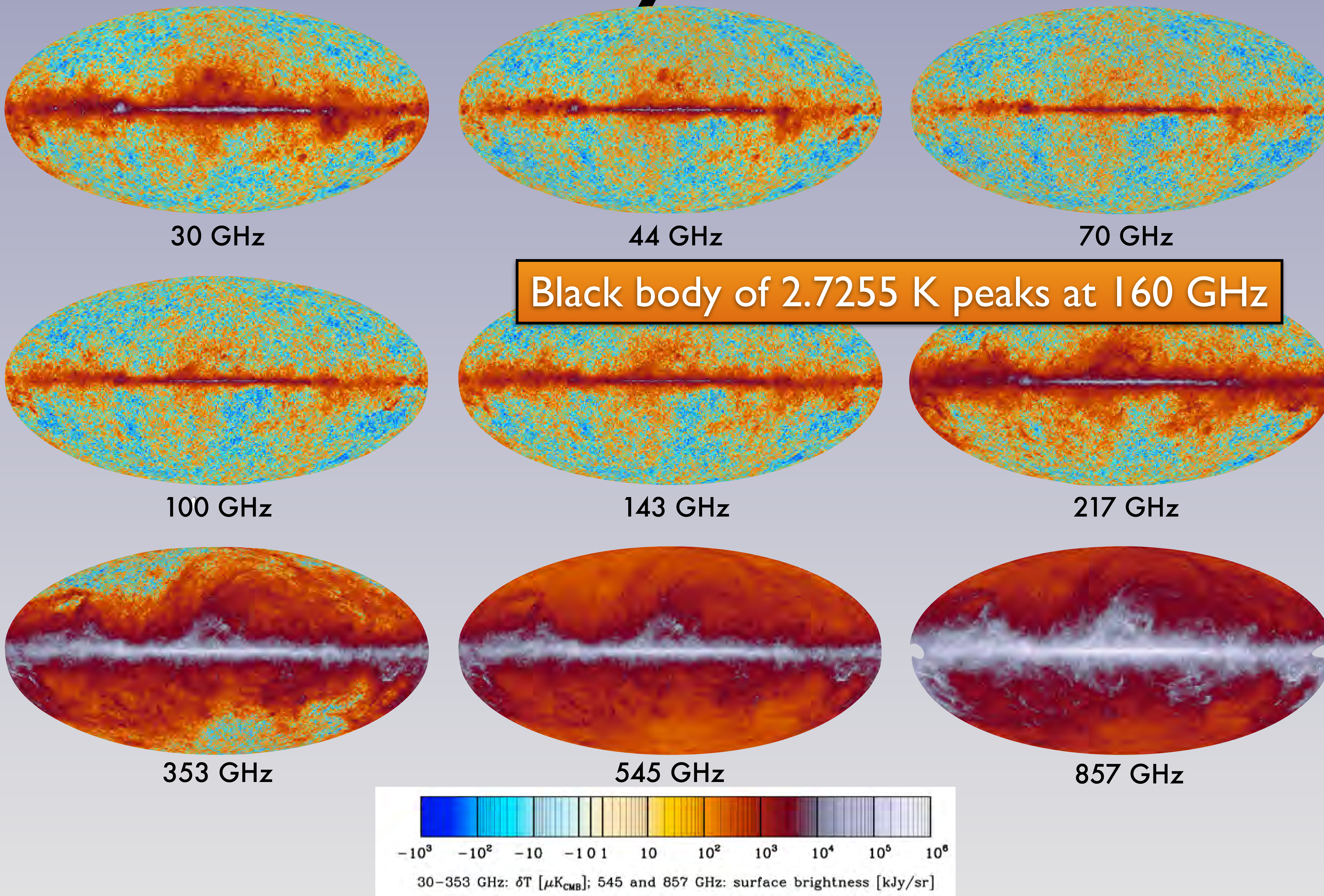
✓ Summary of previous.

- Correlation functions
- Perturbation theory of CMB
- CMB lensing
- Cosmological parameters in CMB
- Experimental constraints

# CMB by Planck

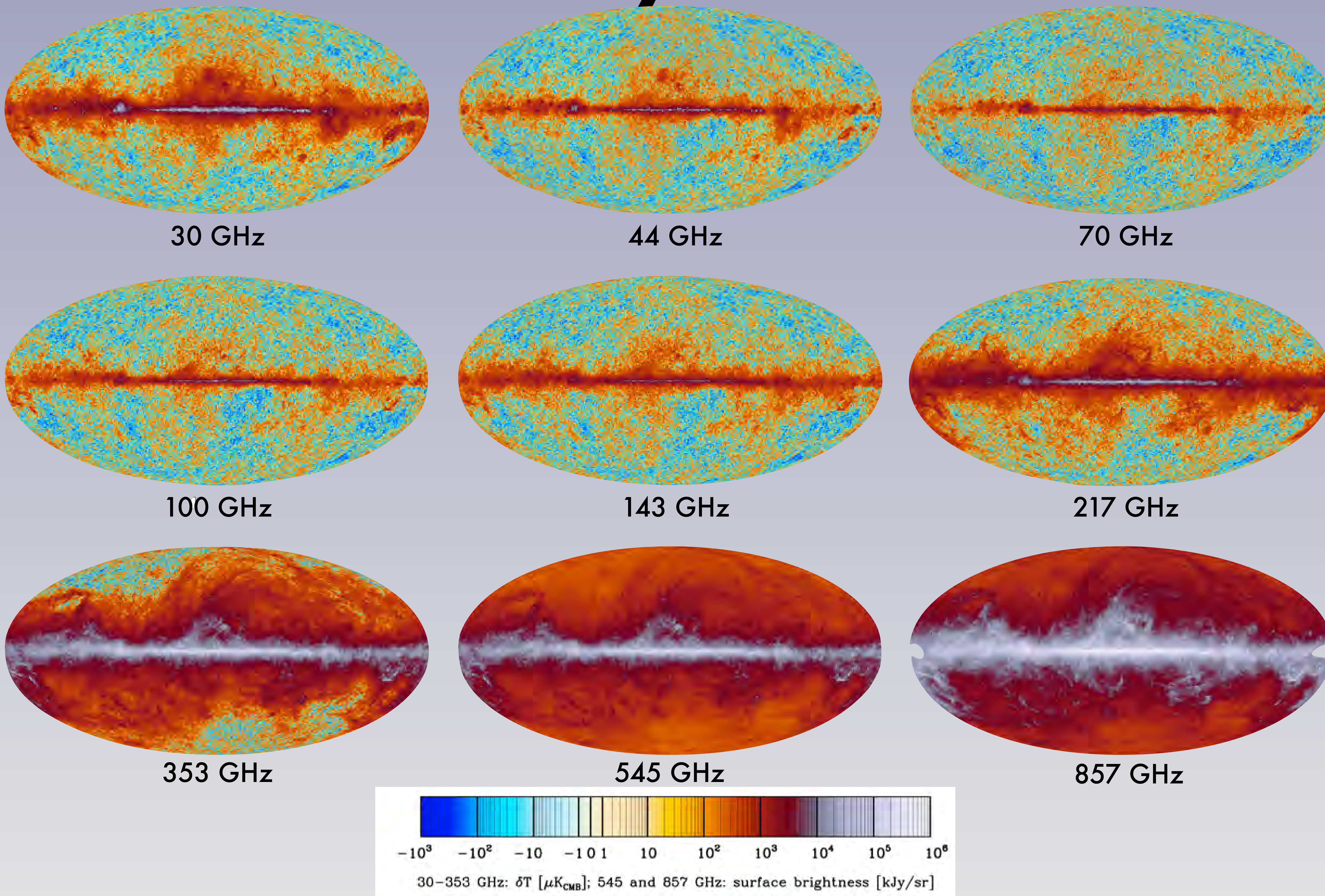


# CMB by Planck

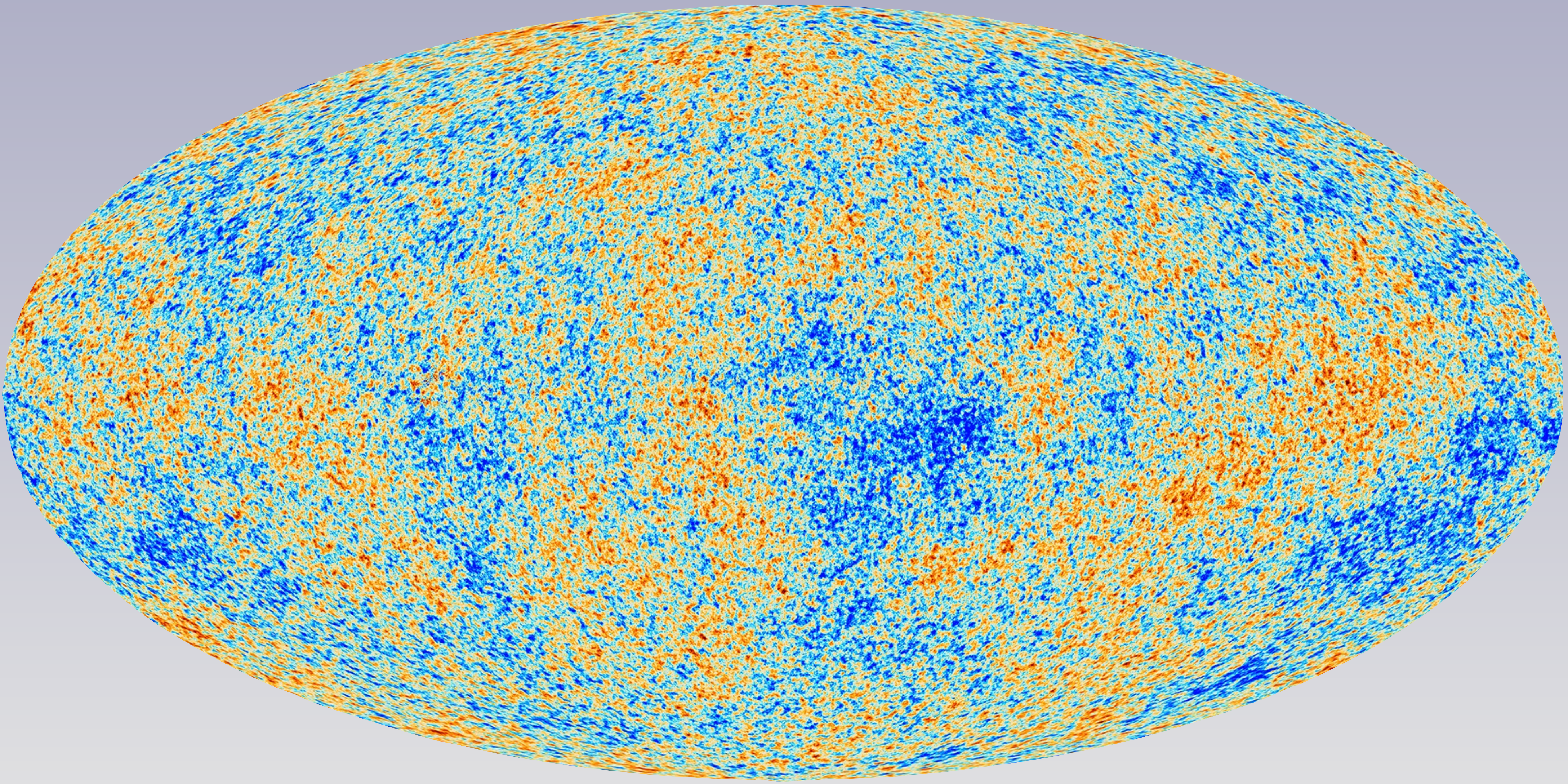



Black body of 2.7255 K peaks at 160 GHz

# CMB by Planck



# CMB by Planck



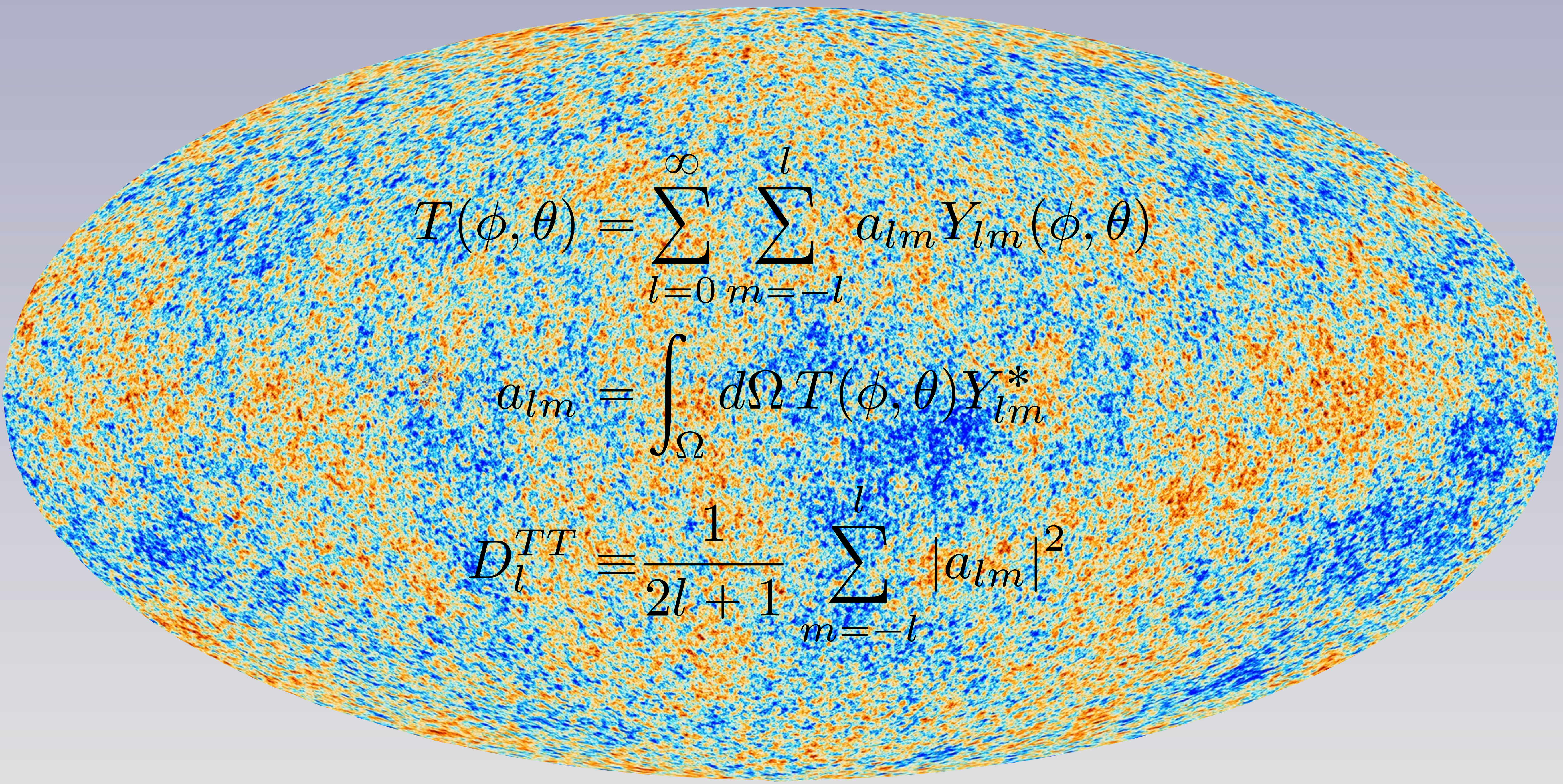
# Spherical harmonics

$$T(\phi,\theta) = \sum_{l=0}^{\infty}\sum_{m=-l}^{l} a_{lm} Y_{lm}(\phi,\theta)$$

$$a_{lm} = \int_\Omega d\Omega\, T(\phi,\theta) Y_{lm}^*$$

$$D_l^{TT} \equiv \frac{1}{2l+1}\sum_{m=-l}^{l} |a_{lm}|^2$$

# Spherical harmonics

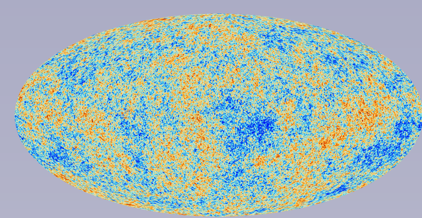


$$T(\phi,\theta) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} a_{lm} Y_{lm}(\phi,\theta)$$

$$a_{lm} = \int_{\Omega} d\Omega \, T(\phi,\theta) Y_{lm}^{*}$$

$$D_l^{TT} \equiv \frac{1}{2l+1} \sum_{m=-l}^{l} |a_{lm}|^2$$

# Spherical harmonics



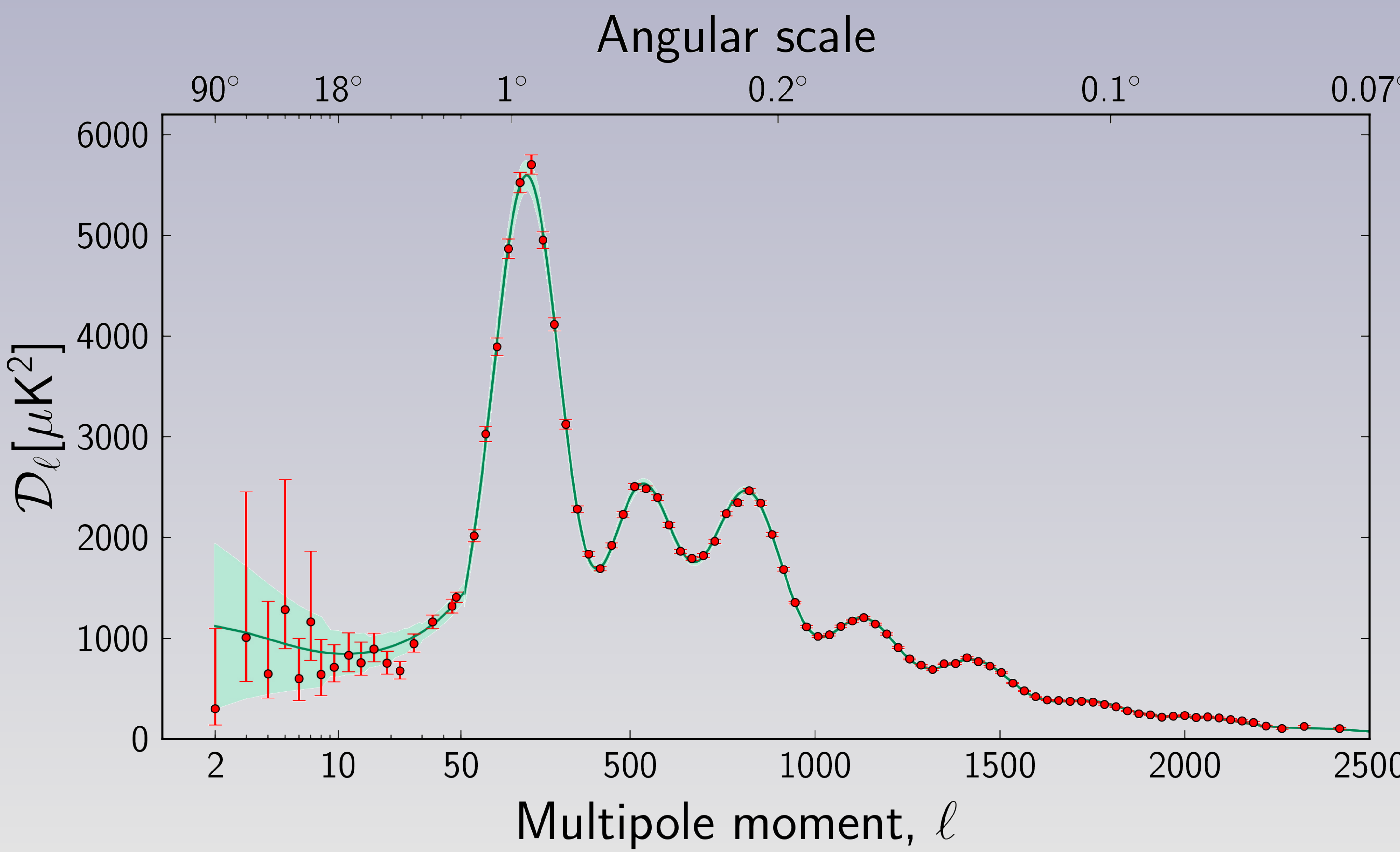
+

$$T(\phi,\theta) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} a_{lm} Y_{lm}(\phi,\theta)$$

$$a_{lm} = \int_{\Omega} d\Omega \, T(\phi,\theta) Y_{lm}^{*}$$

$$D_l^{TT} \equiv \frac{1}{2l+1} \sum_{m=-l}^{l} |a_{lm}|^2$$

=



# Correlations?

# Correlations?

- Important to remember meaning of *correlation*

# Correlations?

- Important to remember meaning of *correlation*

**correlation** | ˌkɒrəˈleɪʃ(ə)n, -rɪ- |
noun
a mutual relationship or connection between two or more things: *research showed a clear correlation between recession and levels of property crime* | [ mass noun ] *: there was no correlation between the number of visits to the clinic and the treatment outcome*.
• [ mass noun ] the process of establishing a relationship or connection between two or more things. *the increasingly similar basis underlying national soil maps allows correlation to take place more easily*.
• [ mass noun ] Statistics interdependence of variable quantities.
• Statistics a quantity measuring the extent of the interdependence of variable quantities.

# Correlations?

- Important to remember meaning of *correlation*
- Cosmological problem: we measure only one universe, one realization
- Fundamentally impossible to measure correlations in cosmological quantities

# Correlations?

# Correlations?

- Definition of correlation of variables:

$$\langle ab \rangle \equiv \lim_{N \to \infty} \frac{1}{N} \sum_{i=0}^{N} a_i b_i$$

# Correlations?

- Definition of correlation of variables:

$$\langle ab \rangle \equiv \lim_{N \to \infty} \frac{1}{N} \sum_{i=0}^{N} a_i b_i$$

- Application to CMB temperature

$$\langle T(\phi_1, \theta_1) T(\phi_2, \theta_2) \rangle \equiv \lim_{N \to \infty} \frac{1}{N} \sum_{i=0}^{N} T_i(\phi_1, \theta_1) T_i(\phi_2, \theta_2)$$

# Correlations?

- Definition of correlation of variables:
$$\langle ab \rangle \equiv \lim_{N \to \infty} \frac{1}{N} \sum_{i=0}^{N} a_i b_i$$
- Application to CMB temperature
$$\langle T(\phi_1, \theta_1) T(\phi_2, \theta_2) \rangle \equiv \lim_{N \to \infty} \frac{1}{N} \sum_{i=0}^{N} T_i(\phi_1, \theta_1) T_i(\phi_2, \theta_2)$$
- But, for one human lifetime
T(φ,ϑ) is always T(φ,ϑ,r(z=1100),t(z=1100)), always same remote spot in universe.

# Correlations?

- Definition of correlation of variables:

$$\langle ab \rangle \equiv \lim_{N\to\infty} \frac{1}{N} \sum_{i=0}^{N} a_i b_i$$

- Application to CMB temperature

$$\langle T(\phi_1,\theta_1)T(\phi_2,\theta_2)\rangle \equiv \lim_{N\to\infty} \frac{1}{N} \sum_{i=0}^{N} T_i(\phi_1,\theta_1)T_i(\phi_2,\theta_2)$$

- But, for one human lifetime
  T(φ,ϑ) is always T(φ,ϑ,r(z=1100),t(z=1100)), always same remote spot in universe.

- We are stuck at i=0 in the sum that gives the correlation...

# Correlations?

- No, we do not see correlations in the CMB.
- We see a spectrum of anisotropies.
- So what is the relation to correlations?

# Correlations

$$T(\phi,\theta) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} a_{lm} Y_{lm}(\phi,\theta)$$

$$a_{lm} = \int_{\Omega} d\Omega\, T(\phi,\theta) Y_{lm}^{*}$$

$$D_l^{TT} \equiv \frac{1}{2l+1} \sum_{m=-l}^{l} |a_{lm}|^2$$

# Correlations

$$T(\phi,\theta) = \sum_{l=0}^{\infty}\sum_{m=-l}^{l} a_{lm} Y_{lm}(\phi,\theta)$$

$$a_{lm} = \int_\Omega d\Omega\, T(\phi,\theta) Y^*_{lm}$$

$$D_l^{TT} \equiv \frac{1}{2l+1}\sum_{m=-l}^{l} |a_{lm}|^2$$

$$\langle T(\phi_1,\theta_1) T^*(\phi_2,\theta_2)\rangle \equiv \lim_{N\to\infty}\frac{1}{N}\sum_{i=0}^{N} T_i(\phi_1,\theta_1) T_i(\phi_2,\theta_2)$$

$$= \sum_{l=0}^{\infty}\sum_{m=-l}^{l}\sum_{l'=0}^{\infty}\sum_{m'=-l'}^{l'} \langle a_{lm} a^*_{l'm'}\rangle\, Y_{lm}(\phi_1,\theta_1) Y^*_{l'm'}(\phi_2,\theta_2)$$

# Correlations

$$\langle T(\phi_1,\theta_1)T^*(\phi_2,\theta_2)\rangle \equiv \lim_{N\to\infty}\frac{1}{N}\sum_{i=0}^{N}T_i(\phi_1,\theta_1)T_i(\phi_2,\theta_2)$$

$$=\sum_{l=0}^{\infty}\sum_{m=-l}^{l}\sum_{l'=0}^{\infty}\sum_{m'=-l'}^{l'}\langle a_{lm}a^*_{l'm'}\rangle\, Y_{lm}(\phi_1,\theta_1)Y^*_{l'm'}(\phi_2,\theta_2)$$

For a gaussian random field:

$$\langle a_{lm}a^*_{l'm'}\rangle \equiv C_l\delta_{ll'}\delta_{mm'}$$

# Correlations

$$
\begin{aligned}
\langle T(\phi_1,\theta_1)T^*(\phi_2,\theta_2)\rangle &\equiv \lim_{N\to\infty}\frac{1}{N}\sum_{i=0}^{N} T_i(\phi_1,\theta_1)T_i(\phi_2,\theta_2) \\
&= \sum_{l=0}^{\infty}\sum_{m=-l}^{l}\sum_{l'=0}^{\infty}\sum_{m'=-l'}^{l'} \langle a_{lm}a^*_{l'm'}\rangle\, Y_{lm}(\phi_1,\theta_1)Y^*_{l'm'}(\phi_2,\theta_2) \\
&= \sum_{l=0}^{\infty}\sum_{m=-l}^{l}\sum_{l'=0}^{\infty}\sum_{m'=-l'}^{l'} C_l\delta_{ll'}\delta_{mm'}Y_{lm}(\phi_1,\theta_1)Y^*_{l'm'}(\phi_2,\theta_2) \\
&= \sum_{l=0}^{\infty} C_l \sum_{m=-l}^{l} Y_{lm}(\phi_1,\theta_1)Y^*_{lm}(\phi_2,\theta_2) \\
&= \sum_{l=0}^{\infty} C_l\,\frac{2l+1}{4\pi}P_l(\hat{n}_1\cdot\hat{n}_2)
\end{aligned}
$$

# Correlations

$$\langle T(\phi_1,\theta_1)T^*(\phi_2,\theta_2)\rangle = \sum_{l=0}^{\infty} C_l \,\frac{2l+1}{4\pi} P_l(\hat{n}_1\cdot\hat{n}_2)$$

For a gaussian random field:

$$\langle a_{lm}a^*_{l'm'}\rangle \equiv C_l\delta_{ll'}\delta_{mm'}$$

independent of $m$!

So, $D_l^{TT} \equiv \frac{1}{2l+1}\sum_{m=-l}^{l}|a_{lm}|^2$ measures $2l+1$ independent random variables, drawn from the same gaussian distribution with variance $C_l$.

# Correlations

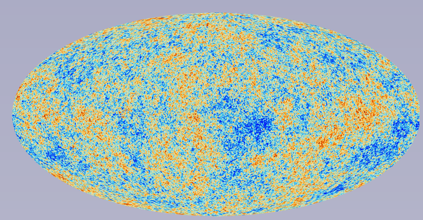
$$\langle T(\phi_1,\theta_1)T^*(\phi_2,\theta_2)\rangle = \sum_{l=0}^{\infty} C_l \, \frac{2l+1}{4\pi} P_l(\hat{n}_1 \cdot \hat{n}_2)$$

So, $D_l^{TT} \equiv \frac{1}{2l+1} \sum_{m=-l}^{l} |a_{lm}|^2$ measures $2l+1$ independent random variables, drawn from the same gaussian distribution with variance $C_l$.

Often heard: $D_l$ called *pseudo* $C_l$.

Large $l$: $D_l$ gives a fair measurement of $C_l$.
Small $l$: $D_l$ has can have large deviation from $C_l$.

# Spherical harmonics



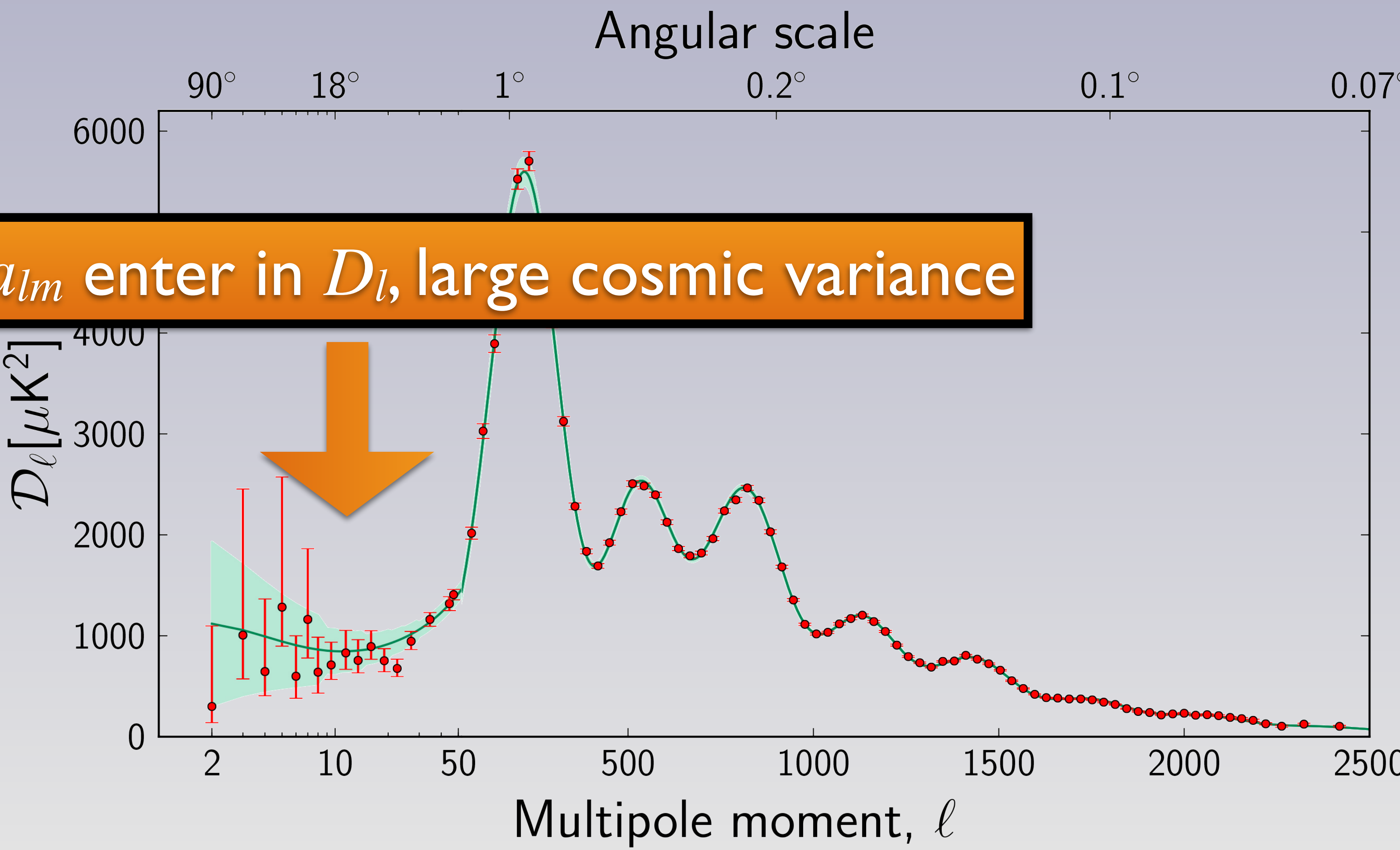
+

$$T(\phi,\theta) = \sum_{l=0}^{\infty}\sum_{m=-l}^{l} a_{lm} Y_{lm}(\phi,\theta)$$

$$a_{lm} = \int_{\Omega} d\Omega\, T(\phi,\theta) Y^{*}_{lm}$$

$$D_l^{TT} \equiv \frac{1}{2l+1}\sum_{m=-l}^{l} |a_{lm}|^2$$

=




Few $a_{lm}$ enter in $D_l$, large cosmic variance

# Keep in mind

- Fundamentally, all $D_l$ have cosmic variance with respect to $C_l$
- Variance scales as $\sqrt{l}$
- Planck satellite:
  - cosmic variance limited $2 < l < 1000$
  - instrument limited $1000 \leq l$

# Perturbation Theory

$$\frac{\Delta T(\mathbf{n})}{T} = \left[\frac{1}{4}D_g^{(r)} + V_j^{(b)}n^j + \Psi - \Phi\right](\eta_{dec}, \mathbf{x}_{dec}) + \int_{\eta_{dec}}^{\eta_0} (\dot{\Psi} - \dot{\Phi})(\eta, \mathbf{x}(\eta))d\eta$$

see [Durrer, astro-ph/0109522] or
[text book “The Cosmic Microwave Background” by Durrer]

# Perturbation Theory

$$\frac{\Delta T(\mathbf{n})}{T} = \left[\frac{1}{4}D_g^{(r)} + V_j^{(b)}n^j + \Psi - \Phi\right](\eta_{dec}, \mathbf{x}_{dec}) + \int_{\eta_{dec}}^{\eta_0} (\dot{\Psi} - \dot{\Phi})(\eta, \mathbf{x}(\eta))d\eta$$

Photon density

see [Durrer, astro-ph/0109522] or
[text book "The Cosmic Microwave Background" by Durrer]

# Perturbation Theory

$$\frac{\Delta T(\mathbf{n})}{T} = \left[\frac{1}{4}D_g^{(r)} + V_j^{(b)}n^j + \Psi - \Phi\right](\eta_{dec}, \mathbf{x}_{dec}) + \int_{\eta_{dec}}^{\eta_0} (\dot{\Psi} - \dot{\Phi})(\eta, \mathbf{x}(\eta))d\eta$$

Photon density

Velocity (Doppler)
*negligible*

see [Durrer, astro-ph/0109522] or
[text book "The Cosmic Microwave Background" by Durrer]

# Perturbation Theory

$$\frac{\Delta T(\mathbf{n})}{T} = \left[\frac{1}{4}D_g^{(r)} + V_j^{(b)}n^j + \Psi - \Phi\right](\eta_{dec}, \mathbf{x}_{dec}) + \int_{\eta_{dec}}^{\eta_0} (\dot{\Psi} - \dot{\Phi})(\eta, \mathbf{x}(\eta))d\eta$$

Photon density

Gravitational potential

Velocity (Doppler)
*negligible*

see [Durrer, astro-ph/0109522] or
[text book "The Cosmic Microwave Background" by Durrer]

# Perturbation Theory

$$\frac{\Delta T(\mathbf{n})}{T} = \left[\frac{1}{4}D_g^{(r)} + V_j^{(b)} n^j + \Psi - \Phi\right](\eta_{dec}, \mathbf{x}_{dec}) + \int_{\eta_{dec}}^{\eta_0} (\dot{\Psi} - \dot{\Phi})(\eta, \mathbf{x}(\eta)) d\eta$$

Photon density

Velocity (Doppler)
*negligible*

Gravitational potential

Time change of gravitational potential along the photon path

see [Durrer, astro-ph/0109522] or
[text book "The Cosmic Microwave Background" by Durrer]

$$\frac{\Delta T(\mathbf{n})}{T} = \left[\frac{1}{4}D_g^{(r)} + V_j^{(b)}n^j + \Psi - \Phi\right](\eta_{dec}, \mathbf{x}_{dec}) + \int_{\eta_{dec}}^{\eta_0}(\dot{\Psi} - \dot{\Phi})(\eta, \mathbf{x}(\eta))d\eta$$

# For adiabatic perturbations:

$$D_g^{(r)}(k,\eta) = -20/3\Psi(k,\eta)$$

$$\left(\frac{\Delta T(\mathbf{n})}{T}\right)^{(OSW)}_{\text{adiabatic}} = \frac{1}{3}\Psi(\eta_{dec}, \mathbf{x}_{dec})$$

see [Durrer, astro-ph/0109522] or
[text book “The Cosmic Microwave Background” by Durrer]

Ordinary Sachs-Wolfe effect Integrated Sachs-Wolfe effect

$$\frac{\Delta T(\mathbf{n})}{T} = \left[\frac{1}{4}D_g^{(r)} + V_j^{(b)}n^j + \Psi - \Phi\right](\eta_{dec}, \mathbf{x}_{dec}) + \int_{\eta_{dec}}^{\eta_0} (\dot{\Psi} - \dot{\Phi})(\eta, \mathbf{x}(\eta))d\eta$$

## For adiabatic perturbations:

$$D_g^{(r)}(k,\eta) = -20/3\Psi(k,\eta)$$

$$\left(\frac{\Delta T(\mathbf{n})}{T}\right)^{(OSW)}_{\text{adiabatic}} = \frac{1}{3}\Psi(\eta_{dec}, \mathbf{x}_{dec})$$

see [Durrer, astro-ph/0109522] or
[text book “The Cosmic Microwave Background” by Durrer]

$$\frac{\Delta T(\mathbf{n})}{T} = \left[\frac{1}{4}D_g^{(r)} + V_j^{(b)}n^j + \Psi - \Phi\right](\eta_{dec}, \mathbf{x}_{dec}) + \int_{\eta_{dec}}^{\eta_0} (\dot{\Psi} - \dot{\Phi})(\eta, \mathbf{x}(\eta))d\eta$$

**Gravity wins!**
**Hot = underdense**
**Cold=overdense**

$$\left(\frac{\Delta T(\mathbf{n})}{T}\right)^{(OSW)}_{\text{adiabatic}} = \frac{1}{3}\Psi(\eta_{dec}, \mathbf{x}_{dec})$$

see [Durrer, astro-ph/0109522] or
[text book “The Cosmic Microwave Background” by Durrer]

Ordinary Sachs-Wolfe effect

Integrated Sachs-Wolfe effect

$$\frac{\Delta T(\mathbf{n})}{T} = \left[\frac{1}{4}D_g^{(r)} + V_j^{(b)}n^j + \Psi - \Phi\right](\eta_{dec}, \mathbf{x}_{dec}) + \int_{\eta_{dec}}^{\eta_0} (\dot{\Psi} - \dot{\Phi})(\eta, \mathbf{x}(\eta))d\eta$$

**Gravity wins!**

**Hot = underdense**

**Cold=overdense**

$$\left(\frac{\Delta T(\mathbf{n})}{T}\right)^{(OSW)}_{\text{adiabatic}} = \frac{1}{3}\Psi(\eta_{dec}, \mathbf{x}_{dec})$$

see [Durrer, astro-ph/0109522] or
[text book "The Cosmic Microwave Background" by Durrer]

Einstein equations

$$\frac{\Delta T(\mathbf{n})}{T} = \left[\frac{1}{4}D_g^{(r)} + V_j^{(b)}n^j + \Psi - \Phi\right](\eta_{dec}, \mathbf{x}_{dec}) + \int_{\eta_{dec}}^{\eta_0} (\dot{\Psi} - \dot{\Phi})(\eta, \mathbf{x}(\eta))d\eta$$

$$\begin{aligned} 4\pi G a^2 \rho D &= (k^2 - 3\kappa)\Phi & (00) \\ 4\pi G a^2 (\rho + p) V &= k\left(\left(\tfrac{\dot{a}}{a}\right)\Psi - \dot{\Phi}\right) & (0i) \end{aligned}$$

$$-k^2(\Phi + \Psi) = 8\pi G a^2 p \Pi^{(S)} \qquad \text{(ii)}$$

$\Pi$: scalar part of stress tensor perturbation projected on synchronous comoving spatial slices

see [Durrer, astro-ph/0109522] or
[text book "The Cosmic Microwave Background" by Durrer]

Ordinary Sachs-Wolfe effect

Integrated Sachs-Wolfe effect

$$\frac{\Delta T(\mathbf{n})}{T} = \left[\frac{1}{4}D_g^{(r)} + V_j^{(b)}n^j + \Psi - \Phi\right](\eta_{dec}, \mathbf{x}_{dec}) + \int_{\eta_{dec}}^{\eta_0} (\dot{\Psi} - \dot{\Phi})(\eta, \mathbf{x}(\eta))d\eta$$

Einstein equations

$$\begin{aligned} 4\pi G a^2 \rho D &= (k^2 - 3\kappa)\Phi && (00) \\ 4\pi G a^2 (\rho + p) V &= k\left(\left(\frac{\dot{a}}{a}\right)\Psi - \dot{\Phi}\right) && (0i) \end{aligned}$$

$$-k^2(\Phi + \Psi) = 8\pi G a^2 p \Pi^{(S)} \qquad \text{(ii)}$$

$\Pi$: scalar part of stress tensor perturbation projected on synchronous comoving spatial slices

see [Durrer, astro-ph/0109522] or
[text book "The Cosmic Microwave Background" by Durrer]

$$\frac{\Delta T(\mathbf{n})}{T} = \left[\frac{1}{4}D_g^{(r)} + V_j^{(b)}n^j + \Psi - \Phi\right](\eta_{dec}, \mathbf{x}_{dec}) + \int_{\eta_{dec}}^{\eta_0} (\dot{\Psi} - \dot{\Phi})(\eta, \mathbf{x}(\eta))d\eta$$

$$\begin{aligned} 4\pi G a^2 \rho D &= (k^2 - 3\kappa)\Phi && (00) \\ 4\pi G a^2 (\rho + p) V &= k\left(\left(\tfrac{\dot{a}}{a}\right)\Psi - \dot{\Phi}\right) && (0i) \end{aligned}$$

$$-k^2(\Phi + \Psi) = 8\pi G a^2 p \Pi^{(S)}$$

# Conservation equations

$$\dot{D}_g + 3(c_s^2 - w)\left(\tfrac{\dot{a}}{a}\right)D_g + (1+w)kV + 3w\left(\tfrac{\dot{a}}{a}\right)\Gamma = 0$$

$$\dot{V} + \left(\tfrac{\dot{a}}{a}\right)(1 - 3c_s^2)V = k(\Psi - 3c_s^2\Phi) + \tfrac{c_s^2 k}{1+w}D_g + \tfrac{wk}{1+w}\left[\Gamma - \tfrac{2}{3}\left(1 - \tfrac{3\kappa}{k^2}\right)\Pi\right]$$

see [Durrer, astro-ph/0109522] or
[text book “The Cosmic Microwave Background” by Durrer]

Ordinary Sachs-Wolfe effect

Integrated Sachs-Wolfe effect

$$\frac{\Delta T(\mathbf{n})}{T} = \left[\frac{1}{4}D_g^{(r)} + V_j^{(b)}n^j + \Psi - \Phi\right](\eta_{dec}, \mathbf{x}_{dec}) + \int_{\eta_{dec}}^{\eta_0} (\dot{\Psi} - \dot{\Phi})(\eta, \mathbf{x}(\eta))d\eta$$

$$\begin{aligned} 4\pi G a^2 \rho D &= (k^2 - 3\kappa)\Phi && (00) \\ 4\pi G a^2 (\rho + p) V &= k\left(\left(\frac{\dot{a}}{a}\right)\Psi - \dot{\Phi}\right) && (0i) \end{aligned}$$

$$-k^2(\Phi + \Psi) = 8\pi G a^2 p \Pi^{(S)}$$

Conservation equations

$$\dot{D}_g + 3\left(c_s^2 - w\right)\left(\frac{\dot{a}}{a}\right) D_g + (1 + w)kV + 3w\left(\frac{\dot{a}}{a}\right)\Gamma = 0$$

$$\dot{V} + \left(\frac{\dot{a}}{a}\right)\left(1 - 3c_s^2\right)V = k\left(\Psi - 3c_s^2\Phi\right) + \frac{c_s^2 k}{1+w}D_g + \frac{wk}{1+w}\left[\Gamma - \frac{2}{3}\left(1 - \frac{3\kappa}{k^2}\right)\Pi\right]$$

see [Durrer, astro-ph/0109522] or [text book “The Cosmic Microwave Background” by Durrer]

Conservation equations

$$\frac{\Delta T(\mathbf{n})}{T} = \left[\frac{1}{4}D_g^{(r)} + V_j^{(b)}n^j + \Psi - \Phi\right](\eta_{dec}, \mathbf{x}_{dec}) + \int_{\eta_{dec}}^{\eta_0} (\dot{\Psi} - \dot{\Phi})(\eta, \mathbf{x}(\eta))d\eta$$

$$\begin{aligned} 4\pi G a^2 \rho D &= (k^2 - 3\kappa)\Phi && (00) \\ 4\pi G a^2 (\rho + p) V &= k\left(\left(\tfrac{\dot{a}}{a}\right)\Psi - \dot{\Phi}\right) && (0i) \end{aligned}$$

$$-k^2(\Phi + \Psi) = 8\pi G a^2 p \Pi^{(S)}$$

$$\dot{D}_g + 3\left(c_s^2 - w\right)\left(\tfrac{\dot{a}}{a}\right)D_g + (1+w)kV + 3w\left(\tfrac{\dot{a}}{a}\right)\Gamma = 0$$

$$\dot{V} + \left(\tfrac{\dot{a}}{a}\right)\left(1 - 3c_s^2\right)V = k\left(\Psi - 3c_s^2\Phi\right) + \tfrac{c_s^2 k}{1+w}D_g + \tfrac{wk}{1+w}\left[\Gamma - \tfrac{2}{3}\left(1 - \tfrac{3\kappa}{k^2}\right)\Pi\right]$$

see [Durrer, astro-ph/0109522] or
[text book “The Cosmic Microwave Background” by Durrer]

Ordinary Sachs-Wolfe effect

Integrated Sachs-Wolfe effect

$$\frac{\Delta T(\mathbf{n})}{T} = \left[\frac{1}{4}D_g^{(r)} + V_j^{(b)} n^j + \Psi - \Phi\right](\eta_{dec}, \mathbf{x}_{dec}) + \int_{\eta_{dec}}^{\eta_0} (\dot{\Psi} - \dot{\Phi})(\eta, \mathbf{x}(\eta)) d\eta$$

$$\begin{aligned} 4\pi G a^2 \rho D &= (k^2 - 3\kappa)\Phi && (00) \\ 4\pi G a^2 (\rho + p) V &= k\left(\left(\frac{\dot{a}}{a}\right)\Psi - \dot{\Phi}\right) && (0i) \end{aligned}$$

$$-k^2(\Phi + \Psi) = 8\pi G a^2 p \Pi^{(S)}$$

Conservation equations

$$\dot{D}_g + 3(c_s^2 - w)\left(\frac{\dot{a}}{a}\right) D_g + (1+w)kV + 3w\left(\frac{\dot{a}}{a}\right)\Gamma = 0$$

$$\dot{V} + \left(\frac{\dot{a}}{a}\right)(1 - 3c_s^2) V = k(\Psi - 3c_s^2\Phi) + \frac{c_s^2 k}{1+w} D_g + \frac{wk}{1+w}\left[\Gamma - \frac{2}{3}\left(1 - \frac{3\kappa}{k^2}\right)\Pi\right]$$

see [Durrer, astro-ph/0109522] or [text book “The Cosmic Microwave Background” by Durrer]

$$\frac{\Delta T(\mathbf{n})}{T} = \left[\frac{1}{4}D_g^{(r)} + V_j^{(b)}n^j + \Psi - \Phi\right](\eta_{dec}, \mathbf{x}_{dec}) + \int_{\eta_{dec}}^{\eta_0} (\dot{\Psi} - \dot{\Phi})(\eta, \mathbf{x}(\eta))d\eta$$

$$\begin{aligned} D_g' &= -\frac{4}{3}V \\ V' &= 2\Psi + \frac{1}{4}D_g \\ -2x^2\Psi &= 3D_g + 12\Psi + \frac{12}{x}V \end{aligned}$$

# Pure radiation fluid

see [Durrer, astro-ph/0109522] or
[text book "The Cosmic Microwave Background" by Durrer]

Ordinary Sachs-Wolfe effect

Integrated Sachs-Wolfe effect

$$\frac{\Delta T(\mathbf{n})}{T} = \left[\frac{1}{4}D_g^{(r)} + V_j^{(b)}n^j + \Psi - \Phi\right](\eta_{dec}, \mathbf{x}_{dec}) + \int_{\eta_{dec}}^{\eta_0}(\dot{\Psi} - \dot{\Phi})(\eta, \mathbf{x}(\eta))d\eta$$

Pure radiation fluid

$$\begin{aligned} D_g' &= -\frac{4}{3}V \\ V' &= 2\Psi + \frac{1}{4}D_g \\ -2x^2\Psi &= 3D_g + 12\Psi + \frac{12}{x}V \end{aligned}$$

see [Durrer, astro-ph/0109522] or
[text book "The Cosmic Microwave Background" by Durrer]

Pure radiation fluid

$$\frac{\Delta T(\mathbf{n})}{T} = \left[\frac{1}{4}D_g^{(r)} + V_j^{(b)}n^j + \Psi - \Phi\right](\eta_{dec}, \mathbf{x}_{dec}) + \int_{\eta_{dec}}^{\eta_0} (\dot{\Psi} - \dot{\Phi})(\eta, \mathbf{x}(\eta))d\eta$$

$$\begin{aligned}
D_g' &= -\frac{4}{3}V \\
V' &= 2\Psi + \frac{1}{4}D_g \\
-2x^2\Psi &= 3D_g + 12\Psi + \frac{12}{x}V
\end{aligned}$$

$$\begin{aligned}
D_g &= D_2\left[\cos\left(\frac{x}{\sqrt{3}}\right) - 2\frac{\sqrt{3}}{x}\sin\left(\frac{x}{\sqrt{3}}\right)\right] \\
&\quad + D_1\left[\sin\left(\frac{x}{\sqrt{3}}\right) + 2\frac{\sqrt{3}}{x}\cos\left(\frac{x}{\sqrt{3}}\right)\right] \\
V &= -\frac{3}{4}D_g' \\
\Psi &= \frac{-3D_g - (12/x)V}{12 + 2x^2} \;.
\end{aligned}$$

$$x = k\eta$$

$$a(\eta)d\eta = dt$$

see [Durrer, astro-ph/0109522] or
[text book “The Cosmic Microwave Background” by Durrer]

Ordinary Sachs-Wolfe effect

Integrated Sachs-Wolfe effect

$$\frac{\Delta T(\mathbf{n})}{T} = \left[\frac{1}{4}D_g^{(r)} + V_j^{(b)} n^j + \Psi - \Phi\right](\eta_{dec}, \mathbf{x}_{dec}) + \int_{\eta_{dec}}^{\eta_0} (\dot{\Psi} - \dot{\Phi})(\eta, \mathbf{x}(\eta))d\eta$$

Pure radiation fluid

$$\begin{aligned} D_g' &= -\frac{4}{3}V \\ V' &= 2\Psi + \frac{1}{4}D_g \\ -2x^2\Psi &= 3D_g + 12\Psi + \frac{12}{x}V \end{aligned}$$

$$\begin{aligned} D_g &= D_2\left[\cos\left(\frac{x}{\sqrt{3}}\right) - 2\frac{\sqrt{3}}{x}\sin\left(\frac{x}{\sqrt{3}}\right)\right] \\ &\quad + D_1\left[\sin\left(\frac{x}{\sqrt{3}}\right) + 2\frac{\sqrt{3}}{x}\cos\left(\frac{x}{\sqrt{3}}\right)\right] \\ V &= -\frac{3}{4}D_g' \\ \Psi &= \frac{-3D_g - (12/x)V}{12 + 2x^2}\,. \end{aligned}$$

$$x = k\eta$$

$$a(\eta)d\eta = dt$$

see [Durrer, astro-ph/0109522] or
[text book “The Cosmic Microwave Background” by Durrer]

# Pure radiation fluid

$$\frac{\Delta T(\mathbf{n})}{T} = \left[\frac{1}{4}D_g^{(r)} + V_j^{(b)}n^j + \Psi - \Phi\right](\eta_{dec}, \mathbf{x}_{dec}) + \int_{\eta_{dec}}^{\eta_0} (\dot{\Psi} - \dot{\Phi})(\eta, \mathbf{x}(\eta))d\eta$$

$x \ll 1$

$$\Psi = \Psi_0, \quad D_g = D_0 - \frac{2}{3}V_0 x^2, \quad V = V_0 x$$

$x \gg 1$

$$\begin{aligned} V &= V_2 \sin\left(\frac{x}{\sqrt{3}}\right) \\ D_g &= D_2 \cos\left(\frac{x}{\sqrt{3}}\right), \quad \Psi = -\frac{3}{2}x^{-2}D_g \end{aligned}$$

$$D_2 = \frac{4V_2}{\sqrt{3}}.$$

$$x = k\eta$$

$$a(\eta)d\eta = dt$$

see [Durrer, astro-ph/0109522] or
[text book “The Cosmic Microwave Background” by Durrer]

Ordinary Sachs-Wolfe effect

Integrated Sachs-Wolfe effect

$$\frac{\Delta T(\mathbf{n})}{T} = \left[\frac{1}{4}D_g^{(r)} + V_j^{(b)}n^j + \Psi - \Phi\right](\eta_{dec}, \mathbf{x}_{dec}) + \int_{\eta_{dec}}^{\eta_0} (\dot{\Psi} - \dot{\Phi})(\eta, \mathbf{x}(\eta))d\eta$$

Pure radiation fluid

$x \ll 1$ $\quad \Psi = \Psi_0, \quad D_g = D_0 - \frac{2}{3}V_0 x^2, \quad V = V_0 x$

$x \gg 1$

$$\begin{aligned} V &= V_2 \sin\left(\frac{x}{\sqrt{3}}\right) \\ D_g &= D_2 \cos\left(\frac{x}{\sqrt{3}}\right), \quad \Psi = -\frac{3}{2}x^{-2}D_g \\ D_2 &= \frac{4V_2}{\sqrt{3}}. \end{aligned}$$

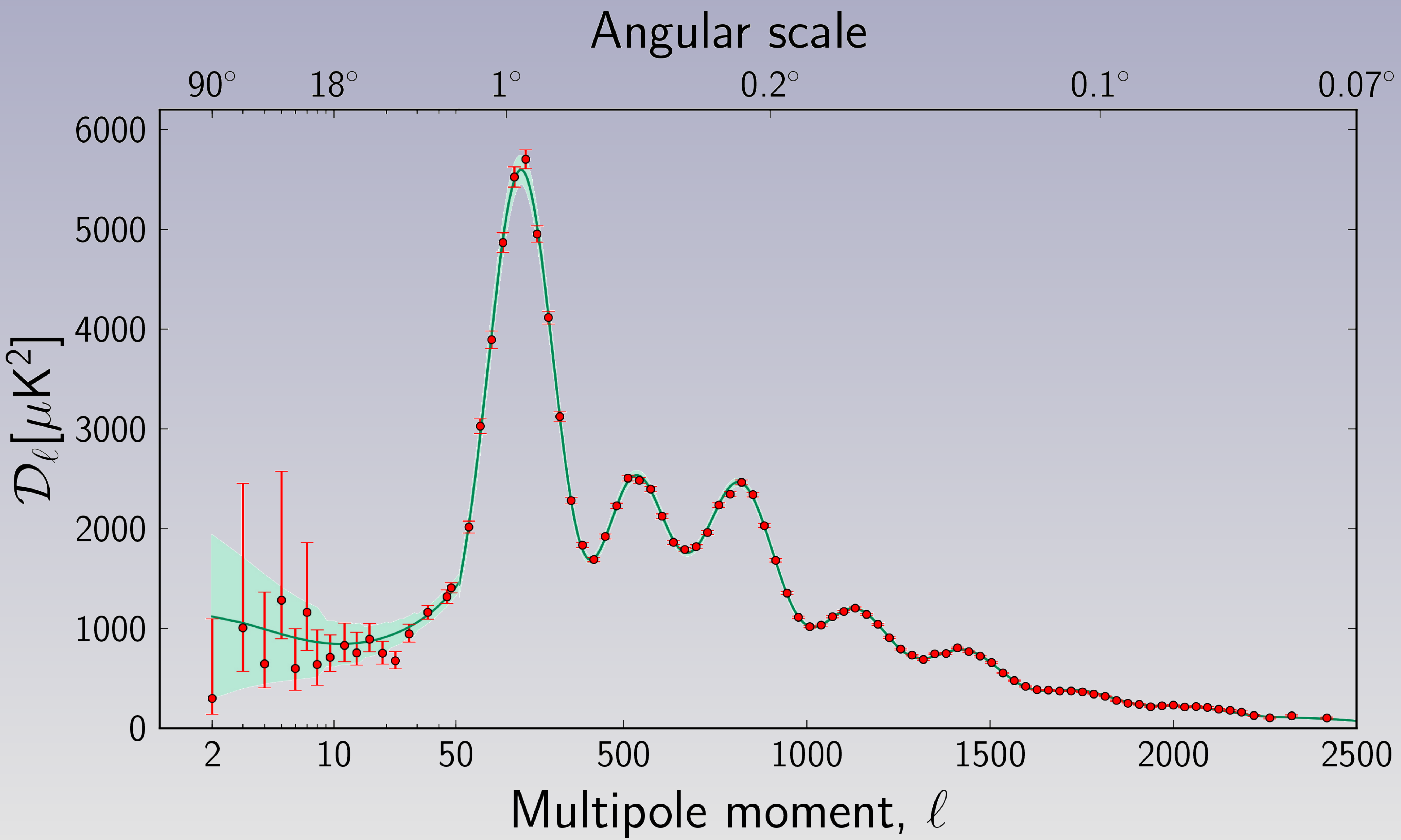
$x = k\eta$

$a(\eta)d\eta = dt$

see [Durrer, astro-ph/0109522] or

[text book “The Cosmic Microwave Background” by Durrer]

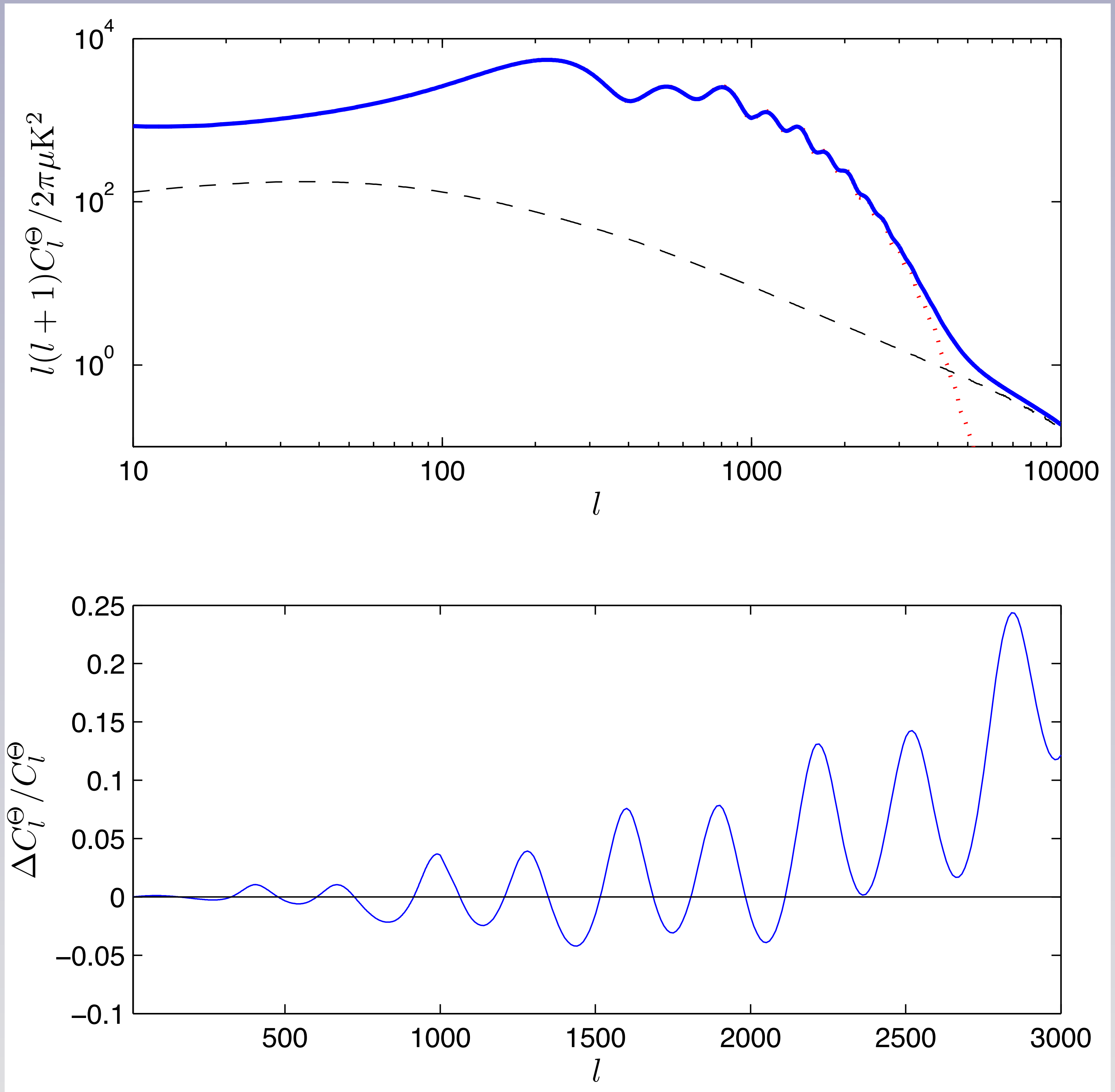
# Shortcut



Angular scale
90°
18°
1°
0.2°
0.1°
0.07°
6000
5000
4000
3000
2000
1000
0
$\mathcal{D}_\ell [\mu K^2]$
2
10
50
500
1000
1500
2000
2500
Multipole moment, $\ell$

Secondary anisotropy

# Lensing




$$\tilde{T}(\hat{\mathbf{n}}) = T(\hat{\mathbf{n}}') = T(\hat{\mathbf{n}} + \boldsymbol{\alpha})$$

- Late time universe
- Dark Energy from CMB only!!
- Massive Neutrinos

[Lewis & Challinor, 2006]

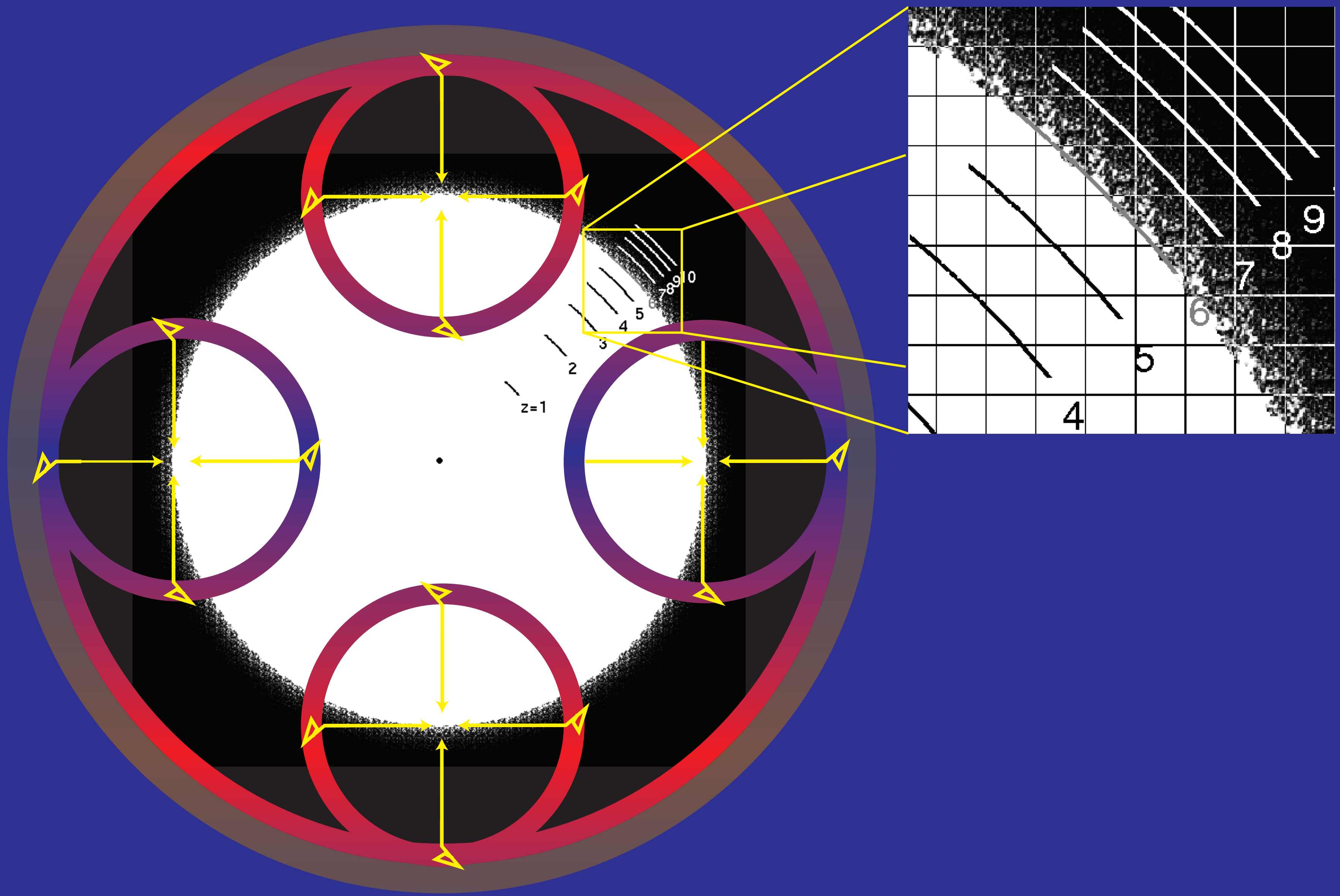
Secondary anisotropy

# Reionization

- Photons rescatter at late time
- Mixes CMB photons from different directions
- Hence smoothes the anisotropies

# Inhomogeneous Ionization

- As reionization completes, ionization regions grow and fill the space



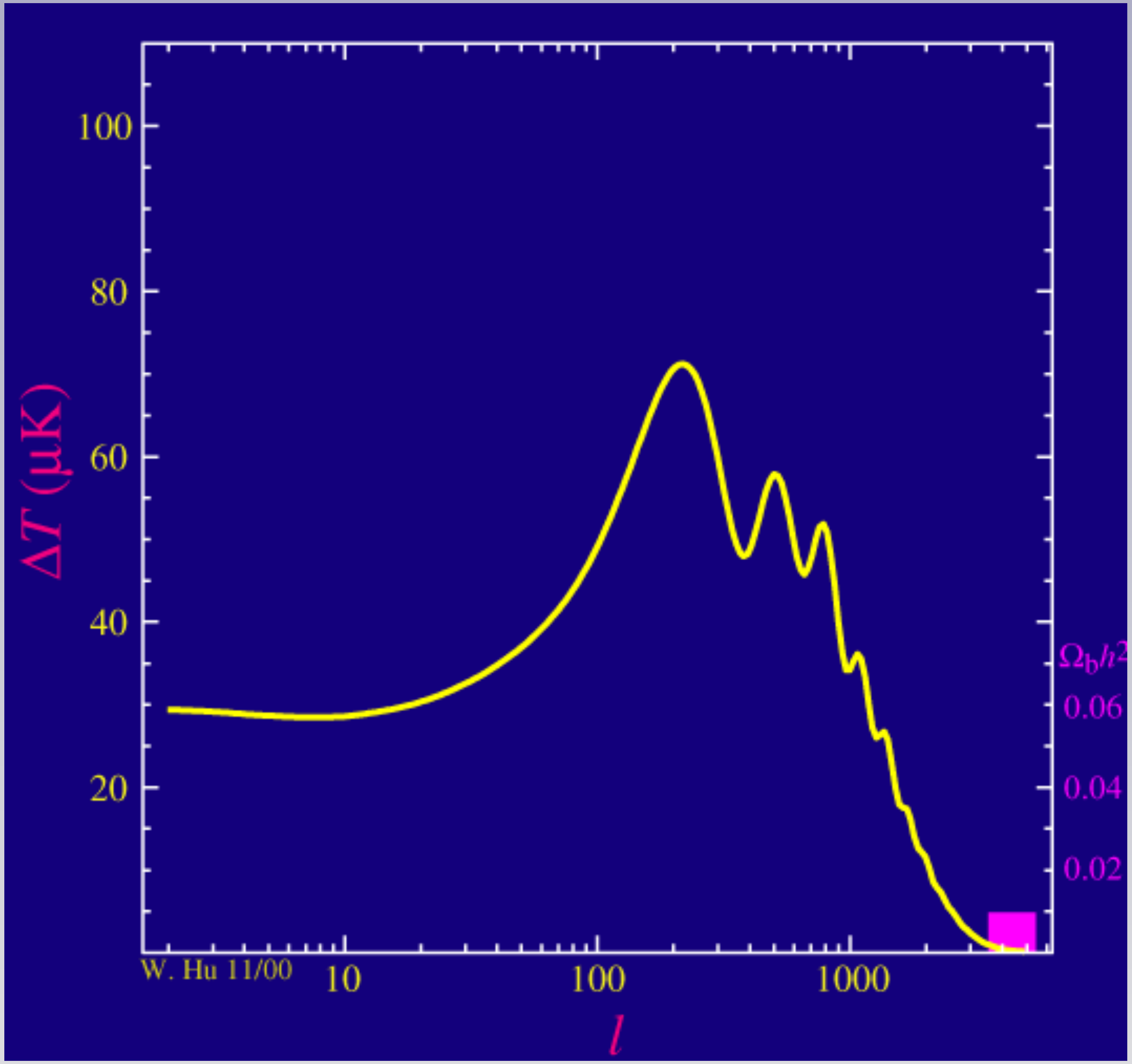

Zahn et al. (2006) [Mortonson et al (2009)]

Secondary anisotropy

# Sunyaev-Zel'dovich effect

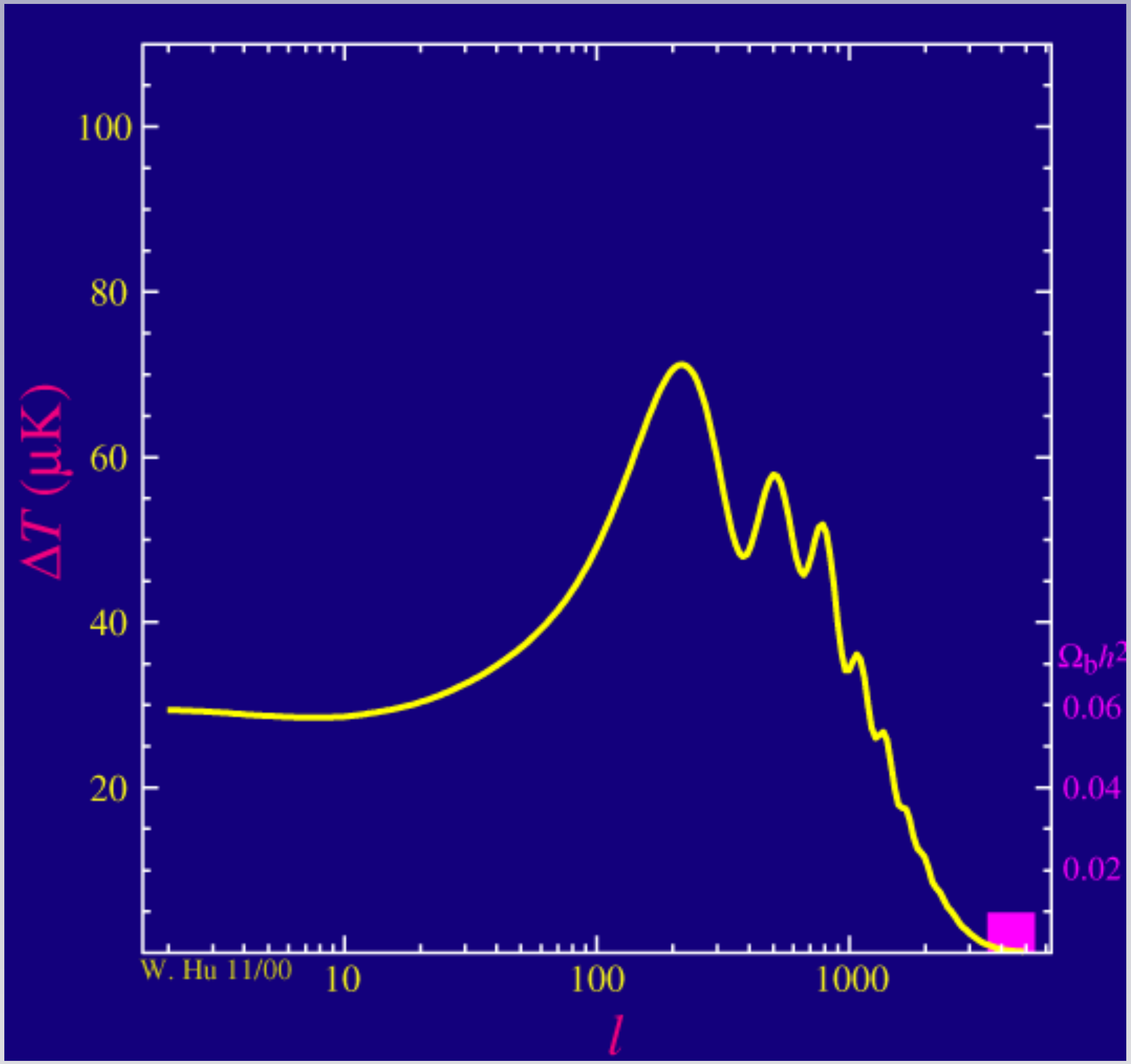
- Photons rescatter in hot gas in galaxies / clusters
- Produces spots in CMB
- If cluster resolved: cut it out
- If not: source of error
- Anyaway source of information

# Baryons



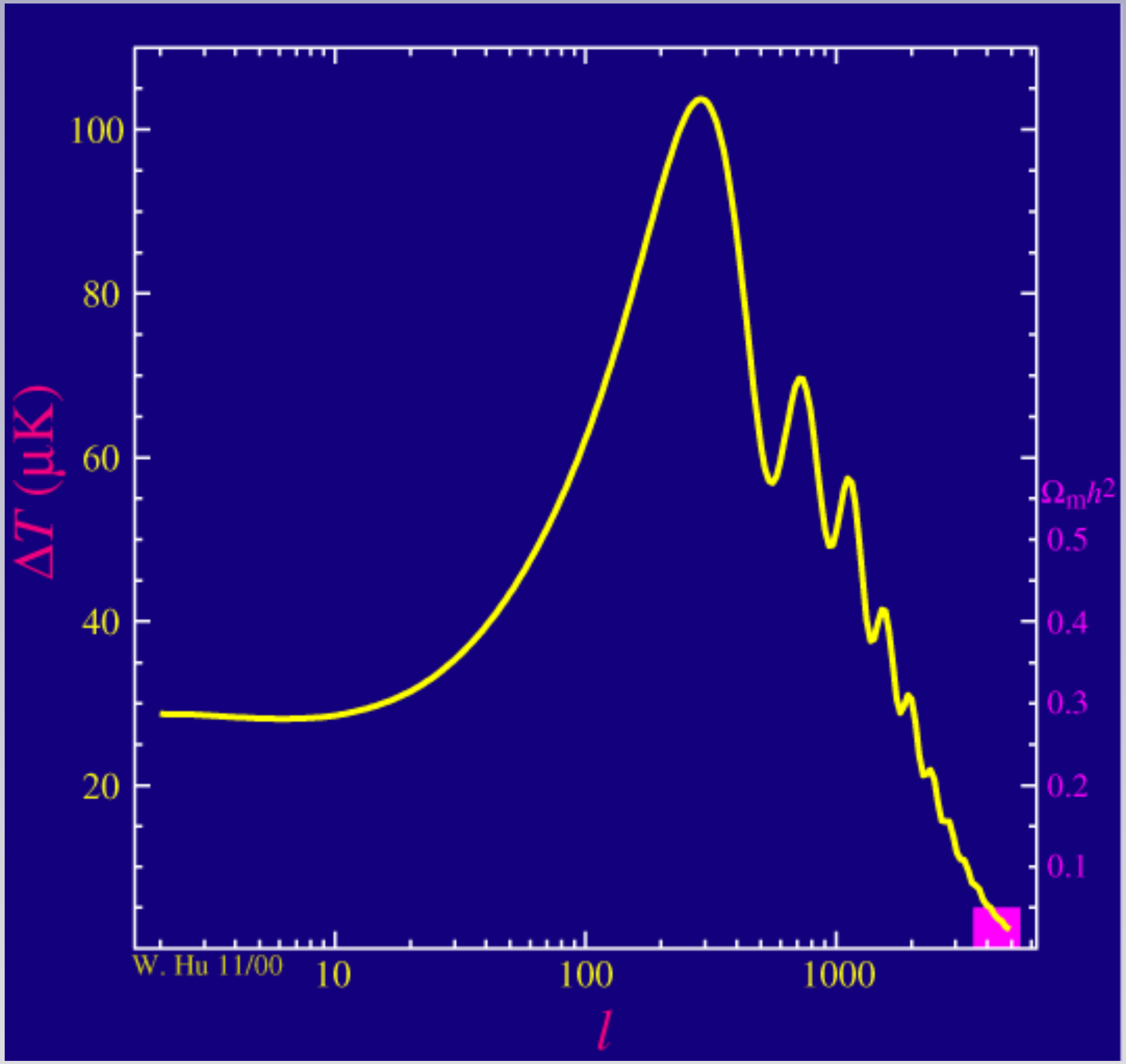

from Wayne Hu [http://background.uchicago.edu/~whu/animbut/anim4.html]

# Baryons



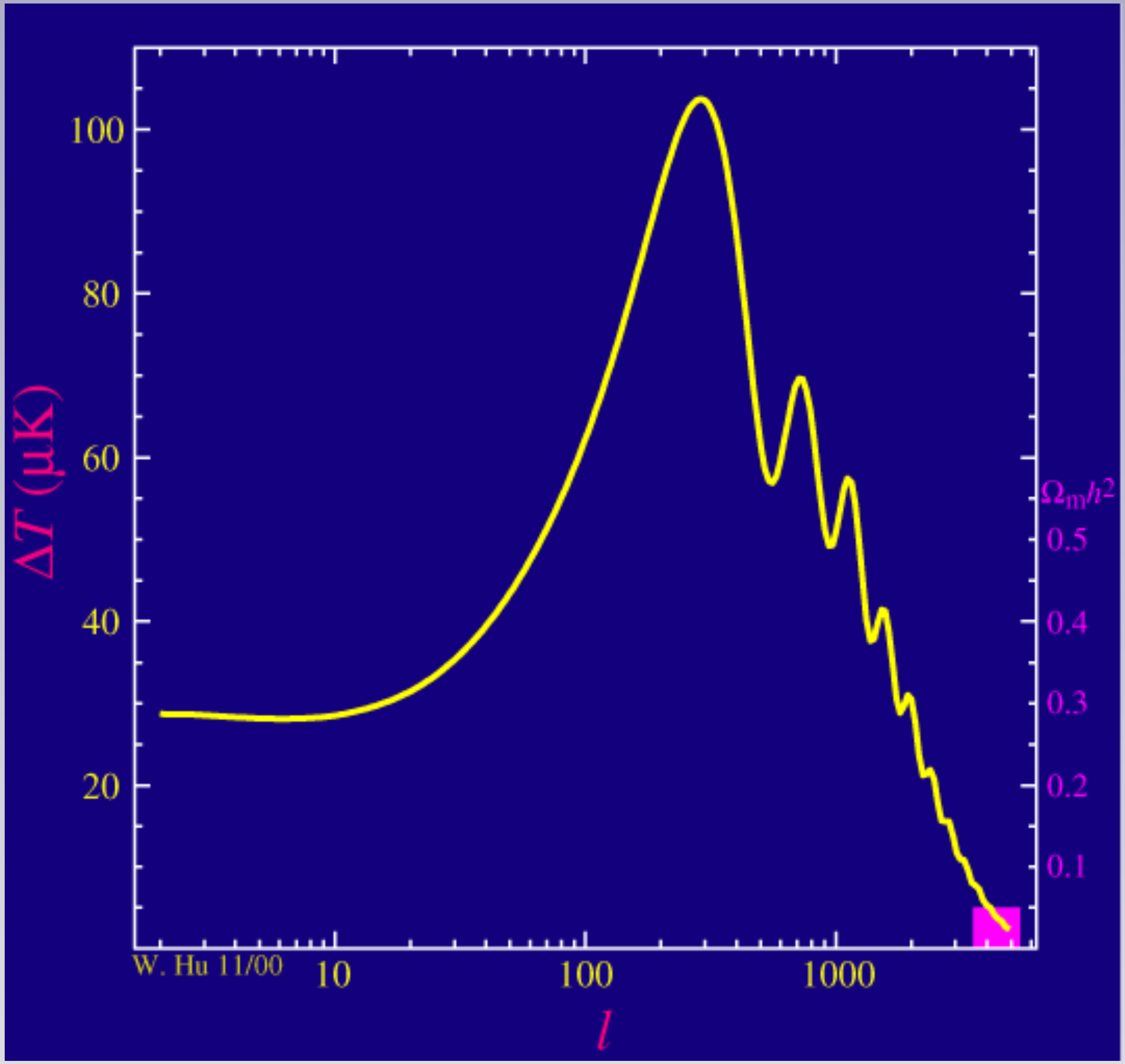

from Wayne Hu [http://background.uchicago.edu/~whu/animbut/anim4.html]

# Dark Matter




from Wayne Hu [http://background.uchicago.edu/~whu/animbut/anim4.html]

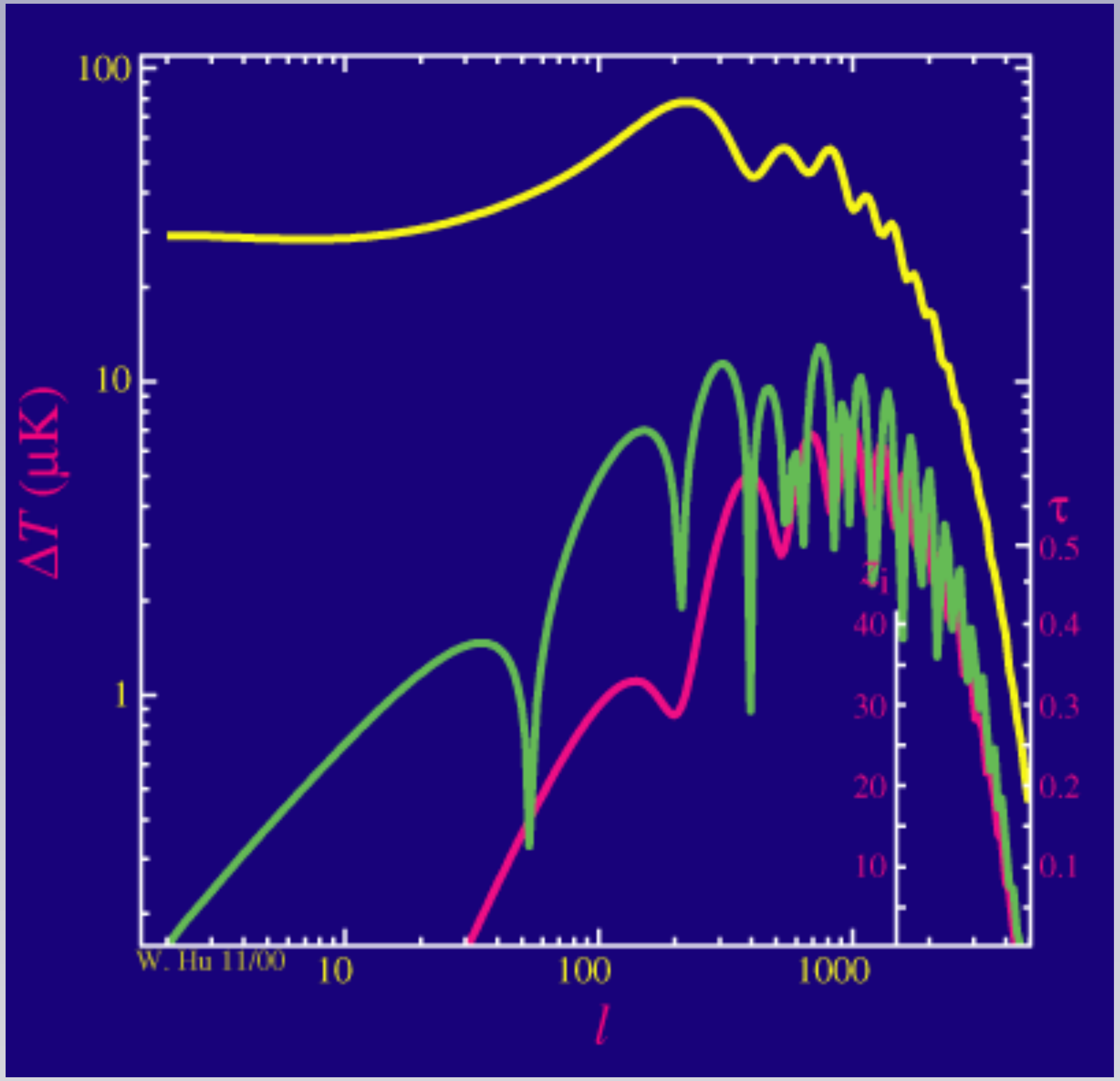
# Dark Matter




from Wayne Hu [http://background.uchicago.edu/~whu/animbut/anim4.html]

Secondary anisotropy

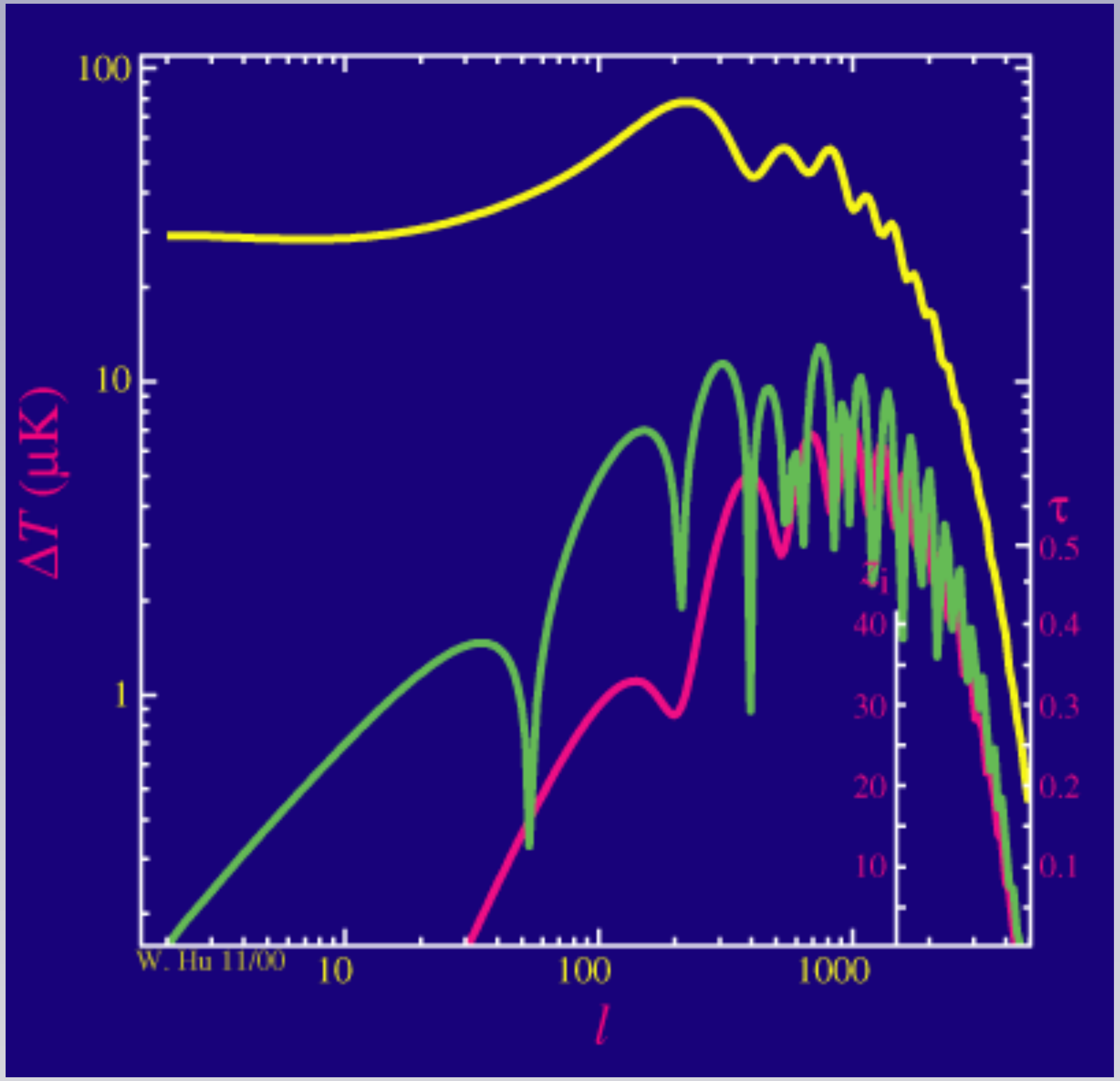
# Reionization




from Wayne Hu [http://background.uchicago.edu/~whu/animbut/anim4.html]

Secondary anisotropy

# Reionization

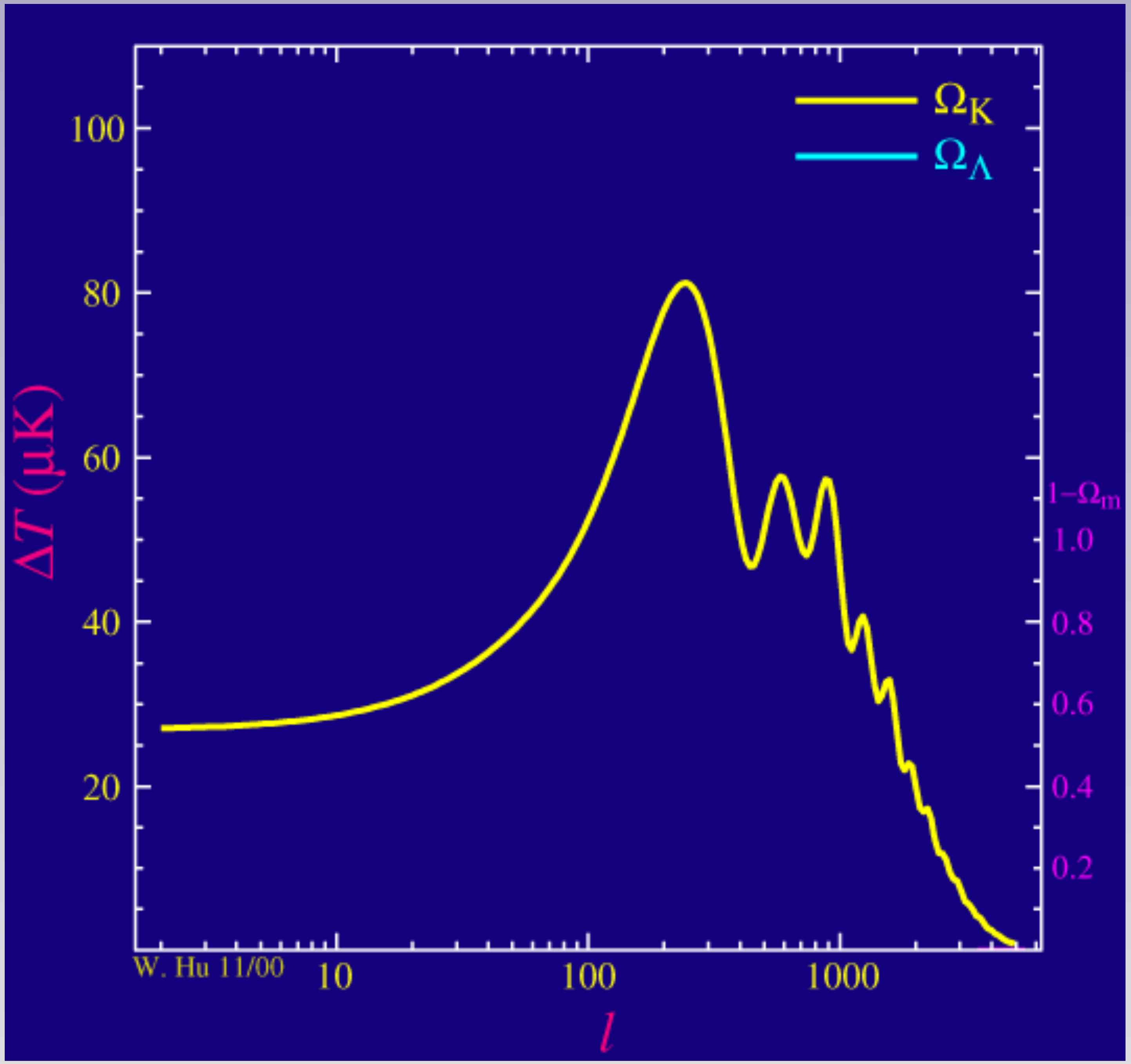



from Wayne Hu [http://background.uchicago.edu/~whu/animbut/anim4.html]

# Age of the universe

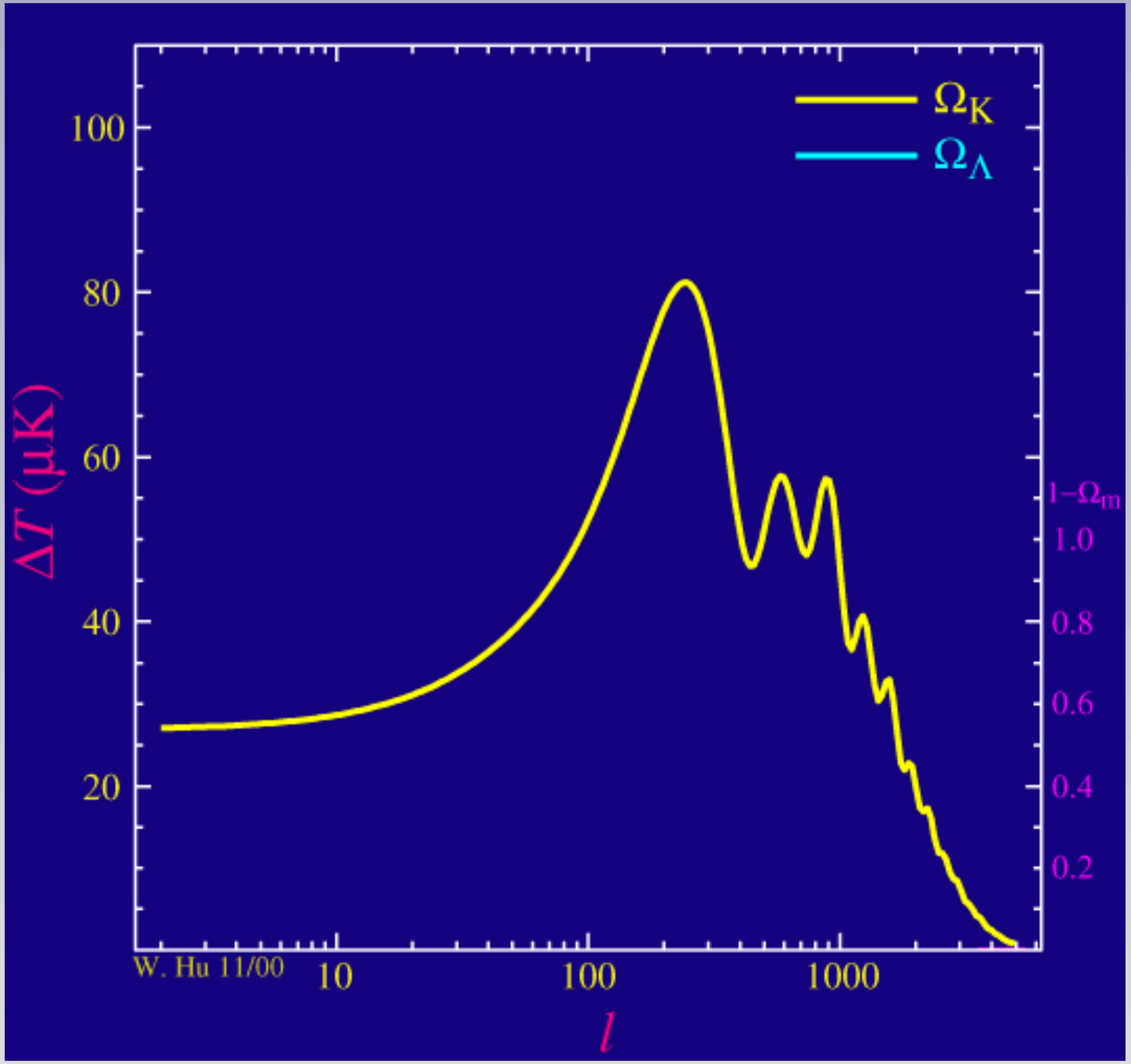
- Observe CMB peaks at some angle
- Compute original length scale: ca 100 Mpc = $3 \times 10^8$ lightyear
- Angular diameter distance to CMB
- Remember its relation to expansion rate
- That gives strong handle on age of universe

# Age of the universe



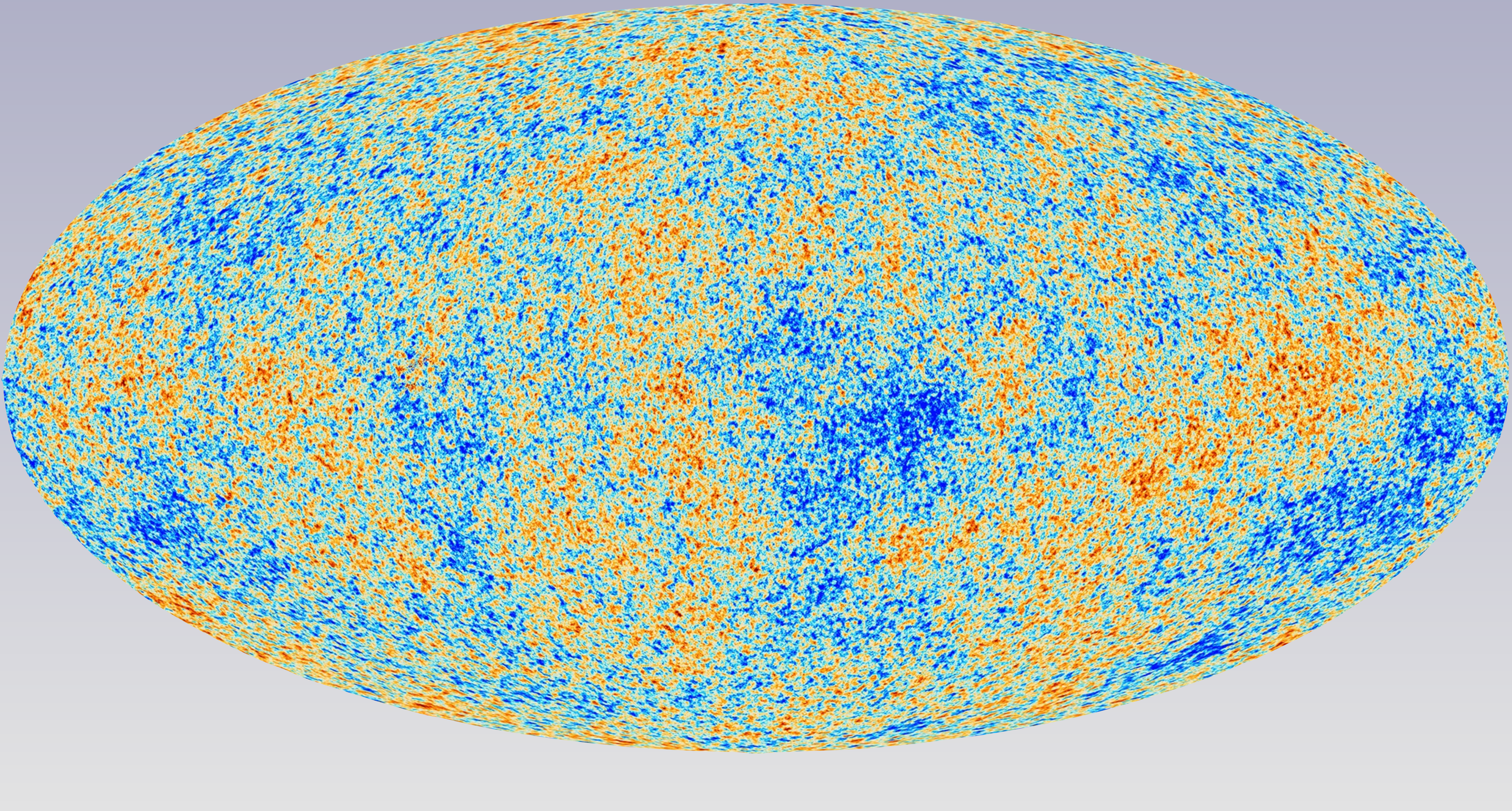

from Wayne Hu [http://background.uchicago.edu/~whu/animbut/anim4.html]

# Age of the universe

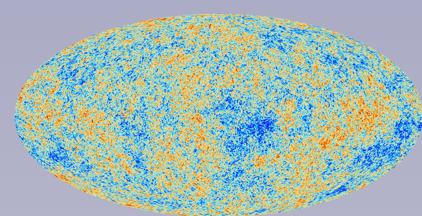



from Wayne Hu [http://background.uchicago.edu/~whu/animbut/anim4.html]

# Parameter constraints from Planck



# Parameter constraints from Planck



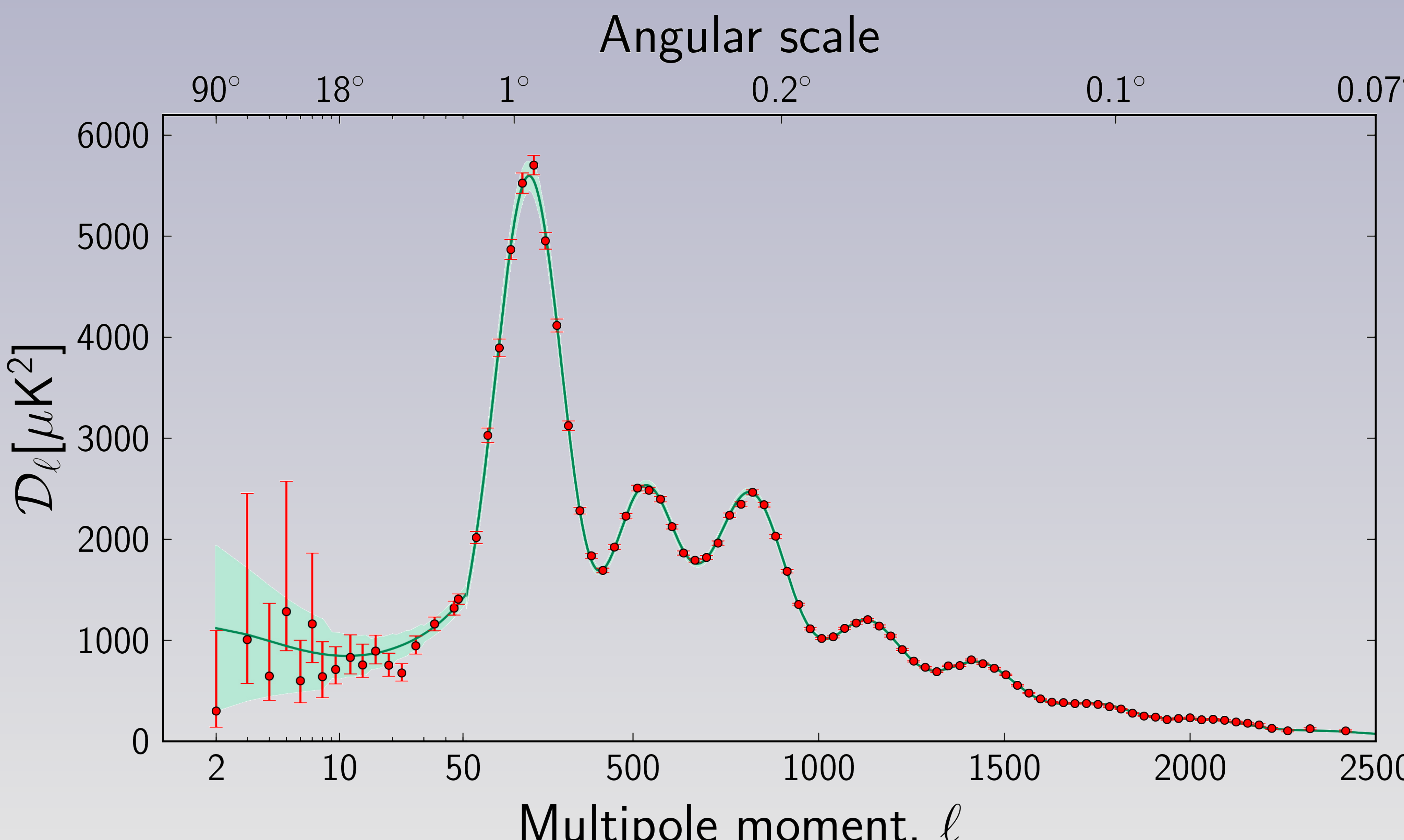
+

$$T(\phi,\theta) = \sum_{l=0}^{\infty}\sum_{m=-l}^{l} a_{lm} Y_{lm}(\phi,\theta)$$

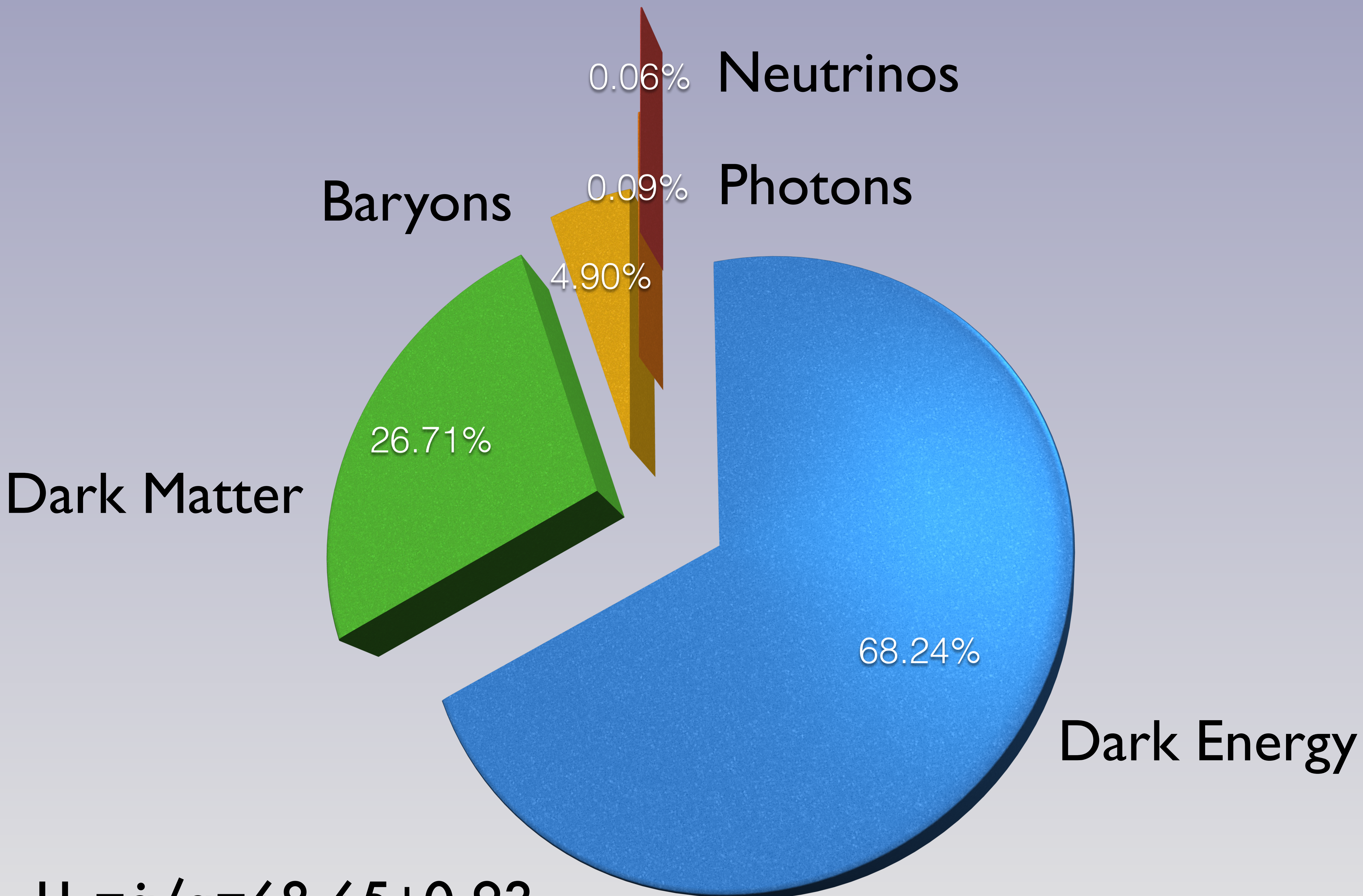
$$a_{lm} = \int_{\Omega} d\Omega\, T(\phi,\theta) Y^{*}_{lm}$$

$$D_l^{TT} \equiv \frac{1}{2l+1}\sum_{m=-l}^{l} |a_{lm}|^2$$

=



# Parameter constraints from Planck




$H_0 = \dot{a}/a = 68.65 \pm 0.93$