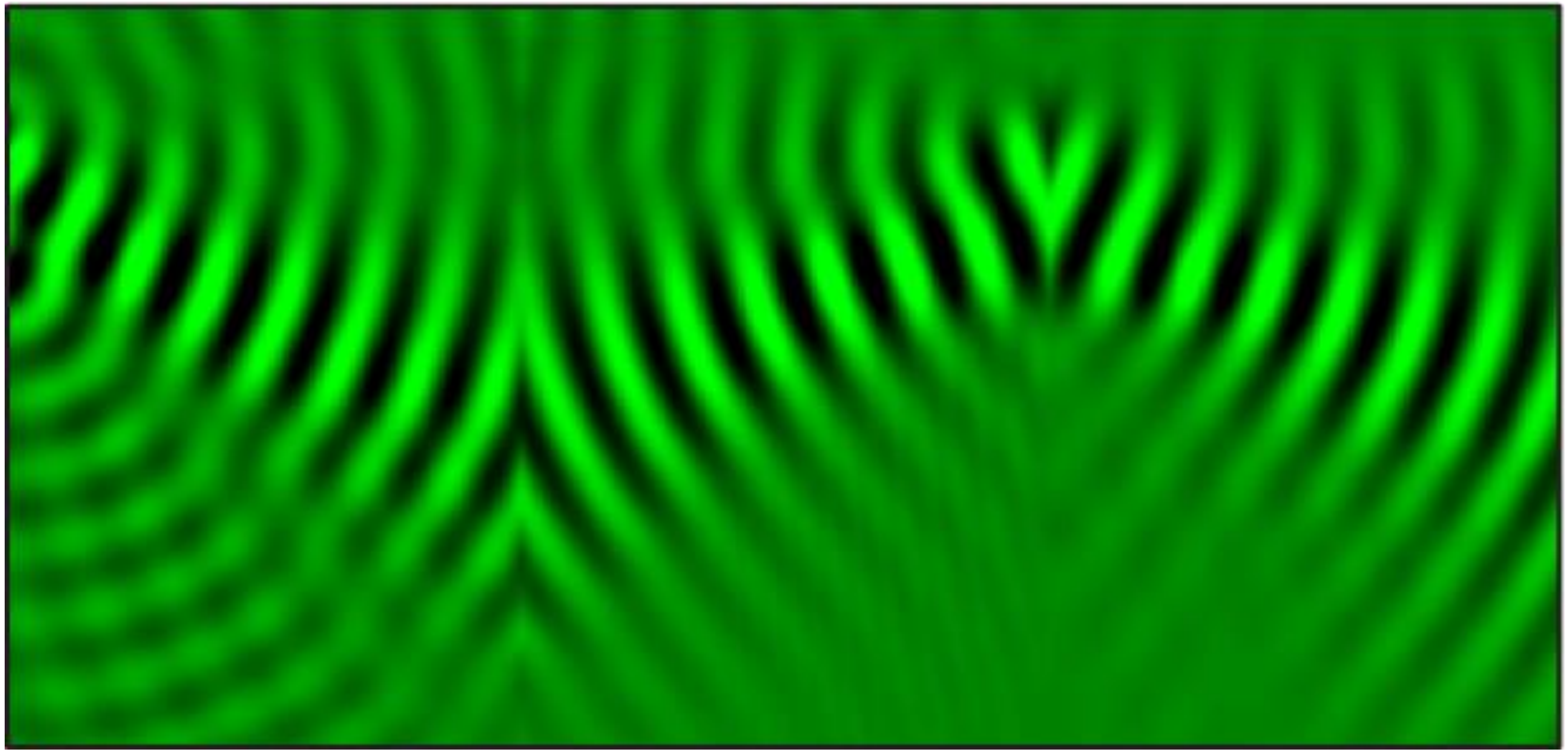
Negative refraction:
Bending light the wrong way



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References

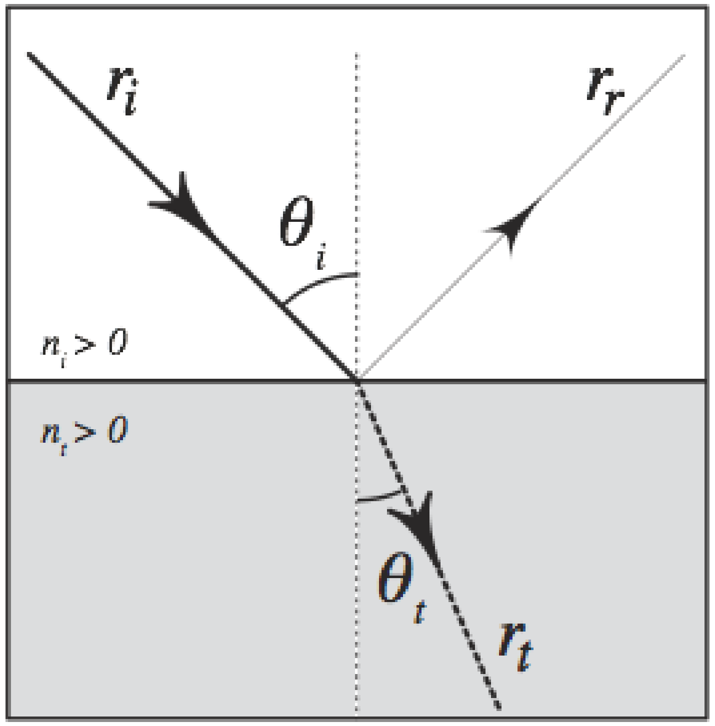
Physics of Negative Refraction and Negative Index Materials
Krowne, Clifford, Zhang, Yong
Springer 2007
http://91.216.243.21/fachbuch/leseprobe/9783540721314_Excerpt_001.pdf

Introduction

- *Negative Refraction?*

Snell-Descarte law:

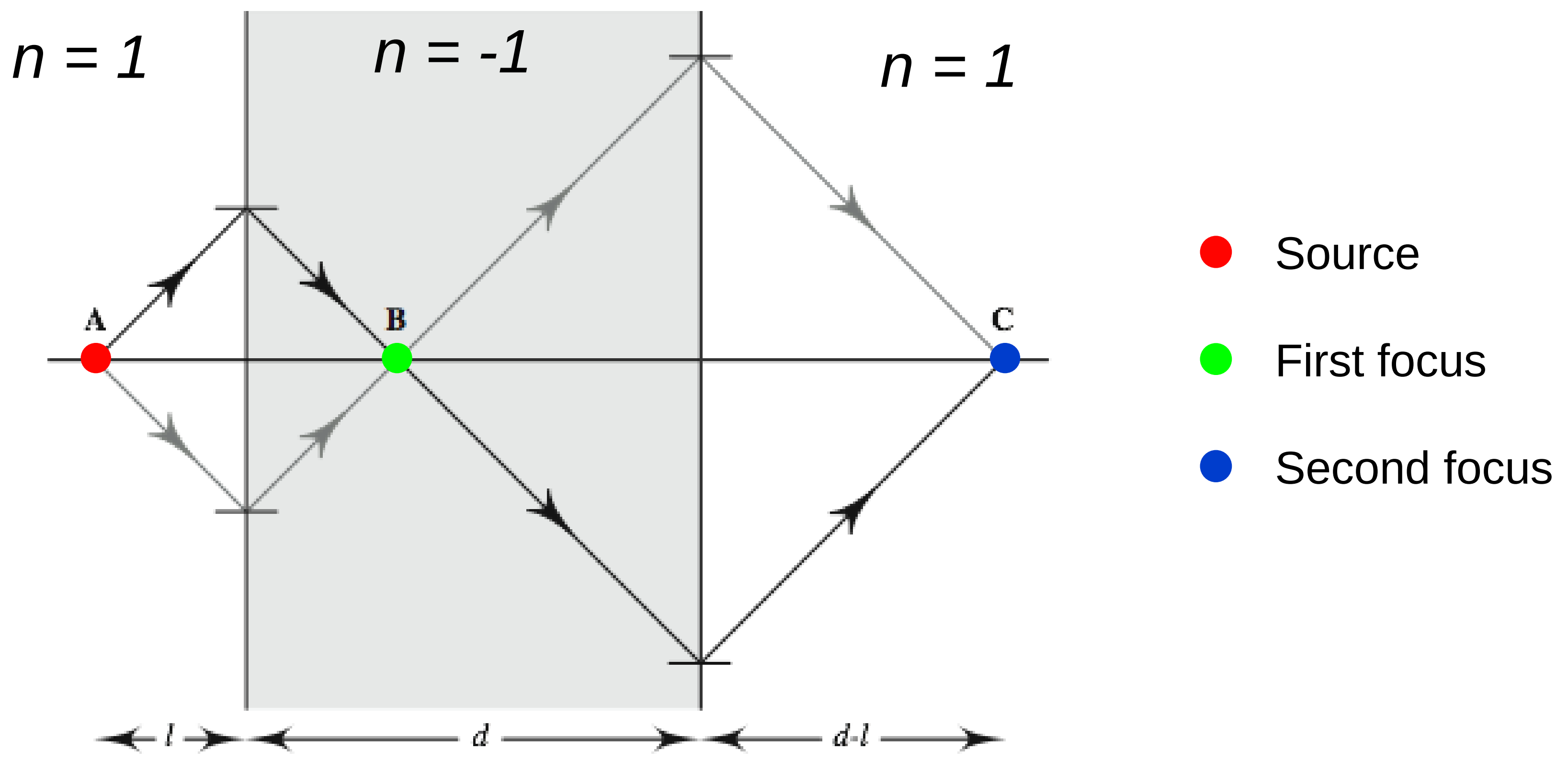
$$n_i \sin(\theta_i) = n_t \sin(\theta_t)$$



Classical refraction

Introduction

- *Consequence:*



A slab of negative refractive material in free space
Can be used to achieve a lens

(Numerical aperture ~ 1 !)

Introduction

- *Condition to have a material with a negative refractive index:*

  ε < 0

  et

  μ < 0

- *What is the refractive index?*

$$n = \sqrt{\epsilon \, \mu}$$

- *What happens if ε or μ are negative?*

Introduction

- *What can happen?*

ε

μ

	+	**-**
+	Transparent material	Electric plasma (absorbion)
-	Magnetic plasma (absorbtion)	Material with negative refractive index

History

V. G. Veselago - (1968)

- *Theoretical introduction of the negative refraction concept*

D.R. Smith - (2000)

- *First proposal how to achieve a material with a negative refraction*

J.B. Pendry - (2000)

- *Superlens concept*

J.B. Pendry - (2006)

- *Invisible Cloak concept [Science 312, 1780 (2006)]*

M. Wegener - (2010)

- *First invisible cloak for optical wavelength [Science 328, 337 (2010)]*

Content

1- Negative refraction index concept

- *Materials with:* $\varepsilon < 0$ **et** $\mu < 0$
- *Propagation equation:* $$\nabla^2\mathbf{E}(\mathbf{r},t) - \epsilon\mu\,\frac{\partial^2}{\partial t^2}\mathbf{E}(\mathbf{r},t) = 0$$
- *Solution:*

  Monochromatic plane waves: $$E(z,t) = E^0\,e^{\,i\,(-\omega\,t\,\pm\,|k|\,z)}$$

  Dispersion relation: $$\omega^2\varepsilon\mu = k^2$$

  Phase velocity: $$v_\varphi = \frac{\omega}{k} = \pm\frac{1}{\sqrt{\varepsilon\mu}}$$

  Refractive index: $$n = \frac{c}{v_\varphi} = \pm\sqrt{\varepsilon_r\mu_r}$$

The refractive index is linked to the phase velocity

What sign has to be chosen?

1- Negative refraction index concept

Phase velocity (vector)

$$\mathbf{v}_\varphi = \frac{\omega}{|\mathbf{k}|^2}\,\mathbf{k}$$

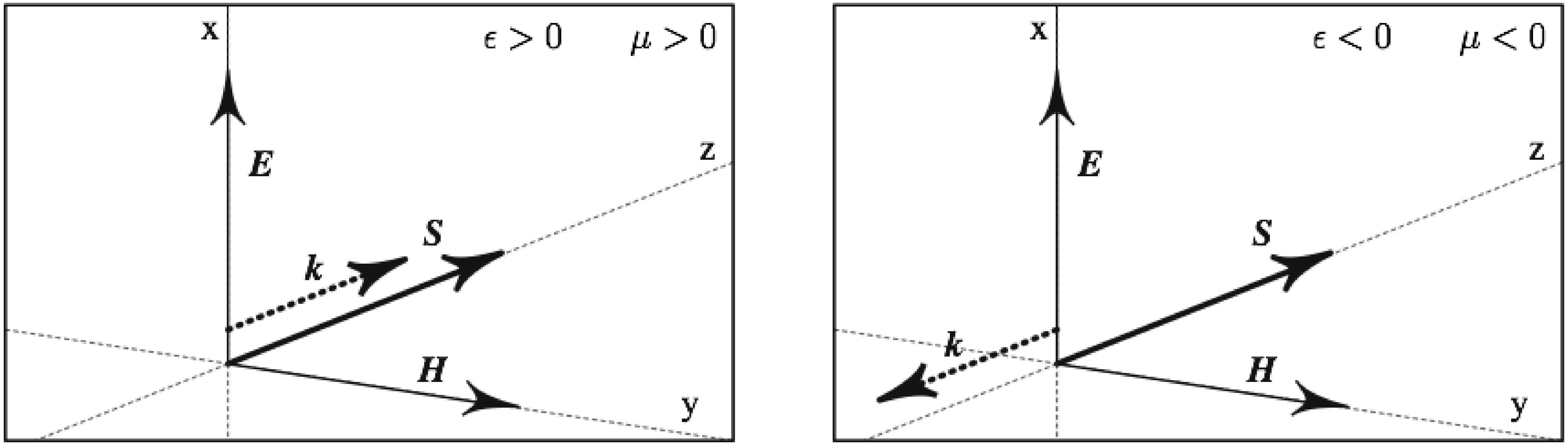
Poynting Vector - *(energy propagation)*

$$\mathbf{S} = \mathbf{E} \times \mathbf{H}$$

For a monochromatic wave:

$$\mathbf{S} = \frac{E^2}{\mu\,\omega}\,\mathbf{k}$$

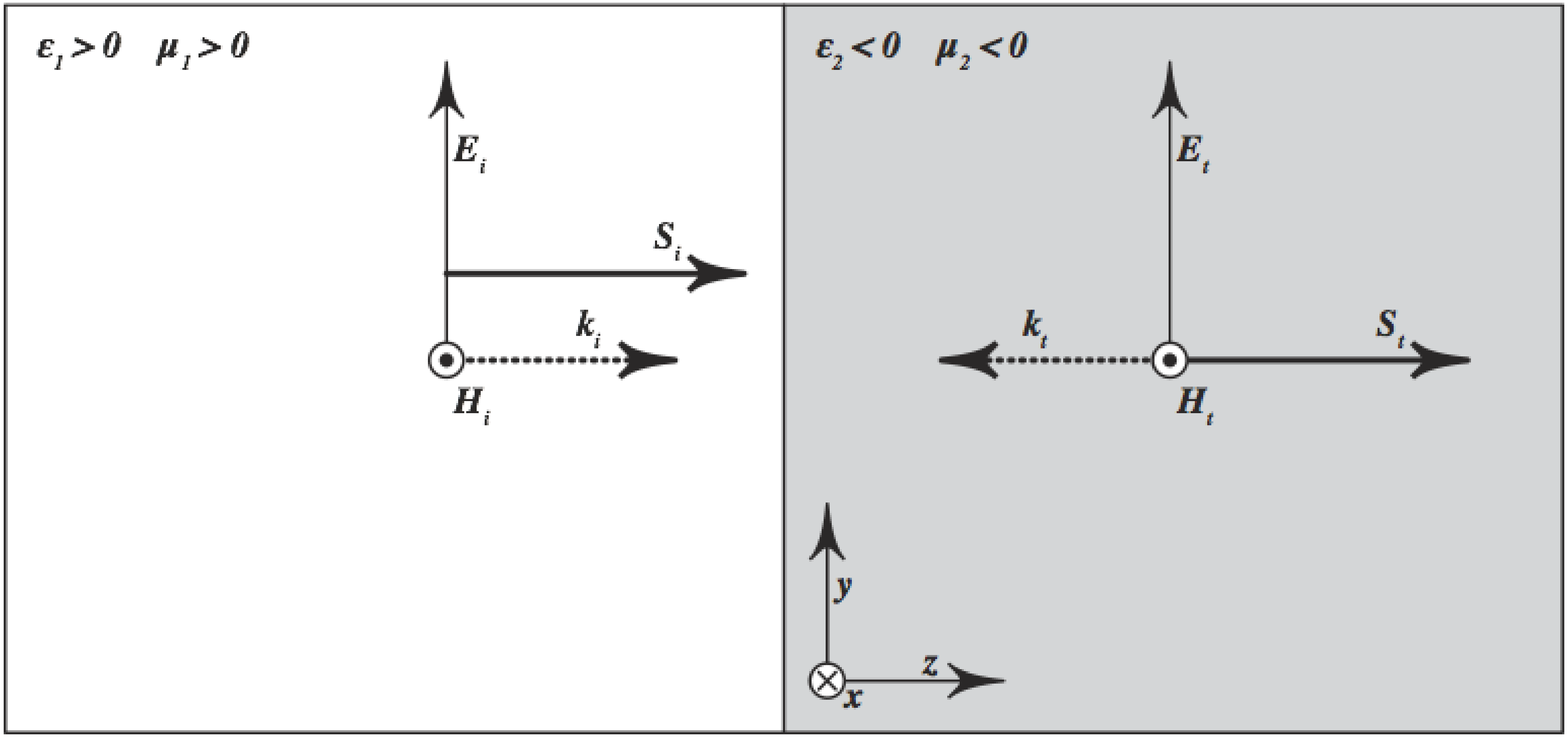
*If $\mu<0$ the energy (**S**) propagates in the opposite direction of the phase (**k**)*



1- Negative refraction index concept

Interface between free space and a material with ε < 0 et μ < 0

- ***Use of the causality principle***



2- Why negative refraction material are dispersive

- ***Group velocity***

$$|v_g| = \frac{d\omega}{dk}$$

If the material were non dispersive, its group velocity would be negative!

- ***Energy density***

In a non dispersive material:

$$W = \epsilon < \mathbf{E}^2 > + \mu < \mathbf{H}^2 >$$

In a dispersive material:

$$W = \frac{\partial(\omega\,\epsilon(\omega))}{\partial\omega} < \mathbf{E}^2 > + \frac{\partial(\omega\,\mu(\omega))}{\partial\omega} < \mathbf{H}^2 >$$

If the material were non dispersive, its
energy density and group velocity would be negative!

3- FDTD simulations

FDTD: ***F**inite-**D**ifference **T**ime-**D**omain*

Numerical method using finite-differences to solve Maxwell equation and to simulate light propagation in a discretized area

Principle:

An iterative algorithm which computes the new electromagnetic field at a certain moment by using the previous values of the electromagnetic in the neighboring region

Used dispersion model:

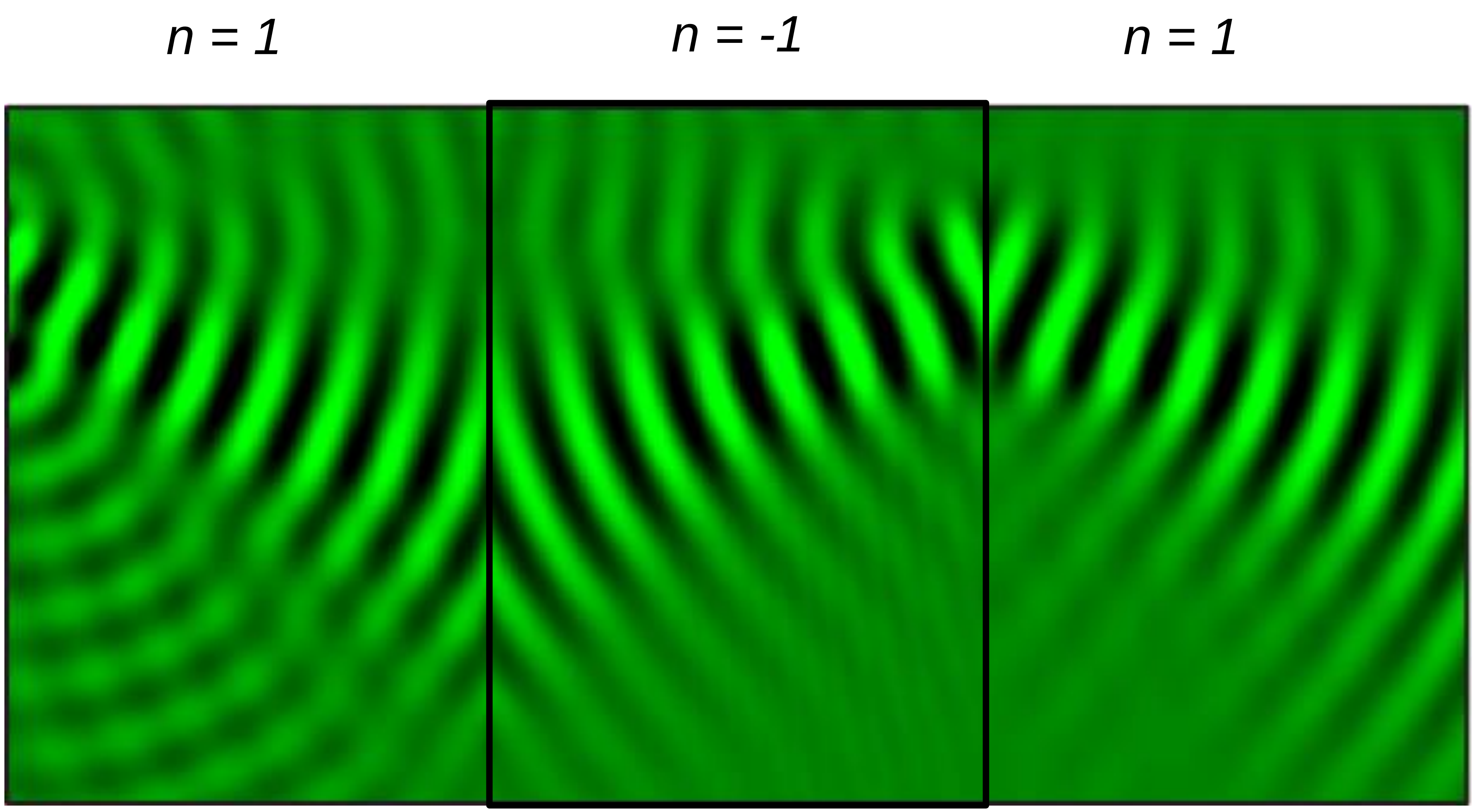
$$\epsilon(\omega) = \epsilon_0 \left(1 - \frac{w_p^2}{w^2}\right)$$

$$\mu(\omega) = \mu_0 \left(1 - \frac{w_p^2}{w^2}\right)$$

No absorption

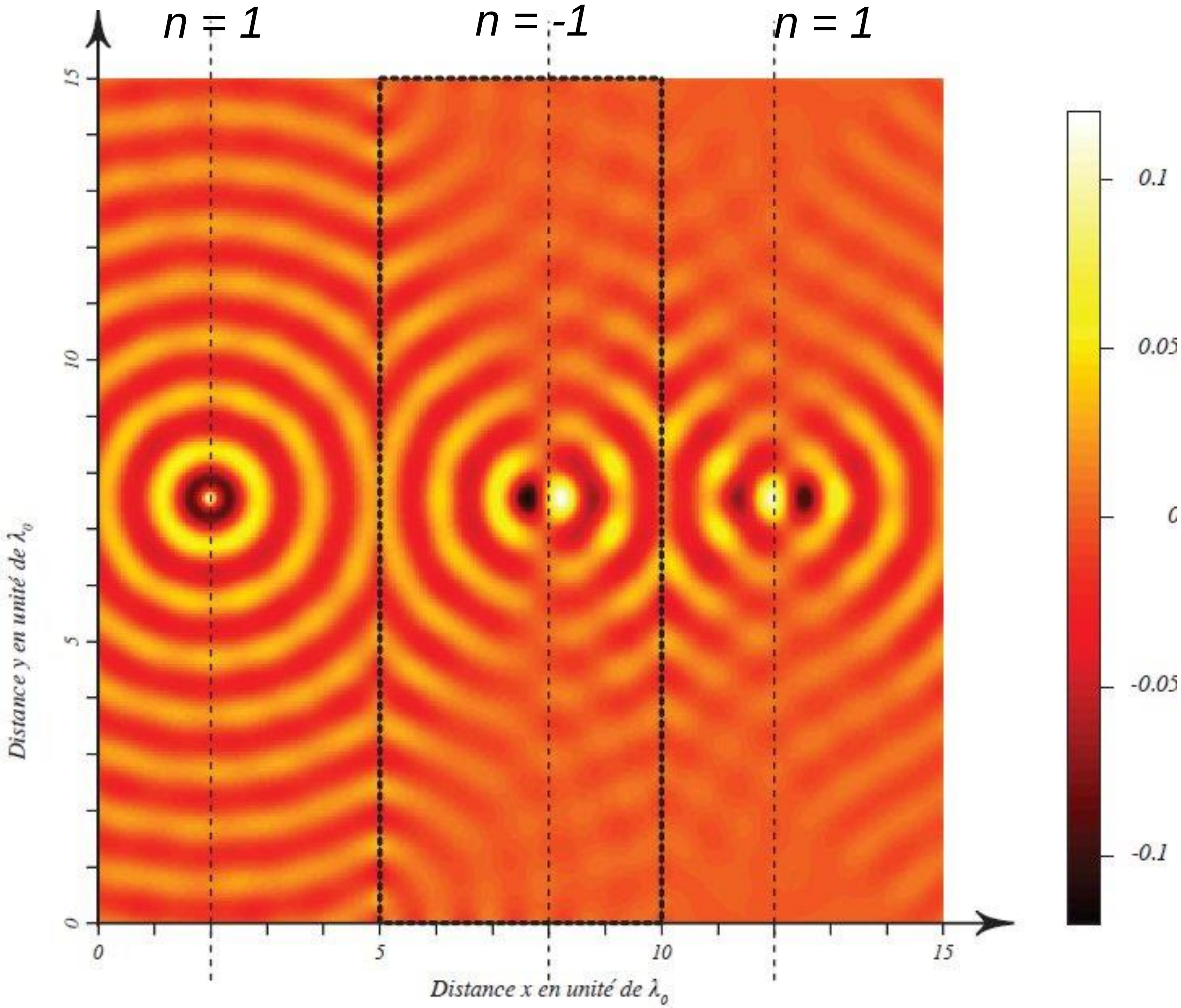
3- FDTD simulations

Negative refraction



3- FDTD simulations

Super lens



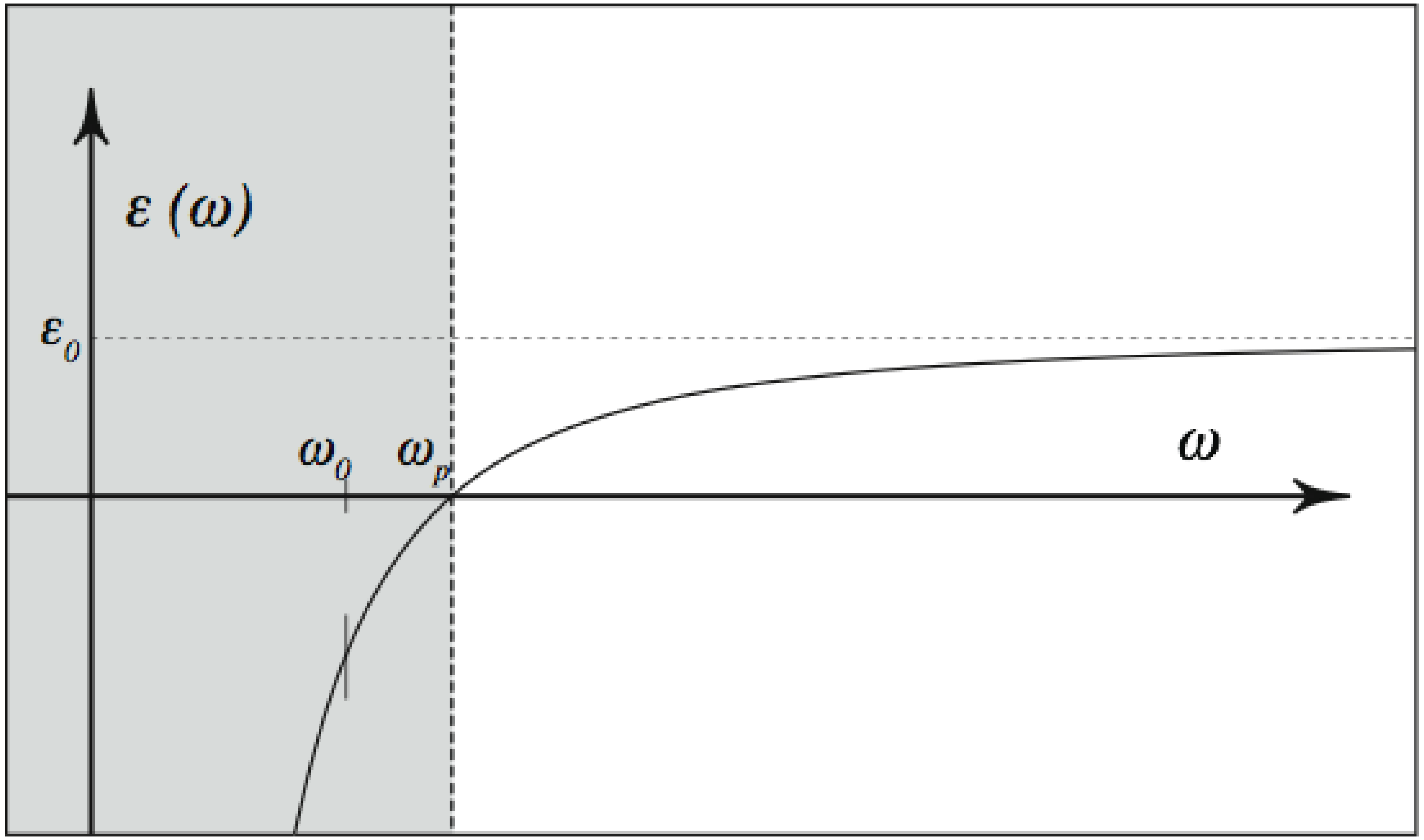
4- *How to achieve such a materials?*

Example of negative ε: *high conductive material, electric plasma*

Drude model: electron gas

Real part:

$$\epsilon(\omega) = \epsilon_0 \left(1 - \frac{w_p^2}{w^2}\right)$$



If $\omega<\omega_{p,}$ $\varepsilon < 0$
But $\mu > 0$

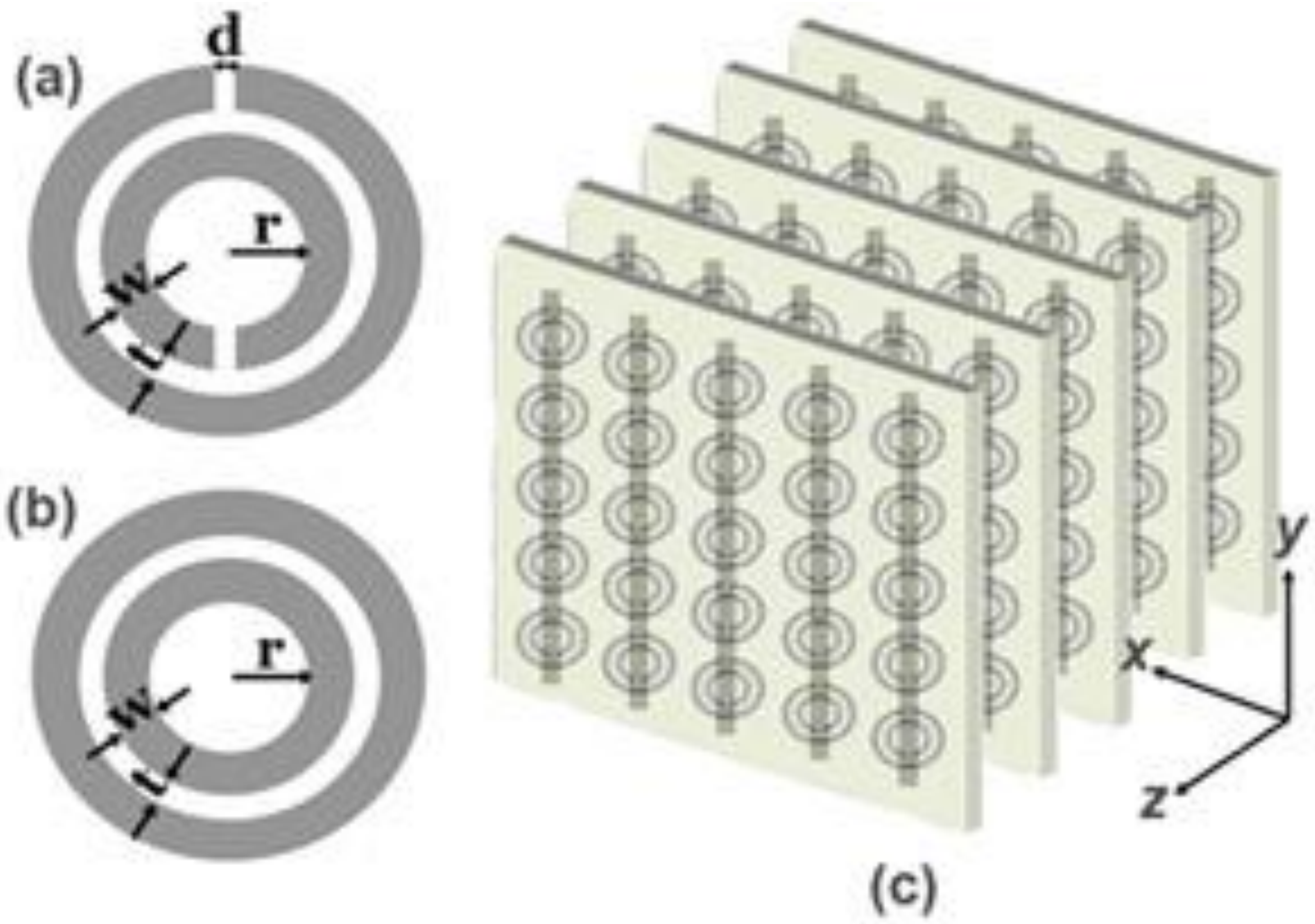
$n = (\varepsilon_r \mu_r)^{1/2}$ *complex:*
no propagative waves

What's about μ? $\mu = \mu_0$ in the visible range …

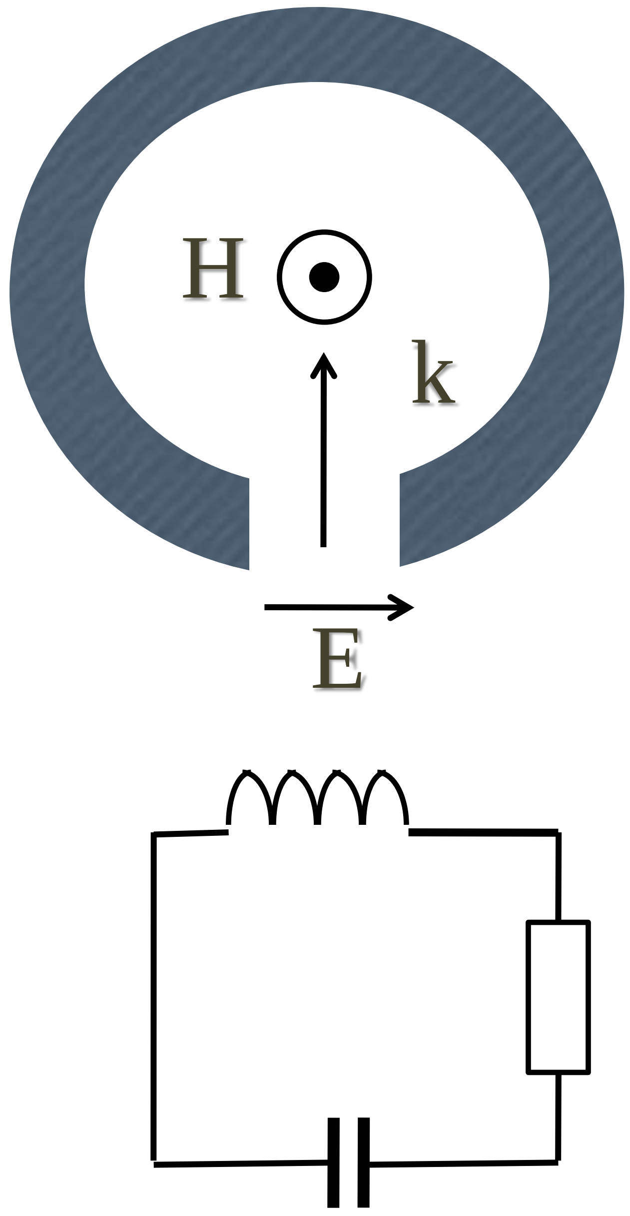
4- *How to achieve such a materials?*

Metamaterial:

structure that behaves such as an homogeneous material with new "effective properties"



« Split ring resonators » to increase μ and create « magnetic dipoles »

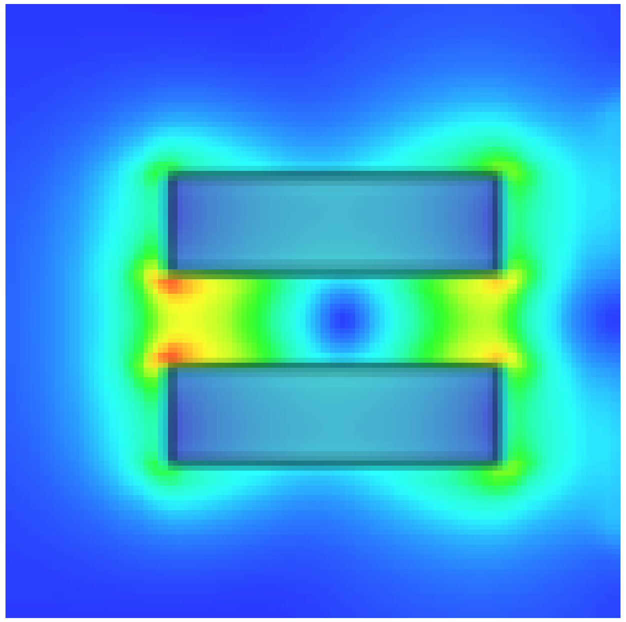


RLC circuit: possible resonance

4- *How to achieve such a materials?*

Question:
Sub-λ structures are better to achieved metamaterial.
Is it possible to have resonance with a sub-λ structures ?
(no resonance in a cavity smaller than λ/2)

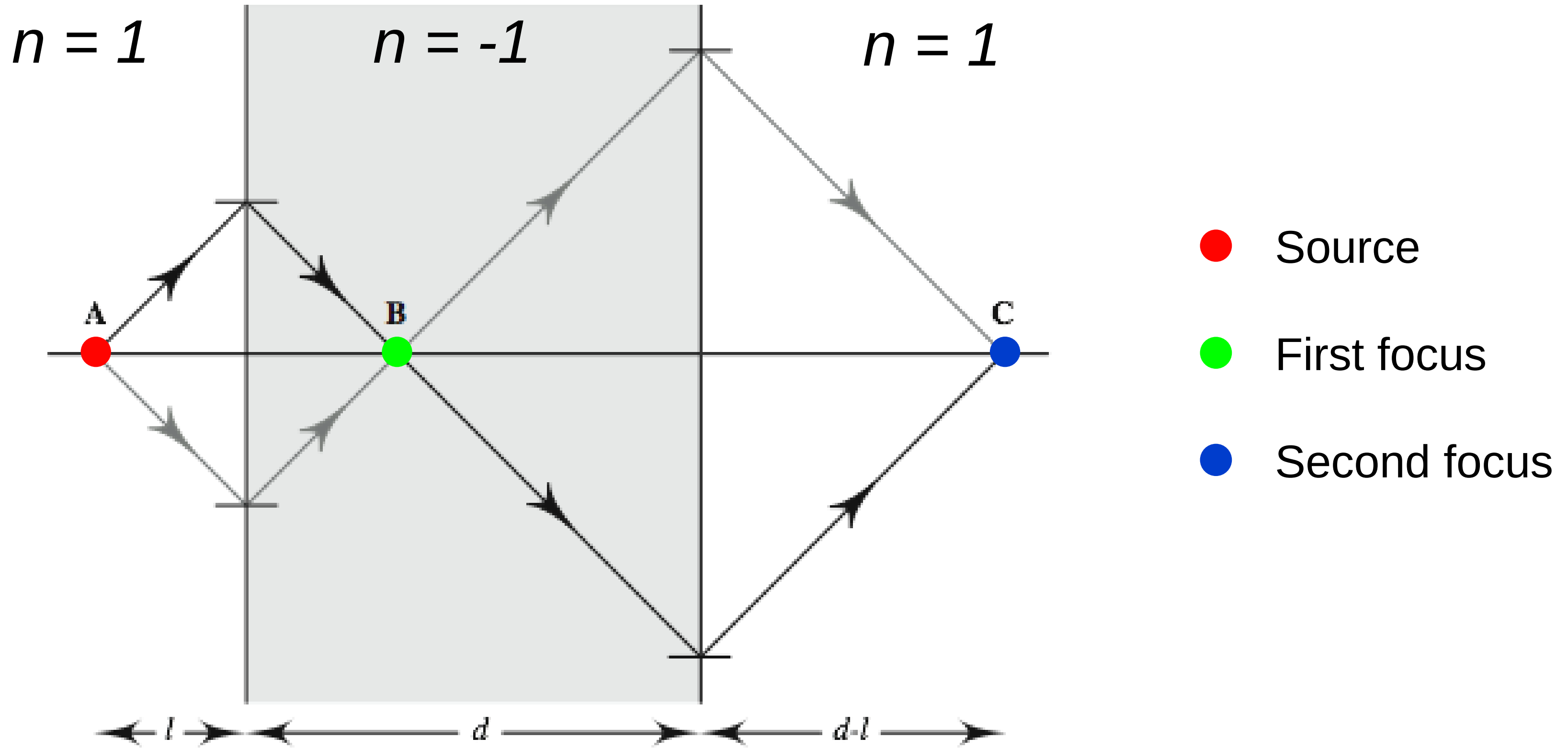
Yes using plasmonic resonance



[Zhang Phys. Rev. Lett. **101**, 047401(2008)]

5- Super lens application

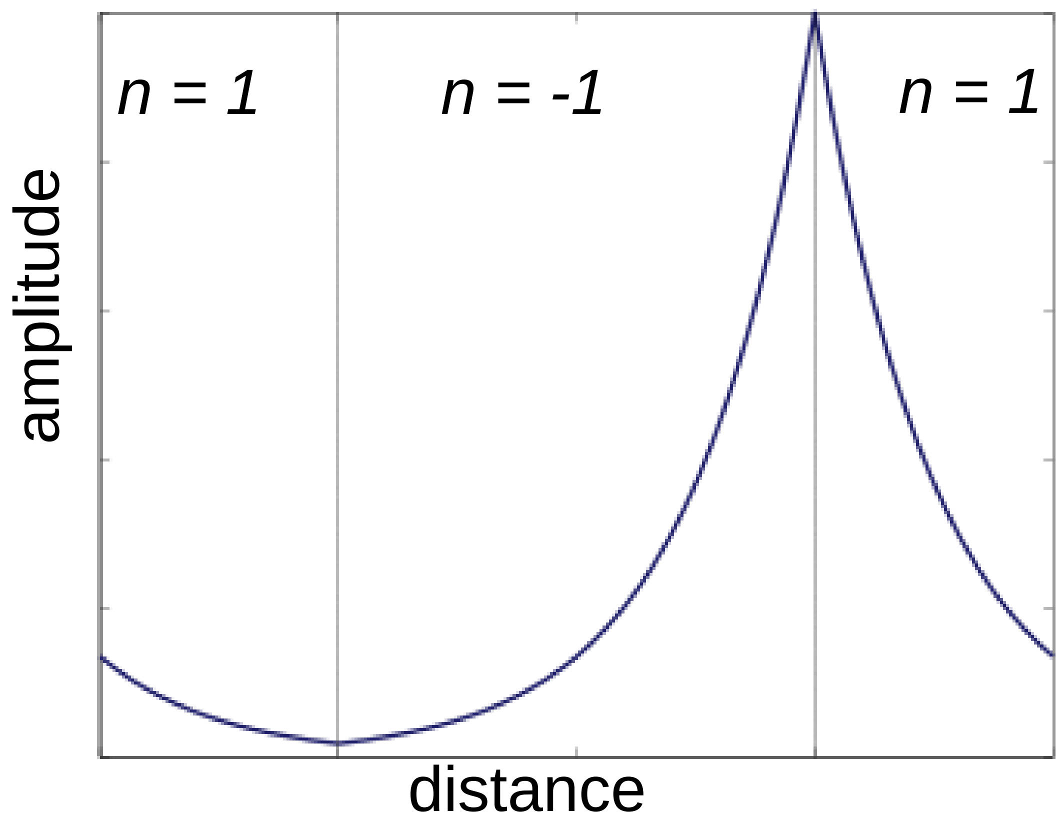
- *Super lens: propagative waves*



A slab of negative refractive material in free space

5- Super lens application

- *Super lens: evanescent waves*



Evanescent waves are enhanced

Energy creation?

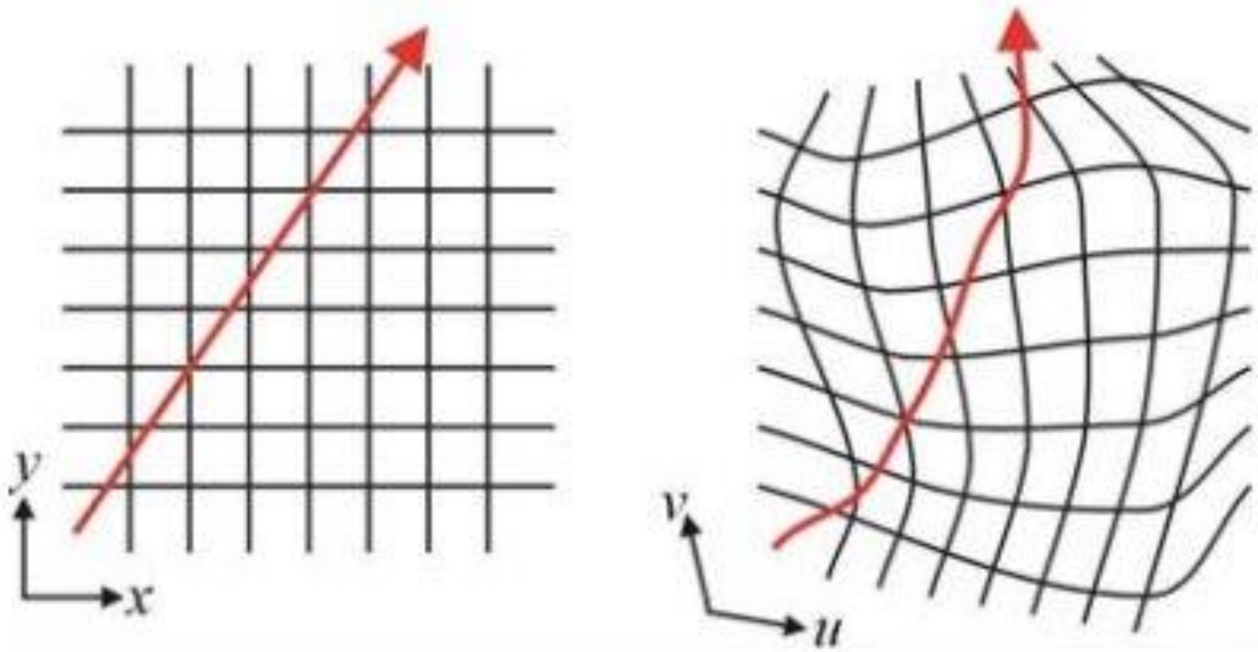
No: stationnary state inside the slab

No propagation through the slab but along the slab

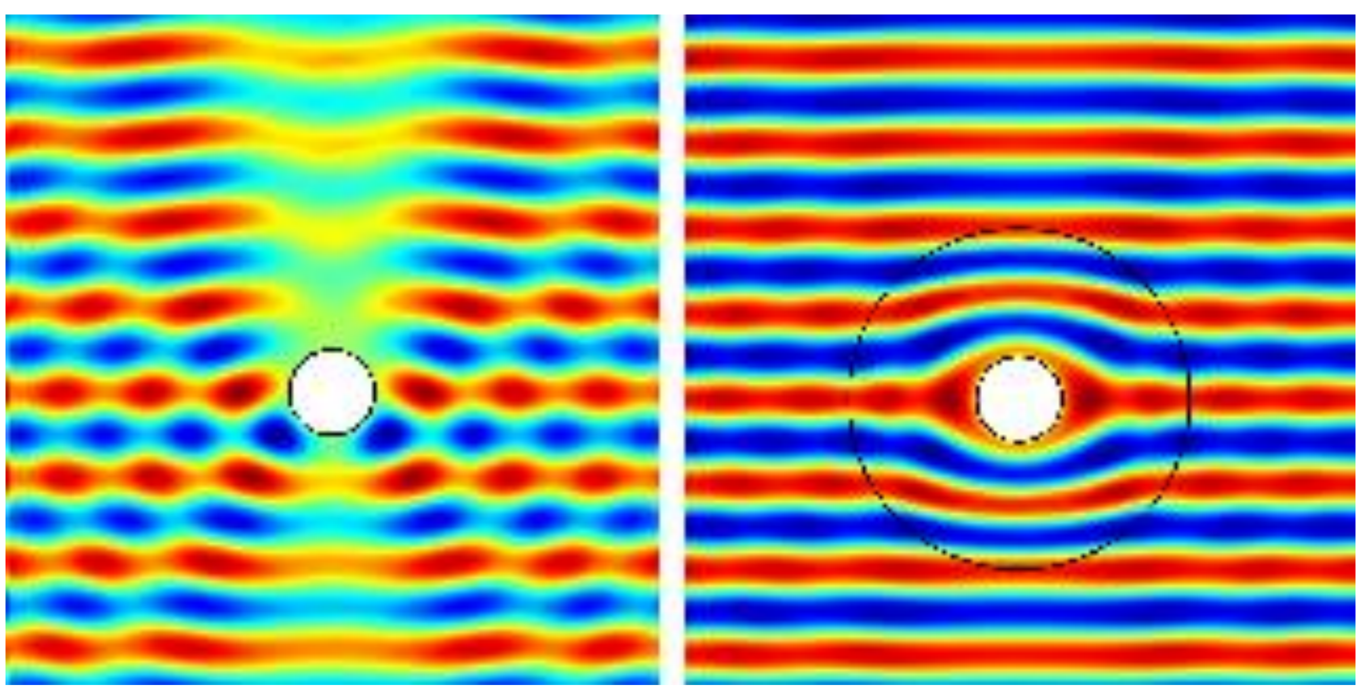
Evanescent waves are also imaged
-> possibility to have a sub-diffraction limit

6- Invisible cloak application

•*Spacial deformation (x,y) -> (u,v)*



•*Application to invisible cloak*



•*Equivalent to EM-properties modifications*

$$\varepsilon'_u = \varepsilon_u \frac{Q_u Q_v Q_w}{Q_u^2},$$

$$\mu'_u = \mu_u \frac{Q_u Q_v Q_w}{Q_u^2}, \quad \text{etc.}$$

$$E'_u = Q_u E_u, \quad H'_u = Q_u H_u, \quad \text{etc.}$$

where,

$$Q_u^2 = \left(\frac{\partial x}{\partial u}\right)^2 + \left(\frac{\partial y}{\partial u}\right)^2 + \left(\frac{\partial z}{\partial u}\right)^2$$

$$Q_v^2 = \left(\frac{\partial x}{\partial v}\right)^2 + \left(\frac{\partial y}{\partial v}\right)^2 + \left(\frac{\partial z}{\partial v}\right)^2$$

$$Q_w^2 = \left(\frac{\partial x}{\partial w}\right)^2 + \left(\frac{\partial y}{\partial w}\right)^2 + \left(\frac{\partial z}{\partial w}\right)^2$$

As usual,

$$\mathbf{B}' = \mu_0 \boldsymbol{\mu}' \mathbf{H}', \qquad \mathbf{D}' = \varepsilon_0 \boldsymbol{\varepsilon}' \mathbf{E}'$$

Conclusion

- *ε and μ are macroscopic descriptions of sub-λ behaviors*

- *Sub-λ structures can be achived to create metamaterials with new effective properties ε and μ (absorption, anisotropies, dispersion, etc.)*

- *Light can be manipulated using these metamaterials such as we already do with material*

- *Maxwell-equations have not finished to show us what we can do with light*

Thank you for your attention

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