

Cosmic Ray transport in magnetohydrodynamic turbulence

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Context

Transport from sources to earth

- MIS → complex fluid
 - Turbulent magnetised fluid (unknown properties)
 - Various phases
- Handling Cosmic Ray (CR) transport is a difficult task

How to proceed?

- Modeling turbulence : two possibilities
 - Model dependent (prescribed spectra, Casse & al, 2002)
 - Direct Numerical Simulation (DNS)
 - Decaying turbulence (Hennebelle, 2013)
 - Large scale forcing (Beresnyak, 2011; Yan & Lazarian, 2002...)
- Modeling transport
 - Leaky Box models (Gaisser, 1990)
 - Stochastic differential equation (diffusions coefficients prescribed) (Kruells & Achterberg, 1994)
 - Tracking experiment (Particule-in-Cells) (Beresnyak & al., 2011, ...)

Direct numerical simulation (DNS)

Ramses MHD

- MHD AMR code, in FORTRAN 90 and full Message Passing Interface (MPI) ready.
- Based on MUSCL-Hancock scheme, second order in both space and time.

History

- Created in 2002 by R. Teyssier, aim to cosmological simulation and self graviting objects (R. Tessier (2002))
- In 2006, Implementation of the induction equation by S. Fromang, P. Hennebelle et R. Teyssier (P. Hennebelle S. Fromang (2007))

Numerical simulation of CR transport

- This work is based on a fully dynamical large scale driven turbulence model. (Beresnyak & al. 2011, Wisniewski & al. 2012).
 - Driving turbulence → MHD Module: add a source term in both Euler and energy equations
- PIC Module: Solve Lorentz equation
 - Gather electromagnetic fields at particule position → CIC interpolation (first order)

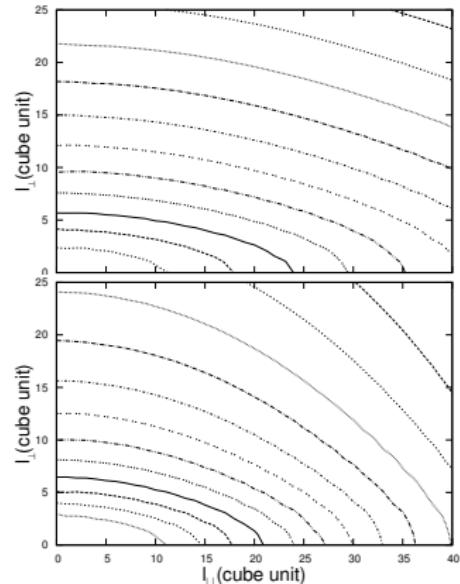
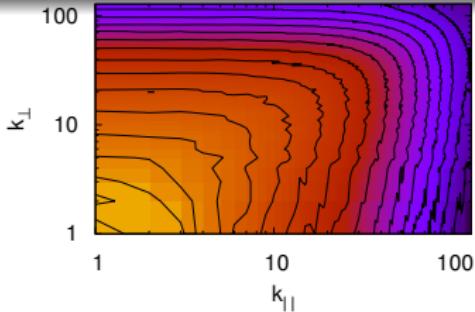
MHD turbulence properties

MHD Anisotropy

- Strong mean magnetic field $\mathbf{B}_0 \rightarrow$ 2D energy cascade
 - Opposite directed waves produce small perturbations perpendicular to $\mathbf{B}_0 \rightarrow$ Aflvénic turbulence (Goldreich & Sridhar, 1995)

Caracterisation

- 2D Fourier spectra
 - $k_{\perp} \propto k_{\parallel}^{3/2}$
- Second order structure function (Cho & al., 2000)
 - $s_2(\vec{r}) = \left\langle \left(\vec{u}(\vec{r} + \vec{r}') - \vec{u}(\vec{r}') \right)^2 \right\rangle$



MHD module

Forcing momentum field

$$\frac{\partial \rho u_i}{\partial t} + \partial_j (\rho u_j u_i - B_j B_i) + \partial_i P_{tot} = \rho f_i \quad i = x, y, z$$

$$\frac{\partial E}{\partial t} + \partial_i [(E + P_{tot}) u_i - B_i (B_j u_j)] = \rho f_i u_i \quad j = x, y, z$$

The Forcing term f_i is calculated from its Fourier spectrum

$$f_i = \sum_m \hat{f}_{i,m} \cos(2\pi k_{j,m} x_j)$$

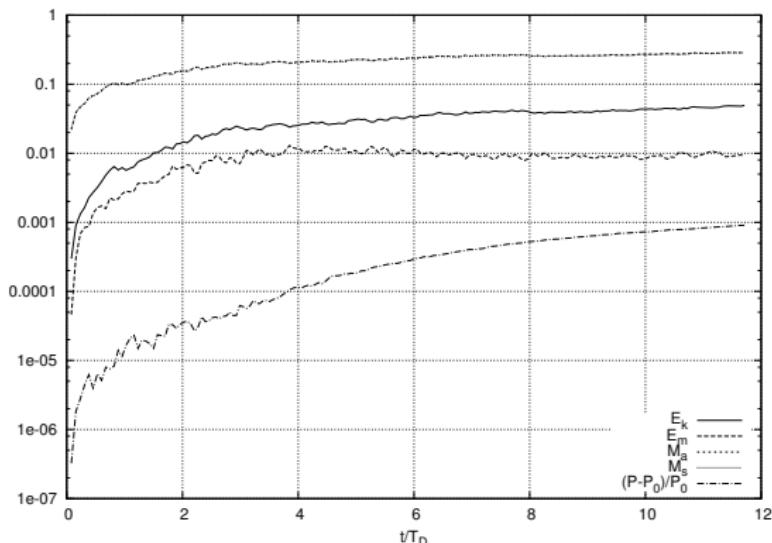
With a forcing range set by $2 < k < 2\sqrt{3}$

Stochastic differential process: Ornstein-Uhlenbeck process

Ornstein-Uhlenbeck process (Schmidt & al, 2008)

$$d\hat{f}_{i,m} = g_\chi \left[-\alpha \hat{f}_{i,m} \frac{dt}{T} + \beta \frac{c_s}{T} \sqrt{\frac{2w^2}{T}} P_{ij,m}^\chi d\xi_j \right]$$

Time evolution of MHD turbulence



Resolution 256^3

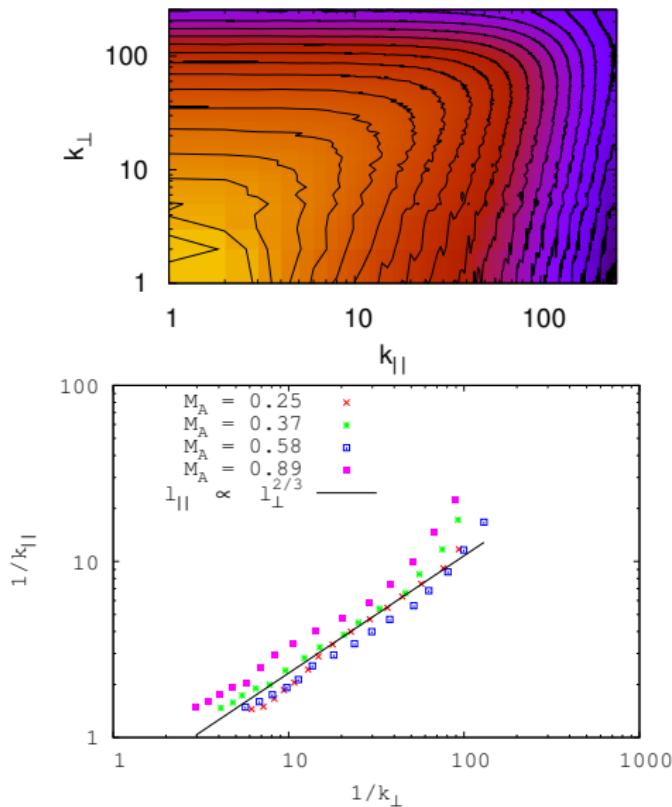
$$\chi = 0.5$$
$$T_D = \frac{L}{c_s} = 1.2$$

Stationnary Turbulence

$T \equiv T_D$ Dynamical Turbulent Time

- when $t = 4T_D$, turbulence is fully developed.
- Start characterisation.
- Particles can be injected.

2D kinetic spectrum

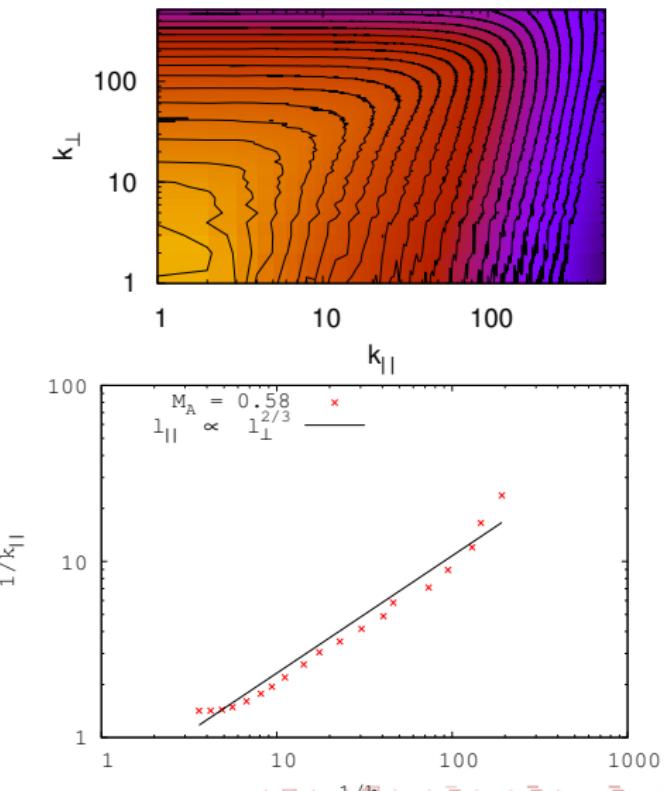


Kinetic energy spectrum
Resolution 512^3

$\chi = 0.5$

2D kinetic spectrum

Kinetic energy spectrum
Resolution 1024^3
 $\chi = 0.5$



Conclusion

Goldreich & Sridhar
scaling is well verified.

PIC module : Equations of motion

Main assumptions

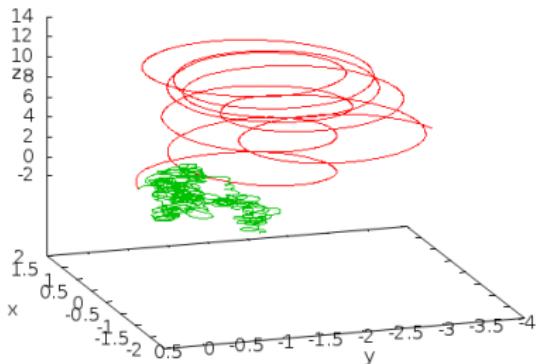
- particles tests approximation (no retroaction)
- Approximation $v_p \gg u_{mhd}$ (snapshot)
- Magnetostatic Approximation $\mathbf{E} = 0$

Parameters and equations

Solved Equations :

- $\frac{d\hat{x}_i}{d\tau} = \frac{\hat{v}_i}{\hat{\gamma}}$
- $\frac{d\hat{v}_i}{d\tau} = \epsilon^{ijk} \frac{\hat{v}_j}{\hat{\gamma}} \mathbf{b}_k$

Runge Kutta 4th order quality controlled implemented



Mean free path and diffusive regime

 λ_{\parallel}

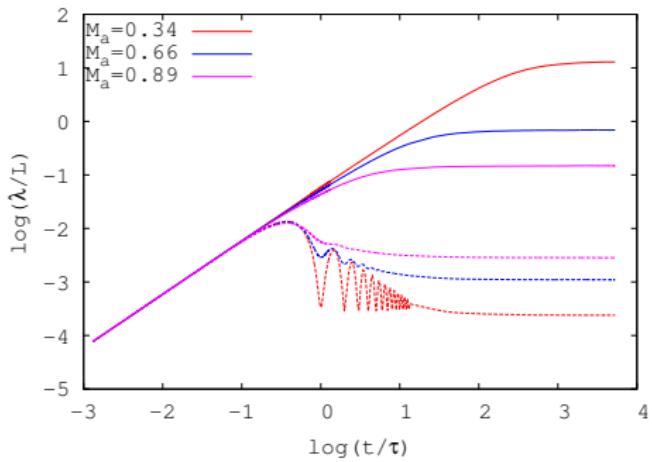
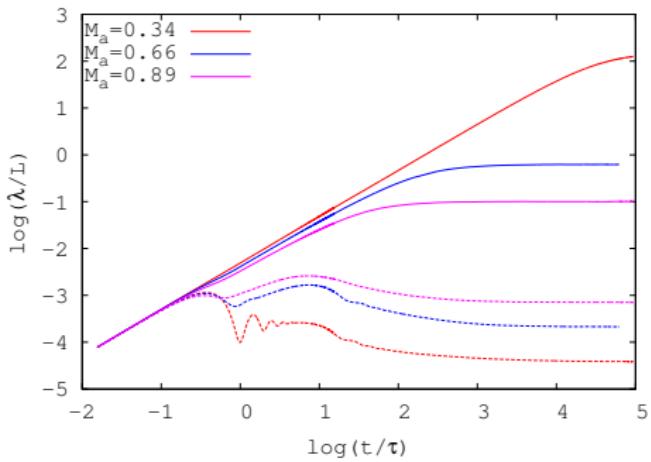
$$\lambda_{\parallel} = \frac{3D_{\parallel}}{v} = \frac{3}{v} \frac{\langle (z(t) - z(0))^2 \rangle}{4\Delta t}$$

 λ_{\perp}

$$\lambda_{\perp} = \frac{3D_{\perp}}{v} = \frac{3}{v} \frac{\langle (x(t) - x(0))^2 + (y(t) - y(0))^2 \rangle}{2\Delta t}$$

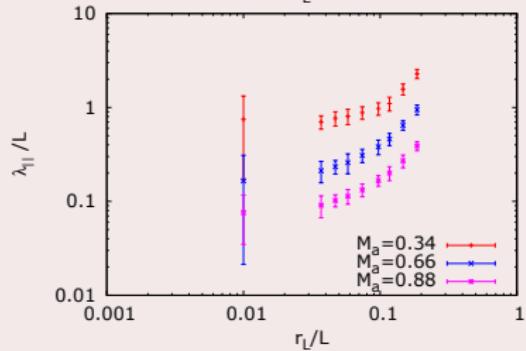
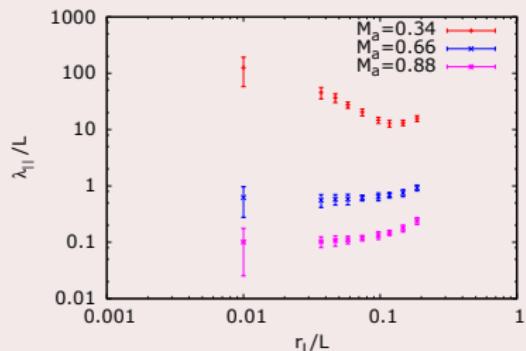
$$\frac{r_L}{L} = 0.01$$

$$\frac{r_L}{L} = 0.117$$

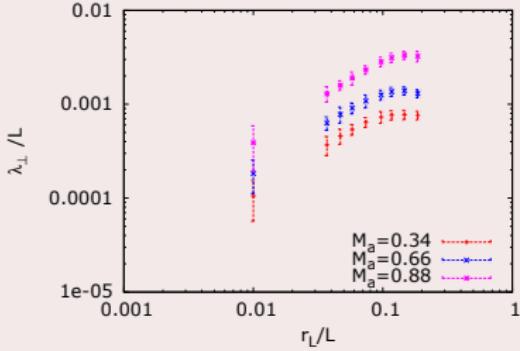
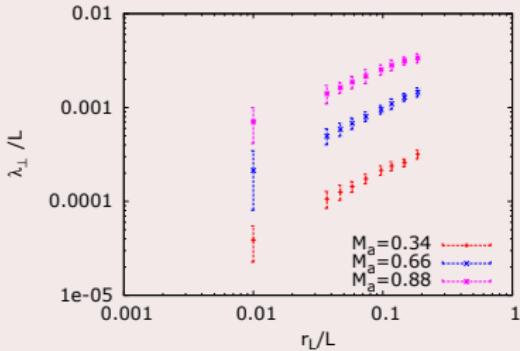


Mean free path and energy dependencies

λ_{\parallel}

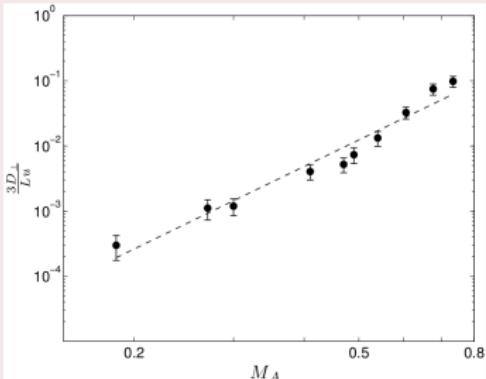


λ_{\perp}



Comparison with previous results

Perpendicular MFP with M_a : Xu & Yan, 2013



Parameters

$$r_L/L = 0.01$$

$\chi = 1$ (solenoidal forcing)

Produced a power-law

$$\lambda_{\perp} \propto M_a^{4.21 \pm 0.75}$$

Our work produced a χ -dependent power law

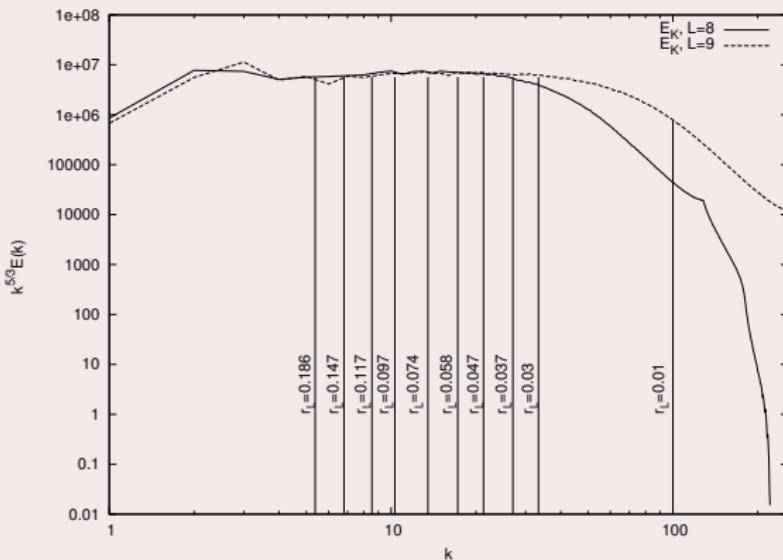
$\chi = 0$ Compressive forcing

- $r_L/L = 0.01 \rightarrow \lambda_{\perp} \propto M_a^{1.36 \pm 1.15}$
- $r_L/L = 0.037 \rightarrow \lambda_{\perp} \propto M_a^{1.32 \pm 0.44}$

$\chi = 1$ Solenoidal forcing

- $r_L/L = 0.01 \rightarrow \lambda_{\perp} \propto M_a^{3.0 \pm 1.0}$
- $r_L/L = 0.037 \rightarrow \lambda_{\perp} \propto M_a^{2.58 \pm 0.35}$

Comparison with previous results



$\chi = 0$ Compressive forcing

- $r_L/L = 0.01 \rightarrow \lambda_\perp \propto M_a^{1.36 \pm 1.15}$
- $r_L/L = 0.037 \rightarrow \lambda_\perp \propto M_a^{1.32 \pm 0.44}$

$\chi = 1$ Solenoidal forcing

- $r_L/L = 0.01 \rightarrow \lambda_\perp \propto M_a^{3.0 \pm 1.0}$
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Conclusions

Mean free path and energy dependencies

We showed two regions.

- $r_L < 0.1L$
 - $\lambda_{\perp} \rightarrow \propto r_L \forall \{\chi, M_a\}$
 - $\lambda_{\parallel} \rightarrow$ strong energy and Alfvénic Mach number dependencies
- $r_L > 0.1L$
 - λ_{\perp} is χ dependent
 - λ_{\parallel} is M_a dependent

→ The forcing geometry may be a way to investigate some ISM phases

Problem : The forcing module does not (yet!) provide an imbalanced forcing

Next steps

- Investigate pitch angle diffusion
- Include stochastic acceleration (cancel the magnetostatic assumption)
- Include radiative losses and back reaction on fields