

in**v**isibles

# One loop effective non-linear Lagrangian with a light H boson

B. Gavela, KK, P. Machado, S. Saa

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**Kirill Kanshin  
Padova Uni / INFN**

**Rencontres de Moriond  
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# Non-linear effective Lagrangian aka Electroweak Chiral Lagrangian

For linear effective Lagrangian see F. Riva's talk.

For some class of UV completions of the scalar sector of the SM, in particular for strongly interacting ones, EFT with nonlinearly realized EW symmetry might be more appropriate than with linear one.

Building blocks:

- 1) Dimensionless matrix of SM goldstones  $\mathbf{U}(\boldsymbol{\pi})$ , nonlinear under  $SU_L(2) \times U(1)$ ,
- 2) Scalar  $h$ , as a singlet of EW symmetry group, entering as generic polynomials  $\mathcal{F}(h)$ .

$$\mathcal{L}_0 = -\lambda_1 v^3 h - \frac{1}{2} m_h^2 h^2 - \frac{\lambda_3}{3!} v h^3 - \frac{\lambda_4}{4!} h^4$$

$$\mathcal{L}_2 = \frac{v^2}{4} \text{Tr}[\partial_\mu \mathbf{U} \partial^\mu \mathbf{U}^\dagger] \mathcal{F}_C(h) + \frac{1}{2} \partial_\mu h \partial^\mu h \mathcal{F}_H(h)$$

Similar language to Chiral Lagrangian in QCD, but now with scalar. The SM case is recovered for some particular choice of parameters.

# Renormalizability?

1. ChL is renormalizable order by order, finite number of counterterms required at each step.
2. On shell these counterterms obey the symmetries.
3. Loop calculation is a consistency check of NLO Lagrangian.

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More operators in:

Contino et al. '10

Alonso, Gavela, Merlo, Rigolin, and Yepes '12

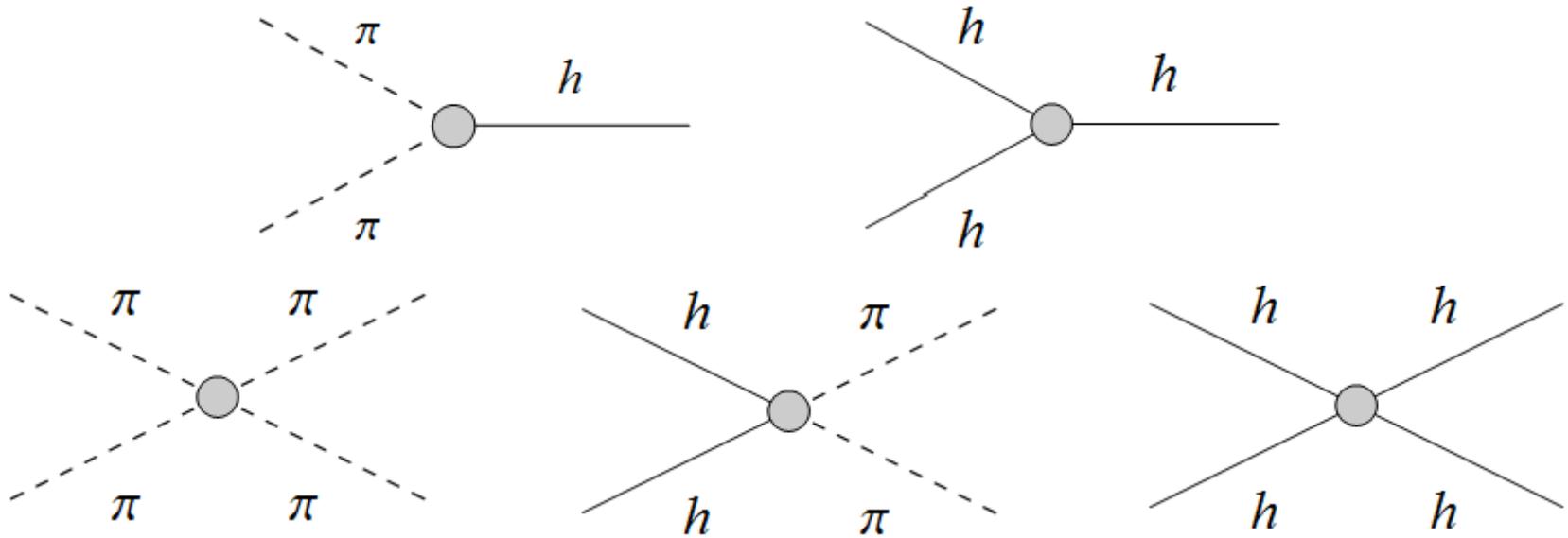
Buchalla, Catà, Krause '13

$$\mathbf{V}_\mu = (D_\mu \mathbf{U}) \mathbf{U}^\dagger$$

$$\mathcal{F}_i(h) = 1 + 2a_i h/v + b_i h^2/v^2$$

$$\begin{aligned} \mathcal{L}_4 = & (\text{Tr}[\mathbf{V}_\mu \mathbf{V}^\mu])^2 c_6 \mathcal{F}_6(h) + (\text{Tr}[\mathbf{V}_\mu \mathbf{V}_\nu])^2 c_{11} \mathcal{F}_{11}(h) + \text{Tr}[\mathcal{D}_\mu \mathbf{V}^\mu \mathcal{D}_\nu \mathbf{V}^\nu] c_9 \mathcal{F}_9(h) + \\ & + \text{Tr}[\mathbf{V}_\nu \mathcal{D}_\mu \mathbf{V}^\mu] \frac{\partial^\nu h}{v} c_{10} \mathcal{F}_{10}(h) + \\ & + \text{Tr}[\mathbf{V}_\mu \mathbf{V}^\mu] \left( \frac{\square h}{v} c_7 \mathcal{F}_7(h) + \frac{\partial_\nu h \partial^\nu h}{v^2} c_{20} \mathcal{F}_{20}(h) \right) + \text{Tr}[\mathbf{V}_\mu \mathbf{V}_\nu] \frac{\partial^\mu h \partial^\nu h}{v^2} c_8 \mathcal{F}_8(h) + \\ & + \frac{\square h \square h}{v^2} c_{\square H} \mathcal{F}_{\square H}(h) + \frac{\square h \partial_\mu h \partial^\mu h}{v^3} c_{h2} \mathcal{F}_{h2}(h) + \frac{(\partial_\mu h \partial^\mu h)^2}{v^4} c_{DH} \mathcal{F}_{DH}(h) \end{aligned}$$

# Off shell renormalization, only div parts



On shell calculations of these types:

...

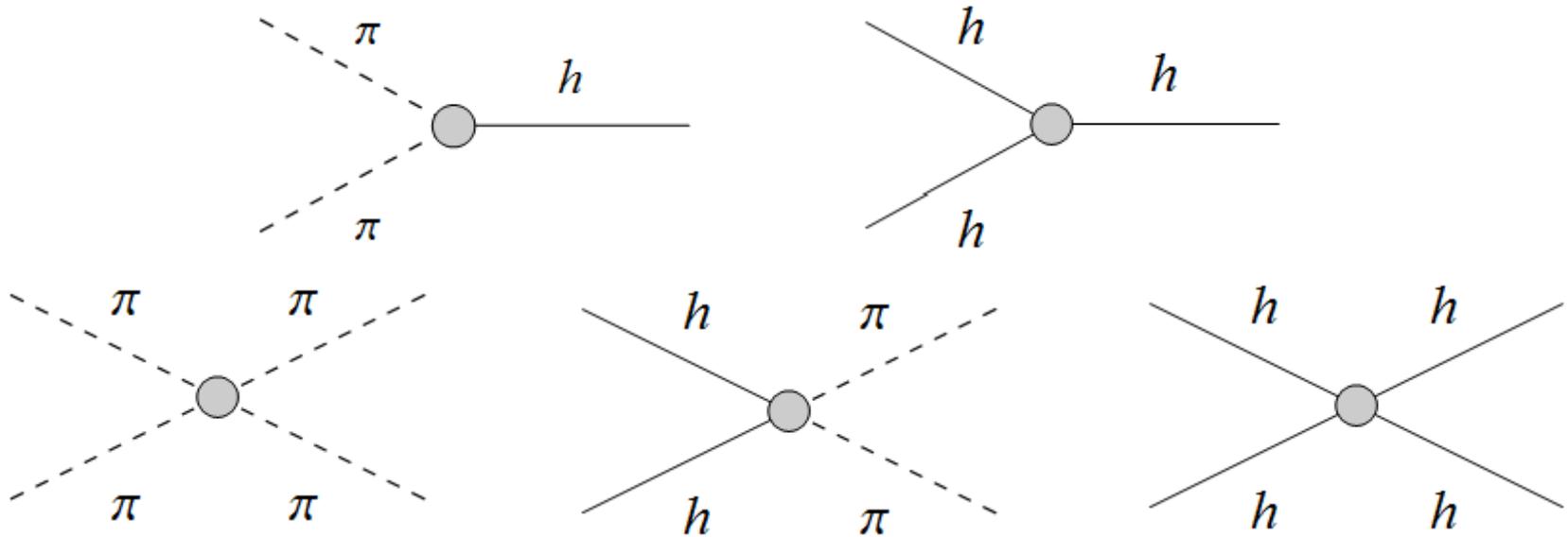
Delgado, Dobado, and Llanes-Estrada '13

Esprui, Mescia, Yencho '13

Delgado, Dobado, Herrero, and Sanz-Cillero '14

...

## Off shell renormalization, only div parts



1.  $L_{0,2,4}$  parameters renormalized as  $x_b = x_{\text{ren}} + \delta x$ , all  $\delta x$  have been defined
2. Off-shell **non-chiral-invariant divergences** are generated!!!  
Appelquist, Bernard '81
3. They cannot be absorbed into  $L_{0,2,4}$  parameters.
4. But no impact on-shell.

# Field redefinition and Non Invariant Divergences (NID)

$$\pi \rightarrow \pi + \delta\pi, \quad \mathcal{L} \rightarrow \mathcal{L} + \delta\pi \left( \frac{\delta\mathcal{L}}{\delta\pi} - \partial_\mu \frac{\delta\mathcal{L}}{\delta\partial_\mu\pi} \right)$$

Field redefinition generate additional piece in Lagrangian, which is proportional to EOM.  
It vanish if EOM is satisfied.

Ostrogradskiy 1850; Grosse-Knetter '94; Scherer, Fearing '94; Arzt '95

# Field redefinition and NID

$$\pi_i \rightarrow \pi_i \left( 1 + \frac{\alpha_1}{v^4} \boldsymbol{\pi} \square \boldsymbol{\pi} + \frac{\alpha_2}{v^4} \partial_\mu \boldsymbol{\pi} \partial^\mu \boldsymbol{\pi} + \frac{\beta}{v^3} \square h + \frac{\tilde{\gamma}_1}{v^4} h \square h + \frac{\gamma_2}{v^4} \partial_\mu h \partial^\mu h \right) + \frac{\alpha_3}{v^4} \square \pi_i (\boldsymbol{\pi} \boldsymbol{\pi}) + \frac{\alpha_4}{v^4} \partial_\mu \pi_i (\boldsymbol{\pi} \partial^\mu \boldsymbol{\pi}),$$

# Field redefinition and NID

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$$\mathbf{U} = 1 - \frac{\boldsymbol{\pi}^2}{2v^2} - \left( \eta + \frac{1}{8} \right) \frac{\boldsymbol{\pi}^4}{v^4} + \frac{i\boldsymbol{\tau} \boldsymbol{\pi}}{v} \left( 1 + \eta \frac{\boldsymbol{\pi}^2}{v^2} \right) + O(\boldsymbol{\pi}^5)$$

$$\begin{aligned} \eta = 0 &\Rightarrow \mathbf{U} = \sqrt{1 - \boldsymbol{\pi}^2/v^2} + i\boldsymbol{\tau} \boldsymbol{\pi}/v \\ \eta = -1/6 &\Rightarrow \mathbf{U} = e^{i\boldsymbol{\tau} \boldsymbol{\pi}/v} \end{aligned}$$

# Field redefinition and NID

$$\pi_i \rightarrow \pi_i \left( 1 + \frac{\alpha_1}{v^4} \pi \square \pi + \frac{\alpha_2}{v^4} \partial_\mu \pi \partial^\mu \pi + \frac{\beta}{v^3} \square h + \frac{\tilde{\gamma}_1}{v^4} h \square h + \frac{\gamma_2}{v^4} \partial_\mu h \partial^\mu h \right) + \frac{\alpha_3}{v^4} \square \pi_i (\pi \pi) + \frac{\alpha_4}{v^4} \partial_\mu \pi_i (\pi \partial^\mu \pi),$$

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$$\eta = -1/6 \Rightarrow \mathbf{U} = e^{i\tau\pi/v}$$

$$\begin{aligned} \alpha_1 &= \left( 9\eta^2 + 5\eta + \frac{3}{4} \right) \Delta_\varepsilon & \gamma_1 &= \left( 5\eta + \frac{3}{2} \right) (2a_C^2 - b_C) \Delta_\varepsilon \\ \alpha_2 &= \left[ (a_C^2 + 4)\eta + \frac{a_C^2}{2} + 1 \right] \Delta_\varepsilon & \gamma_2 &= \left( 5\eta + \frac{3}{2} \right) (a_C^2 - b_C) \Delta_\varepsilon \\ \alpha_3 &= 2\eta^2 \Delta_\varepsilon & \beta &= - \left( 5\eta + \frac{3}{2} \right) a_C \Delta_\varepsilon \\ \alpha_4 &= 2(a_C^2 - 1)\eta \Delta_\varepsilon \end{aligned}$$

**Only non physical parameters depend on  $\eta$ .**

# Field redefinition and NID

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$$\alpha_1 = \left( 9\eta^2 + 5\eta + \frac{3}{4} \right) \Delta_\epsilon$$

$$\alpha_2 = \left[ (a_C^2 + 4) \eta + \frac{a_C^2}{2} + 1 \right] \Delta_\epsilon$$

$$\alpha_3 = 2\eta^2 \Delta_\epsilon$$

$$\alpha_4 = 2(a_C^2 - 1)\eta \Delta_\epsilon$$

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$$\beta = - \left( 5\eta + \frac{3}{2} \right) a_C \Delta_\epsilon$$

$\eta = -3/10$  eliminate parameters of pion-through-higgs field redefinition.

(to our knowledge) it does not correspond to any known U-matrix parameterization.

# Summary

1. We performed one loop renormalization of scalar sector of EWChL with a scalar
2. Identified the complete set of 10 NLO operators needed
3. Non-chiral-invariant divergences have been eliminated by field redefinition
4. Some of parameters may have significantly large coefficients of beta-functions in RGE