

ON THE ORIGIN OF SCALES AND INFLATION

**Models of all interactions
in the absence of fundamental scales (agravity)**

Alberto Salvio



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March ..., 2015

Based on

- ▶ Salvio, Strumia, [arXiv:1403.4226](https://arxiv.org/abs/1403.4226) (JHEP)
- ▶ Kannike, Hütsi, Pizza, Racioppi, Raidal, Salvio, Strumia, [arXiv:1502.01334](https://arxiv.org/abs/1502.01334)
- ▶ Giudice, Isidori, Salvio, Strumia [arXiv:1412.2769](https://arxiv.org/abs/1412.2769)

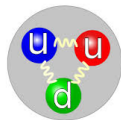
Main motivations for agravity

Motivation 1: naturalness

*Agravity provides an alternative solution of the hierarchy problem:
there are no Λ^2 contributions because there are no masses.*

This in turn leads to dynamical generation of masses

Like for the proton: its mass is mostly dynamical generated



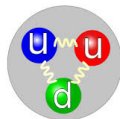
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Motivation 2: inflation

Cosmological observations suggest inflation. However, it requires flat potentials.

What is the reason for this flatness? Agravity gives us an explanation:

The Einstein frame potential of a scalar s in agravity is

$$U(s) = \frac{\lambda_S s^4}{(2\xi_S s^2)^2} \bar{M}_{\text{Pl}}^4 = \frac{\lambda_S}{4\xi_S^2} \bar{M}_{\text{Pl}}^4$$

The potential is flat at tree-level, but at quantum level λ_S and ξ_S depend on s
this effect (due to the RGEs) gives some slope ... which is small if couplings are perturbative

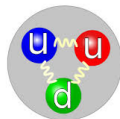
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what we need to have inflation!

Agravity scenario

The most general agravity Lagrangian:

$$\mathcal{L} = \frac{R^2}{6f_0^2} + \frac{\frac{1}{3}R^2 - R_{\mu\nu}^2}{f_2^2} + \mathcal{L}_{\text{SM}}^{\text{adim}} + \mathcal{L}_{\text{BSM}}^{\text{adim}}$$

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Non-gravitational sectors

- ▶ $\mathcal{L}_{\text{SM}}^{\text{adim}}$ is the no-scale part of the SM Lagrangian (without $m^2|H|^2/2$):

$$\mathcal{L}_{\text{SM}}^{\text{adim}} = -\frac{F_{\mu\nu}^2}{4} + \bar{\psi}i\not{D}\psi + |D_\mu H|^2 - (yH\psi\psi + \text{h.c.}) - \lambda_H|H|^4 - \xi_H|H|^2 R$$

- ▶ $\mathcal{L}_{\text{BSM}}^{\text{adim}}$ describes physics beyond the SM (BSM). It generates the weak scale

$$\text{adding a scalar } s \rightarrow \mathcal{L}_{\text{BSM}}^{\text{adim}} = \dots + \lambda_{HS}s^2|H|^2/2 - \xi_S s^2 R/2$$

vectors in the s -sector can be dark matter [*Hambye, Strumia (2013)*]

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Gravity sector

- ▶ $\langle s \rangle$ generates \bar{M}_{Pl}^2 : $\xi_S s^2 R \rightarrow \bar{M}_{\text{Pl}}^2 = \xi_S |\langle s \rangle|^2$
- ▶ Agravity is renormalizable, however, looking at the spectrum:
 - massless graviton
 - scalar z with mass $M_0^2 \sim \frac{1}{2}f_0^2 \bar{M}_{\text{Pl}}^2$
 - massive graviton with mass $M_2^2 = \frac{1}{2}f_2^2 \bar{M}_{\text{Pl}}^2$ and negative norm, but with energy bounded from below

Quantum corrections

They are mostly encoded in the RGEs

They are important to obtain n_s and r and to dynamically generate \bar{M}_{Pl} and m

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▶ We computed the 1-loop RGEs for all couplings of the most general agravity

$$\frac{R^2}{6f_0^2} + \frac{\frac{1}{3}R^2 - R_{\mu\nu}^2}{f_2^2} - \frac{(F_{\mu\nu}^A)^2}{4} + \frac{(D_\mu \phi_a)^2}{2} - \frac{\xi_{ab}}{2} \phi_a \phi_b R - \frac{\lambda_{abcd}}{4!} \phi_a \phi_b \phi_c \phi_d + \bar{\psi}_j i \not{D} \psi_j - Y_{ij}^a \psi_i \psi_j \phi_a + \text{h.c.}$$

Without gravity this was done before
[Machacek and Vaughn (1983,1984,1985)]

Dynamical generation of the Planck scale

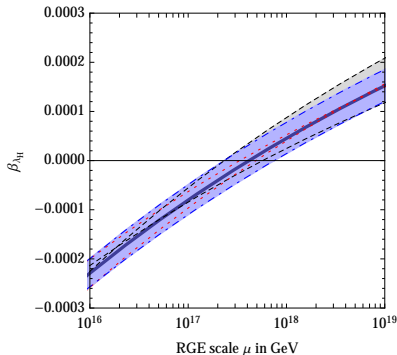
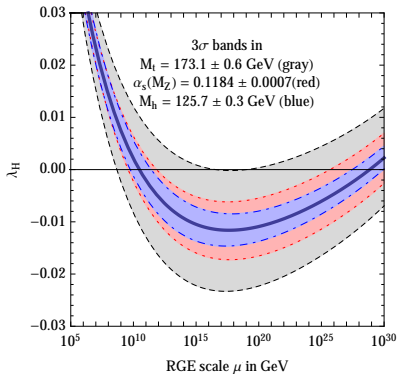
Agravity successfully generates the Planck scale if

$$\left\{ \begin{array}{ll} \lambda_S(s) \simeq 0 & \leftrightarrow \text{nearly vanishing cosmological constant (dark energy)} \\ \frac{d\lambda_S}{ds}(s) = 0 & \leftrightarrow \text{minimum condition} \\ \xi_S(s)s^2 = \bar{M}_{\text{Pl}}^2 & \leftrightarrow \text{observed Planck mass} \end{array} \right.$$

Is the dynamical generation of the Planck scale possible?

Are these conditions realized in the physics we know (the SM)?

RGE running of the $\overline{\text{MS}}$ quartic Higgs coupling in the SM



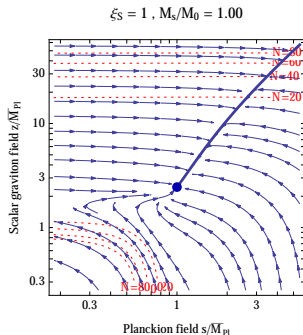
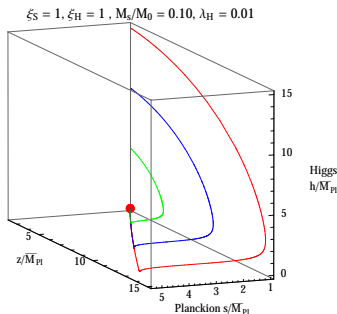
Thus the dynamical generation of the Planck scale is possible!

Predictions for inflation (generically a multifield inflation)

The minimal realistic model has at least 3 scalars:

the SM scalar h
the Planckion s
the graviscalar z

M_s = mass of s
 M_0 = mass of z

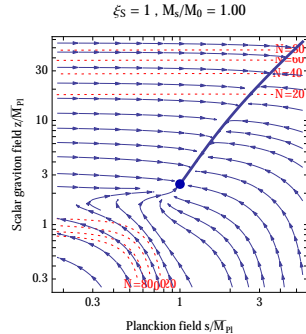
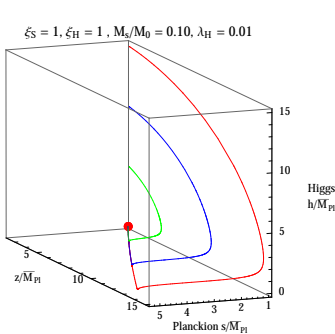


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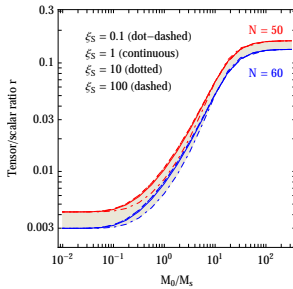
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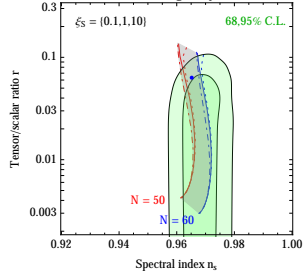


Predictions of agravity inflation



- left: when $M_s \ll (\gg) M_0$, the inflaton is s (z)
- right: comparison with a global fit of PLANCK and BICEP2/KECK

Predictions of agravity inflation



Natural dynamical generation of the weak scale

1) Low energies ($\mu < M_{0,2}$): gravity can be neglected and the SM RGE apply:

$$(4\pi)^2 \frac{dm^2}{d \ln \mu} = m^2 \beta_m^{\text{SM}}, \quad \beta_m^{\text{SM}} = 12\lambda_H + 6y_t^2 - \frac{9g_2^2}{2} - \frac{9g_1^2}{10}$$

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2) Intermediate energies ($M_{0,2} < \mu < \bar{M}_{\text{Pl}}$): Both m and \bar{M}_{Pl} appear and we find

$$(4\pi)^2 \frac{d}{d \ln \mu} \frac{m^2}{\bar{M}_{\text{Pl}}^2} = -\xi_H [5f_2^4 + f_0^4 (1 + 6\xi_H)] + \dots$$

The **red term** is a non-multiplicative potentially dangerous correction to m

$$m^2 \sim \bar{M}_{\text{Pl}}^2 f_{0,2}^4, \quad \text{naturalness} \rightarrow f_0, f_2 \sim \sqrt{\frac{4\pi m}{M_{\text{Pl}}}} \sim 10^{-8}$$

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- 3) Large energies ($\mu > \bar{M}_{\text{Pl}}$):

$$\lambda_{HS} |H|^2 s^2 \rightarrow m^2 = \lambda_{HS} \langle s \rangle^2$$

λ_{HS} can be naturally small (looking at the RGE of λ_{HS}):

$$\rightarrow \lambda_{HS} \sim f_{0,2}^4$$

Natural weak scale and unification

It is possible to preserve a natural weak scale if

- ▶ a semi-simple gauge group is used: e.g. the Pati-Salam $SU(4) \times SU(2) \times SU(2)$
 - ▶ all SM Landau poles are removed (we require this as we want to go up to infinite energy and in the SM the experiments tell us that e.g. g_Y diverges at 10^{42} GeV)
- ▶ Models of this type have been found and predict a lot of new physics not far above the weak scale
(In the SM the elimination of such poles requires unrealistic conditions: $g_Y = 0$, ...)

Conclusions

- ▶ *Naturalness and a rationale for inflation can be achieved in no-scale theories of all interactions (including gravity): agravity*
- ▶ *Inflation: the minimal realistic model predicts $n_s \approx 0.967$, $0.003 < r < 0.13$, in agreement with PLANCK and BICEP2/KECK. KECK/BICEP3 may give us more constraints on this scenario.*
- ▶ *Naturalness is also compatible with unification. SM Landau poles can also be eliminated. In this case there is new physics not far above the weak scale (e.g. W').*

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THANK YOU VERY MUCH
FOR YOUR ATTENTION!

Extra slides

Ghosts

Negative literature [*Ostrogradski (1850), Smilga (2009), ...*]

- ▶ Classically the energy is not bounded from below (Ostrogradski instability)
- ▶ At quantum level creation of negative energy \sim destruction of positive energy: the Hamiltonian becomes positive, but some states (“ghosts”) have negative norm

Positive literature

- ▶ [*Lee, Wick (1969)*] the introduction of negative norms can lead to a unitary S-matrix, provided that all stable particle states have positive norm
- ▶ [*Hawking, Hertog (2001)*] at least in a simple scalar field ϕ theory, the problem comes from regarding ϕ and $\square\phi$ as independent and can be overcome by using the path integral, where they are dependent.

Results for RGEs

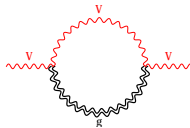
Gauge couplings

Their contributions to the RGEs cancel!

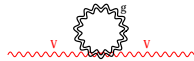
This was previously noticed in
[Narain, Anishetty (2013)]

Possible explanation:
 the graviton is not charged

Possible new gravity contributions



(Rainbow)

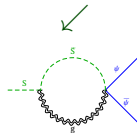
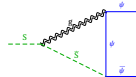
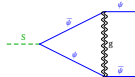
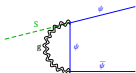


(Seagull)

Yukawa couplings

We find the one-loop RGE (where $C_{2F} \equiv t^A t^A$ and $t^A \equiv$ "fermion gauge generators"):

$$(4\pi)^2 \frac{dY^a}{d \ln \mu} = \frac{1}{2} (Y^{\dagger b} Y^b Y^a + Y^a Y^{\dagger b} Y^b) + 2Y^b Y^{\dagger a} Y^b + Y^b \text{Tr}(Y^{\dagger b} Y^a) - 3\{C_{2F}, Y^a\} + \frac{15}{8} f_2^2 Y^a$$



All remaining RGEs

We also computed the RGEs for

▸ λ_{abcd}

▸ ξ_{ab}

▸ f_0 and f_2

RGEs for the quartic couplings

Tens of Feynman diagrams contribute to these RGEs ... we obtain

$$(4\pi)^2 \frac{d\lambda_{abcd}}{d \ln \mu} = \sum_{\text{perms}} \left[\frac{1}{8} \lambda_{abef} \lambda_{efcd} + \frac{3}{8} \{\theta^A, \theta^B\}_{ab} \{\theta^A, \theta^B\}_{cd} - \text{Tr} Y^a Y^{\dagger b} Y^c Y^{\dagger d} + \right. \\ \left. + \frac{5}{8} f_2^4 \xi_{ab} \xi_{cd} + \frac{f_0^4}{8} \xi_{ae} \xi_{cf} (\delta_{eb} + 6\xi_{eb}) (\delta_{fd} + 6\xi_{fd}) \right. \\ \left. + \frac{f_0^2}{4!} (\delta_{ae} + 6\xi_{ae}) (\delta_{bf} + 6\xi_{bf}) \lambda_{efcd} \right] + \lambda_{abcd} \left[\sum_k (Y_2^k - 3C_{2S}^k) + 5f_2^2 \right],$$

where the first sum runs over the 4! permutations of $abcd$ and the second sum over $k = \{a, b, c, d\}$, with Y_2^k and C_2^k defined by

$$\text{Tr}(Y^{\dagger a} Y^b) = Y_2^a \delta^{ab}, \quad \theta_{ac}^A \theta_{cb}^A = C_{2S}^a \delta_{ab}$$

(θ^A are the scalar gauge generators)

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RGEs for the quartic couplings: SM case

For the SM H plus the complex scalar singlet S the RGEs become:

$$\begin{aligned}(4\pi)^2 \frac{d\lambda_S}{d\ln\mu} &= 20\lambda_S^2 + 2\lambda_{HS}^2 + \frac{\xi_S^2}{2} [5f_2^4 + f_0^4(1 + 6\xi_S)^2] + \lambda_S [5f_2^2 + f_0^2(1 + 6\xi_S)^2] \\(4\pi)^2 \frac{d\lambda_{HS}}{d\ln\mu} &= -\xi_H\xi_S [5f_2^4 + f_0^4(6\xi_S + 1)(6\xi_H + 1)] - 4\lambda_{HS}^2 + \lambda_{HS} \{8\lambda_S + 12\lambda_H + 6y_t^2 \\&\quad + 5f_2^2 + \frac{f_0^2}{6} [(6\xi_S + 1)^2 + (6\xi_H + 1)^2 + 4(6\xi_S + 1)(6\xi_H + 1)] \} \\(4\pi)^2 \frac{d\lambda_H}{d\ln\mu} &= \frac{9}{8}g_2^4 + \frac{9}{20}g_1^2g_2^2 + \frac{27}{200}g_1^4 - 6y_t^4 + 24\lambda_H^2 + \lambda_{HS}^2 + \frac{\xi_H^2}{2} [5f_2^4 + f_0^4(1 + 6\xi_H)^2] \\&\quad + \lambda_H \left(5f_2^2 + f_0^2(1 + 6\xi_H)^2 + 12y_t^2 - 9g_2^2 - \frac{9}{5}g_1^2 \right).\end{aligned}$$

▶ back to main slides

RGEs for the scalar/graviton couplings

Complicated calculation (but computer algebra helps!)

$$(4\pi)^2 \frac{d\xi_{ab}}{d \ln \mu} = \frac{1}{6} \lambda_{abcd} (6\xi_{cd} + \delta_{cd}) + (6\xi_{ab} + \delta_{ab}) \sum_k \left[\frac{Y_2^k}{3} - \frac{C_{2S}^k}{2} \right] + \\ - \frac{5f_2^4}{3f_0^2} \xi_{ab} + f_0^2 \xi_{ac} \left(\xi_{cd} + \frac{2}{3} \delta_{cd} \right) (6\xi_{db} + \delta_{db})$$

For the SM H plus the complex scalar singlet S the RGEs become:

$$(4\pi)^2 \frac{d\xi_S}{d \ln \mu} = (1 + 6\xi_S) \frac{4}{3} \lambda_S - \frac{2\lambda_{HS}}{3} (1 + 6\xi_H) + \frac{f_0^2}{3} \xi_S (1 + 6\xi_S) (2 + 3\xi_S) - \frac{5}{3} \frac{f_2^4}{f_0^2} \xi_S \\ (4\pi)^2 \frac{d\xi_H}{d \ln \mu} = (1 + 6\xi_H) (2y_t^2 - \frac{3}{4} g_2^2 - \frac{3}{20} g_1^2 + 2\lambda_H) - \frac{\lambda_{HS}}{3} (1 + 6\xi_S) + \\ + \frac{f_0^2}{3} \xi_H (1 + 6\xi_H) (2 + 3\xi_H) - \frac{5}{3} \frac{f_2^4}{f_0^2} \xi_H$$

RGE for the gravitational couplings

Huge calculation ... (computer algebra practically needed!!)

$$(4\pi)^2 \frac{df_2^2}{d \ln \mu} = -f_2^4 \left(\frac{133}{10} + \frac{N_V}{5} + \frac{N_f}{20} + \frac{N_s}{60} \right)$$
$$(4\pi)^2 \frac{df_0^2}{d \ln \mu} = \frac{5}{3} f_2^4 + 5 f_2^2 f_0^2 + \frac{5}{6} f_0^4 + \frac{f_0^4}{12} (\delta_{ab} + 6\xi_{ab})(\delta_{ab} + 6\xi_{ab})$$

Here N_V , N_f , N_s are the number of vectors, Weyl fermions and real scalars.

In the SM $N_V = 12$, $N_f = 45$, $N_s = 4$.

We confirmed the calculations of *[Avramidi (1995)]*
rather than those of *[Fradkin and Tseytlin (1981,1982)]*

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Agravity inflation

All scalar fields in agravity are inflaton candidates

Agravity inflation

All scalar fields in agravity are inflaton candidates

example (the minimal model): h , the Planckion s , the scalar σ in $g_{\mu\nu}$

To see σ

$$\frac{R^2}{6f_0^2} \rightarrow \frac{R^2}{6f_0^2} - \underbrace{\frac{(R + 3f_0^2\sigma/2)^2}{6f_0^2}}_{\text{zero on-shell}}$$

By redefining $g_{\mu\nu}^E = g_{\mu\nu} \times f / \bar{M}_{\text{Pl}}^2$ with $f = \xi_S s^2 + \xi_H h^2 + \sigma$ one obtains ...

$$\sqrt{|\det g_E|} \left\{ \frac{\bar{M}_{\text{Pl}}^2}{2} R_E + \bar{M}_{\text{Pl}}^2 \left[\frac{(\partial_\mu s)^2 + (\partial_\mu h)^2}{2f} + \frac{3(\partial_\mu f)^2}{4f^2} \right] - U \right\} + \dots$$

as well as their effective potential:

$$U = \frac{\bar{M}_{\text{Pl}}^4}{f^2} \left(V + \frac{3f_0^2}{8} \sigma^2 \right)$$

Agravity inflation: a simple single field case

We identify inflaton = s by taking the other scalar fields heavy ...

Then we can easily convert s into a scalar s_E with canonical kinetic term and find

$$\epsilon \equiv \frac{\bar{M}_{\text{Pl}}^2}{2} \left(\frac{1}{U} \frac{\partial U}{\partial s_E} \right)^2 = \frac{1}{2} \frac{\xi_S}{1 + 6\xi_S} \left(\frac{\beta_{\lambda_S}}{\lambda_S} - 2 \frac{\beta_{\xi_S}}{\xi_S} \right)^2$$

$$\eta \equiv \bar{M}_{\text{Pl}}^2 \frac{1}{U} \frac{\partial^2 U}{\partial s_E^2} = \frac{\xi_S}{1 + 6\xi_S} \left(\frac{\beta(\beta_{\lambda_S})}{\lambda_S} - 2 \frac{\beta(\beta_{\xi_S})}{\xi_S} + \frac{5 + 36\xi_S}{1 + 6\xi_S} \frac{\beta_{\xi_S}^2}{\xi_S^2} - \frac{7 + 48\xi_S}{1 + 6\xi_S} \frac{\beta_{\lambda_S} \beta_{\xi_S}}{2\lambda_S \xi_S} \right)$$

The slow-roll parameters are given by the β -functions ...

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Then we can easily convert s into a scalar s_E with canonical kinetic term and find

$$\epsilon \equiv \frac{\bar{M}_{\text{Pl}}^2}{2} \left(\frac{1}{U} \frac{\partial U}{\partial s_E} \right)^2 = \frac{1}{2} \frac{\xi_S}{1 + 6\xi_S} \left(\frac{\beta_{\lambda_S}}{\lambda_S} - 2 \frac{\beta_{\xi_S}}{\xi_S} \right)^2$$

$$\eta \equiv \bar{M}_{\text{Pl}}^2 \frac{1}{U} \frac{\partial^2 U}{\partial s_E^2} = \frac{\xi_S}{1 + 6\xi_S} \left(\frac{\beta(\beta_{\lambda_S})}{\lambda_S} - 2 \frac{\beta(\beta_{\xi_S})}{\xi_S} + \frac{5 + 36\xi_S}{1 + 6\xi_S} \frac{\beta_{\xi_S}^2}{\xi_S^2} - \frac{7 + 48\xi_S}{1 + 6\xi_S} \frac{\beta_{\lambda_S} \beta_{\xi_S}}{2\lambda_S \xi_S} \right)$$

The slow-roll parameters are given by the β -functions ...

We can insert them in the formulae for the observable parameters A_s , n_s and $r = \frac{A_t}{A_s}$:

$$n_s = 1 - 6\epsilon + 2\eta, \quad A_s = \frac{U/\epsilon}{24\pi^2 \bar{M}_{\text{Pl}}^4}, \quad r = 16\epsilon$$

where everything is evaluated at about $N \approx 60$ e-foldings when the inflaton $s_E(N)$ was

$$N = \frac{1}{\bar{M}_{\text{Pl}}^2} \int_0^{s_E(N)} \frac{U(s_E)}{U'(s_E)} ds_E$$

A Pati-Salam model without Landau poles

	Fields	spin	generations	$SU(2)_L$	$SU(2)_R$	$SU(4)_{PS}$
skeleton model	$\psi_L = \begin{pmatrix} \nu_L & e_L \\ u_L & d_L \end{pmatrix}$	1/2	3	$\bar{2}$	1	4
	$\psi_R = \begin{pmatrix} \nu_R & u_R \\ e_R & d_R \end{pmatrix}$	1/2	3	1	2	$\bar{4}$
	ϕ_R	0	1	1	2	$\bar{4}$
	$\phi = \begin{pmatrix} H_U^0 & H_D^+ \\ H_U^- & H_D^0 \end{pmatrix}$	0	1	2	$\bar{2}$	1
extra fields	ψ	1/2	$N_\psi \leq 3$	2	$\bar{2}$	1
	Q_L	1/2	2	1	1	10
	Q_R	1/2	2	1	1	$\bar{10}$
	Σ	0	1	1	1	15