ON THE ORIGIN OF SCALES AND INFLATION

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We give an overview of theories of all interactions (including gravity) in the absence of fundamental scales: *agravity.* All observed masses, such as the Planck and the weak scales, are generated by quantum corrections. The main observational implications for the physics at colliders as well as for cosmology are outlined.

1 Introduction

Two types of motivations for the theories defined in the abstract come to mind.

First, the generation of the Planck and weak scales can be achieved naturally, thereby solving the (fine-tuning) hierarchy problem. As we will see, the mechanism employed to do so differs from standard solutions, such as supersymmetry, where an extra symmetry is introduced to protect the mass M_h of the Standard Model (SM) physical scalar h: within agravity all possible new particles that acquire masses $M \gg M_h$ can have tiny couplings to the SM; this avoids large quantum corrections to M_h.

Second, theories of this sort that remain perturbative at the inflationary scales provide naturally flat inflaton potentials. To explain this point in simple terms let us assume that inflation is driven by a single field I (as we will see this is not necessarily the case). The most general classical action for the self-interactions of I and its couplings to gravity has only two terms:

$$\int d^4x\sqrt{g}\left(-\frac{f(I)}{2}R - V(I)\right), \qquad f(I) = \xi_I I^2, \quad V(I) = \frac{\lambda_I}{4}I^4 \tag{1}$$

where g is the modulus of the determinant of the metric $g_{\mu\nu}$ and R is the Ricci scalar. Notice that the functions f and V are fixed by the no-scale principle. The first term is a non-minimal coupling to gravity, which would make the analysis non-standard, but one can go to a frame where the couplings to gravity are minimal (i.e. Einstein frame), by redefining the metric as $g^E_{\mu\nu} \equiv g_{\mu\nu} f(I)/\bar{M}^2_{\rm Pl}$, where $\bar{M}_{\rm Pl} \simeq 2.4 \times 10^{18}$ GeV is the reduced Planck mass. The potential of I in the Einstein frame is

$$V_E(I) = \frac{\bar{M}^4_{\rm Pl} V(I)}{f(I)^2} = \frac{\bar{M}^4_{\rm Pl}\lambda_I}{4\xi_I^2}, \tag{2}$$

which is flat. Once quantum corrections are included ξ_I and λ_I acquire a dependence on I that is encoded in the renormalization group equations (RGEs). However, perturbativity at the inflationary scales (which we can obtain [1]) ensures that such dependence is small.

This contribution to the proceedings of the $50^{\rm th}$ Rencontres de Moriond is mainly based on three articles [1,2,3], which I will refer to as I, II and III respectively.

2 Agravity scenario

The *most general* agravity action is

$$S=\int d^4x\sqrt{g}\left(\mathcal{L}_{\rm SM}^{\rm adim}+\mathcal{L}_{\rm BSM}^{\rm adim}+\frac{R^2}{6f_0^2}+\frac{\frac{1}{3}R^2-R_{\mu\nu}^2}{f_2^2}\right). \tag{3}$$

It consists of two parts.

The first one is the non-gravitational part, described by $\mathcal{L}_{\rm SM}^{\rm adim}+\mathcal{L}_{\rm BSM}^{\rm adim}$. Here $\mathcal{L}_{\rm SM}^{\rm adim}$ is the no-scale SM Lagrangian: namely the SM Lagrangian without the H mass term $m_h^2|H|^2/2$ plus a possible non-minimal coupling $-\xi_H|H|^2R$, where H is the SM doublet; this represents the most general no-scale Lagrangian constructed with only SM fields and their interactions with gravity. $\mathcal{L}_{\rm BSM}^{\rm adim}$ is a beyond the SM (BSM) no-scale Lagrangian, which should account for the observational problems of the SM (e.g. neutrino oscillations, dark matter, baryon asymmetry, the strong CP problem, etc [4]) and generate the scales we see in nature. In order to do so one should include at least one extra real scalar s. Indeed, the possible terms in the Lagrangian $\lambda_{HS}|H|^2s^2/2-\xi_Ss^2R/2$ respectively generate the H mass term and the Planck scale once s acquires a vacuum expectation value (VEV) at quantum level.

The second part of the action is the gravity sector, described by the third and fourth terms in Eq. (3), where $R_{\mu\nu}$ is the Ricci tensor. A remarkable property of this theory is its renormalizability, which allows us to obtain predictions up to infinite energies [a]. However, this comes with a price. By studying the spectrum one finds three states: a massless graviton, which is responsible for the large distance gravitational interactions we are used to, an extra real scalar (graviscalar henceforth) with mass $|f_0|\bar{M}_{\rm Pl}/\sqrt{2}+\ldots$, where the dots are corrections coming from the mixing with other possible scalars, and a massive graviton with mass $M_2=|f_2|\bar{M}_{\rm Pl}/\sqrt{2}$ and negative norm [5] (i.e. a ghost). Although the energy of the theory is bounded from below there is currently no proof that the latter state can have a sensible physical interpretation. However, a no-go theorem is also not known; thus in the rest of this article we will simply assume that such interpretation exists and discuss the main phenomenological implications.

3 Quantum corrections and generation of scales

The quantum corrections are mostly encoded in the RGEs, which are important to generate the scales we observe and to extract inflationary predictions. For this reason, in Article I we obtained the full set of one-loop RGEs for the most general agravity theory, Eq. (3). This generalizes previous computations [6,7,8] performed without gravity.

3.1 Dynamical generation of the Planck scale

Once the RGEs are known one can study the possible generation of scales, starting from the one we are familiar with, the gravitational constant of Newton's law (i.e. the Planck scale).

Agravity successfully generates $\bar{M}_{\rm Pl}$ if the VEV $\bar{s}$ of the scalar s fulfills the following conditions:

$$\left\{\begin{array}{rcll}\lambda_S(\bar{s}) & \simeq & 0 & \leftrightarrow \quad \text{nearly vanishing cosmological constant (dark energy)}\\ \frac{d\lambda_S}{ds}(\bar{s}) & = & 0 & \leftrightarrow \quad \text{minimum condition}\\ \xi_S(\bar{s})\bar{s}^2 & = & \bar{M}_{\rm Pl}^2 & \leftrightarrow \quad \text{observed Planck mass}\end{array}\right.$$

where λ_S is the quartic self-coupling of s and ξ_S is its non-minimal coupling to gravity: in the Lagrangian these couplings appear as $-\lambda_Ss^4/4-\xi_Ss^2R/2$.

[a] In fact, even if we do not introduce those terms in the classical Lagrangian, they are generated by quantum corrections.

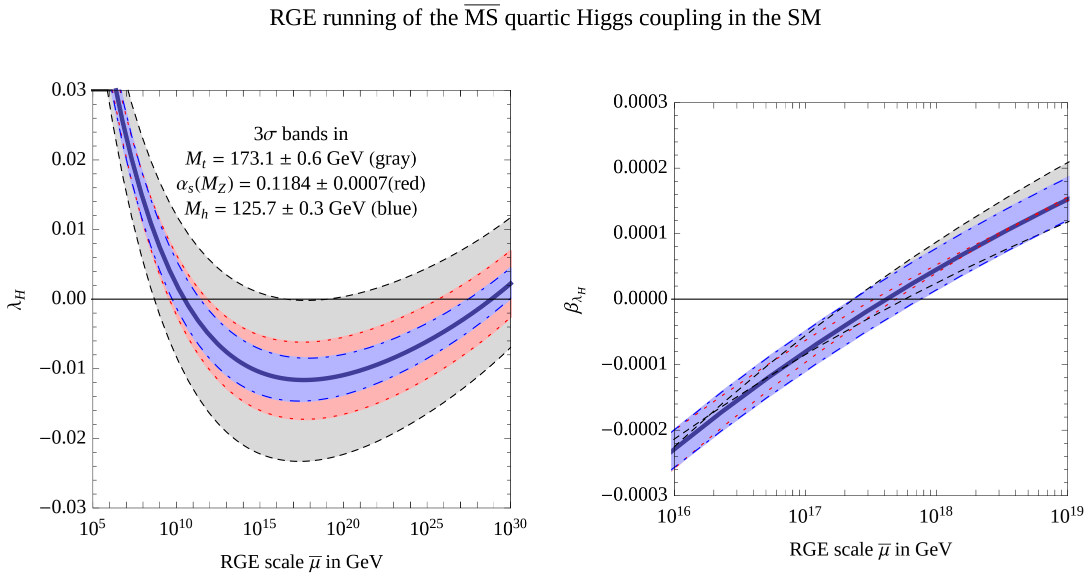

Figure 1 – Running of λ_H (left) and its β-function (right) in the SM [10]: λ_H appears in the potential in a term $\lambda_H|H|^4$, while its β-function is defined by $\beta_{\lambda_H} \equiv d\lambda_H/d\ln\bar\mu$. Figure reproduced from Article I.

It is possible to fulfill these conditions: remarkably, this is what happens in the SM for h and top masses around $M_h \approx 125\,\text{GeV}$ and $M_t \approx 171\,\text{GeV}$ (see [b] Fig. 1). Although s and h cannot be identified as their VEVs should differ by several orders of magnitude, this argument clearly shows that we can build concrete models (in fact many models) that generate the Planck scale.

3.2 Observational implications for cosmology

After generating $\bar M_{\rm Pl}$ one can study inflation in agravity. How the weak scale is obtained will be discussed later on because M_h is negligible compared to inflationary scales.

Generically inflation in this theory is a multifield process: there are at least three scalar fields (the SM scalar h, the Planckion s and the graviscalar z). By studying the dynamics of h, s and z in the minimal realistic model [2], we found that inflation occurs once an attractor in the plane of s and z is reached [c]. This has two consequences. First, h never dominates inflation: the reason is that the h quartic self-coupling (assumed to be positive) is unavoidably larger than the other scalar couplings, taking into account its RGE running [9,10,11,12]. Second, the presence of an attractor fortunately ensures that the observable predictions do not depend on the chosen initial field values for a given number of e-folds N. Another important parameter is the ratio between $M_0^2 = f_0^2 \bar M_{\rm Pl}^2(1+6\xi_S)/2$ and $M_s^2 = \partial^2 V/\partial s^2$, where V is the potential of s: when $M_0/M_s \ll 1$ the scalar s becomes very massive and gets frozen to $\bar s$; in this case we recover Starobinsky inflation, which gives a small value of the tensor-to-scalar ratio r, of order of 0.001; in the opposite limit it is the other scalar that becomes very massive and we find a sizable value, $r \sim 0.1$. All the intermediate values of r can be obtained for an appropriate M_0/M_s, while the prediction for the scalar spectral index n_s is steadily close to $n_s \approx 0.97$. These findings are in good agreement with a global fit of the most recent studies by PLANCK and BICEP2/KECK [13,14,15,16]. In Fig. 2 there is a more quantitative description of the predictions and the comparison with data. Moreover, in Article II one can see how matching the scalar amplitude P_R with observations requires generically $f_0 > 10^{-5}$.

[b] The RGEs used in this article are defined in the $\overline{\rm MS}$ renormalization scheme and the RGE sliding scale is denoted by $\bar\mu$.

[c] See Fig. 4 of Article II.

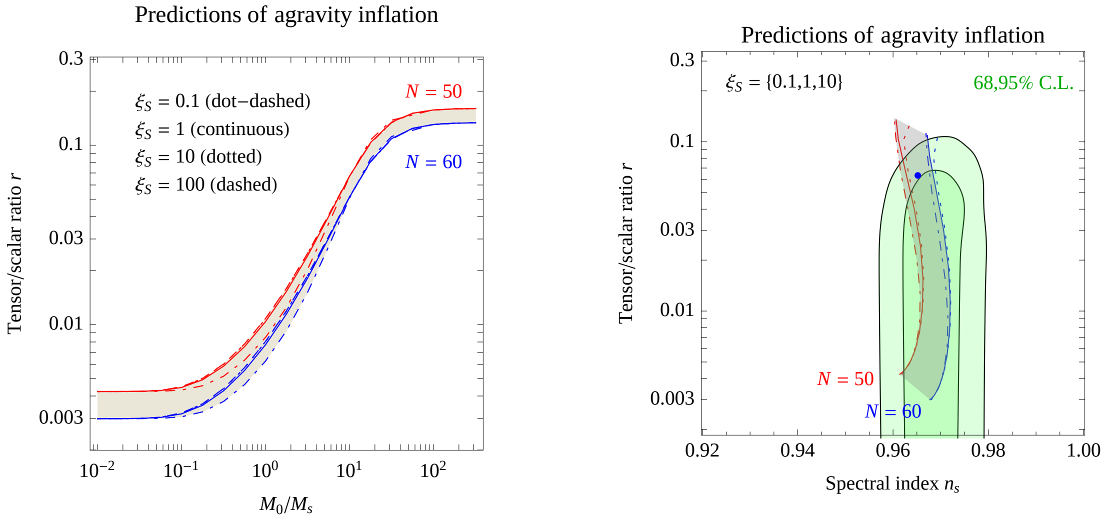

Figure 2 – **Left:** predictions for the tensor-to-scalar ratio r after $N = 50$ or 60 e-folds of inflation for various values of ξ_S as function of M_0/M_s. In the limit where this ratio is large (small) inflation is dominated by the Planckion (the graviscalar). **Right:** predictions for the scalar spectral index n_s and r with the same coding. The green area is favored by a global fit of PLANCK, BICEP2/KECK [13,14,15,16]. Figure reproduced from Article II.

Article II also provided a study of cosmology after inflation. The inflaton decays occur via Planck-suppressed interactions (it couples to a combination of the trace of the energy momentum tensor and of the divergence of the dilatation current) producing a reheating temperature $T_{\rm RH} \sim 10^{7-9}$ GeV. Also, the s-sector must contain fermions that either behave as right-handed neutrinos (if they have no gauge interactions) or are stable. In the latter case, they might be light enough that the inflaton can decay into them, providing the observed Dark Matter abundance with adiabatic primordial inhomogeneities if their mass is around $10 - 200\,$TeV.

3.3 Natural dynamical generation of the weak scale

The VEV of s, besides generating the Planck scale, also induces the weak scale. We here describe how these scales can coexist naturally (i.e. avoiding the hierarchy problem). Let us divide the discussion in three energy ranges.

First, focus on $\bar\mu < M_{0,2}$. In this case the RGE of m_h is well approximated by the SM one, where m_h is the only mass parameter and therefore one does not see any unnaturally large corrections to m_h.

The situation is more complicated for $M_{0,2} < \bar\mu < \bar M_{\rm Pl}$, where the RGE for $m_h/\bar M_{\rm Pl}$ shows a potentially dangerous term,

$$(4\pi)^2 \frac{d}{d\ln\bar\mu}\frac{m_h^2}{\bar M_{\rm Pl}^2} = -\xi_H[5 f_2^4 + f_0^4(1+6\xi_H)] + \ldots, \tag{4}$$

where the dots represent terms that are harmless from the point of view of naturalness [1]. By looking at the RGEs for f_0, f_2 and the ξ couplings, presented in Articles I and II, one finds that a way to obtain the small ratio $m_h/\bar M_{\rm Pl} \sim 10^{-16}$ naturally is to impose $f_2 \sim 10^{-8}$ and $1 + 6\xi_H \sim f_2^4/(4\pi f_0)^2$, which can accommodate the value of $f_0 > 10^{-5}$ generically required to match the (inflationary) scalar amplitude.

In the large energy range, $\bar\mu > \bar M_{\rm Pl}$, the theory is dimensionless and m_h arises from the interaction $\lambda_{HS}|H|^2 s^2/2$, which leads to

$$m_h^2 = \lambda_{HS}\bar s^2. \tag{5}$$

Since $\bar s \sim \bar M_{\rm Pl}$, this requires a tiny value of $\lambda_{HS} \sim 10^{-32}$. The structure of the RGE of λ_{HS} (presented in Article I) are naturally compatible with this small number and the measured scalar amplitude as long as the setting for f_2 and $1 + 6\xi$ discussed in the previous paragraph holds.

Intuitively, Eqs. (4) and (5) tell us that there is a contribution to m_h^2 of the form

$$\delta m_h^2 \sim f^4 M^2, \tag{6}$$

where f is a coupling constant and M is a new mass scale. Therefore, although the difference between m_h and $\bar{M}_{\rm Pl}$ is large, naturalness can be preserved if the smallness of f is compatible with the RGE running. A novel feature here is that a symmetry is not required to protect m_h even if $M \gg m_h$ because some couplings can be small [17]. Possible quadratic divergences with respect to some particular regulator, such as the lattice, are not regarded here as a problem: they do not appear for other regulators, e.g. dimensional regularization [d], and we therefore do not attribute to them a physical meaning.

4 Unification, final theories and experimental consequences at colliders

Agravity provides an alternative solution of the hierarchy problem. Since this problem is most evident in unified models, it is also interesting to ask whether unification can be achieved in this scenario. Here "unification" means embedding the SM into a BSM model without gauge U(1) factors, leading to an explanation for the observed charge quantization. This typically requires new physics with non-negligible couplings to the SM particles. Thus, in practice naturalness tells us that the masses of these new states should not be too far from m_h (see Eq. (6)). Barring symmetries protecting m_h, simple gauge groups, such as SU(5) or SO(10), are not natural as the experimental bounds on proton decay imply large vector boson masses, $M \gg m_h$, that contribute too much to m_h. A possible solution is to use semi-simple gauge groups: the Pati-Salam SU(4)×SU(2)×SU(2) or the trinification SU(3)×SU(3)×SU(3) groups. The new states contained in these theories can be accessible at the LHC or at future colliders.

In the context of agravity there is another reason for unification. The action in (3) is renormalizable and, therefore, can be used to study arbitrarily high energies. A problem in this case, however, is the presence of Landau poles in the SM: e.g. the hypercharge gauge coupling g_Y diverges within perturbation theory at $\sim 10^{42}$ GeV. A necessary condition to avoid these poles perturbatively is to eliminate any gauge U(1) factor, in other words to have unification. Indeed, in the SM the Landau poles are eliminated only for the unphysical choice [3] $g_Y = 0$ (as well as $M_t = 186$ GeV, $M_\tau = 0$, $M_h = 163$ GeV). Realistic theories without Landau poles can be considered as candidate final theories as they describe physics without any energy cut-off. Article III provided a general technique to search for such theories and found examples based on the Pati-Salam group. These examples are, however, affected by a residual little fine-tuning: limits from precision and flavor physics imply [e] that the mass of some vector leptoquarks W'_μ, of charge $\pm 2/3$, corresponding to the broken generators in SU(4)/SU(3)$_c$, is above the TeV scale. An interesting outlook is the quest for other explicit examples where all masses are generated by quantum corrections (as required by the agravity principle).

5 Conclusions

A natural hierarchy between the weak and the Planck scales and a rationale for inflation can be obtained in theories of all interactions (including gravity) in the absence of fundamental scales; we referred to this class of theories as agravity.

Regarding inflation, we found $n_s \approx 0.967$ and $0.003 < r < 0.13$, in agreement with Planck and BICEP2/Keck [13,14,15,16]. We observe that a future measurement of r by Keck/Bicep3 would give us more constraints on this scenario.

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[d] Divergences of the form $m^2/(d-4)$, where m is a mass and d is the space-time dimension, do not appear simply because there are no masses.

[e] The existence of other examples where this situation is improved is not excluded [3].

The mechanism used here to have naturalness differs from standard solutions, based on symmetries: here a naturally small m_h is obtained by requiring the new physics to be light and/or weakly coupled to h. This is also compatible with unification in the case of Pati-Salam or trinification models, which predict new particles (e.g. W', Z', ...) that are accessible at the LHC or future colliders.

Unification here is also motivated by the requirement to have a consistent theory that is valid up to infinite energies. Indeed, at the perturbative level, such a requirement tells us that the SM should be embedded in a BSM model without Landau poles, which excludes, among other things, any gauge U(1) factor.

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