On the Origin of Scales and Inflation

Models of all interactions in the absence of fundamental scales (agraavity)

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Based on
- Salvio, Strumia, arXiv:1403.4226 (JHEP)
- Giudice, Isidori, Salvio, Strumia arXiv:1412.2769
Main motivations for agravity

Motivation 1: naturalness

Agravity provides an alternative solution of the hierarchy problem: there are no $\Lambda^2$ contributions because there are no masses. This in turn leads to dynamical generation of masses.

Like for the proton: its mass is mostly dynamical generated.
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Motivation 2: inflation

Cosmological observations suggest inflation. However, it requires flat potentials. What is the reason for this flatness? Agravity gives us an explanation:
The Einstein frame potential of a scalar $s$ in agravity is

$$U(s) = \frac{\lambda_S s^4}{(2\xi_S s^2)^2} \bar{M}_{P1}^4 = \frac{\lambda_S}{4\xi_S^2} \bar{M}_{P1}^4$$

The potential is flat at tree-level, but at quantum level $\lambda_S$ and $\xi_S$ depend on $s$
this effect (due to the RGEs) gives some slope ... which is small if couplings are perturbative
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The potential is flat at tree-level, but at quantum level $\lambda_S$ and $\xi_S$ depend on $s$

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what we need to have inflation!
Agravity scenario

The **most general** agravity Lagrangian:

\[
\mathcal{L} = \frac{R^2}{6f_0^2} + \frac{\frac{1}{3} R^2 - R_{\mu\nu}^2}{f_2^2} + \mathcal{L}_{\text{adim}}^{\text{SM}} + \mathcal{L}_{\text{adim}}^{\text{BSM}}
\]
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Non-gravitational sectors

- \( \mathcal{L}_{\text{SM}}^{\text{adim}} \) is the no-scale part of the SM Lagrangian (without \( m^2|H|^2/2 \)):
  \[ \mathcal{L}_{\text{SM}}^{\text{adim}} = -\frac{F_{\mu\nu}^2}{4} + \bar{\psi}iD\psi + |D_\mu H|^2 - (y_H\psi\psi + \text{h.c.}) - \lambda_H|H|^4 - \xi_H|H|^2R \]

- \( \mathcal{L}_{\text{BSM}}^{\text{adim}} \) describes physics beyond the SM (BSM). It generates the weak scale

  \[ \text{adding a scalar } s \rightarrow \mathcal{L}_{\text{BSM}}^{\text{adim}} = \ldots + \lambda_{HS}s^2|H|^2/2 - \xi_s s^2 R/2 \]

  vectors in the \( s \)-sector can be dark matter \([\text{Hambye, Strumia (2013)}]\)
Gravity scenario

The most general gravity Lagrangian:

\[ \mathcal{L} = \frac{R^2}{6f_0^2} + \frac{1}{3} \frac{R^2 - R_{\mu\nu}}{f_2^2} + \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{BSM}} \]

Non-gravitational sectors

- \( \mathcal{L}_{\text{SM}} \) is the no-scale part of the SM Lagrangian (without \( m^2|H|^2/2 \)):
  \[ \mathcal{L}_{\text{SM}} = -\frac{F^2_{\mu\nu}}{4} + \bar{\psi}i\partial\psi + |D_\mu H|^2 - (yH\psi\psi + \text{h.c.}) - \lambda_H|H|^4 - \xi_H|H|^2R \]

- \( \mathcal{L}_{\text{BSM}} \) describes physics beyond the SM (BSM). It generates the weak scale by adding a scalar \( s \):
  \[ \mathcal{L}_{\text{BSM}} = ... + \lambda_Hs^2|H|^2/2 - \xi_s s^2 R/2 \]

  vectors in the \( s \)-sector can be dark matter [Hambye, Strumia (2013)]

Gravity sector

- \( \langle s \rangle \) generates \( \tilde{M}_{\text{P1}} \):
  \[ \xi_s s^2 R \to \tilde{M}_{\text{P1}}^2 = \xi_s |\langle s \rangle|^2 \]

- Agravity is renormalizable, however, looking at the spectrum:
  (i) massless graviton
  (ii) scalar \( z \) with mass \( M_0^2 \sim \frac{1}{2} f_0^2 \tilde{M}_{\text{P1}}^2 \)
  (iii) massive graviton with mass \( M_2^2 = \frac{1}{2} f_2^2 \tilde{M}_{\text{P1}}^2 \) and negative norm, but with energy bounded from below

The literature is controversial
Quantum corrections

They are mostly encoded in the RGEs

*They are important to obtain $n_s$ and $r$ and to dynamically generate $\bar{M}_{P1}$ and $m$*
Quantum corrections

They are mostly encoded in the RGEs

_They are important to obtain n_s and r and to dynamically generate \( \bar{M}_{Pl} \) and m_

We computed the 1-loop RGEs for all couplings of the most general agravity

\[
\frac{R^2}{6f_0^2} + \frac{1}{3} \frac{R^2 - R_{\mu\nu}^2}{f_2^2} - \frac{(F_{\mu\nu}^A)^2}{4} + \frac{(D_{\mu} \phi_a)^2}{2} - \frac{\xi_{ab}}{2} \phi_a \phi_b R - \frac{\lambda_{abcd}}{4!} \phi_a \phi_b \phi_c \phi_d + \bar{\psi}_j i \slashed{D} \psi_j - Y^a_{ij} \psi_i \psi_j \phi_a + \text{h.c.}
\]

_Without gravity this was done before_

Dynamical generation of the Planck scale

Agravity successfully generates the Planck scale if

\[
\begin{align*}
\lambda_S(s) & \simeq 0 \quad \Leftrightarrow \quad \text{nearly vanishing cosmological constant (dark energy)} \\
\frac{d\lambda_S}{ds}(s) & = 0 \quad \Leftrightarrow \quad \text{minimum condition} \\
\xi_S(s)s^2 & = \bar{M}^2_{\text{Pl}} \quad \Leftrightarrow \quad \text{observed Planck mass}
\end{align*}
\]
Is the dynamical generation of the Planck scale possible?

Are these conditions realized in the physics we know (the SM)?

Thus the dynamical generation of the Planck scale is possible!
Predictions for inflation (generically a multifield inflation)

The minimal realistic model has at least 3 scalars:

- the SM scalar $h$
- the Planckion $s$
- the graviscalar $z$

$M_s = \text{mass of } s$
$M_0 = \text{mass of } z$

$\xi_S = 1$, $\xi_H = 1$, $M_s/M_0 = 0.10$, $\lambda_H = 0.01$
Predictions for inflation (generically a multifield inflation)

The minimal realistic model has at least 3 scalars:

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\[ M_s = \text{mass of } s \]
\[ M_0 = \text{mass of } z \]

\[ N = 20 \]
\[ N = 20 \]
\[ N = 40 \]
\[ N = 40 \]
\[ N = 60 \]
\[ N = 60 \]
\[ N = 80 \]

\[ 10^{-2}, 10^{-1}, 10^0, 10^1, 10^2 \]

- left: when \( M_s \ll (\gg) M_0 \), the inflaton is \( s \) (\( z \))
- right: comparison with a global fit of PLANCK and BICEP2/KECK
Natural dynamical generation of the weak scale

1) **Low energies** ($\mu < M_{0,2}$): gravity can be neglected and the SM RGE apply:

\[ (4\pi)^2 \frac{dm^2}{d\ln \mu} = m^2 \beta_m^{\text{SM}}, \quad \beta_m^{\text{SM}} = 12\lambda_H + 6y_t^2 - \frac{9g_2^2}{2} - \frac{9g_1^2}{10} \]
Natural dynamical generation of the weak scale

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2) **Intermediate energies** ($M_{0,2} < \mu < \bar{M}_{P1}$): Both $m$ and $\bar{M}_{P1}$ appear and we find

$$(4\pi)^2 \frac{d}{d \ln \mu} \frac{m^2}{\bar{M}_{P1}^2} = -\xi_H [5f_2^4 + f_0^4 (1 + 6\xi_H)] + ...$$

The **red term** is a non-multiplicative potentially dangerous correction to $m$

$$m^2 \sim \bar{M}_{P1}^2 f_{0,2}^4, \quad \text{naturalness} \rightarrow f_0, f_2 \sim \sqrt{\frac{4\pi m}{M_{P1}}} \sim 10^{-8}$$
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2) **Intermediate energies** \((M_{0,2} < \mu < \bar{M}_{\text{Pl}})\): Both \(m\) and \(\bar{M}_{\text{Pl}}\) appear and we find

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\]

3) **Large energies** \((\mu > \bar{M}_{\text{Pl}})\):

\[
\lambda_{HS} |H|^2 s^2 \rightarrow m^2 = \lambda_{HS} \langle s \rangle^2
\]

\(\lambda_{HS}\) can be naturally small (looking at the RGE of \(\lambda_{HS}\)):

\[
\rightarrow \lambda_{HS} \sim f_{0,2}^4
\]
Natural weak scale and unification

It is possible to preserve a natural weak scale if

- a semi-simple gauge group is used: e.g. the Pati-Salam $SU(4) \times SU(2) \times SU(2)$
- all SM Landau poles are removed (we require this as we want to go up to infinite energy and in the SM the experiments tell us that e.g. $g_Y$ diverges at $10^{42}$ GeV)

Models of this type have been found and predict a lot of new physics not far above the weak scale (In the SM the elimination of such poles requires unrealistic conditions: $g_Y = 0$, ...
Conclusions

- **Naturalness and a rationale for inflation can be achieved in no-scale theories of all interactions (including gravity): agravity**

- **Inflation: the minimal realistic model predicts** $n_s \approx 0.967$, $0.003 < r < 0.13$, *in agreement with Planck and BICEP2/Keck*. Keck/Bicep3 *may give us more constraints on this scenario.*

- **Naturalness is also compatible with unification.** *SM Landau poles can also be eliminated.* In this case there is new physics not far above the weak scale (e.g. $W'$).
Conclusions

- Naturalness and a rationale for inflation can be achieved in no-scale theories of all interactions (including gravity): agravity

- Inflation: the minimal realistic model predicts $n_s \approx 0.967$, $0.003 < r < 0.13$, in agreement with Planck and BICEP2/Keck. Keck/Bicep3 may give us more constraints on this scenario.

- Naturalness is also compatible with unification. SM Landau poles can also be eliminated. In this case there is new physics not far above the weak scale (e.g. $W'$).

Thank you very much for your attention!
Extra slides
Ghosts

**Negative literature** [Ostrogradski (1850), Smilga (2009), ...]

- *Classically* the energy is not bounded from below (Ostrogradski instability)
- *At quantum level* creation of negative energy $\sim$ destruction of positive energy: the Hamiltonian becomes positive, but some states ("ghosts") have negative norm

**Positive literature**

- *[Lee, Wick (1969)]* the introduction of negative norms can lead to a unitary S-matrix, provided that all stable particle states have positive norm
- *[Hawking, Hertog (2001)]* at least in a simple scalar field $\phi$ theory, the problem comes from regarding $\phi$ and $\Box \phi$ as independent and can be overcome by using the path integral, where they are dependent.
Results for RGEs

Gauge couplings

Their contributions to the RGEs cancel!

This was previously noticed in [Narain, Anishetty (2013)]

Possible explanation:
the graviton is not charged

Yukawa couplings

We find the one-loop RGE (where \( C_{2F} \equiv t^A t^A \) and \( t^A \equiv \) “fermion gauge generators”):

\[
(4\pi)^2 \frac{dY^a}{d \ln \mu} = \frac{1}{2} (Y^b Y^b Y^a + Y^a Y^b Y^b) + 2 Y^b Y^{\dagger a} Y^b + Y^b \text{Tr}(Y^\dagger Y^a) - 3 \{ C_{2F}, Y^a \} + \frac{15}{8} f_2^2 Y^a
\]

All remaining RGEs

We also computed the RGEs for \( \lambda_{abcd} \), \( \xi_{ab} \), \( f_0 \) and \( f_2 \)
RGEs for the quartic couplings

Tens of Feynman diagrams contribute to these RGEs ... we obtain

\[
(4\pi)^2 \frac{d\lambda_{abcd}}{d \ln \mu} = \sum_{\text{perms}} \left[ \frac{1}{8} \lambda_{abef} \lambda_{efcd} + \frac{3}{8} \{\theta^A, \theta^B\}_{ab} \{\theta^A, \theta^B\}_{cd} - \operatorname{Tr} Y^a Y^b_{\dagger} Y^c Y^d_{\dagger} + \right. \\
+ \frac{5}{8} f_2^4 \xi_{ab} \xi_{cd} + \frac{f_0^4}{8} \xi_{ae} \xi_{cf} (\delta_{eb} + 6 \xi_{eb})(\delta_{fd} + 6 \xi_{fd}) \\
+ \frac{f_2^2}{4!} (\delta_{ae} + 6 \xi_{ae})(\delta_{bf} + 6 \xi_{bf}) \lambda_{efcd} \right] + \lambda_{abcd} \left[ \sum_k (Y^k_2 - 3 C^k_{2S}) + 5 f_2^2 \right],
\]

where the first sum runs over the 4! permutations of \( abcd \) and the second sum over \( k = \{a, b, c, d\} \), with \( Y^k_2 \) and \( C^k_2 \) defined by

\[ \operatorname{Tr}(Y^a_{\dagger} Y^b) = Y^a_2 \delta^{ab}, \quad \theta^A_{ac} \theta^A_{cb} = C^a_{2S} \delta_{ab} \]

(\( \theta^A \) are the scalar gauge generators)
RGEs for the quartic couplings: SM case

For the SM $H$ plus the complex scalar singlet $S$ the RGEs become:

\[
\begin{align*}
(4\pi)^2 \frac{d\lambda_S}{d\ln \mu} &= 20\lambda_S^2 + 2\lambda_{HS}^2 + \frac{\xi_S^2}{2} \left[5f_2^4 + f_0^4(1 + 6\xi_S)^2\right] + \lambda_S \left[5f_2^2 + f_0^2(1 + 6\xi_S)^2\right] \\
(4\pi)^2 \frac{d\lambda_{HS}}{d\ln \mu} &= -\xi_H\xi_S \left[5f_2^4 + f_0^4(6\xi_S + 1)(6\xi_H + 1)\right] - 4\lambda_{HS}^2 + \lambda_{HS} \left\{8\lambda_S + 12\lambda_H + 6y_t^2 + 5f_2^2 + \frac{f_0^2}{6} \left[(6\xi_S + 1)^2 + (6\xi_H + 1)^2 + 4(6\xi_S + 1)(6\xi_H + 1)\right] \right\} \\
(4\pi)^2 \frac{d\lambda_H}{d\ln \mu} &= \frac{9}{8}g_2^4 + \frac{9}{20}g_1^2g_2^2 + \frac{27}{200}g_1^4 - 6y_t^4 + 24\lambda_H^2 + \lambda_{HS}^2 + \frac{\xi_H^2}{2} \left[5f_2^4 + f_0^4(1 + 6\xi_H)^2\right] + \lambda_H \left(5f_2^2 + f_0^2(1 + 6\xi_H)^2 + 12y_t^2 - 9g_2^2 - \frac{9}{5}g_1^2\right).
\end{align*}
\]
RGEs for the scalar/graviton couplings

Complicated calculation (but computer algebra helps!)

\[
(4\pi)^2 \frac{d\xi_{ab}}{d \ln \mu} = \frac{1}{6} \lambda_{abcd} (6\xi_{cd} + \delta_{cd}) + (6\xi_{ab} + \delta_{ab}) \sum_k \left[ \frac{Y_k^2}{3} - \frac{C_k^{2S}}{2} \right] +
\]

\[-\frac{5f_2^4}{3f_0^2} \xi_{ab} + f_0^2 \xi_{ac} \left( \xi_{cd} + \frac{2}{3} \delta_{cd} \right) (6\xi_{db} + \delta_{db})\]

For the SM $H$ plus the complex scalar singlet $S$ the RGEs become:

\[
(4\pi)^2 \frac{d\xi_S}{d \ln \mu} = \left(1 + 6\xi_S\right) \frac{4}{3} \lambda_S - \frac{2\lambda_{HS}}{3} (1 + 6\xi_H) + \frac{f_0^2}{3} \xi_S (1 + 6\xi_S)(2 + 3\xi_S) - \frac{5}{3} \frac{f_2^4}{f_0^2} \xi_S
\]

\[
(4\pi)^2 \frac{d\xi_H}{d \ln \mu} = \left(1 + 6\xi_H\right) \left(2y_t^2 - \frac{3}{4} g_2^2 - \frac{3}{20} g_1^2 + 2\lambda_H\right) - \frac{\lambda_{HS}}{3} (1 + 6\xi_S) +
\]

\[+ \frac{f_0^2}{3} \xi_H (1 + 6\xi_H)(2 + 3\xi_H) - \frac{5}{3} \frac{f_2^4}{f_0^2} \xi_H\]
RGE for the gravitational couplings

Huge calculation ... (computer algebra practically needed!!)

\[
(4\pi)^2 \frac{df_2^2}{d\ln \mu} = -f_2^4 \left( \frac{133}{10} + \frac{N_V}{5} + \frac{N_f}{20} + \frac{N_s}{60} \right)
\]

\[
(4\pi)^2 \frac{df_0^2}{d\ln \mu} = \frac{5}{3} f_2^4 + 5 f_2^2 f_0^2 + \frac{5}{6} f_0^4 + \frac{f_0^4}{12} (\delta_{ab} + 6\xi_{ab})(\delta_{ab} + 6\xi_{ab})
\]

Here \( N_V, N_f, N_s \) are the number of vectors, Weyl fermions and real scalars.

In the SM \( N_V = 12, N_f = 45, N_s = 4 \).

We confirmed the calculations of [Avramidi (1995)]
rather than those of [Fradkin and Tseytlin (1981,1982)]
Agravity inflation

All scalar fields in agravity are inflaton candidates
Agravity inflation

All scalar fields in agravity are inflaton candidates

example (the minimal model): \( h \), the Planckion \( s \), the scalar \( \sigma \) in \( g_{\mu \nu} \)

To see \( \sigma \)

\[
\frac{R^2}{6f_0^2} \rightarrow \frac{R^2}{6f_0^2} - \left( \frac{R + 3f_0^2\sigma/2}{6f_0^2} \right)^2
\]

zero on-shell

By redefining \( g^E_{\mu \nu} = g_{\mu \nu} \times f/\bar{M}^2_{\text{Pl}} \) with \( f = \xi_S s^2 + \xi_H h^2 + \sigma \) one obtains ...

\[
\sqrt{|\det g_E|} \left\{ \frac{\bar{M}^2_{\text{Pl}}}{2} R_E + \bar{M}^2_{\text{Pl}} \left[ \frac{(\partial_\mu s)^2 + (\partial_\mu h)^2}{2f} + \frac{3(\partial_\mu f)^2}{4f^2} \right] - U \right\} + \cdots
\]

as well as their effective potential:

\[
U = \frac{\bar{M}^4_{\text{Pl}}}{f^2} \left( V + \frac{3f_0^2}{8} \sigma^2 \right)
\]
Agravity inflation: a simple single field case

We identify inflaton = s by taking the other scalar fields heavy ...

Then we can easily convert s into a scalar $s_E$ with canonical kinetic term and find

$$
\epsilon \equiv \frac{\tilde{M}_{P1}^2}{2} \left( \frac{1}{U} \frac{\partial U}{\partial s_E} \right)^2 = \frac{1}{2} \frac{\xi_S}{1 + 6 \xi_S} \left( \frac{\beta \lambda_S}{\lambda_S} - 2 \frac{\beta \xi_S}{\xi_S} \right)^2
$$

$$
\eta \equiv \frac{\tilde{M}_{P1}^2}{U} \frac{1}{\partial^2 \frac{U}{s_E^2}} = \frac{\xi_S}{1 + 6 \xi_S} \left( \frac{\beta (\beta \lambda_S)}{\lambda_S} - 2 \frac{\beta (\beta \xi_S)}{\xi_S} + \frac{5 + 36 \xi_S}{1 + 6 \xi_S} \frac{\beta^2 \xi_S}{\xi_S^2} - \frac{7 + 48 \xi_S}{1 + 6 \xi_S} \frac{\beta \lambda_S \beta \xi_S}{2 \lambda_S \xi_S} \right)
$$

The slow-roll parameters are given by the $\beta$-functions ...
Agravity inflation: a simple single field case

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$$\eta \equiv \frac{M^2_{P1}}{U} \frac{1}{1 + 6\xi_S} \left( \frac{\beta (\beta \lambda_S)}{\lambda_S} - 2 \frac{\beta (\beta \xi_S)}{\xi_S} + \frac{5 + 36\xi_S}{1 + 6\xi_S} \frac{\beta^2}{\xi_S^2} - \frac{7 + 48\xi_S}{1 + 6\xi_S} \frac{\beta \lambda_S \beta \xi_S}{2\lambda_S \xi_S} \frac{\xi_S}{\xi_S} \right)$$

The slow-roll parameters are given by the $\beta$-functions ...

We can insert them in the formulae for the observable parameters $A_s$, $n_s$ and $r = \frac{A_t}{A_s}$:

$$n_s = 1 - 6\epsilon + 2\eta, \quad A_s = \frac{U/\epsilon}{24\pi^2 M^4_{P1}}, \quad r = 16\epsilon$$

where everything is evaluated at about $N \approx 60$ e-foldings when the inflaton $s_E(N)$ was

$$N = \frac{1}{M^2_{P1}} \int_{0}^{s_E(N)} \frac{U(s_E)}{U'(s_E)} \, ds_E$$
A Pati-Salam model without Landau poles

<table>
<thead>
<tr>
<th>Fields</th>
<th>spin</th>
<th>generations</th>
<th>SU(2)$_L$</th>
<th>SU(2)$_R$</th>
<th>SU(4)$_{PS}$</th>
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