RENORMALISATION OF THE NMSSM IN SLOOPS

V. BIZOUARD
LAPTh, 9 Chemin de Bellevue,
74940 Annecy-le-Vieux, France

The renormalisation of the chargino and neutralino sector of the NMSSM as implemented in SloopS is presented together with a few numerical examples.

1 Introduction

The Standard Model (SM) of particle physics is a successful theory that predicts with a very high precision most of measured observables. Even if no significant deviations from the SM have been observed so far, several theoretical arguments as well as cosmological and astrophysics observations indicate that it cannot be complete. In particular, the hierarchy problem, dark matter as well as neutrino masses cannot be explained in this model. This motivates the need to go beyond the Standard Model (BSM). One of the best motivated and studied theory is supersymmetry (SUSY). Its minimal incarnation, the Minimal Supersymmetric Standard Model (MSSM) presents some problems such as the so-called μ problem or the difficulty to get a 125 GeV Higgs boson without fine-tuning radiative corrections. Adding a new singlet Higgs chiral superfield to the MSSM, called then the Next to Minimal Supersymmetric extension of the SM (NMSSM), will solve these problems [1].

1.1 The NMSSM

In the MSSM, two Higgs superfields $\hat{H}_u$ and $\hat{H}_d$, which are weak doublets are necessary to generate quarks and gauge-boson masses via the usual Higgs mechanism and also to avoid gauge anomaly. These two superfields are linked in the superpotential :

$$W^{\text{MSSM}}_{\text{Higgs}} = \mu \hat{H}_u.\hat{H}_d, \tag{1}$$

The μ parameter has the dimension of a mass and is a free parameter. In this sense, its mass scale can vary *a priori* between 0 and the Planck mass. However, μ is related to other observables, such as the Z-boson mass,

$$M_Z^2 \simeq -2\mu^2 + 2\frac{m^2_{H_d} - t^2_\beta m^2_{H_u}}{t_\beta - 1}, \tag{2}$$

where m_{H_u} and m_{H_d} are the soft mass terms for $\hat{H}_u$ and $\hat{H}_d$, and t_β is defined via their vacuum expectation values (vev), $t_\beta = \frac{v_u}{v_d}$. Therefore, μ has to be of the order of the electroweak or SUSY breaking scale, even if, in the MSSM, it has no reason to be so. This is called the μ-problem.

In the NMSSM, the addition of a new gauge singlet Higgs superfield $\hat{S}$ will modify the Higgs superpotential, Eq. 1, as,

$$W_{\text{Higgs}}^{\text{NMSSM}} = \lambda \hat{S}\hat{H}_u.\hat{H}_d + \frac{\kappa}{3}\hat{S}^3, \tag{3}$$

where λ and κ are now dimensionless couplings. The scalar (and neutral) part of $\hat{S}$ gets a vev s, generated by soft mass terms, of the order of the SUSY breaking scale. This will generate a new effective μ term, $\mu_{eff} = \lambda s$, at the desired scale, thus solving the μ-problem.

Another challenge for the MSSM is the Higgs mass, the upper bound is given by :

$$m^2_{h,\text{MSSM}} = m_Z^2\cos(2\beta) + \frac{3}{(4\pi)^2}\frac{m_t^4}{v^2}\left[\ln\frac{m^2_{\tilde{t}}}{m_t^2} + \frac{X_t^2}{m^2_{\tilde{t}}}\left(1 - \frac{X_t^2}{12m_Z^2}\right)\right], \tag{4}$$

where m_Z is the Z-boson mass, m_t the top mass, $m^2_{\tilde{t}} = m_{\tilde{t}_1}m_{\tilde{t}_2}$ the product of both stop masses, $X_t = A_t - \frac{\mu}{t_\beta}$ the off-diagonal term in the stop mass matrix and $v^2 = v_u^2 + v_d^2$. In Eq. 4, the first term is the tree level mass term, whose value is bounded by the Z-boson mass. To get a 125 GeV Higgs boson, one then has to fine tune the second term, which is the dominant top sector contribution to the radiative corrections of the Higgs mass. Consequently, the MSSM is strongly constrained by the Higgs sector.

In the NMSSM, this is not the case anymore. Indeed, the modification of the Higgs superpotential, Eq. 3, will add a new tree-level mass term to the upper bound shown in Eq. 4 :

$$m^2_{h,\text{NMSSM}} = m^2_{h,\text{MSSM}} + \lambda^2 v^2\sin^2(2\beta). \tag{5}$$

With this new contribution, it is possible to reach already at tree-level a Higgs boson mass of 125 GeV thus relaxing the fine-tuning of radiative corrections. This feature renders the NMSSM more natural than the MSSM.

The NMSSM features an extended Higgs sector with tree CP-even Higgs bosons that are mixture of the two doublets and the singlet Higgs gauge eigenstates. The lightest or the second lightest ones could correspond to the Higgs observed at the LHC. Indeed, if the lightest one is dominantly singlet, all its couplings to other particles, except to the other Higgses, are suppressed and it could escape detection. Moreover, the singlino, the fermionic content of the superfield $\hat{S}$, mixes with the higgsinos, bino and neutral wino to give five neutralinos rather than four in the MSSM. The phenomenology of neutralinos, especially for Dark Matter searches, is then broadened.

2 Renormalisation of charginos-neutralinos

Precise predictions for observables require to be able to compute radiative corrections, for this the renormalisation of the model is required. We will describe here the renormalisation of the chargino/neutralino sectors of the NMSSM.

The non standard fermionic particles of the NMSSM are the 2 charginos, combination of charged winos and Higgsinos, and the 5 neutralinos, combinations of bino, wino, neutral higgsinos and the pure NMSSM singlino :

$$\psi_c^R = \begin{pmatrix} -i\tilde{W}^- \\ \tilde{H}_d^- \end{pmatrix}, \quad \psi_c^L = \begin{pmatrix} -i\tilde{W}^+ \\ \tilde{H}_u^+ \end{pmatrix}, \quad \psi_n^R = \psi_n^L = \psi^0 = \begin{pmatrix} -i\tilde{B}^0 \\ -i\tilde{W}_3^0 \\ \tilde{H}_d^0 \\ \tilde{H}_u^0 \\ \tilde{S}^0 \end{pmatrix} \tag{6}$$

The mass matrix for the charginos (X) are the same as in the MSSM while the mass matrix for neutralinos reads

$$Y = \begin{pmatrix} M_1 & 0 & -M_Z s_W c_\beta & M_Z s_W s_\beta & 0 \\ 0 & M_2 & M_Z c_W c_\beta & -M_Z c_W s_\beta & 0 \\ -M_Z s_W c_\beta & M_Z c_W c_\beta & 0 & -\mu & -\lambda v_u \\ M_Z s_W s_\beta & -M_Z c_W s_\beta & -\mu & 0 & -\lambda v_d \\ 0 & 0 & -\lambda v_u & -\lambda v_d & 2\kappa s \end{pmatrix}, \tag{7}$$

where M_1 and M_2 are the U(1) and SU(2) gaugino masses, λ the coupling between the three Higgs superfields in the NMSSM, κ the trilinear coupling of the singlet Higgs superfield with itself. There is also a dependence on s and t_β. All these parameters have to be renormalised to be able to compute radiative corrections. We also recall that $\mu = \lambda s$, which means that we need to extract the counterterms for only 2 of these 3 parameters.

The transition to mass eigenstates is realized with unitary matrices (U, V, and N),

$$\chi^R = U\psi_c^R, \quad \chi^L = V\psi_c^L, \quad \chi^0 = N\psi^0 \quad . \tag{8}$$

and the mass eigenstates read

$$\tilde{X} = U^* X V^\dagger = diag(m_{\tilde{\chi}_1^+}, m_{\tilde{\chi}_2^+}), \quad \tilde{Y} = N^* Y N^\dagger = diag(m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0}, m_{\tilde{\chi}_3^0}, m_{\tilde{\chi}_4^0}, m_{\tilde{\chi}_5^0}). \tag{9}$$

In the code `SloopS`, presented below, we choose to not renormalize the rotation matrices, which then remain the same at one-loop order. Therefore only the shifts of the mass matrices and wave functions are needed to specify the counterterms :

$$M_0 = M + \delta M, \tag{10}$$

$$\chi_{i_0}^{R,L} = \left(\delta_{ij} + \frac{1}{2}\delta Z_{ij}^{R,L}\right)\chi_j^{R,L}. \tag{11}$$

For charginos, which are Dirac fermions, the left and right-handed parts are not shifted in the same manner, Eq. 11. The renormalised self-energies are then given by (P_L and P_R are the left and right projection operator) :

$$\hat{\Sigma}_{ij}(q) = \Sigma_{ij}(q) - P_L \delta m_{ij} - P_R \delta m_{ji}^* + \frac{1}{2}(\not{q} - m_{\tilde{\chi}_i})\left[\delta Z_{ij}^L P_L + \delta Z_{ij}^{R*}\right] + \frac{1}{2}\left[\delta Z_{ji}^{L*} P_R + \delta Z_{ji}^R P_L\right](\not{q} - m_{\tilde{\chi}_j}) \tag{12}$$

To fix the wave function and mass counterterms and then extract the counterterms for parameters, we use on-shell renormalisation conditions. Requiring that the masses of particles chosen as input do not receive any one-loop corrections, is equivalent to imposing the following condition on the renormalised self energies : $Re\hat{\Sigma}_{ii}(m_{\chi_i}^2) = 0$. This gives the mass counterterms. We also demand that the propagators of all charginos and neutralinos are properly normalised with a residue equal to 1 at the pole mass, $Re\hat{\Sigma}'_{ii}(m_{\chi_i}^2) = 0$. From this we extract the diagonal wave function renormalisation constants. Finally, we require no mixing between fields when on-shell, $Re\hat{\Sigma}_{ij}(m_{\chi_i}^2) = 0$. This gives the non diagonal wave function renormalisation constants and completes the renormalisation of the chargino-neutralino sector.

3 SloopS

`SloopS` is a code developed at LAPTh to compute cross-sections and other observables at one-loop in SUSY. The full renormalisation of the MSSM was done some years ago [2,3]. In this code, the complete spectrum and set of vertices are generated at the tree-level through the LanHEP package [4]. The complete set of Feynman rules is then derived automatically and passed to the bundle FeynArts/FormCalc/LoopTools [5,6,7]. A powerful feature of SloopS is the ability to check not only the UV finiteness , but also IR convergence and gauge independence of the results through a generalized gauge fixing Lagrangian. Here we present the renormalisation of the NMSSM, whose implementation is still in progress in SloopS.

4 Numerical results

In the chargino-neutralino sector, one needs to renormalize six parameters. In an on-shell scheme, this means that the masses of six particles should be chosen as input. Different schemes are implemented and compared, for example we can take choose both charginos and four neutralinos as input. A look at the mass matrices, Eq. 7, shows that the charginos will basically reconstruct $\delta\mu$ and δM_2 and the neutralinos δM_1, $\delta\kappa$, δs and δt_β. Since there are five neutralinos, we apparently have the freedom to choose any set of four neutralinos as input. In practice, some choices lead to numerical instability. In fact, to reconstruct δM_1, it is essential to take the dominantly bino neutralino as input. Indeed, we have to invert a system of six equations to get the counterterms of the six parameters. If the bino-like neutralino is not in input, this will imply division by small elements of the mixing matrix and yield a bad reconstruction. The same argument shows that the dominantly singlino and at least one of the two higgisnos should be used as input. For each point of the parameter space, a procedure to choose the best scheme must be implemented. To illustrate the scheme dependence, we compare two schemes, one with the four lightest neutralinos as input (OS_{1234}) and one with the four heaviest ones (OS_{2345}) for three points with different neutralino hierarchy and compute the corrected mass of the remaining neutralino. The results are presented in table 1.

Table 1: (Left) Hierachy of neutralinos for three benchmark points, only the dominant part for each neutralino is listed. (Right) Corrected mass of the remaining neutralino in both schemes

	χ_1^0	χ_2^0	χ_3^0	χ_4^0	χ_5^0
Point 1	$\tilde{W}_3^0$	$\tilde{H}^0$	$\tilde{H}^0$	$\tilde{S}^0$	$\tilde{B}^0$
Point 2	$\tilde{S}^0$	$\tilde{B}^0$	$\tilde{H}^0$	$\tilde{H}^0$	$\tilde{W}_3^0$
Point 3	$\tilde{W}_3^0$	$\tilde{S}^0$	$\tilde{B}^0$	$\tilde{H}^0$	$\tilde{H}^0$

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Scheme	Masses	point 1	Point 2	Point 3
OS_{1234}	$m_{\tilde{\chi}_5}^{tree}$	1002.17	614.78	573.89
	$m_{\tilde{\chi}_5}^{1-loop}$	710.69	614.82	574.92
OS_{2345}	$m_{\tilde{\chi}_1}^{tree}$	125.68	123.51	139.40
	$m_{\tilde{\chi}_1}^{1-loop}$	125.55	-114.38	139.43

Following the discussions above, we expect OS_{1234} to be a good scheme for point 2 and 3, but not for point 1 where the bino is not in input while OS_{2345} should be a good scheme only for points 1 and 3. Indeed, for a good choice of scheme, the corrected mass of the neutralino not chosen as input receives only a small correction neutralino, below 0.1%, but is wrong otherwise.

These few results illustrate the importance of the choice of renormalisation scheme to obtain reliable results. The procedure described here allows to compute automatically radiative corrections for physical observables such as decay widths and cross sections in the NMSSM. More complete results will be presented in an upcoming paper we well as the renormalisation of the sfermion and Higgs sectors.

References

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