Lattice Inputs to Flavor Physics

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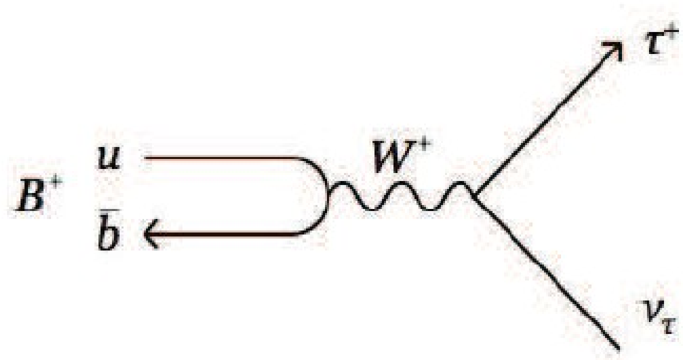
Outline

- Flavour Lattice Averaging Group (FLAG)
  Scope and review as of end of 2013
- $\mathcal{R}(D^{(*)})$ status and perspective for lattice
- QED corrections to hadronic processes
  Spectrum [BMW, 1406.4088]
  Leptonic decays [Carrasco et al., 1502.00257]
- $K \to \pi\pi$ and perspective for hadronic decays on the lattice [RBC-UKQCD, 1502.00263] and [Hansen and Sharpe, Briceno and Davoudi, 2012]

Several hadronic processes depend on hadronic contributions. E.g.

(Charged) Decay constants

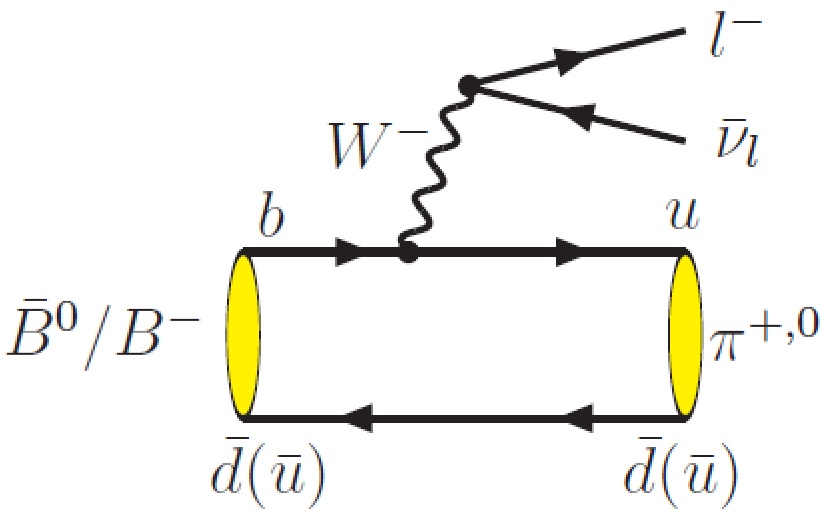
$\langle 0|\overline{u}_f \gamma_\mu \gamma_5 d_{f'}|P(p)\rangle = F_P p_\mu$ are the hadronic parameters entering leptonic decays of pseudoscalar mesons



$$\Gamma(B \to \ell \overline{\nu}_\ell) = \frac{G_F^2}{8\pi}|V_{ub}|^2 F_B^2 \left(\frac{m_\ell}{m_B}\right)^2 m_B^3 \left(1 - \frac{m_\ell^2}{m_B^2}\right)$$

Form factors

Parameterizing semileptonic decay. Simplest: $B \to \pi \ell \nu$



Ignoring the lepton mass:

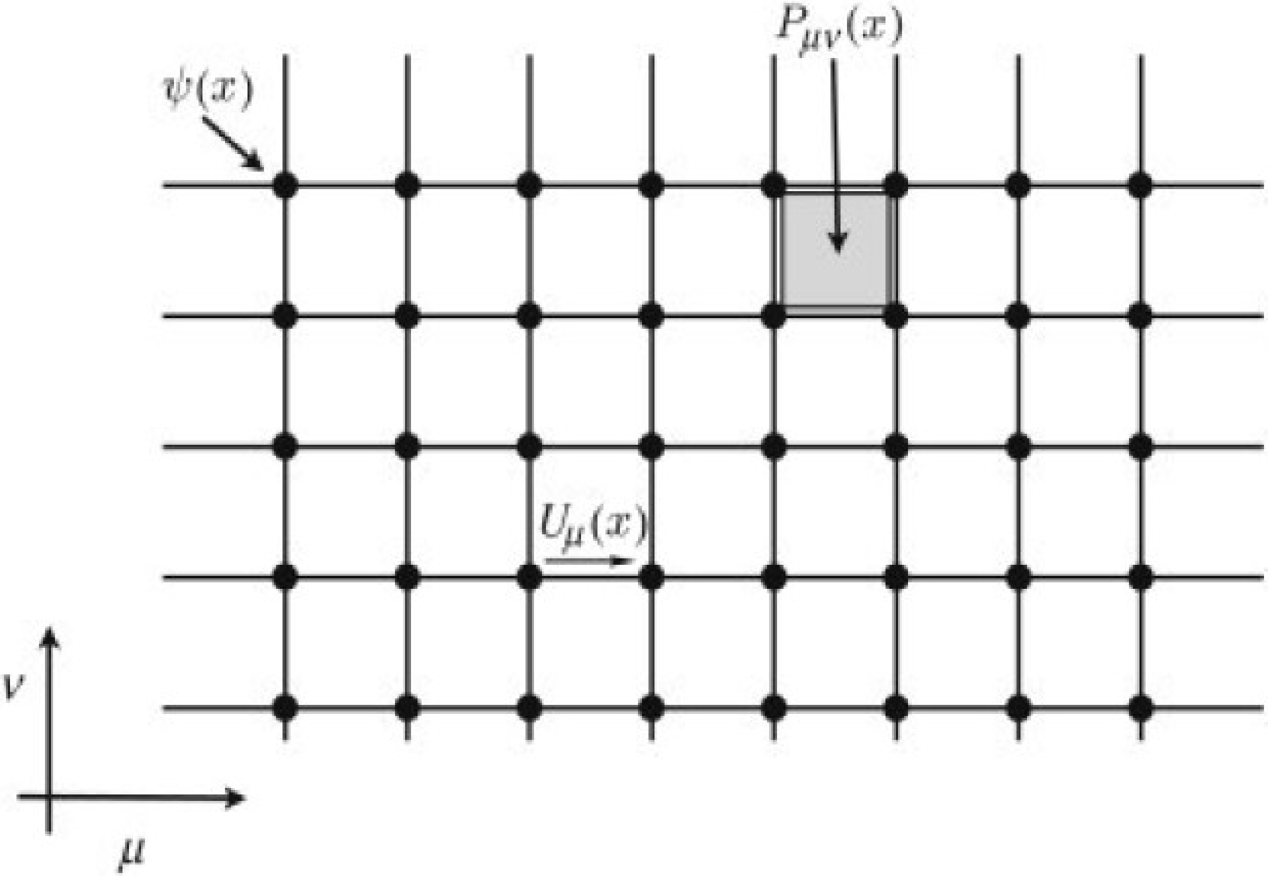
$$\frac{d\Gamma}{dq^2} = \frac{G_F^2}{24\pi^3} p_\pi^3 |V_{ub}|^2 |f_+(q^2)|^2$$

The hadronic matrix element is from a quark bilinear

$$\langle \pi(p_\pi)|V^\mu|B(p_B)\rangle = f_+(q^2)(p_\pi + p_B - q\Delta_{m^2})^\mu + f_0(q^2)q^\mu$$

with $\Delta_{m^2} = (m_B^2 - m_\pi^2)/q^2$

QCD on the lattice

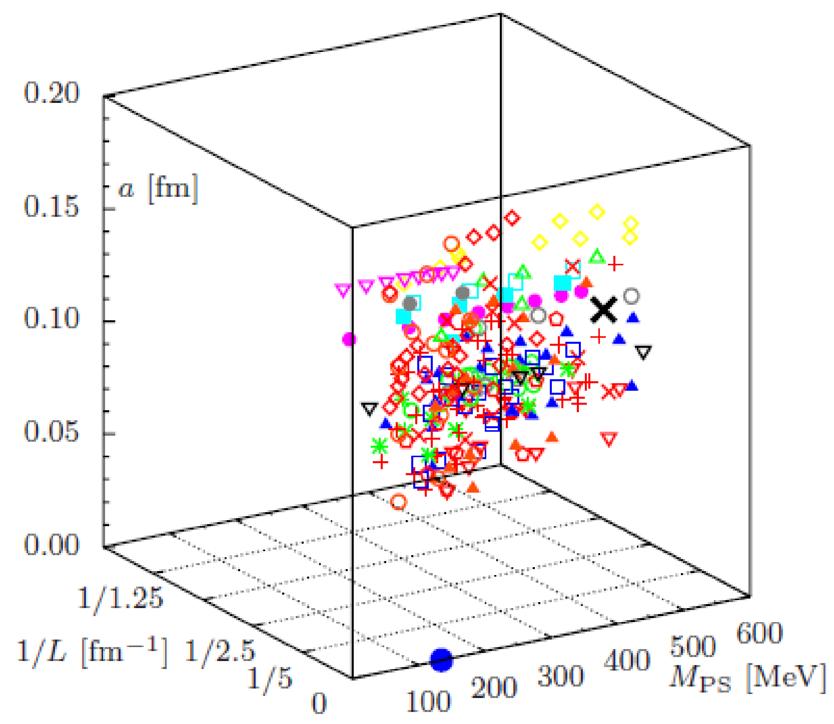


$$S_{\mathrm{W}}^{\mathrm{QCD}} = \overline{\psi}_x D_{xy}^{\mathrm{W}}(U)\psi_y + \beta \sum_{\square} \left(1 - \frac{1}{N}\mathrm{Re}\,\mathrm{Tr}\; U_{\mu\nu}(x)\right)$$

Lattice can provide first principle – systematically improvable determinations of such parameters. However they are not free from approximations / systematics

- Number of dynamical flavours
- Unphysical quark masses (and no isospin breaking)
- Finite lattice spacing
- Finite volume
- Renormalization

FLAG's goal is to walk users of lattice results through systematics and the way they have been addressed



Gregorio Herdoíza

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THE EUROPEAN PHYSICAL JOURNAL C

Review

Review of lattice results concerning low-energy particle physics

FLAG Working Group

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Criteria, as of now

- Chiral extrapolation:
  ★ $M_{\pi,\mathrm{min}} < 200$ MeV
  ○ $200 \text{ MeV} \leq M_{\pi,\mathrm{min}} \leq 400$ MeV
  ■ $400 \text{ MeV} < M_{\pi,\mathrm{min}}$

- Continuum extrapolation:
  ★ 3 or more lattice spacings, at least 2 points below 0.1 fm
  ○ 2 or more lattice spacings, at least 1 point below 0.1 fm
  ■ otherwise

- Finite-volume effects:
  ★ $M_{\pi,\mathrm{min}}L > 4$ or at least 3 volumes
  ○ $M_{\pi,\mathrm{min}}L > 3$ and at least 2 volumes
  ■ otherwise

- Renormalization (where applicable):
  ★ non-perturbative
  ○ 1-loop perturbation theory or higher with a reasonable estimate of truncation errors
  ■ otherwise

For heavy-light quantities, in order to deal with the different approaches, we used data-driven criteria. We introduced

$$D(a) = \frac{Q(a) - Q(0)}{Q(a)}$$

for a=a_{min}

$$\delta(a) = \frac{Q(a) - Q(0)}{\sigma_Q}$$

and then used

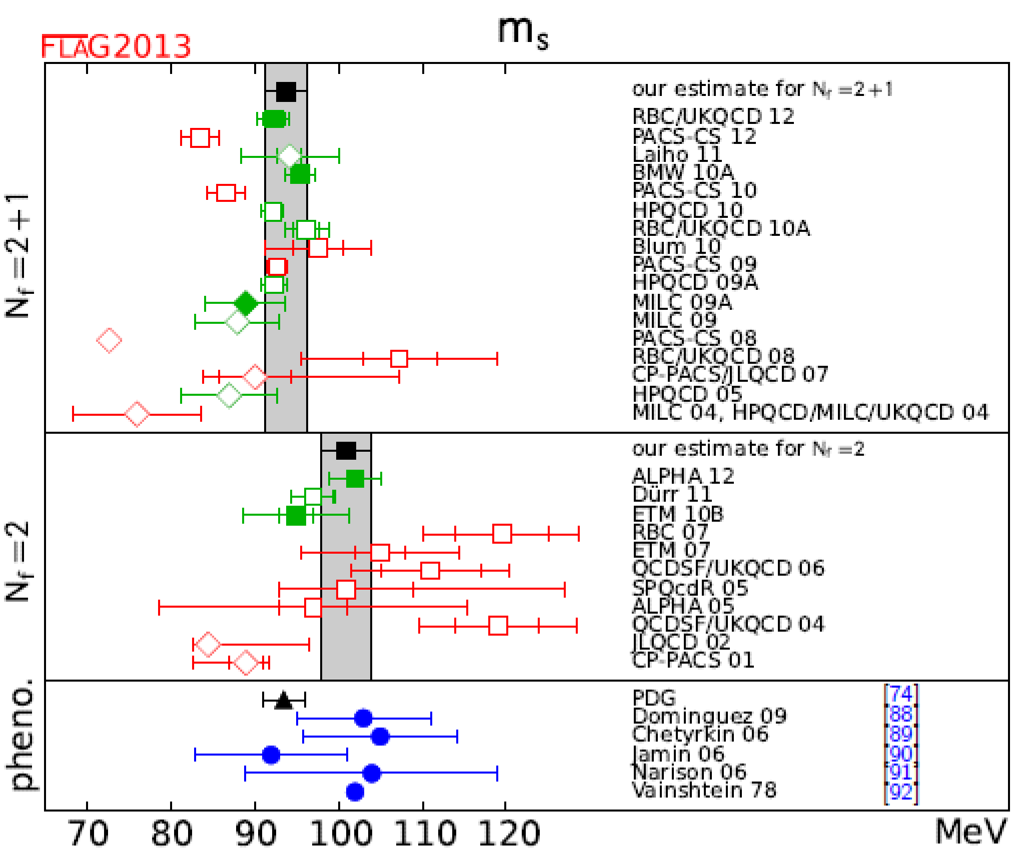
★ (i) Three or more lattice spacings, and
(ii) $a^2_{\mathrm{max}}/a^2_{\mathrm{min}} \geq 2$, and
(iii) $D(a_{\mathrm{min}}) \leq 2\%$, and
(iv) $\delta(a_{\mathrm{min}}) \leq 1$

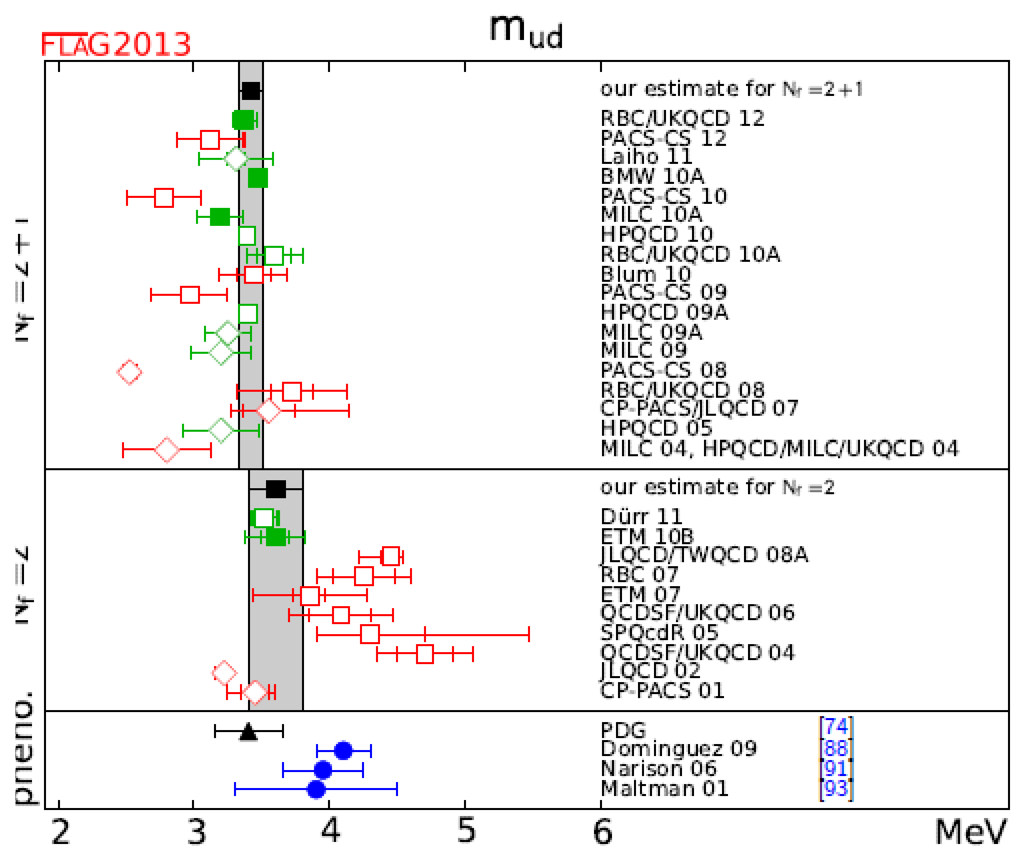
○ (i) Two or more lattice spacings, and
(ii) $a^2_{\mathrm{max}}/a^2_{\mathrm{min}} \geq 1.4$, and
(iii) $D(a_{\mathrm{min}}) \leq 10\%$, and
(iv) $\delta(a_{\mathrm{min}}) \leq 2$

■ otherwise

Only results with no red symbols enter the final estimates

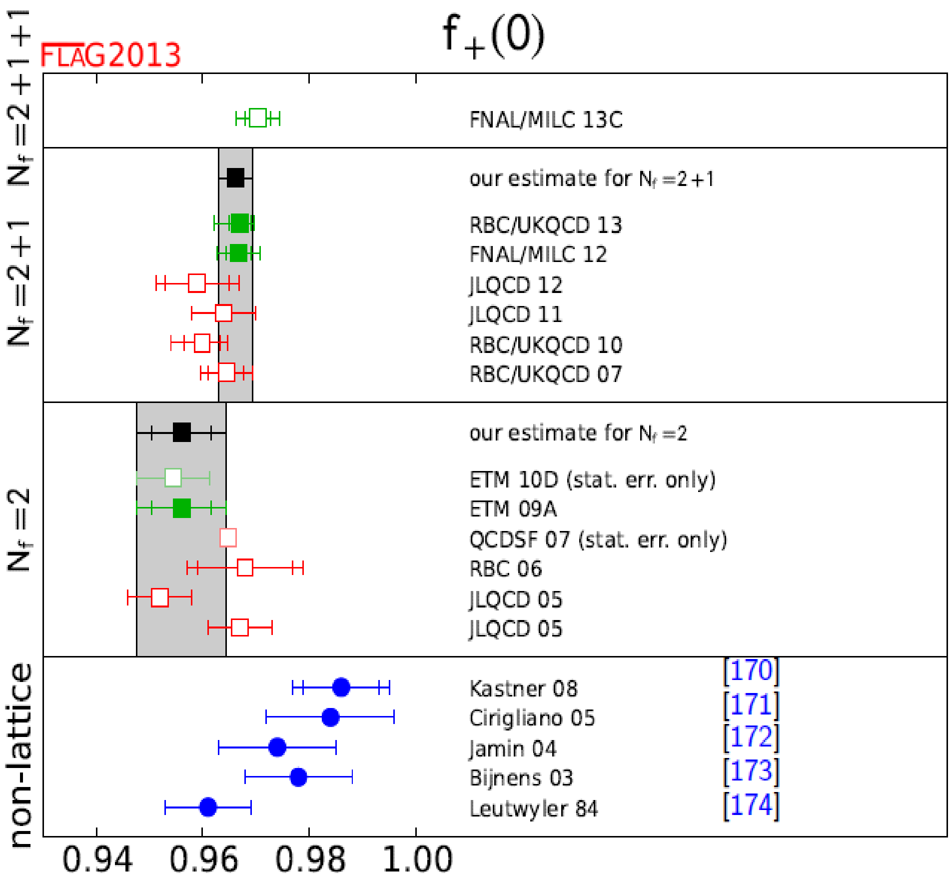
Light quark masses

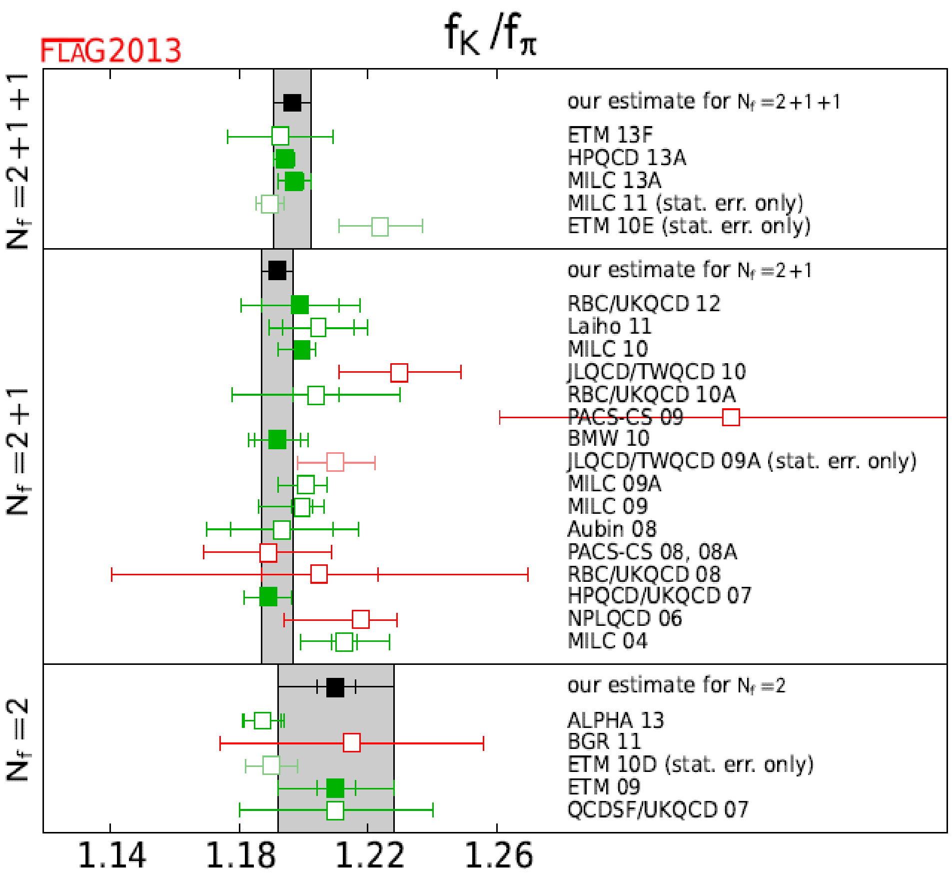


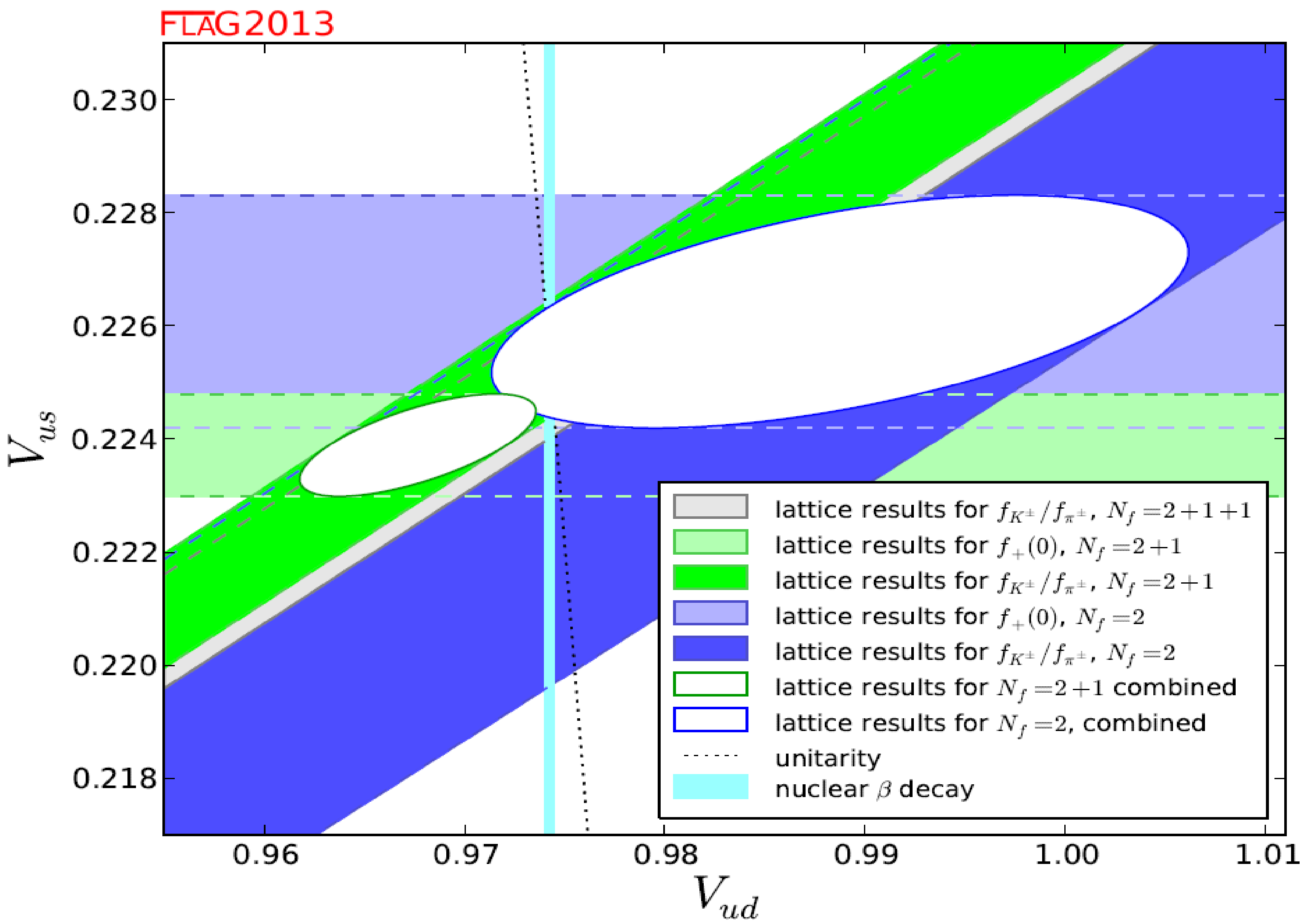


- '*Estimates*' differ from '*averages*'. For N_f=2+1 an error coming from quenching of the charm has been included

Leptonic and semileptonic Kaon and pion decays

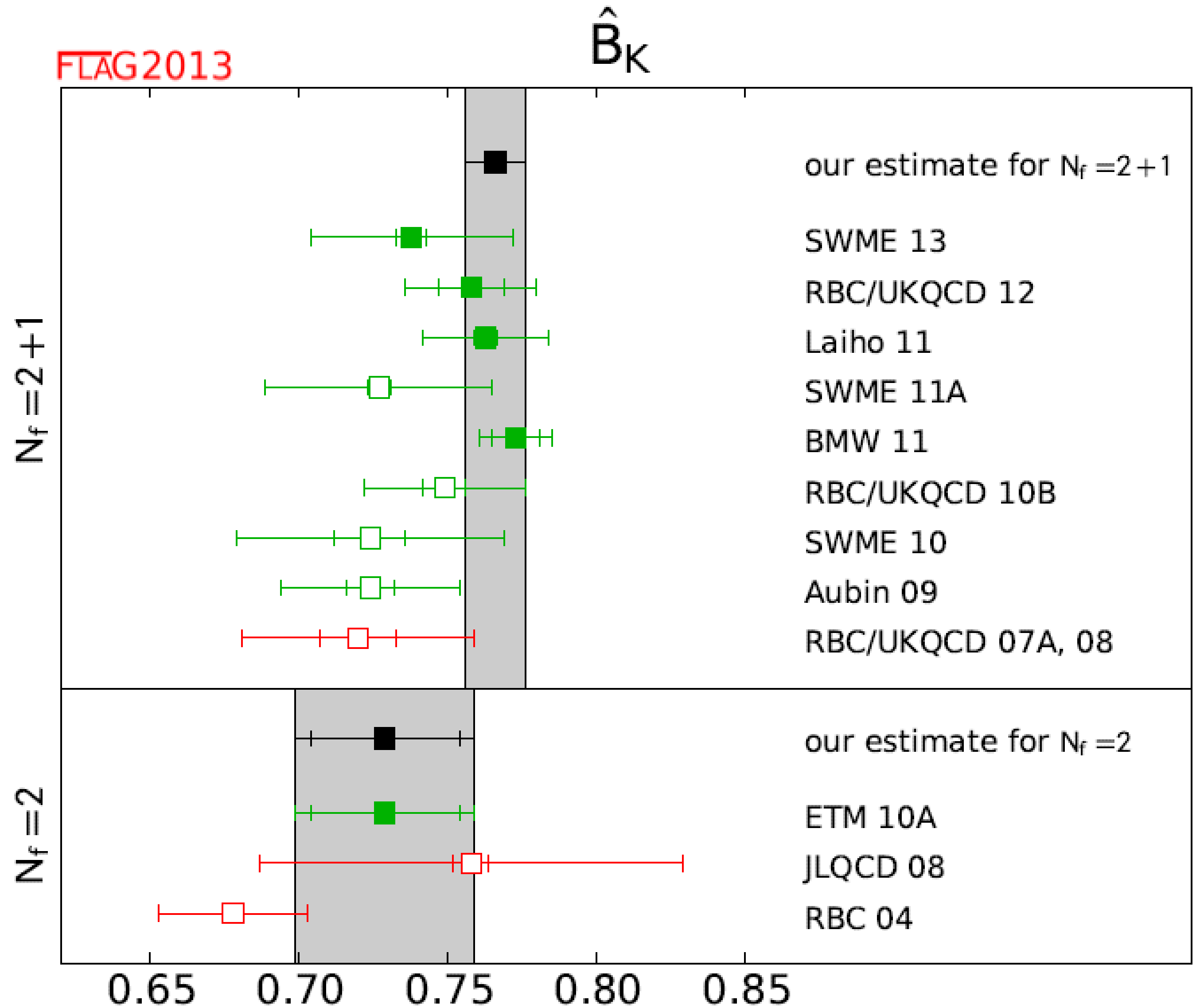






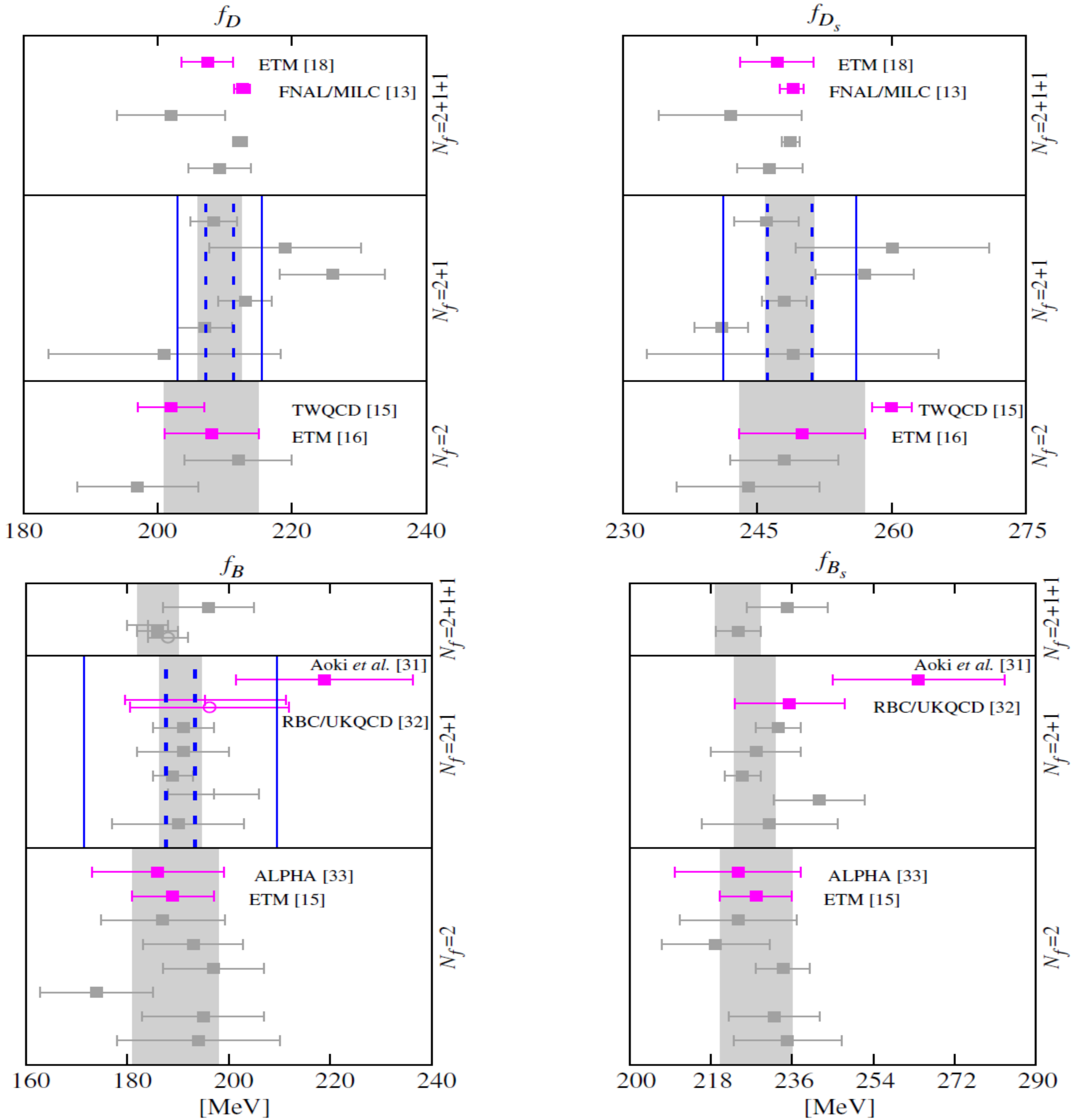
$|V_u|^2$=0.987(10). The consistency with leptonic and semi-leptonic determinations of $|V_{us}|$ is a check of the equality of the Fermi constant describing interactions among leptons and the one describing interactions among leptons and quarks (may not be in BSM).

The hadronic parameter in ε_K at LO in the EWH



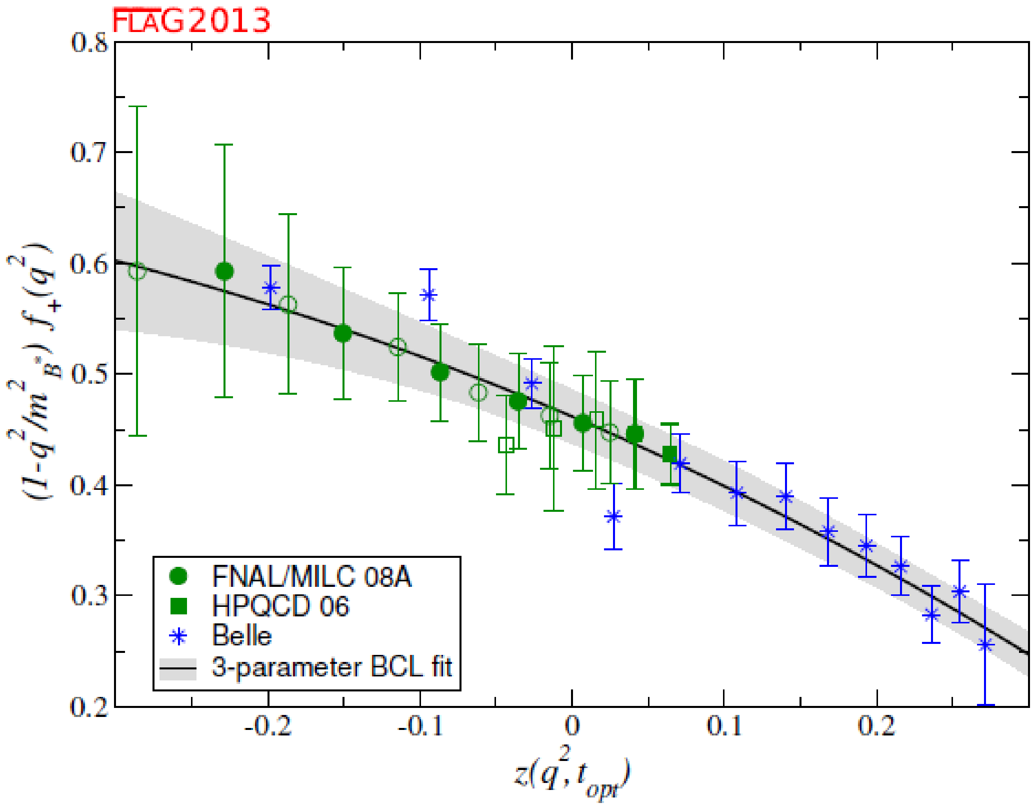
New quantities in FLAG II

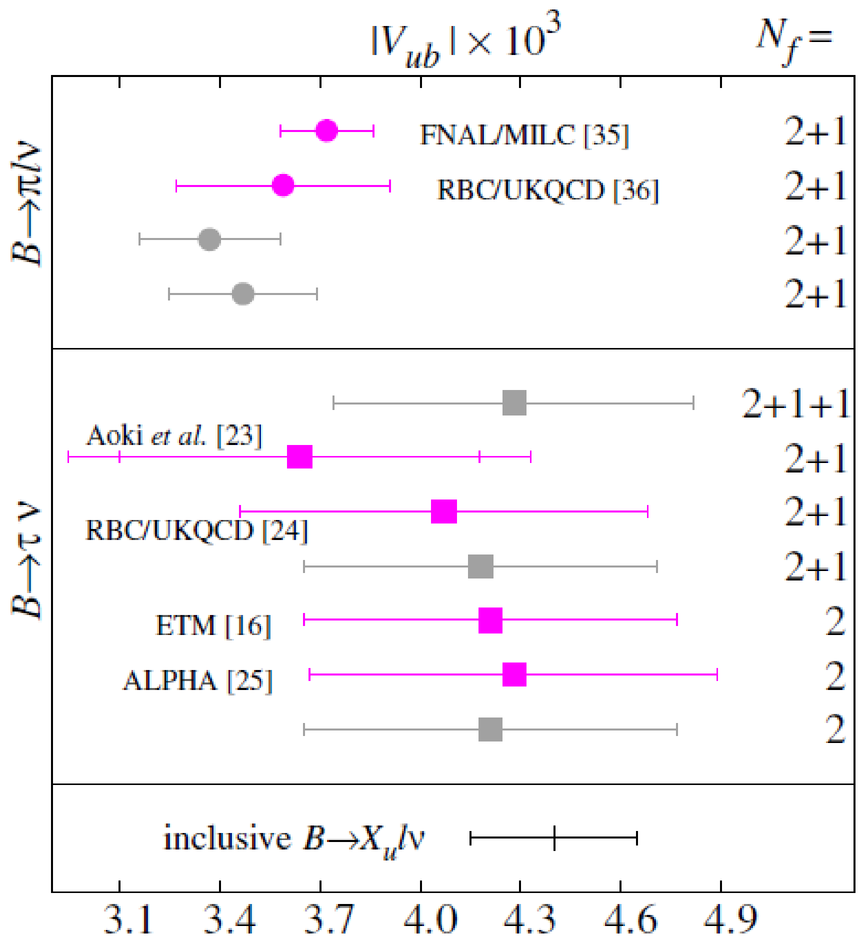
- For light flavors, lattice computations are quite precise, mature and advanced, to the point that isospin breaking and QED effects have to be included soon (see later).
- "Heavy quantities" included in FLAG-II are less advanced. Fewer computations (sometimes one only) passing the criteria.
- Results have been updated in [C. Bouchard, LAT14, arXiv:1501.03204], where errors are compared to the expected experimental improvements from Belle II, BES III and LHCb (e.g. 50 ab^{-1} by Belle II by 2020). The comparison shows once again that what is easy for lattice is difficult for experiment and vice-versa.



f_D
ETM [18]
FNAL/MILC [13]
TWQCD [15]
ETM [16]
N_f=2+1+1
N_f=2+1
N_f=2
180
200
220
240
f_{D_s}
ETM [18]
FNAL/MILC [13]
TWQCD [15]
ETM [16]
230
245
260
275
f_B
Aoki *et al.* [31]
RBC/UKQCD [32]
ALPHA [33]
ETM [15]
160
180
200
220
240
[MeV]
f_{B_s}
Aoki *et al.* [31]
RBC/UKQCD [32]
ALPHA [33]
ETM [15]
200
218
236
254
272
290
[MeV]

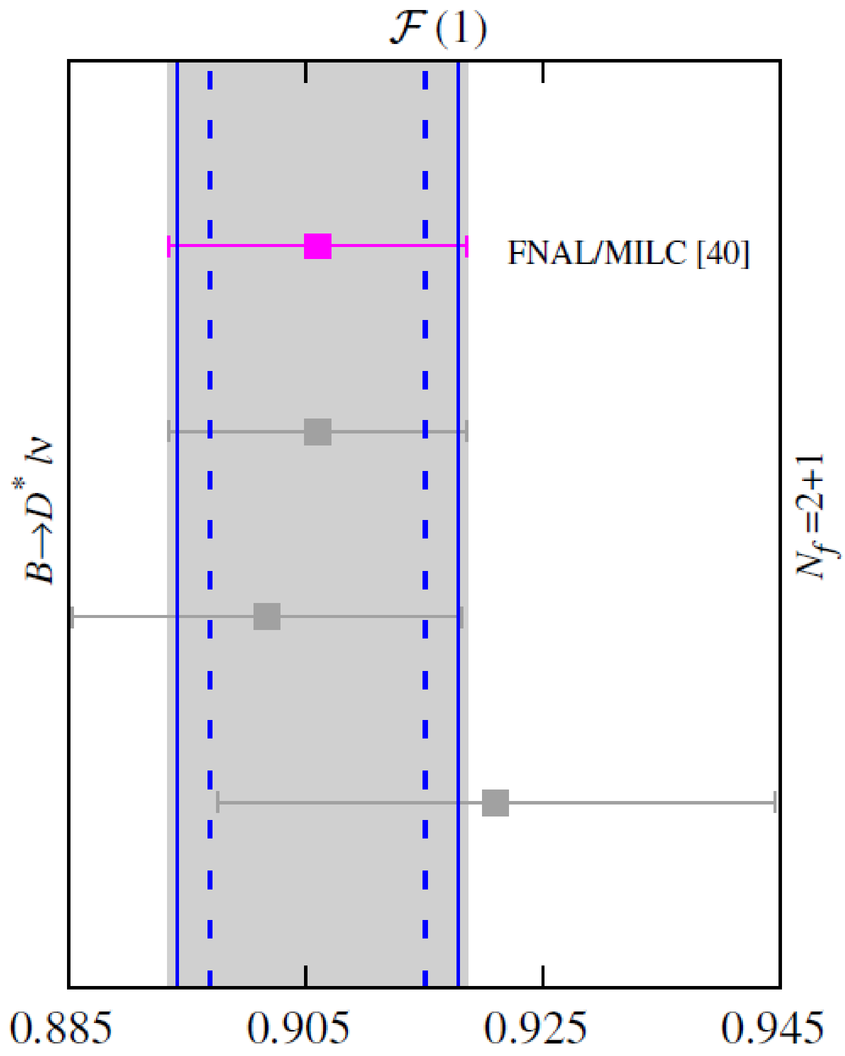
B-> πlν form factors

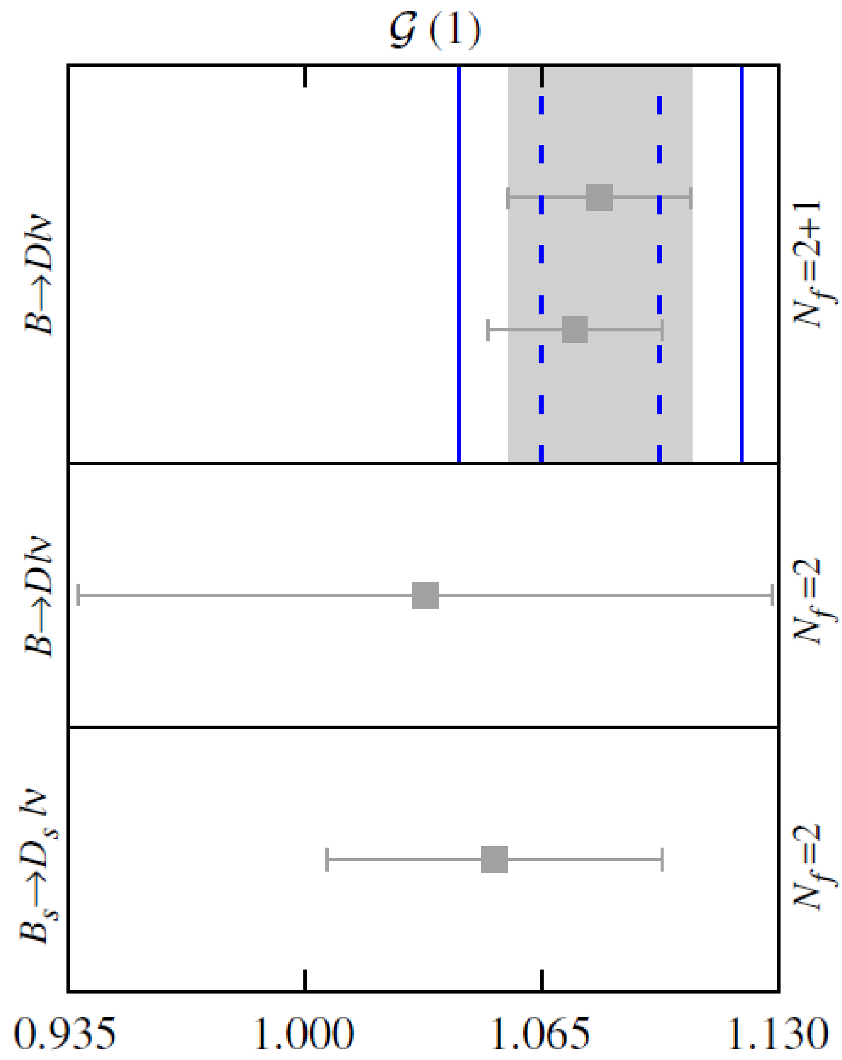




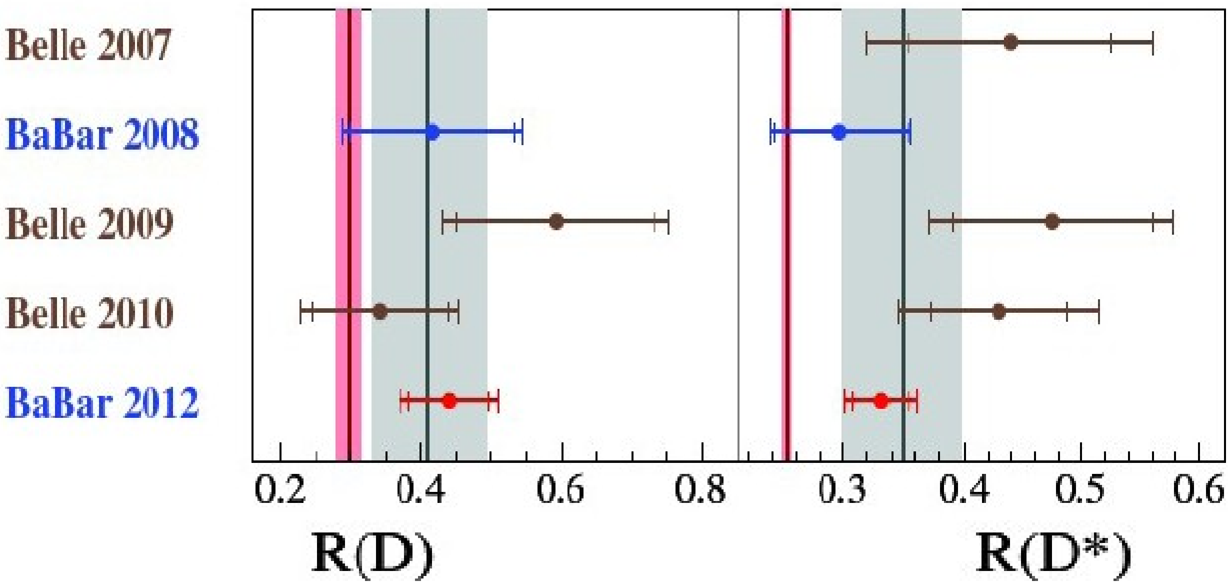
- Few lattice results, in the region complementary to experiments
- CKM matrix element fitted to normalize exp data

B->D$^{(*)}$lv and R(D$^{(*)}$)





- PS -> V form factors are usually computed at zero recoil. In that limit only one form factor is relevant (others are helicity suppressed).

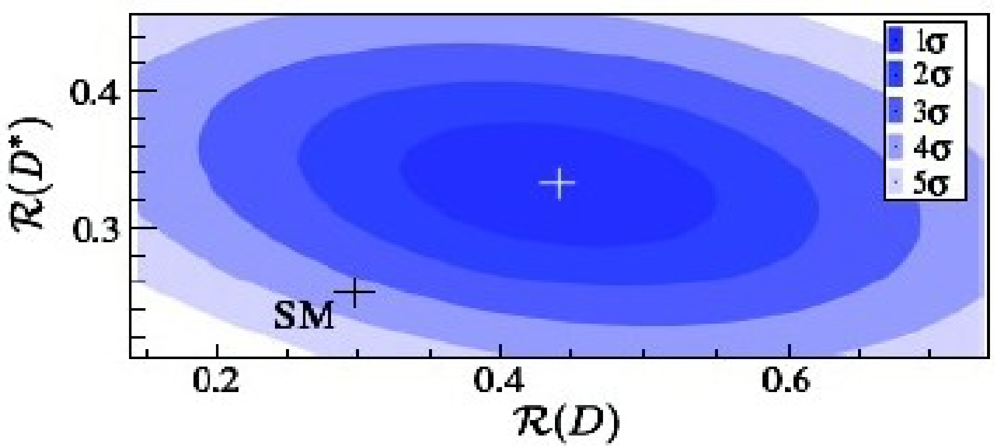


$$\mathcal{R}(D) \equiv \frac{\mathcal{B}(B \to D\tau^- \bar{\nu}_\tau)}{\mathcal{B}(B \to D l^- \bar{\nu}_l)}$$

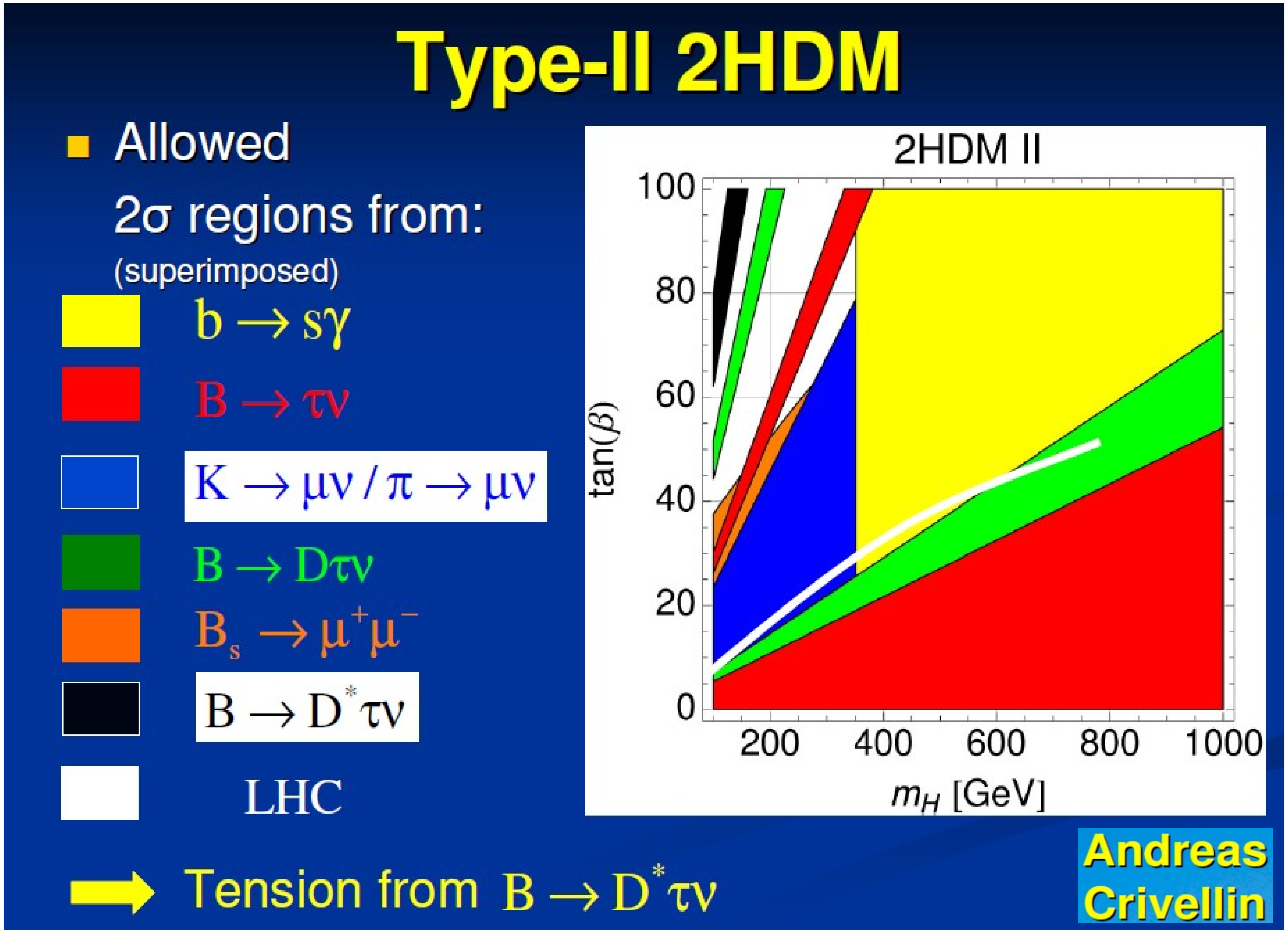
$$\mathcal{R}(D^*) \equiv \frac{\mathcal{B}(B \to D^* \tau^- \bar{\nu}_\tau)}{\mathcal{B}(B \to D^* l^- \bar{\nu}_l)}$$

$$\mathcal{R}(D^0) = \mathcal{R}(D^+) = \mathcal{R}(D)$$

$$\mathcal{R}(D^{*0}) = \mathcal{R}(D^{*+}) = \mathcal{R}(D^*)$$



- $\mathcal{R}^0(D)_{\mathrm{SM}} = 0.324(22)$, $\mathcal{R}^0(D^*)_{\mathrm{SM}} = 0.250(3)$ from combination of phenomenological inputs, HQET and sum-rules [Biancofiore, Colangelo and De Fazio, 2013].
- Only one result, $\mathcal{R}(D)_{\mathrm{SM}} = 0.316(12)(7)$ from lattice [FNAL/MILC, Bailey et al., 2012].
- Values of form factors at zero recoil cancel out in the ratios.



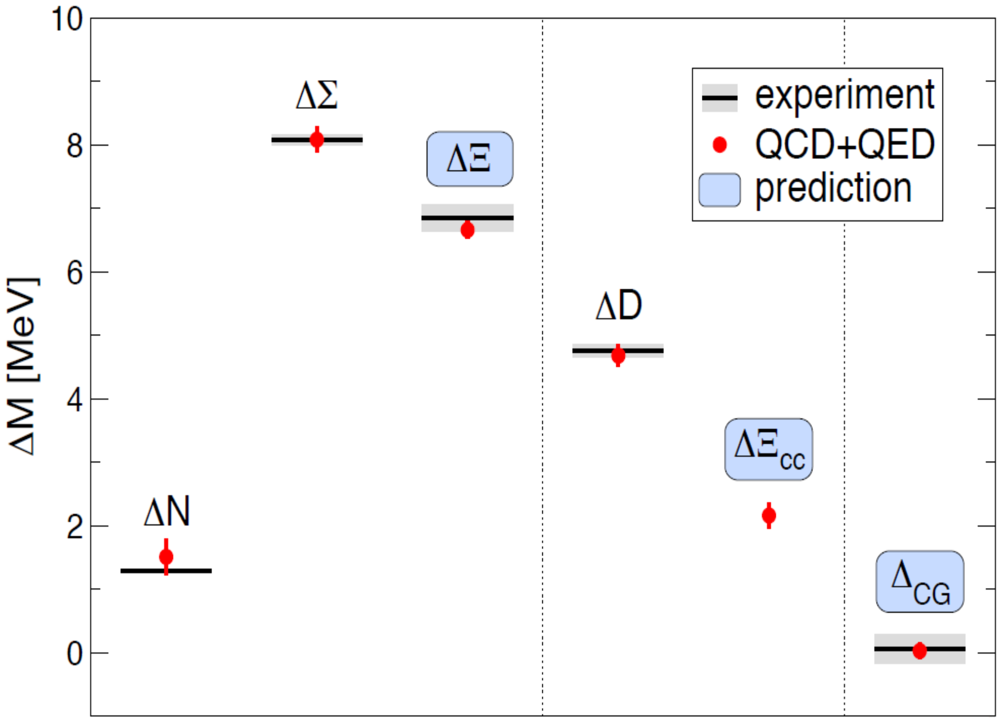
- Natural idea would be new contributions from charged scalar exchanges.
- However, those would enhance the leptonic channel as well.
- It is difficult to accommodate the discrepancy in 2HDM without FCNC at tree level [Celis, Crivellin]. Otherwise, some leptoquark models can explain it [Tanaka, Watanabe, 2013].

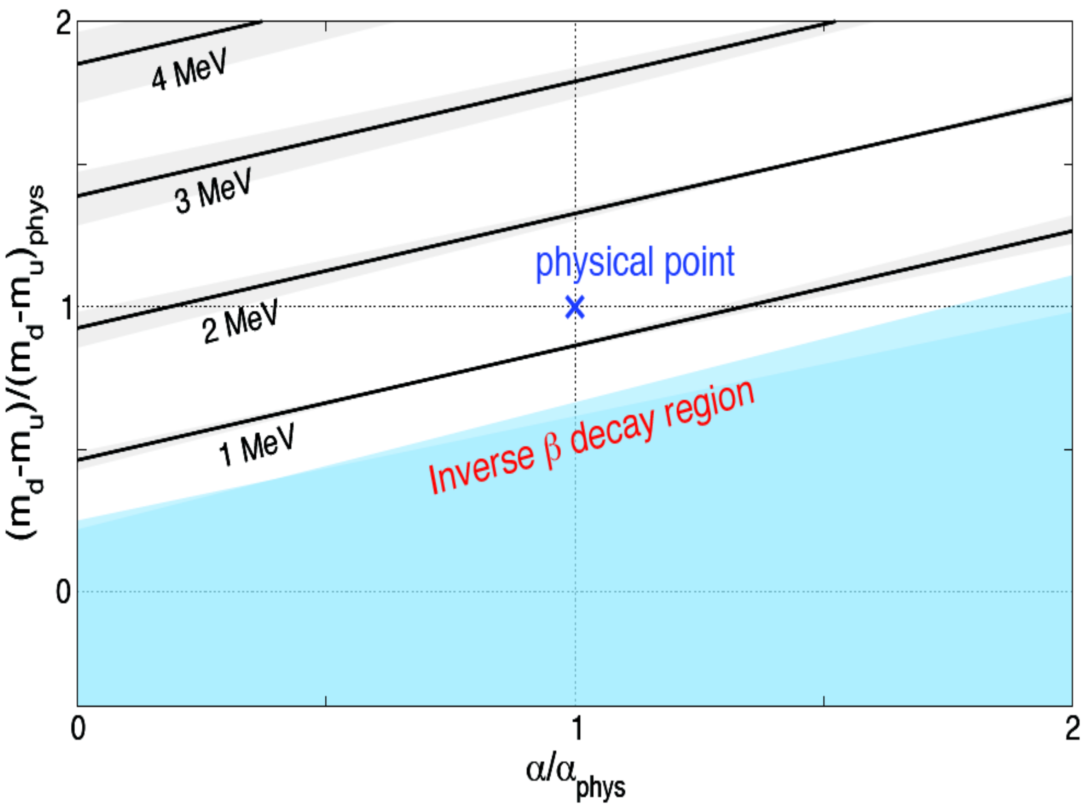
In general, in BSM theories other form factors (S, PS, T) may appear [Nierste, Trine, Westhoff '08, Kamenik, Mescia '08, Kamenik, Fajfer, Nisandzic '12, Biancofiore …].

Clear opportunities for lattice

QED effects are becoming relevant for *light* quantities.

QCD + QED direct simulations [Borsanyi et al., BMW group, 2014]





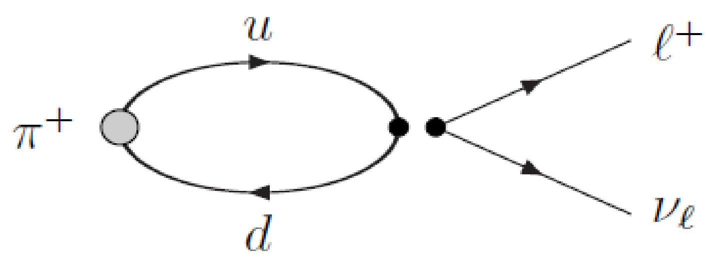
- Large volume 1+1+1+1 simulations of QCD + QED (at unphysical *e* due to noise to signal problem). 300 times more expensive than N_f=2 QCD. Pilot and benchmark computation concerning the setup.

- Separation of effects using $\Delta M_{\Sigma}^{QED}=0$

QED corrections to hadronic processes

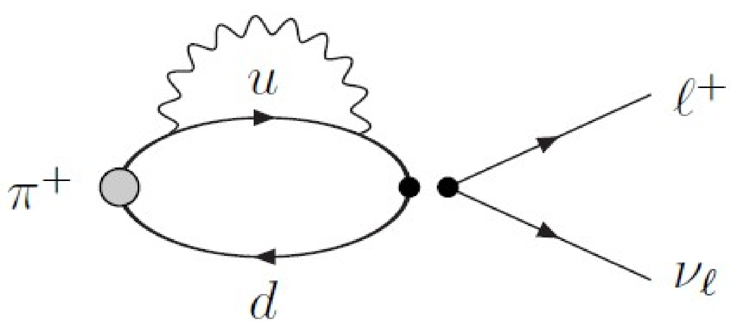
Let's consider the leptonic decay at $O(\alpha)$ in the WEH [N. Carrasco et al., 1502.00257]

Pure QCD

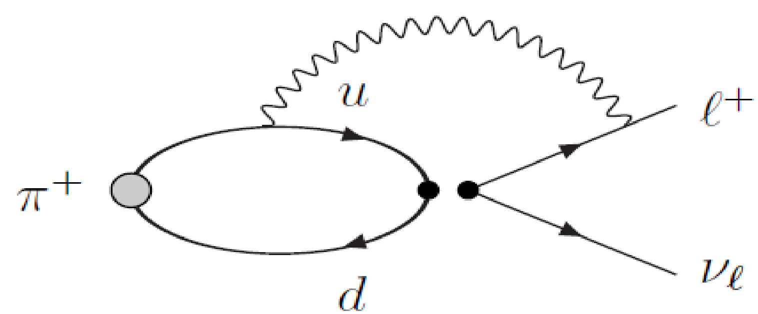


factorizable. Hadronic part $\to f_P$. Then Γ_0 at $O(\alpha)$

||
Number external photons



Still factorizable.



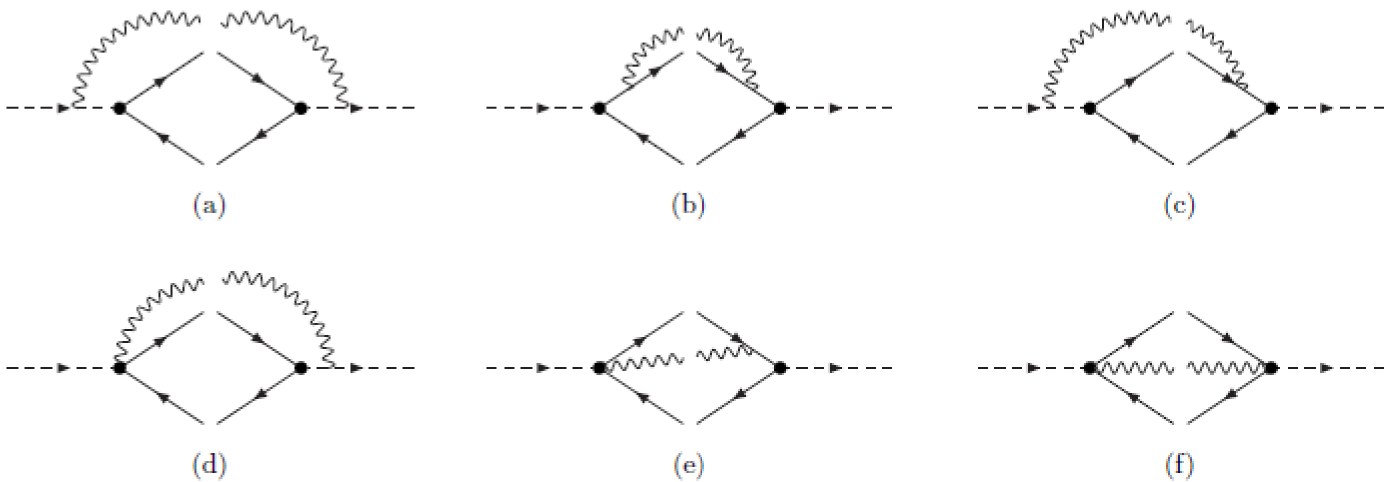
NON factorizable.

Also, Γ_0 is infrared divergent, one needs to consider (one) real photon emission as well. No such problems for spectrum.

F.Bloch, A.Nordsieck Phys.Rev. 52 (1937)
T.D.Lee, M.Nauenberg Phys.Rev. 133 (1964)

$\Rightarrow$ Not much sense of QED corrections to a decay constant ...

$\Gamma_1(\Delta E)$ with $\Delta E = E_\gamma^{\rm max}$



The combination $\Gamma_0 + \Gamma_1(\Delta E)$ is free from IR divergencies at $O(\alpha)$. One can split it as

$$\Gamma(\Delta) = \left\{\Gamma_0 - \Gamma_0^{pt}\right\} + \left\{\Gamma_0^{pt} + \Gamma_1(\Delta)\right\} = \overbrace{\lim_{L\mapsto\infty}\left\{\Gamma_0(L) - \Gamma_0^{pt}(L)\right\}}^{\text{lattice}} + \overbrace{\left\{\Gamma_0^{pt} + \Gamma_1(\Delta)\right\}}^{\text{continuum pt for } \Delta \ll M_P}$$

- pt=pointlike approximation (perturbative). OK for soft photons, they can't resolve the hadron structure. For K and π, $\Delta E \simeq 20$ MeV. Currently main limitation of the approach.
- Both terms are IR-safe and have a $L \to \infty$ limit.
- $\Gamma_0(L)$ is computed on the lattice. It requires rather involved Euclidean correlators, with lepton propagators in the numerical computation of the non-factorizable contributions.

Hadronic decays. A_2 amplitude of $K \to \pi\pi$ [many years of work by RBC/UKQCD, arXiv:1502.00263]

$$K \to (\pi\pi)_{I=2}$$
$$K \to (\pi\pi)_{I=0}$$

$$\frac{\mathrm{Re}(A_0)}{\mathrm{Re}(A_2)} \approx 22.5$$

This $\Delta I = 1/2$ rule is unexplained and must be of non-perturbative nature.

$$A_{2/0} = F\langle(\pi\pi)_{I=2/0} \mid H_W \mid K\rangle$$

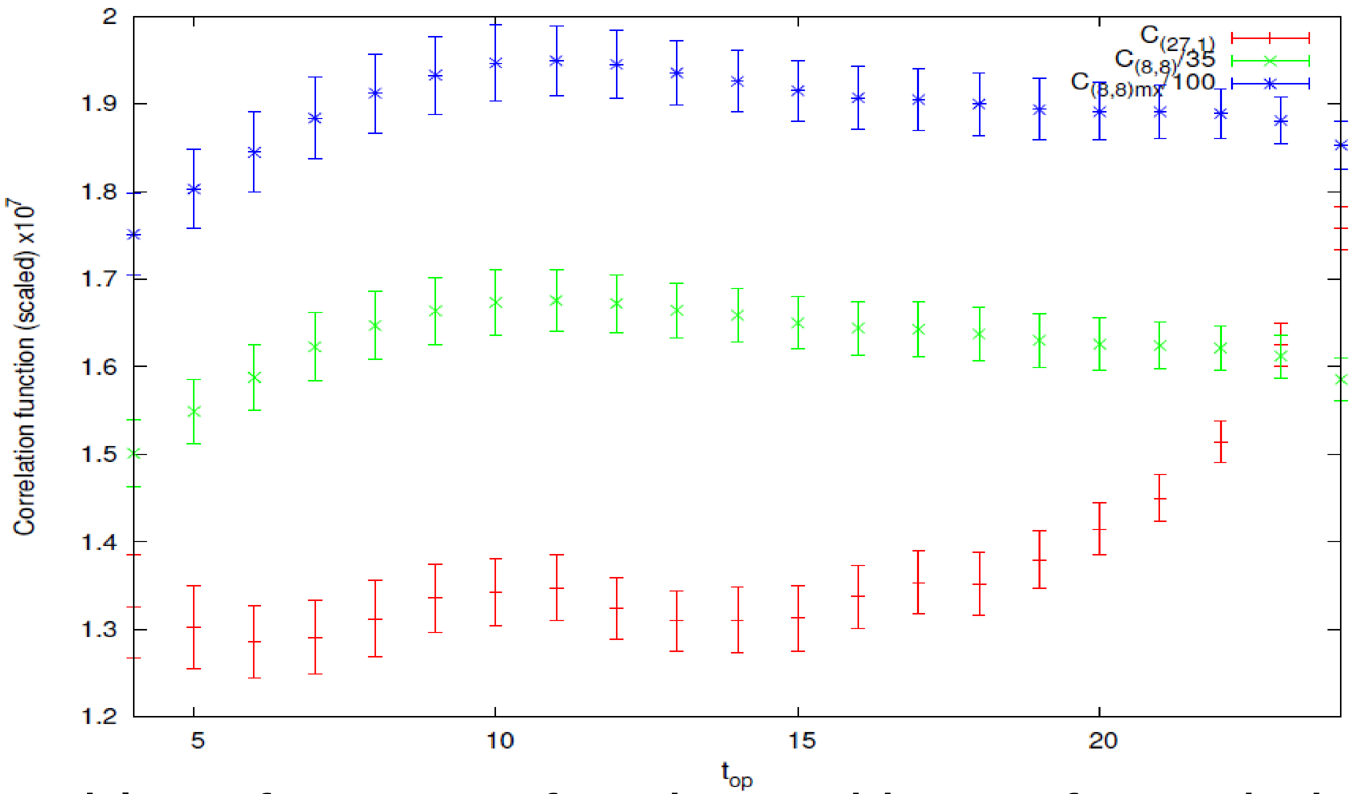
- 3 (four-fermion) operators in the Weak Eff. Hamiltonian contribute.
- F is a factor relating the finite volume matrix elements to the infinite volume ones. It depends on the $\pi\pi$ phase shift [Lellouch and Lüscher, '01]
- Kinematics should be matched, i.e. $E_{\pi\pi} = m_K$. That is achieved using antiperiodic boundary conditions for the d, s.t. $p = \pm\pi/L$....

64^3 K$\rightarrow \pi\pi$ 3-point correlation functions

Kaon - 2 pion separation 26

Tadeusz Janowski

$C_i^{K\rightarrow\pi\pi}(t) = N_{\pi\pi} N_K M_i e^{-(m_K - E_{\pi\pi})t_{op}} e^{-E_{\pi\pi} t_{\pi\pi}}$



- 2 ensembles of 2+1 DW fermions with L~5 fm and physical m_π.
- NP renormalization in RI-SMOM scheme. Matching to $\overline{\text{MS}}$ at 1-loop. Currently dominating error budget.

$$\text{Re}(A_2) = 1.50(4)_{\text{stat}}(14)_{\text{syst}} \times 10^{-8}\ \text{GeV}; \quad \text{Im}(A_2) = -6.99(20)_{\text{stat}}(84)_{\text{syst}} \times 10^{-13}\ \text{GeV}\,.$$

experimental value $1.570(53) \times 10^{-8}$ GeV from neutral kaon decays

Hadronic decays. Multiple-channel generalization of the LL approach [Sharpe and Hansen, 2012, Briceno and Davoudi, 2012]

- The LL method, derived in Minkowski pace, first relates the finite volume dependence of the energy levels of two-particle states (accessible in Euclidean) to the (∞-L) S-matrix and phase shifts (not accessible, due to Maiani-Testa no-go theorem, '90).
- In a second step a new state (e.g. K) is introduced with a perturbative interaction term H_W with $\pi\pi$. Matching the kinematic and considering degenerate PT, the finite L correction to the energy levels is related to the ∞-L scattering amplitude (i.e. the finite and ∞ L, matrix elements of $\langle K|H_W|\pi\pi\rangle$ are related).
- The explicit generalization includes several two-particle states ($\pi\pi$ and $\bar{K}K$).
- Now the S-matrix does not only include phase shifts and different kinematics are needed to determine the parameters. Also, one gets a system of equations relating finite and infinite volumes matrix elements.
- This is a first step towards hadronic decays of e.g. D-mesons.

Conclusions

- I have given an incomplete review of flavor physics on the lattice.

- I did not cover tensions as those still present in V_{ub} and V_{cb} (excl. vs incl.).

- If the keywords are precise and rare, we are getting there. Approaches to include sub-leading systematics being developed (QED, multi-hadron channels).

- Belle II, LHCb (run II) and BES-III put pressure on us.