Lattice Inputs to Flavor Physics

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Rencontres de Moriond
March 14th - 21st, 2015
● Flavour Lattice Averaging Group (FLAG) Scope and review as of end of 2013
● $\mathcal{R}(D^{(*)})$ status and perspective for lattice
● QED corrections to hadronic processes
  Spectrum [BMW, 1406.4088]
  Leptonic decays [Carrasco et al., 1502.00257]
● $K \rightarrow \pi\pi$ and perspective for hadronic decays on the lattice [RBC-UKQCD, 1502.00263] and [Hansen and Sharpe, Briceno and Davoudi, 2012]
Several hadronic processes depend on hadronic contributions. E.g.

(Charged) Decay constants

\[ \langle 0 | \bar{u}_f \gamma_\mu \gamma_5 d_f | P(p) \rangle = F_P p_\mu \] are the hadronic parameters entering leptonic decays of pseudoscalar mesons

\[ \Gamma (B \to \ell \bar{\nu}_\ell) = \frac{G_F^2}{8\pi} |V_{ub}|^2 F_B^2 \left( \frac{m_\ell}{m_B} \right)^2 m_B^3 \left( 1 - \frac{m_\ell^2}{m_B^2} \right) \]
Form factors

Parameterizing semileptonic decay. Simplest: $B \rightarrow \pi \ell \nu$

Ignoring the lepton mass:

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2}{24\pi^3} p_\pi^3 |V_{ub}|^2 |f_+(q^2)|^2$$

The hadronic matrix element is from a quark bilinear

$$\langle \pi(p_\pi) | V^\mu | B(p_B) \rangle = f_+(q^2)(p_\pi + p_B - q\Delta m^2)^\mu + f_0(q^2)q^\mu$$

with $\Delta m^2 = (m_B^2 - m_\pi^2)/q^2$
\[ S^{\text{QCD}}_W = \bar{\psi}_x D^{W}_{xy}(U) \psi_y + \beta \sum \left( 1 - \frac{1}{N} \text{Re Tr } U_{\mu \nu}(x) \right) \]
Lattice can provide first principle – systematically improvable determinations of such parameters. However they are not free from approximations / systematics

- Number of dynamical flavours
- Unphysical quark masses (and no isospin breaking)
- Finite lattice spacing
- Finite volume
- Renormalization

**FLAG**'s goal is to walk users of lattice results through systematics and the way they have been addressed
Review of lattice results concerning low-energy particle physics

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Criteria, as of now

- **Chiral extrapolation:**
  - ★ $M_{\pi,\text{min}} < 200 \text{ MeV}$
  - ○ $200 \text{ MeV} \leq M_{\pi,\text{min}} \leq 400 \text{ MeV}$
  - ■ $400 \text{ MeV} < M_{\pi,\text{min}}$

- **Continuum extrapolation:**
  - ★ 3 or more lattice spacings, at least 2 points below 0.1 fm
  - ○ 2 or more lattice spacings, at least 1 point below 0.1 fm
  - ■ otherwise

- **Finite-volume effects:**
  - ★ $M_{\pi,\text{min}}L > 4$ or at least 3 volumes
  - ○ $M_{\pi,\text{min}}L > 3$ and at least 2 volumes
  - ■ otherwise

- **Renormalization (where applicable):**
  - ★ non-perturbative
  - ○ 1-loop perturbation theory or higher with a reasonable estimate of truncation errors
  - ■ otherwise
For heavy-light quantities, in order to deal with the different approaches, we used data-driven criteria. We introduced

\[ D(a) = \frac{Q(a) - Q(0)}{Q(a)} \]

for \( a = a_{\text{min}} \)

\[ \delta(a) = \frac{Q(a) - Q(0)}{\sigma_Q} \]

and then used

- (i) Three or more lattice spacings, and
  - (ii) \( a_{\text{max}}^2 / a_{\text{min}}^2 \geq 2 \), and
  - (iii) \( D(a_{\text{min}}) \leq 2\% \), and
  - (iv) \( \delta(a_{\text{min}}) \leq 1 \)
- (i) Two or more lattice spacings, and
  - (ii) \( a_{\text{max}}^2 / a_{\text{min}}^2 \geq 1.4 \), and
  - (iii) \( D(a_{\text{min}}) \leq 10\% \), and
  - (iv) \( \delta(a_{\text{min}}) \leq 2 \)
- otherwise

Only results with no red symbols enter the final estimates.
Light quark masses

- 'Estimates' differ from 'averages'. For $N_f=2+1$ an error coming from quenching of the charm has been included.
Leptonic and semileptonic Kaon and pion decays

\begin{center}
\begin{tabular}{|c|c|}
\hline
\textbf{FLAG2013} & \textbf{f_+ (0)} \\
\hline
\textbf{N_r=2} & FNAL/MILC 13C \\
\hline
\textbf{N_r=2+1} & our estimate for \( N_r=2+1 \) \\
& RBC/UKQCD 13 \\
& FNAL/MILC 12 \\
& JLQCD 12 \\
& JLQCD 11 \\
& RBC/UKQCD 10 \\
& RBC/UKQCD 07 \\
\hline
\end{tabular}
\end{center}

\begin{center}
\begin{tabular}{|c|c|}
\hline
\textbf{N_r=2} & ETM 10D (stat. err. only) \\
& ETM 09A \\
& QCDSF 07 (stat. err. only) \\
& RBC 06 \\
& JLQCD 05 \\
& JLQCD 05 \\
\hline
\end{tabular}
\end{center}

\begin{center}
\begin{tabular}{|c|c|}
\hline
\textbf{N_r=2+1} & our estimate for \( N_r=2 \) \\
& RBC/UKQCD 12 \\
& LATH0 11 \\
& MILC 10 \\
& JLQCD/TWOQCD 10 \\
& RBC/UKQCD 10A \\
& PACS-CS 09 \\
& BMW 10 \\
& JLQCD/TWOQCD 09A (stat. err. only) \\
& MILC 09A \\
& MILC 09 \\
& Aubin 08 \\
& PACS-CS 08, 08A \\
& RBC/UKQCD 08 \\
& HPQCD/UKQCD 07 \\
& NPLQCD 06 \\
& MILC 04 \\
\hline
\end{tabular}
\end{center}

\begin{center}
\begin{tabular}{|c|c|}
\hline
\textbf{N_r=2} & our estimate for \( N_r=2 \) \\
& ALPHA 13 \\
& BGR 11 \\
& ETM 10D (stat. err. only) \\
& ETM 09 \\
& QCDSF/UKQCD 07 \\
\hline
\end{tabular}
\end{center}
$|V_u|^2 = 0.987(10)$. The consistency with leptonic and semi-leptonic determinations of $|V_{us}|$ is a check of the equality of the Fermi constant describing interactions among leptons and the one describing interactions among leptons and quarks (may not be in BSM).
The hadronic parameter in $\varepsilon_K$ at LO in the EWH

![Diagram showing data points for $N_f=2+1$ and $N_f=2$ with corresponding estimates for $\hat{B}_K$.]
For light flavors, lattice computations are quite precise, mature and advanced, to the point that isospin breaking and QED effects have to be included soon (see later).

“Heavy quantities” included in FLAG-II are less advanced. Fewer computations (sometimes one only) passing the criteria.

Results have been updated in [C. Bouchard, LAT14, arXiv:1501.03204], where errors are compared to the expected experimental improvements from Belle II, BES III and LHCb (e.g. 50 ab$^{-1}$ by Belle II by 2020). The comparison shows once again that what is easy for lattice is difficult for experiment and vice-versa.
B-> nlv form factors

- Few lattice results, in the region complementary to experiments
- CKM matrix element fitted to normalize exp data
PS -> V form factors are usually computed at zero recoil. In that limit only one form factor is relevant (others are helicity suppressed).
\[ \mathcal{R}(D) = \frac{B(B \rightarrow D\tau^- \bar{\nu}_\tau)}{B(B \rightarrow Dl^- \bar{\nu}_l)} \]

\[ \mathcal{R}(D^*) = \frac{B(B \rightarrow D^* \tau^- \bar{\nu}_\tau)}{B(B \rightarrow D^*l^- \bar{\nu}_l)} \]

\[ \mathcal{R}(D^+) = \mathcal{R}(D^+ *) = \mathcal{R}(D) \]

\[ \mathcal{R}(D^{*0}) = \mathcal{R}(D^{*+}) = \mathcal{R}(D^*) \]

- \[ \mathcal{R}^0(D)_{SM} = 0.324(22) \]
  \[ \mathcal{R}^0(D^*)_{SM} = 0.250(3) \]
  from combination of phenomenological inputs, HQET and sum-rules [Biancofiore, Colangelo and De Fazio, 2013].

- Only one result, \[ \mathcal{R}(D)_{SM} = 0.316(12)(7) \] from lattice [FNAL/MILC, Bailey et al., 2012].

- Values of form factors at zero recoil cancel out in the ratios.
In general, in BSM theories other form factors (S, PS, T) may appear [Nierste, Trine, Westhoff '08, Kamenik, Mescia '08, Kamenik, Fajfer, Nisandzic '12, Biancofiore …].

Clear opportunities for lattice

- Natural idea would be new contributions from charged scalar exchanges.
- However, those would enhance the leptonic channel as well.
- It is difficult to accommodate the discrepancy in 2HDM without FCNC at tree level [Celis, Crivellin]. Otherwise, some leptoquark models can explain it [Tanaka, Watanabe, 2013].
QED effects are becoming relevant for light quantities.

QCD + QED direct simulations [Borsanyi et al., BMW group, 2014]

- Large volume $1+1+1+1$ simulations of QCD + QED (at unphysical $e$ due to noise to signal problem). 300 times more expensive than $N_f=2$ QCD. Pilot and benchmark computation concerning the setup.

- Separation of effects using $\Delta M_{\Sigma}^{\text{QED}}=0$
QED corrections to hadronic processes

Let’s consider the leptonic decay at $O(\alpha)$ in the WEH [N. Carrasco et al., 1502.00257]

Pure QCD

$\pi^+ \rightarrow u \ell^+ \nu_\ell$
$\pi^+ \rightarrow d \ell^+ \nu_\ell$

factorizable. Hadronic part $\rightarrow f_P$. Then $\Gamma_0$ at $O(\alpha)$

Still factorizable.

NON factorizable.

Also, $\Gamma_0$ is infrared divergent, one needs to consider (one) real photon emission as well. No such problems for spectrum.

$\Rightarrow$ Not much sense of QED corrections to a decay constant ...
\[ \Gamma_1(\Delta E) \text{ with } \Delta E = E_\gamma^{\text{max}} \]

The combination \( \Gamma_0 + \Gamma_1(\Delta E) \) is free from IR divergencies at \( \mathcal{O}(\alpha) \). One can split it as

\[
\Gamma(\Delta) = \{ \Gamma_0 - \Gamma_0^{pt} \} + \{ \Gamma_0^{pt} + \Gamma_1(\Delta) \} = \lim_{L \to \infty} \{ \Gamma_0(L) - \Gamma_0^{pt}(L) \} + \{ \Gamma_0^{pt} + \Gamma_1(\Delta) \}
\]

- \( pt= \) pointlike approximation (perturbative). OK for soft photons, they can’t resolve the hadron structure. For \( K \) and \( \pi \), \( \Delta E \sim 20 \text{ MeV} \). Currently main limitation of the approach.
- Both terms are IR-safe and have a \( L \to \infty \) limit.
- \( \Gamma_0(L) \) is computed on the lattice. It requires rather involved Euclidean correlators, with lepton propagators in the numerical computation of the non-factorizable contributions.
Hadronic decays. $A_2$ amplitude of $K \rightarrow \pi\pi$ [many years of work by RBC/UKQCD, arXiv:1502.00263]

\[
\begin{align*}
K & \rightarrow (\pi\pi)_{I=2} \\
K & \rightarrow (\pi\pi)_{I=0}
\end{align*}
\]

\[
\frac{\text{Re}(A_0)}{\text{Re}(A_2)} \approx 22.5
\]

This $\Delta I = 1/2$ rule is unexplained and must be of non-perturbative nature.

\[
A_{2/0} = F \langle (\pi\pi)_{I=2/0} \mid H_W \mid K \rangle
\]

- 3 (four-fermion) operators in the Weak Eff. Hamiltonian contribute.
- $F$ is a factor relating the finite volume matrix elements to the infinite volume ones. It depends on the $\pi\pi$ phase shift [Lellouch and Lüscher, '01]
- Kinematics should be matched, i.e. $E_{\pi\pi} = m_K$. That is achieved using antiperiodic boundary conditions for the d, s.t. $p = \pm\pi/L$...
\[ 64^3 \mathbf{K \rightarrow \pi\pi} \] 3-point correlation functions

\[ C_i^{K \rightarrow \pi\pi}(t) = N_{\pi\pi} N_K M_i e^{-(m_K - E_{\pi\pi})t_{\text{op}}} e^{-E_{\pi\pi}t_{\pi\pi}} \]

- 2 ensembles of 2+1 DW fermions with L\(\sim\)5 fm and physical \(m_\pi\).
- NP renormalization in RI-SMOM scheme. Matching to \(\overline{\text{MS}}\) at 1-loop. Currently dominating error budget.

\[ \text{Re}(A_2) = 1.50(4)_{\text{stat}}(14)_{\text{syst}} \times 10^{-8} \text{ GeV}; \quad \text{Im}(A_2) = -6.99(20)_{\text{stat}}(84)_{\text{syst}} \times 10^{-13} \text{ GeV}. \]

Experimental value \(1.570(53) \times 10^{-8} \text{ GeV}\) from neutral kaon decays.
Hadronic decays. Multiple-channel generalization of the LL approach [Sharpe and Hansen, 2012, Briceno and Davoudi, 2012]

- The LL method, derived in Minkowski pace, first relates the finite volume dependence of the energy levels of two-particle states (accessible in Euclidean) to the ($\infty$-L) S-matrix and phase shifts (not accessible, due to Maiani-Testa no-go theorem, '90).

- In a second step a new state (e.g. $K$) is introduced with a perturbative interaction term $H_W$ with $\pi\pi$. Matching the kinematic and considering degenerate PT, the finite L correction to the energy levels is related to the $\infty$-L scattering amplitude (i.e. the finite and $\infty$ L, matrix elements of $\langle K|H_W|\pi\pi\rangle$ are related).

- The explicit generalization includes several two-particle states ($\pi\pi$ and $\bar{K}K$).  

- Now the S-matrix does not only include phase shifts and different kinematics are needed to determine the parameters. Also, one gets a system of equations relating finite and infinite volumes matrix elements.

- This is a first step towards hadronic decays of e.g. D-mesons.
Conclusions

• I have given an incomplete review of flavor physics on the lattice.

• I did not cover tensions as those still present in $V_{ub}$ and $V_{cb}$ (excl. vs incl.).

• If the keywords are precise and rare, we are getting there. Approaches to include sub-leading systematics being developed (QED, multi-hadron channels).

• Belle II, LHCb (run II) and BES-III put pressure on us.