I.: Basic facts about the top quark
II: Basic facts about the mass in general
III: Implications on the precision for the top mass
Why do we care about precision on the top mass?

- Obviously, the value of the top mass affects the measured top cross sections.

- Affects searches for new physics with top background, BSM decays into tops, etc.

- Top mass close to the electro-weak breaking scale, impact on precision physics of the Higgs sector.
  If there is new physics associated with electro-weak symmetry breaking top physics is a place to look for.

- Stability of the electro-weak vacuum depends crucially on the precise numerical value of $m_t$. 
Basic facts about top

The essential numbers:

Mass:

\[ m_t = 173.21 \pm 0.51 \pm 0.71 \text{ GeV} \]

Width:

\[ \Gamma = 2.0 \pm 0.5 \text{ GeV} \]

Discovered at the Tevatron in 1995

A $t\bar{t}$ event from CDF.
The top quark is special:

+ The large top mass sets a hard scale.
+ Lifetime shorter than characteristic hadronization time scale.

⇒ Top physics is (mainly) described by perturbative QCD.

But, of course as any quark of the 2\textsuperscript{nd} or 3\textsuperscript{rd} generation:

- The top quark is a colour-charged particle.
- The top quark is not a stable particle.

⇒ There is no asymptotic free top state, non-perturbative effects (might) enter here through the back door.
Basic facts about a fermion mass

Theorists like Lagrangians:

\[ L_{\text{fermion}} = \bar{\psi}_{\text{bare}} (iD\psi - m_{\text{bare}}) \psi_{\text{bare}} \]

Beyond leading-order in perturbation theory: The (one-loop) self-energy:

\[-i \Sigma = \int \frac{d^Dk}{(2\pi)^D} i\gamma_\rho \frac{i}{k_1' - m_{\text{bare}}} i\gamma^\rho \frac{(-i)}{k_0^2} = -i (A p + B m_{\text{bare}})\]

- In four space-time dimensions this integral is divergent.
- \( D = 4 - 2\varepsilon \) is a regulator, divergences show up as poles \( 1/\varepsilon \).
- \( A \) and \( B \) depend on \( p^2, m^2_{\text{bare}} \) and \( \mu^2 \).
Basic facts about a fermion mass

Resummed self-energy insertions:

\[
\frac{i}{p' - m_{\text{bare}} - \Sigma} = \frac{i(1 + A)}{p' - (1 + A + B)m_{\text{bare}}} + O(\alpha_s^2)
\]

Renormalisation:

\[
\psi_{\text{bare}} = \sqrt{Z_2} \psi_{\text{renorm}}
\]

\[
m_{\text{bare}} = Z_m m_{\text{renorm}}
\]

All renormalisation schemes entail:

- **Wave function renormalisation**: Absorb UV-divergences of \((1 + A)\) in the numerator.

- **Mass renormalisation**: Absorb UV-divergences of \((1 + A + B)\).
Absorb only the parts proportional to $\frac{1}{\varepsilon} - \gamma_E + \ln(4\pi)$ and nothing else into $Z_m$:

$$Z_m = 1 - (A + B)_{\text{div}}$$

The propagator is then

$$\frac{i}{\not{p} - m_{\overline{\text{MS}}} - (A + B)_{\text{fin}}m_{\overline{\text{MS}}}}$$

- $m_{\overline{\text{MS}}}$ depends on the scale $\mu$: Running mass.

- Presence of $(A + B)_{\text{fin}}m_{\overline{\text{MS}}}$: The propagator does not have a pole at $m_{\overline{\text{MS}}}$, matrix elements do not factor at $p^2 = m_{\overline{\text{MS}}}^2$.

- $(A + B)_{\text{fin}}$ depends on $p^2$: Propagator does not yield Breit-Wigner shape.
$m_{\overline{\text{MS}}}$ is an example of a short-distance mass.

Can extract $m_{\overline{\text{MS}}}$ from an infrared safe observable for a process like $pp \rightarrow l\bar{\nu} j j b \bar{b}$ at high energies by comparing

$$\sigma_{\text{exp}} \quad \text{with} \quad \sigma_{\text{theo}} (m_{\overline{\text{MS}}})$$

Moch, Langenfeld, Uwer, ’09;
Czakon, Fiedler, Mitov, ’13;
Dowling, Moch, ’13
Define $Z_m$ such that the propagator has a pole at $m_{\text{pole}}$.

The propagator is then by definition

$$
\frac{i}{p^2 - m_{\text{pole}}}
$$

+ $m_{\text{pole}}$ is complex, includes the width.

+ Matrix elements factor at $p^2 = m_{\text{pole}}^2$.

+ Propagator corresponds to a Breit-Wigner shape.

- The pole mass is not a short distance mass.
The pole mass is ambiguous by an amount $O(\Lambda_{\text{QCD}})$:

- In the on-shell scheme, the renormalisation constant $Z_m$ contains contributions from all momentum scales, not just the ultraviolet region.

- In higher orders, subsets of diagrams are dominated by the IR-region.

- Therefore, the full perturbative series can only be summed up to an (infrared) renormalon ambiguity.

- The renormalon ambiguity is of $O(\Lambda_{\text{QCD}})$.

Conversion between the pole mass and the $\overline{\text{MS}}$-mass

In perturbation theory one has with $\tilde{m} = m_{\overline{\text{MS}}} (\mu = m_{\overline{\text{MS}}})$

$$m_{\text{pole}} = \tilde{m} \times \left[ 1 + c_1 \frac{\alpha_s(\tilde{m})}{\pi} + c_2 \left( \frac{\alpha_s(\tilde{m})}{\pi} \right)^2 + c_3 \left( \frac{\alpha_s(\tilde{m})}{\pi} \right)^3 + c_4 \left( \frac{\alpha_s(\tilde{m})}{\pi} \right)^4 + ... \right]$$

Melnikov, van Ritbergen, '99; Chetyrkin, Steinhauser, '99; Marquard, A. Smirnov, V. Smirnov, Steinhauser, '15

Numerically for the top quark:

$$m_{\text{pole}} = \tilde{m} \times \left[ 1 + 0.046 + 0.010 + 0.003 + 0.001 + ... \right]$$

The conversion formula is again only an asymptotic series and has an renormalon ambiguity as well.
Crude estimates of the ambiguity

From the truncation of the conversion formula between \( m_{\text{pole}} \) and \( \bar{m} \):

\[
\delta m_{\text{pole}} \approx O(200 \text{ MeV})
\]

From the estimate of the renormalon:

\[
\delta m_{\text{pole}} \approx O(270 \text{ MeV})
\]

What about determining the non-perturbative effects by comparing two different non-perturbative models?

Engineer A: \( 13^2 = 172 \) (sic)

Engineer B: \( 13^2 = 174 \) (sic)

This does not imply \( 13^2 = 173 \pm 1 \) (sic)
Can one translate a measurement of the peak position into a theoretical well defined short-distance top mass?

Remark: Experimentalists can measure many things to high precision (average number of pions in $pp$ collisions, etc.), the question is if and how a quantity can be related to a quantity depending only on short-distance physics.

Let’s split up this question:

- Which scales are involved?
- How to define a short-distance mass at a given scale?
- How to translate the measurement?
The involved scales

In order to avoid large logarithms:

- Describe physics at a particular scale $\mu$ by an appropriate effective theory.
- Evolution operators sum up large logarithms.

From a study of $e^+e^- \rightarrow t\bar{t}$:

<table>
<thead>
<tr>
<th>Scale</th>
<th>Matrix elements</th>
<th>Effective theory</th>
<th>Affects</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q \ldots m_t$</td>
<td>hard function</td>
<td>QCD</td>
<td>norm of the distribution</td>
<td>depends on $m_t$</td>
</tr>
<tr>
<td>$m_t \ldots \Gamma_t$</td>
<td>jet function</td>
<td>SCET</td>
<td>shape and position</td>
<td>depends on $m_t$</td>
</tr>
<tr>
<td>$\Gamma_t \ldots \Lambda_{QCD}$</td>
<td>soft function</td>
<td>top-HQET</td>
<td>shape and position</td>
<td>independent of $m_t$</td>
</tr>
</tbody>
</table>

$\Rightarrow$ Need a short-distance mass definition for scales down to $\Gamma_t$.

Fleming, Hoang, Mantry, Stewart, '07
The MSR mass

**Short-distance mass**: any mass definition not affected by a renormalon ambiguity.

Idea for construction: *Remove contributions giving rise to this ambiguity* (known from bottomium, potential subtracted mass).

This will involve apart from the UV-renormalisation scale $\mu$ a second scale $R$.

The **\text{MS}-mass** is a short-distance mass, and $R = \bar{m}$ in this case.

The **MSR-mass** (read: $\bar{m}$ substituted by $R$) is the two-scale generalisation with a UV-scale $\mu$ and an IR-scale $R$, such that

$$m_{\text{MSR}}(R = 0) = m_{\text{pole}}, \quad m_{\text{MSR}}(R = \bar{m}) = \bar{m}.$$  

Hoang, Jain, Scimemi, Stewart, '08
Translating the measurement

Theory sneaks in though template method / matrix element method.

Analogy of factorisation:

Effective theory: Hard function / jet function / soft function
Monte Carlo: Hard matrix element / parton shower / hadronisation

Parton shower has a lower cut-off.

⇒ Monte Carlo mass is something like a short-distance mass.

Translation for Pythia:

\[ m_{\text{Pythia}} = m_{\text{MSR}} (R = 1\ldots9 \text{ GeV}) \]

This introduces an uncertainty of the order of 1 GeV on the translation from the Monte Carlo mass to a theoretically well defined short-distance mass.

Hoang, Stewart, '08
Work to do

- Work out in detail factorisation and short-distance mass in $pp$-collisions.
- Compare in detail MC mass with a well defined short-distance mass.
- Consider practical issues.

Active field:

Mainz Institute for Theoretical Physics (MITP) scientific program: *High precision fundamental contants at the TeV scale*, March 2014

TOPLHCWG Open Meeting, CERN, April 2014

TOP 2014 Workshop, Cannes, September 2014
Summary

- The value of the top mass is essential for many precision measurements.

- Want to have a well defined short-distance mass.
  - At high scales the $\overline{\text{MS}}$-mass can be used.
  - The pole mass is not a short-distance mass.
  - The $\text{MSR}$-mass can be used as a short-distance mass at lower scales.

- State of the art: Conceptionally understood, details have to be worked out.

- Outlook below 100 MeV uncertainty: Threshold scan at an $e^+e^-$-machine with a potential subtracted mass or 1S mass.