VACUUM STABILITY DEPENDS ON HIGH ENERGY PHYSICS SCALES[a]

VINCENZO BRANCHINA

Department of Physics and Astronomy, University of Catania, and INFN, Sezione di Catania
via S. Sofia, 63, I-95123, Catania, Italy

The stability analysis of the EW vacuum is usually presented with the help of a phase diagram in the $M_H - M_t$ plane. It has been recently shown that new physics interactions, even if they live at very high energy scales, can strongly affect the stability diagram. This result has far reaching theoretical and phenomenological consequences. In particular, despite claims to the contrary, higher precision measurements of the Higgs and top masses, M_H and M_t, will not tell us whether our universe is in a stable or in a metastable vacuum, nor if we live at the "edge of stability". Moreover, the strong sensitivity to new physics casts serious doubts on speculations and models based on the so called "criticality", the observation that the experimental (M_H, M_t) point lies close to the critical line, the line separating the stability from the instability region in the $M_H - M_t$ plane. In fact, new physics can significantly change the position of the critical line, thus making quite unlikely for our universe to live at the edge of stability. Finally, these results also show that candidate UV completions of the SM need to pass a sort of "stability test": only a model where the EW vacuum is stable or metastable, but with a lifetime larger than the age of the universe, can be considered as a viable UV completion of the SM.

1 Stability diagram: the usual analysis.

For our understanding of physics beyond the Standard Model (BSM), the knowledge of the stability condition of the electroweak (EW) vacuum is of the greatest importance. It is well known that due to the loop corrections coming from the quark top, the Higgs potential $V(\phi)$ turns over for values of $\phi > v$, where $v \sim 246$ GeV is the location of the EW minimum, and develops a second minimum at a very large value $\phi^{(2)}_{min}$. When the usual stability analysis is performed, the potential $V(\phi)$ is obtained by considering SM interactions only[5,6,7,8,9,10]. Depending on the values of the Higgs and top masses, M_H and M_t, the second minimum can be higher or lower than (or at the the same height of) the EW minimum.

When $V(\phi^{(2)}_{min}) < V(v)$, the EW vacuum is a metastable state, a *false vacuum*, and we have to consider its lifetime τ, i.e. the tunneling time from the false vacuum (v) to the true vacuum ($\phi^{(2)}_{min}$). At a certain $\phi = \phi_{inst}$, the potential reaches the same value it has at $\phi = v$,

---

[a] based on work done in collaboration with E. Messina A. Platania, M. Sher [1,2,3,4]

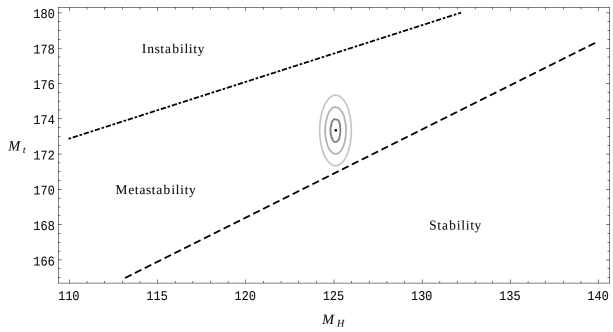

Figure 1 – The stability phase diagram as it results from the usual analysis, i.e. assuming that the presence of new physics interactions at high energy scales can be ignored. The $M_H - M_t$ plane is divided in three sectors, stability, metastability, and instability regions (see text). The dot indicates $M_H \sim 125.09$ GeV and $M_t \sim 173.34$ GeV. The ellipses give the one, two and three sigma errors.

$V(\phi_{inst}) = V(v)$, successively taking lower values: for $\phi > \phi_{inst}$, $V(\phi) < V(v)$. The scale ϕ_{inst} is then the scale where the potential becomes unstable, the *instability scale* for short.

The stability analysis is usually presented with the help of a phase diagram in the $M_H - M_t$ plane. In fig. 1, the stability diagram for the usual analysis is presented. For those values of M_H and M_t such that $V(v) < V(\phi^{(2)}_{min})$, the EW vacuum is the absolute minimum of the potential, and we have the stability region. The instability region is obtained for $V(\phi^{(2)}_{min}) < V(v)$ and $\tau < T_U$, where T_U is the age of the Universe. Finally, the so called metastability region is for $V(\phi^{(2)}_{min}) < V(v)$, but $\tau > T_U$.

It is worth to know that, for the central experimental values $M_H \sim 125.09$ GeV[11] and $M_t \sim 173.34$ GeV[12], $\phi_{inst} \sim 10^{11}$GeV $>> v$, the second minimum is at $\phi^{(2)}_{min} \sim 10^{30}$ GeV, and τ is much larger than T_U. Naturally, new physics interactions are expected to have an effect long before the scale $\phi^{(2)}_{min} \sim 10^{30}$ GeV is reached. Although we do not know where new physics appears, we certainly expect that, at least at very high energy scales (maybe the Planck scale M_P, if not before) new physics shows up. However, despite the presence of these new interactions, it is believed that τ can be calculated with the potential obtained with SM interactions only[5,6]. It is argued, in fact, that the relevant scale for tunneling is the instability scale $\phi_{inst} \sim 10^{11}$ GeV, and that the contribution to τ coming from very high (Planck) scale physics should be suppressed (decoupling)[6].

Contrary to these expectations, it has been shown that the presence of new physics at very high energy scales can strongly modify the stability condition of the EW vacuum[1,2,3]. The analysis presented in these works, however, is realized by parametrizing new physics interactions in terms of few higher order (non-renormalizable) operators. Some people then considered these results with a certain skepticism, suggesting that when the infinite tower higher dimensional operators of the renormalizable UV completion of the SM is taken into account, this effect should disappear, thus recovering the expected decoupling. Actually, the suspect is that this effect takes place above the physical cutoff, where the control of the theory is lost [13].

The introduction of few higher order operators, however, is just a convenient and efficient way of mimicking the presence of new physics, not a (clearly illegitimate) truncation of the UV completion of the SM[b]. Nevertheless, it is understandable that the parametrization of new physics in terms of higher order operators can be the source of a certain confusion and mislead the reader. The effect has nothing to do with this parametrization.

In the following, we investigate the impact of a *fully renormalizable* (toy) UV completion of the SM on the stability condition of the EW vacuum, when new physics interactions live at scales much higher than the instability scale ϕ_{inst}. According to the usual arguments[5,6], the

[b] Naturally, if we consider the expansion of the potential $V(\phi)$ in powers of ϕ, and we want to use this expansion up to very high energies, we have to take into account the whole tower of terms.

stability diagram should not be altered by this very high energy modification of the SM. In the following we show that this is not the case and discuss the origin and the consequences of this apparently unexpected effect.

2 A renormalizable toy UV completion of the Standard Model

The classical potential for the scalar sector of the SM is:

$$U(\Phi) = m^2 \left(\Phi^\dagger \cdot \Phi\right) + \lambda \left(\Phi^\dagger \cdot \Phi\right)^2 , \tag{1}$$

where Φ is the Higgs doublet

$$\Phi = \frac{1}{\sqrt{2}} \left(\begin{array}{c} -i(G_1 - iG_2) \\ \phi + iG_3 \end{array} \right) , \tag{2}$$

ϕ the Higgs field and G_i the Goldstones.

The renormalizable (toy) UV completion of the SM that we use for our analysis is the following. We consider a new scalar field S and a new fermion field ψ that interact in a simple way with Φ, and have masses M_S and M_f well above the instability scale ϕ_{inst}: $M_S, M_f >> \phi_{inst}$. To the SM Lagrangian we add the mass and interaction terms:

$$\Delta\,\mathcal{L} = \frac{M_S^2}{2} S^2 + \frac{\lambda_S}{4} S^4 + 2 g_S \left(\Phi^\dagger \Phi\right) S^2 + M_f \left(\bar\psi_L \psi_R + \bar\psi_R \psi_L\right) + \sqrt{2} g_f \left(\bar\Psi_L \Phi\, \psi_R + \bar\psi_R\, \Phi^\dagger \Psi_L\right) \tag{3}$$

(together with the S and ψ kinetic terms), where λ_S is the self-coupling of the new scalar S, g_S the coupling between Φ and S, ψ_L and ψ_R the left and right components of the Dirac field ψ with mass M_f, Ψ_L the left-handed $SU(2)$ fermion doublet $\Psi_L = (0, \psi_L)^T$ (we are not considering additional neutrinos), and g_f the Yukawa coupling between Ψ and the Higgs doublet.

For our purposes, it is useful to write the Lagrangian in Eq. (3) as:

$$\begin{aligned} \Delta\,\mathcal{L} \;&=\; \frac{M_S^2}{2} S^2 + \frac{\lambda_S}{4} S^4 + g_S \varphi^2 S^2 + M_f \bar\psi\psi + g_f \varphi \bar\psi \psi + g_S \left(G_1^2 + G_2^2 + G_3^2\right) S^2 \\ &+\; g_f G_3 \bar\psi \left[\left(\frac{1+\gamma_5}{2}\right) + i \left(\frac{1-\gamma_5}{2}\right) \right] \psi. \end{aligned} \tag{4}$$

In fact, as we confine ourselves to consider the impact of these additional terms on the Higgs effective potential only at the one-loop level, in the following we do not need to consider further the second and the third lines in the above equation. The one-loop contribution to $V(\phi)$ from the terms in $\Delta\,\mathcal{L}$ is:

$$V_1(\phi) = \frac{(M_S^2 + 2 g_S \phi^2)^2}{64\pi^2} \left[\ln\left(\frac{M_S^2 + 2 g_S \phi^2}{M_S^2}\right) - \frac{3}{2} \right] - \frac{\left(M_f^2 + g_f^2 \phi^2\right)^2}{16\pi^2} \left[\ln\left(\frac{M_f^2 + g_f^2 \phi^2}{M_f^2}\right) - \frac{3}{2} \right] \tag{5}$$

According to the decoupling argument[6], these new physics interactions that live at very high energy scales ($M_S, M_f >> \phi_{inst} \sim 10^{11}$ GeV) should have no impact on the stability diagram of fig. 1. We now proceed with the analysis of the SM with the potential modified by the presence of the term (5), so to verify or disprove this expectation.

3 Stability analysis of the UV completed SM

The tunneling rate Γ, inverse lifetime time τ, is given by[14,15] (for the sake of simplicity, we write the formula with the contribution of the scalar sector of the SM only, the inclusion of the other contributions being straightforward. A more complete expression is given in Ref. 3):

$$\Gamma = \frac{1}{\tau} = T_U^3\, \frac{S[\phi_b]^2}{4\pi^2} \left| \frac{\det' \left[-\partial^2 + V''(\phi_b)\right]}{\det \left[-\partial^2 + V''(v)\right]} \right|^{-1/2} e^{-S[\phi_b]} \tag{6}$$

where $\phi_b(r)$ is the $O(4)$ bounce solution to the euclidean equation of motion, $r = \sqrt{x_\mu x_\mu}$ is the radial coordinate in four euclidean dimensions, $S[\phi_b]$ is the action for the bounce, and $[-\partial^2 + V''(\phi_b)]$ is the fluctuation operator around the bounce (V'' is the second derivative of V with respect to ϕ). The prime in the $\det'$ means that the zero modes are excluded, and $\frac{S[\phi_b]^2}{4\pi^2}$ comes from the translational zero modes.

Before we proceed with the calculation of τ for our UV completed SM, it is worth to consider the corresponding one when the presence of new physics interactions is ignored. For the present central values of M_H and M_t ($M_H = 125.09$ GeV and $M_t = 173.34$ GeV) it gives:

$$\tau \sim 10^{600}\, T_U\,. \tag{7}$$

This result is the basis for the so called metastability scenario. From Eq. (7), in fact, we would conclude that, although the EW minimum is a metastable state (and then a false vacuum), as its lifetime turns out to be much larger than the age of the universe, we may well live in such a state. In fig.1, we have presented the analysis in the whole $M_H - M_t$ plane, performed under the assumption (usually considered in the literature) that new physics interactions at scales $>> \phi_{inst}$ have no impact on the stability condition of the EW vacuum[5,6]. The black dot corresponds to the tunneling time of Eq. (7). The ellipses give the one, two and three sigma errors.

We move now to the computation of the EW vacuum lifetime for our model with new physics at high energy scales, and consider two examples. By starting with taking $M_S = 1.2 \cdot 10^{18}$ GeV, $M_f = 0.6 \cdot 10^{17}$ GeV, $\lambda_S = 0.5$, $g_S = 0.97$, $g_f^2 = 0.48$, $\lambda = \lambda(\mu = M_S) = -0.015$ (where λ is the usual quartic coupling), we find that the Higgs potential $V(\phi)$ develops a new minimum, lower than the EW one, at $\phi_{min}^{(2)} \sim 0.4 \cdot 10^{19}$ GeV. To study the stability condition of the EW vacuum, we have then to calculate the EW vacuum lifetime τ. For the present central experimental values of the Higgs and top masses ($M_H = 125.09$ GeV and $M_t = 173.34$ GeV) we find:

$$\tau \sim 10^{180}\, T_U\,. \tag{8}$$

This result has to be compared with the tunneling time of Eq.(7), obtained by considering the SM potential alone (no new physics included). Although for the example considered here the tunneling time is still much higher than the age of the Universe, Eq. (8) gives a result that is greatly different from the one of Eq.(7).

If we now consider another example, namely we take $M_S = 1.2 \cdot 10^{18}$ GeV, $M_f = 2.4 \cdot 10^{15}$ GeV, $\lambda_S = 0.5$, $g_S = 0.97$, $g_f^2 = 0.48$, and $\lambda = \lambda(\mu = M_S) = -0.015$, by considering the same values for M_H and M_t we find:

$$\tau \sim 10^{-65}\, T_U\,. \tag{9}$$

In this case, the situation is more dramatic than in the previous example: the tunneling time turns out to be much smaller than the age of the Universe. If realistic, the model with these values of the parameters could not be considered as a viable UV completion of the SM.

The lesson from Eqs. (7), (8), and (9) is clear. The expectation that the tunneling time should be insensitive to physics that lives at energies higher than the instability scale, in other words that the result in Eq. (7) should not be modified by the presence of new physics at high energies, is not fulfilled.

The question is then: why the decoupling argument is not operating? The reason is that the decoupling theorem applies when we calculate scattering amplitudes at energies E lower than M_S and M_f. In these cases, the contributions from high energy new physics is suppressed by factors as E/M_S and E/M_f to some appropriate power. In our case, however, we are computing the tunneling time. Tunneling is a non-perturbative phenomenon, and no decoupling applies: in the calculation of τ, no naive suppression factor, ϕ_{inst}/M_S or ϕ_{inst}/M_f, appears. More specifically, the tunneling time τ is essentially given by the exponential $e^{S[\phi_b]}$ (see Eq. (6)). If the Higgs potential is modified by the presence of terms as the one in Eq. (5), the new bounce turns out to be different from the one obtained when this term is absent. The action $S[\phi_b]$ is then modified.

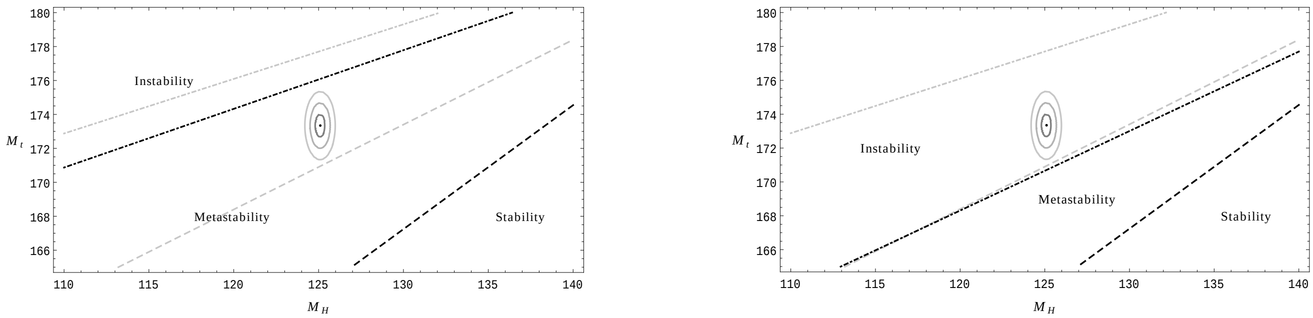

Figure 2 – Left panel: The stability phase diagram for the toy UV completed SM considered in the text and for the following values of the parameters: $M_S = 1.2 \cdot 10^{18}$ GeV, $M_f = 0.6 \cdot 10^{17}$ GeV, $\lambda_S = 0.5$, $g_S = 0.97$, $g_f^2 = 0.48$, $\lambda(\mu = M_S) = -0.015$. As in fig. 1, the $M_H - M_t$ plane is divided in three sectors: stability, metastability, and instability regions. However, compared to the fig. 1 case, the stability and instability lines have moved downwards. Right panel: the same as the left panel, but for different values of the parameters (see text). These two pictures, together with fig. 1, clearly show the main point of the present work, namely that the stability diagram strongly depends on new physics, even when the latter lives at very high energy scales.

Once exponentiated, it gives rise to a value for τ that can be greatly different from the result obtained when new physics is not considered.

We go on now with our analysis. In fig.1 we have shown the stability diagram in the $M_H - M_t$ plane obtained under the assumption that the stability analysis should not depend on new physics that lives at high energy scales. The examples that we have just considered, with the results (7), (8) and (9), indicate that we should on the contrary expect that the stability phase diagram depends on new physics, even if the latter lives at very high energy scales. Still referring to fig.1, we point out that the dashed and the dashed-dotted lines are respectively named the stability line and the instability line. The first one is obtained for those couples of values of M_H and M_t such that the two minima are at the same height, the latter is obtained for the case when $V(\phi_{min}^{(2)}) < V(v)$ and $\tau = T_U$.

Let us repeat this stability analysis when the term (3), i.e. our toy UV completion of the SM, is added to the SM Lagrangian, so that the term (5) is added to the Higgs effective potential. In fig. 2 (left panel), the analysis is performed for the values of the parameters considered in our first example, namely $M_S = 1.2 \cdot 10^{18}$ GeV, $M_f = 0.6 \cdot 10^{17}$ GeV, $\lambda_S = 0.5$, $g_S = 0.97$, $g_f^2 = 0.48$, $\lambda = \lambda(\mu = M_S) = -0.015$.

We first note that the instability line moves downwards. This result had to be expected from the previous results for the tunneling time (see Eqs. (7) and (8)). In fact, we obtained $\tau \sim 10^{180} T_U$ for the UV completed Higgs potential and $\tau \sim 10^{600} T_U$ for the SM Higgs potential. In the case of the UV completed potential, the black dot (experimental point) must be closer to the instability line than in the case of the unmodified potential. The grey lines in fig. 2 are the old instability and stability lines obtained in the case of the unmodified Higgs potential (fig. 1).

Another important result is that even the stability line moves downwards (see fig. 2). When the stability diagram of fig. 1 was thought to be universal (i.e. when it was thought that a decoupling effect assured that new physics at high scales could not modify this diagram), many speculations were triggered by the fact that the experimental point (black dot in the figure), $M_H \sim 125.09$ GeV and $M_t \sim 173.34$ GeV, lies (within 3 sigma) "close" to the stability line. In this respect, it was thought that more refined measurements of M_t and M_H should allow to determine wether the EW vacuum is a stable or a metastable state. Some authors even considered this closeness of the experimental point to the stability line as the most important message from LHC[10], speculating on this closeness and elaborating on it for model building[13].

The results that we have just presented show that the stability condition of the EW vacuum is much more sensitive to high energy new physics than to the values of the Higgs and top masses. Therefore, more refined measurements of M_t and M_H, that are certainly very important for several other reasons, will not allow to determine the stability condition of the EW vacuum.

Speculations and model building based on the so called "criticality condition" seem to be founded on a very unstable result. New physics at high energies actually changes (for the worse) the distance between the experimental point and the critical line, thus greatly weakening (if not excluding) arguments based on this supposed criticality.

In fig. 2 (right panel), the stability diagram for our model with the values of the parameters considered in our second example ($M_S = 1.2 \cdot 10^{18}$ GeV, $M_f = 2.4 \cdot 10^{15}$ GeV, $\lambda_S = 0.5$, $g_S = 0.97$, $g_f^2 = 0.48$, $\lambda = \lambda(\mu = M_S) = -0.015$) is presented. The instability and stability lines move downwards as for the previous case. In this case, however, the tunneling time for the experimental point is much shorter than the age of the Universe, see Eq. (9), and in fact we see that the experimental point is now inside the instability region. This means that the model with these values of the parameters cannot be considered as a viable UV completion of the SM.

This result also contains another important lesson. We have seen that he stability condition of the EW vacuum is strongly sensitive to high energy new physics. Therefore, as we cannot rely on any high energy decoupling, candidate UV completions of the SM models have to pass a sort of stability test: only models with a stable or metastable (but with $\tau > T_U$) EW vacuum can be considered as viable UV completions of the SM.

4 Conclusions

By considering a fully renormalizable (toy) UV completion of the SM, we have definitely shown that new physics interactions, even if they live at very high energy scales, can strongly affect the stability diagram of the SM. This result has far reaching theoretical and phenomenological consequences.

Despite claims to the contrary, it shows that higher precision measurements of M_t and M_H will never tell us whether our universe lives in a stable or in a metastable vacuum, or at the "edge of stability" (near the critical line).

Moreover, as very high energy new physics can significantly modify the position of the critical line, it is quite unlikely that our universe lives at the "edge of stability", i.e. near the critical line. This strongly weakens (if not invalidates) speculations and model building based on this so called "criticality".

Finally, this result shows that candidate UV completions of the SM need to pass a sort of "stability test". Only models with a stable or metastable (but with $\tau > T_U$) vacuum can be considered as viable UV completions of the SM.

References

1. V. Branchina and E. Messina, Phys. Rev. Lett. 111 (2013) 241801.
2. V. Branchina, E. Messina and A. Platania, JHEP 1409 (2014) 182.
3. V. Branchina, E. Messina and M. Sher, Phys.Rev. D91 (2015) 1, 013003.
4. V. Branchina and E. Messina, in preparation.
5. G. Isidori, G. Ridolfi, A. Strumia, Nucl. Phys. B609 (2001) 387.
6. J. R. Espinosa, G. F. Giudice and A. Riotto, JCAP **0805**, 002 (2008).
7. F. Bezrukov, M. Yu. Kalmykov, B. A. Kniehl, M. Shaposhnikov, JHEP 1210 (2012) 140.
8. J. Elias-Miro, J.R. Espinosa, G.F. Giudice, *et al.*, Phys. Lett. B709 (2012) 222.
9. G. Degrassi, S. Di Vita, J. Elias-Miro, *et al.*, JHEP 1208 (2012) 098.
10. D. Buttazzo, G. Degrassi, P. P. Giardino, *et al.*, JHEP 1312 (2013) 089.
11. G. Aad et al. (The ATLAS and CMS Collaborations), Phys.Rev.Lett. 114 (2015) 191803.
12. ATLAS, CDF, CMS, D0 collaborations, arXiv:1403.4427.
13. J. R. Espinosa, G. F. Giudice, E. Morgante, *et al.*, arXiv:1505.04825 [hep-ph].
14. S. Coleman, Phys. Rev. D15 (1977) 2929.
15. C. G. Callan, S. Coleman, Phys. Rev. D16 (1977) 1762.