EW Vacuum Stability depends upon Planck scale physics

Vincenzo Branchina

University of Catania and INFN, Italy

References :

V. Branchina, E. Messina, Phys.Rev.Lett.111, 241801 (2013) (arXiv:1307.5193)

V. Branchina, arXiv:1405.7864, Moriond 2014

V. Branchina, E. Messina, A. Platania JHEP 1409 (2014) 182 (arXiv:1407.4112)

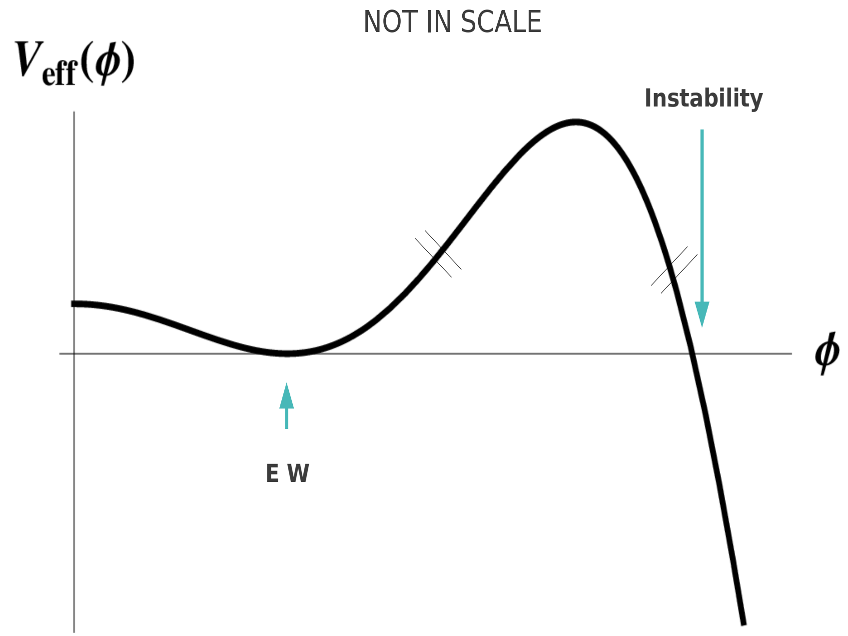
V. Branchina, E. Messina, M. Sher, Phys.Rev.D91 (2015) 1, 013003 (arXiv:1408.5302)

V. Branchina, E. Messina, in preparation

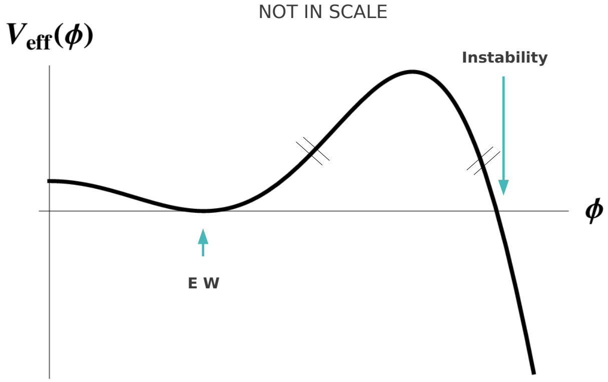
Moriond EW 2015

Top loop-corrections to the Higgs Effective Potential

destabilize the electroweak vacuum...



Higgs One-Loop Effective Potential $V^{1l}(\phi)$

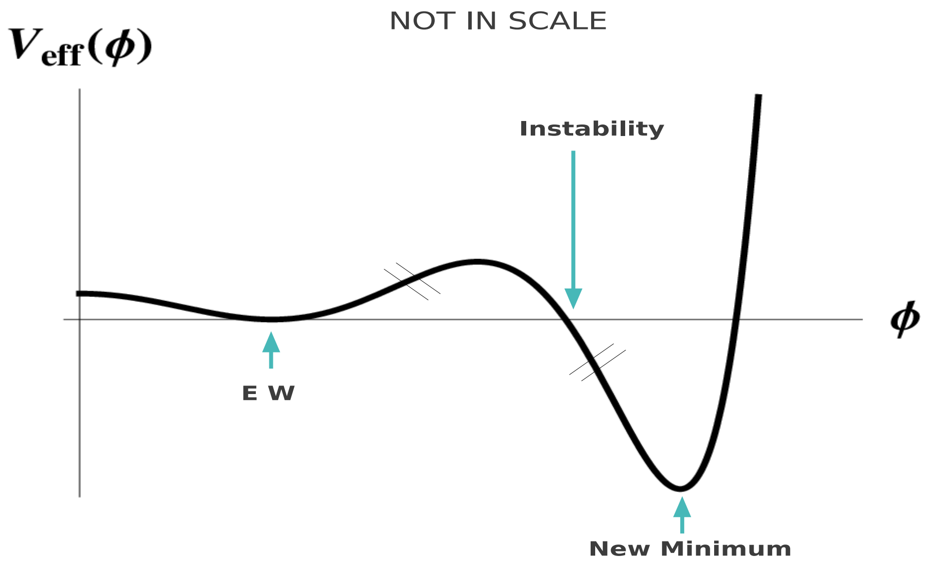


$$V^{1l}(\phi) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{24}\phi^4 + \frac{1}{64\pi^2}\left[\left(m^2 + \frac{\lambda}{2}\phi^2\right)^2\left(\ln\left(\frac{m^2+\frac{\lambda}{2}\phi^2}{\mu^2}\right) - \frac{3}{2}\right)\right.$$

$$+3\left(m^2 + \frac{\lambda}{6}\phi^2\right)^2\left(\ln\left(\frac{m^2+\frac{\lambda}{6}\phi^2}{\mu^2}\right) - \frac{3}{2}\right) + 6\,\frac{{g_1}^4}{16}\phi^4\left(\ln\left(\frac{\frac{1}{4}{g_1}^2\phi^2}{\mu^2}\right) - \frac{5}{6}\right)$$

$$\left.+3\,\frac{\left({g_1}^2+{g_2}^2\right)^2}{16}\phi^4\left(\ln\left(\frac{\frac{1}{4}\left({g_1}^2+{g_2}^2\right)\phi^2}{\mu^2}\right) - \frac{5}{6}\right) - 12\,h_t^4\phi^4\left(\ln\frac{g^2\phi^2}{\mu^2} - \frac{3}{2}\right)\right]$$

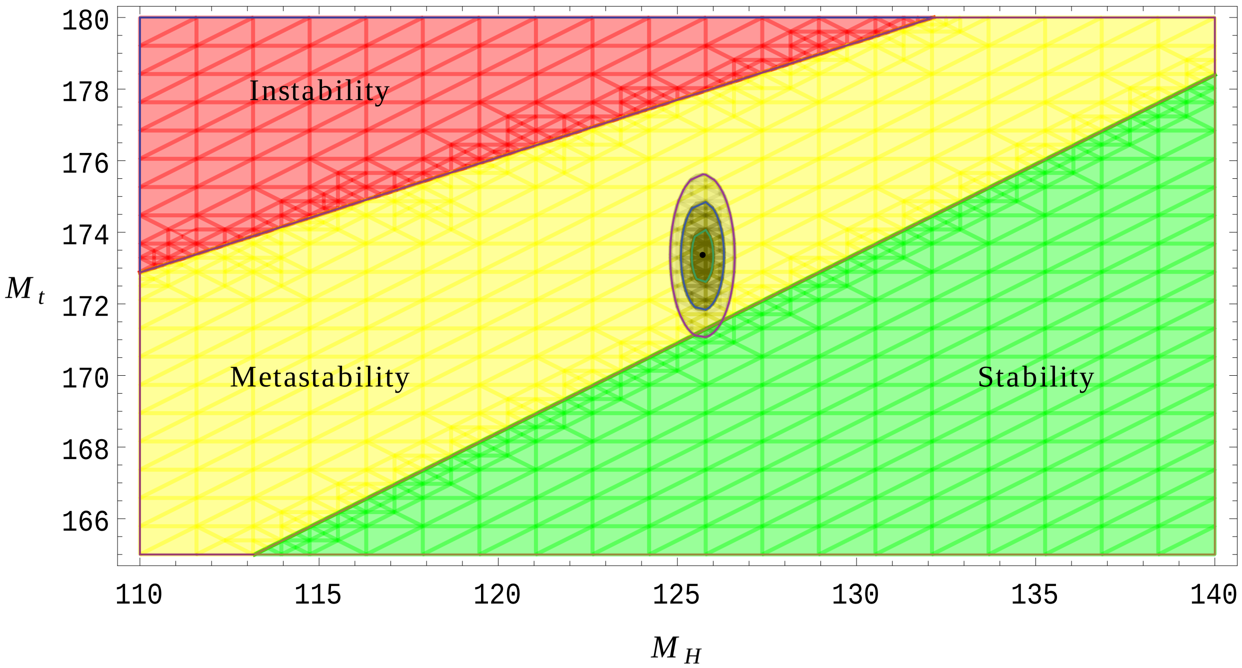
RG Improved Effective Potential $V_{RGI}(\phi)$



Depending on M_H and M_t, the second minimum can be : (1) lower than the EW minimum (as in the figure) ; (2) at the same level of the EW minimum ; (3) higher than the EW minimum.

When the potential at the New Minimum is lower than the potential at the EW Minimum, compute the Tunnelling Time and draw the ...

Stability Diagram in the $M_H - M_t$ plane



Stability region : $V_{eff}(v) < V_{eff}(\phi^{(2)}_{min})$. **Meta-stability** region : $\tau > T_U$. **Instability** region : $\tau < T_U$. Stability line : $V_{eff}(v) = V_{eff}(\phi^{(2)}_{min})$. Instability line : M_H and M_t such that $\tau = T_U$.

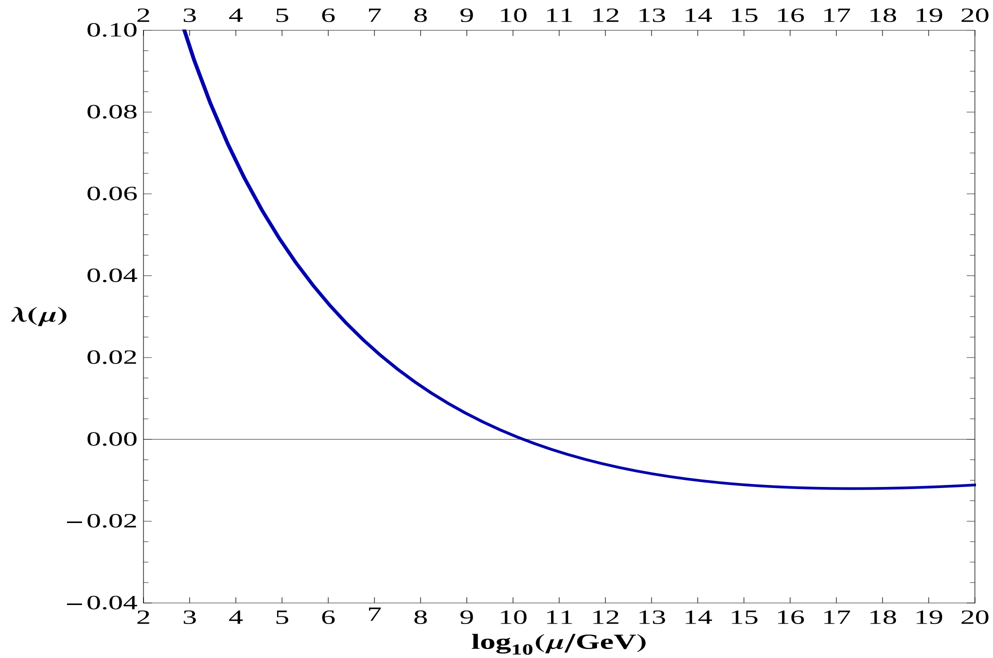
Note : the instability occurs for large values of ϕ

$\Rightarrow$ $V_{_{RGI}}(\phi)$ **well approximated by keeping only the "quartic" term :**

$$V_{_{RGI}}(\phi) \sim \frac{\lambda_{eff}(\phi)}{24}\phi^4$$

Moreover : $\lambda_{eff}(\phi)$ depends on ϕ essentially as the running quartic coupling $\lambda(\mu)$ depends on the running scale μ

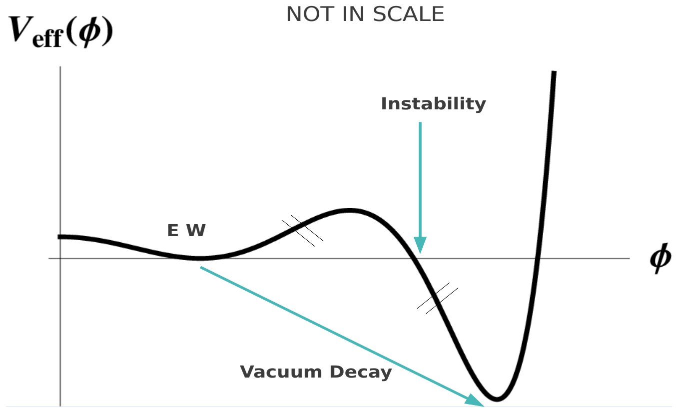
$\Rightarrow$ **we can read the Effective Potential from the $\lambda(\mu)$ flow**



0.10
0.08
0.06
0.04
0.02
0.00
−0.02
−0.04
λ(μ)
2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
$\log_{10}(\mu/\mathrm{GeV})$

Metastability Scenario

When the second minimum is lower than EW



Tunnelling between the Metastable EW Vacuum and the True Vacuum.

As long as EW vacuum lifetime larger than the age of the Universe ...

.... we may well live in the Meta-Stable (EW) Vacuum ....

How do we compute the tunneling time ?

How do we compute the tunneling time ?

Semiclassical calculation - WKB - instantons

EW vacuum lifetime (= Tunneling Time τ)

$$\Gamma = \frac{1}{\tau} = T_U^3 \, \frac{S[\phi_b]^2}{4\pi^2} \left| \frac{\det' \left[-\partial^2 + V''(\phi_b)\right]}{\mathbf{det}\left[-\partial^2 + V''(v)\right]} \right|^{-1/2} e^{-S[\phi_b]}$$

$\phi_b(r)$: **Bounce Solution**

Solution to the Euclidean Equation of Motion with appropriate boundary conditions

S. Coleman, Phys. Rev. D 15 (1977) 2929

C.G.Callan, S.Coleman, Phys. Rev. D 16 (1977) 1762

Tunneling and bounces

Bounce : solution to Euclidean equations of motion

$$-\partial_\mu \partial_\mu \phi + \frac{d\,V(\phi)}{d\,\phi} = -\frac{d^2\phi}{dr^2} - \frac{3}{r}\frac{d\phi}{dr} + \frac{d\,V(\phi)}{d\,\phi} = 0 \ ,$$

Boundary conditions : $\phi'(0) = 0 \ , \qquad \phi(\infty) = v \to 0 \ .$

Potential : $V(\phi) = \frac{\lambda}{4}\phi^4$

with negative λ

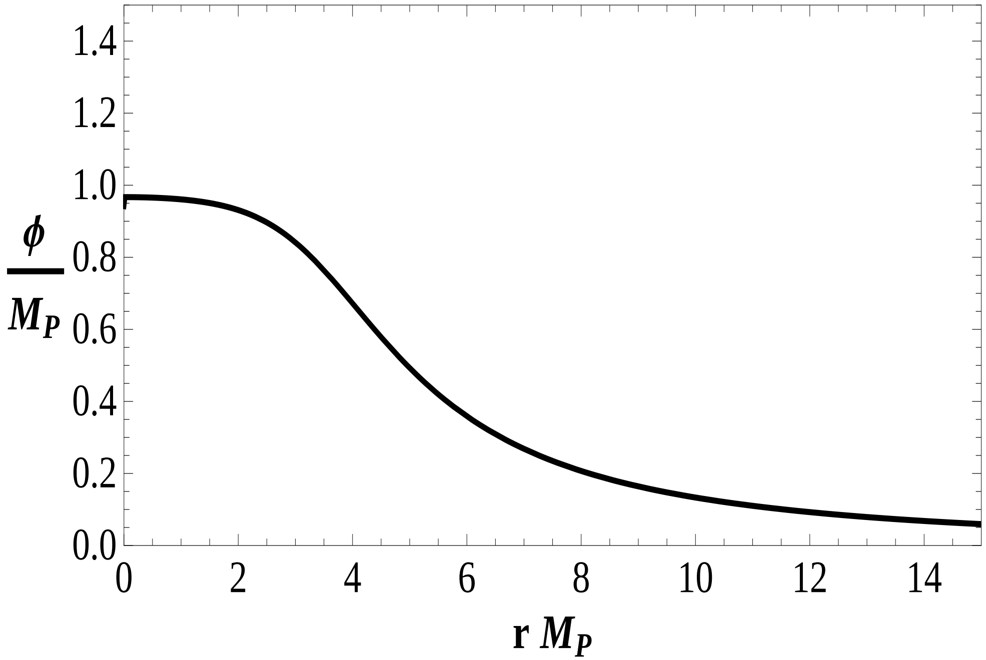
Bounce solutions :

$$\phi_b(r) = \sqrt{\frac{2}{|\lambda|}} \frac{2R}{r^2 + R^2}$$

R **is the size of the bounce**

Bounces : $\phi_b(r) = \sqrt{\frac{2}{|\lambda|}} \frac{2R}{r^2+R^2}$

$R =$ **bounce size** – **Classical degeneracy :** $S[\phi_b] = \frac{8\pi^2}{3|\lambda|}$

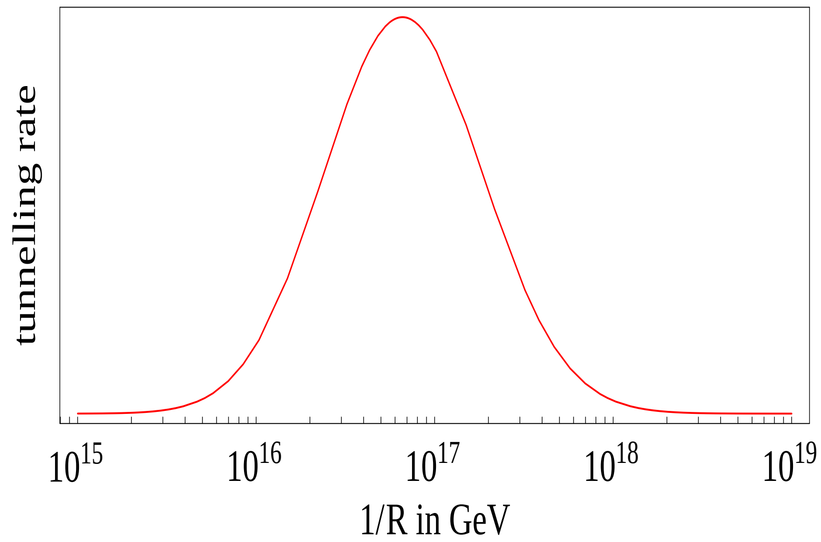


Degeneracy removed at the Quantum Level

Degeneracy removed at the Quantum Level

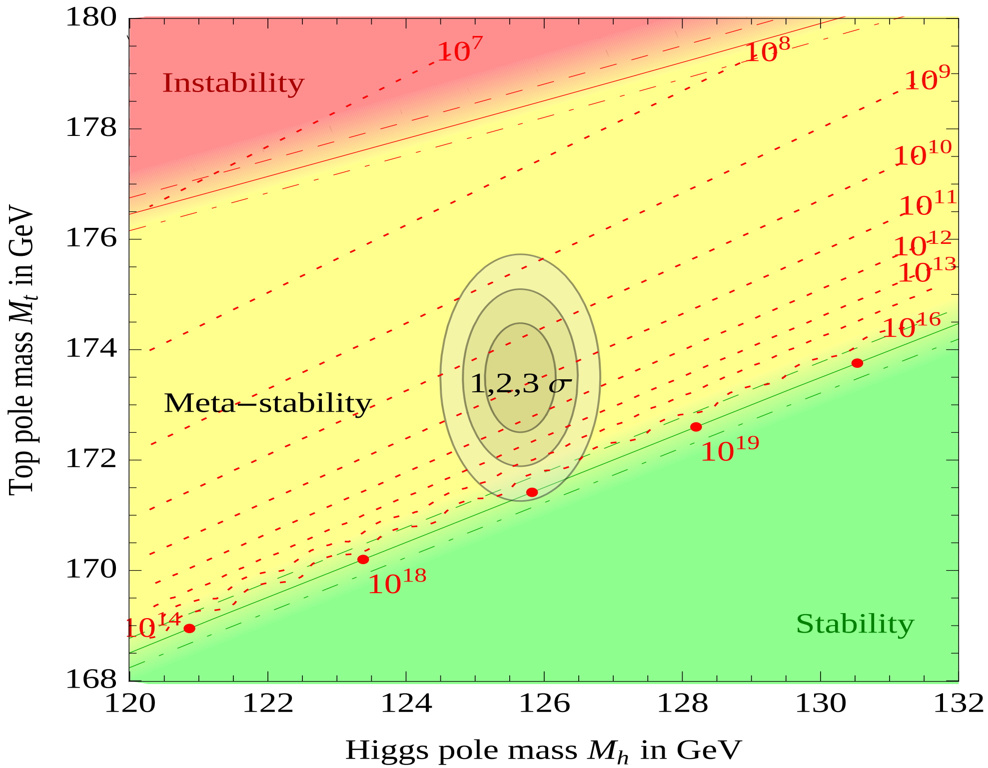
Transition rate as a function of R : ($\mu \sim \frac{1}{R}$)

$$p = \max_R \frac{V_U}{R^4} \exp\left[-\frac{8\pi^2}{3\,|\lambda(\mu)|} - \Delta S\right]$$



from : G. Isidori, G. Ridolfi, A. Strumia, Nucl.Phys.B 609 (2001) 387

... So ... Stability Diagram



Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, Strumia, JHEP 1312 (2013) 089.

... However ...

... Some warnings ...

V. Branchina, E. Messina, Phys.Rev.Lett.111, 241801 (2013) (arXiv:1307.5193)

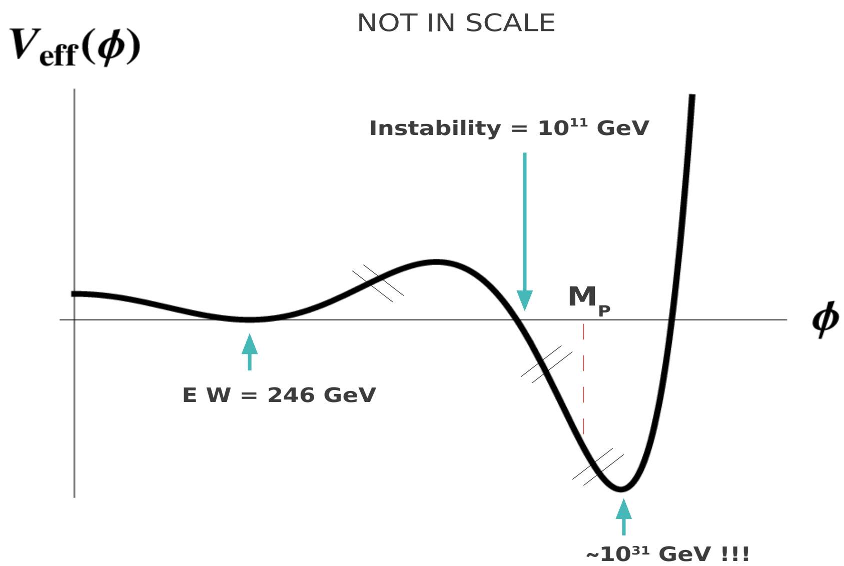
V. Branchina, arXiv:1405.7864, Moriond 2014

V. Branchina, E. Messina, A. Platania JHEP 1409 (2014) 182 (arXiv:1407.4112)

V. Branchina, E. Messina, M. Sher, Phys.Rev.D91 (2015) 1, 013003 (arXiv:1408.5302)

V. Branchina, E. Messina, in preparation

Probably worth to know that for $M_H \sim 126$ GeV and $M_t \sim 173$ GeV



New minimum at $\phi^{(2)}_{min} \sim 10^{30}$ GeV !!!

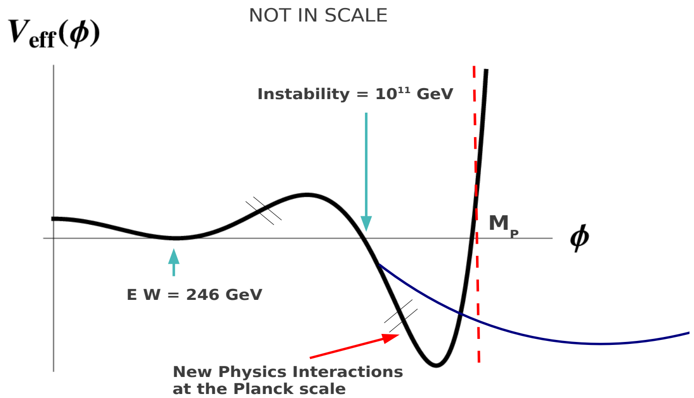
SM Effective Potential extrapolated well above M_P !!!

Remember : you normally hear... "assume SM valid up to M_P"

Does this make any sense ??? Is this a problem or not ???

To make sense out of this potential, people have (had??) arguments ...

1. New Physics Interactions that appear at the Planck scale M_P eventually stabilize the potential around M_P ...



... meaning that if you take into account the presence of these new physics interactions at M_P, given in terms of higher order operators as

$$\frac{\phi^6}{M_P^2} \quad , \quad \frac{\phi^8}{M_P^4} \quad , \ldots .$$

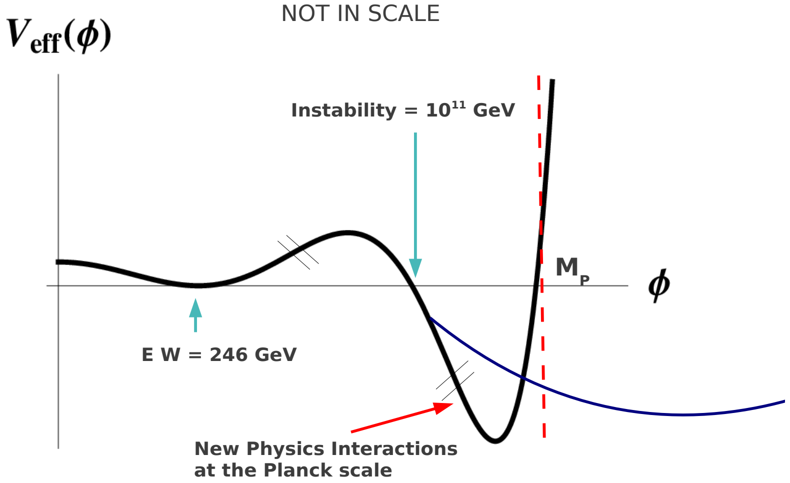
these terms stabilize the Higgs potential around M_P...

2. These New Physics Interactions present at the Planck scale do not affect the EW vacuum lifetime τ (can be neglected when computing τ)

(a) - Instability scale much lower than Planck scale $\Rightarrow$

$$\Rightarrow \quad \text{suppression} \left(\frac{\Lambda_{inst}}{M_P}\right)^n$$

(b) - For tunnelling, only height of the barrier and turning points matter



... All these arguments are incorrect and misleading ...

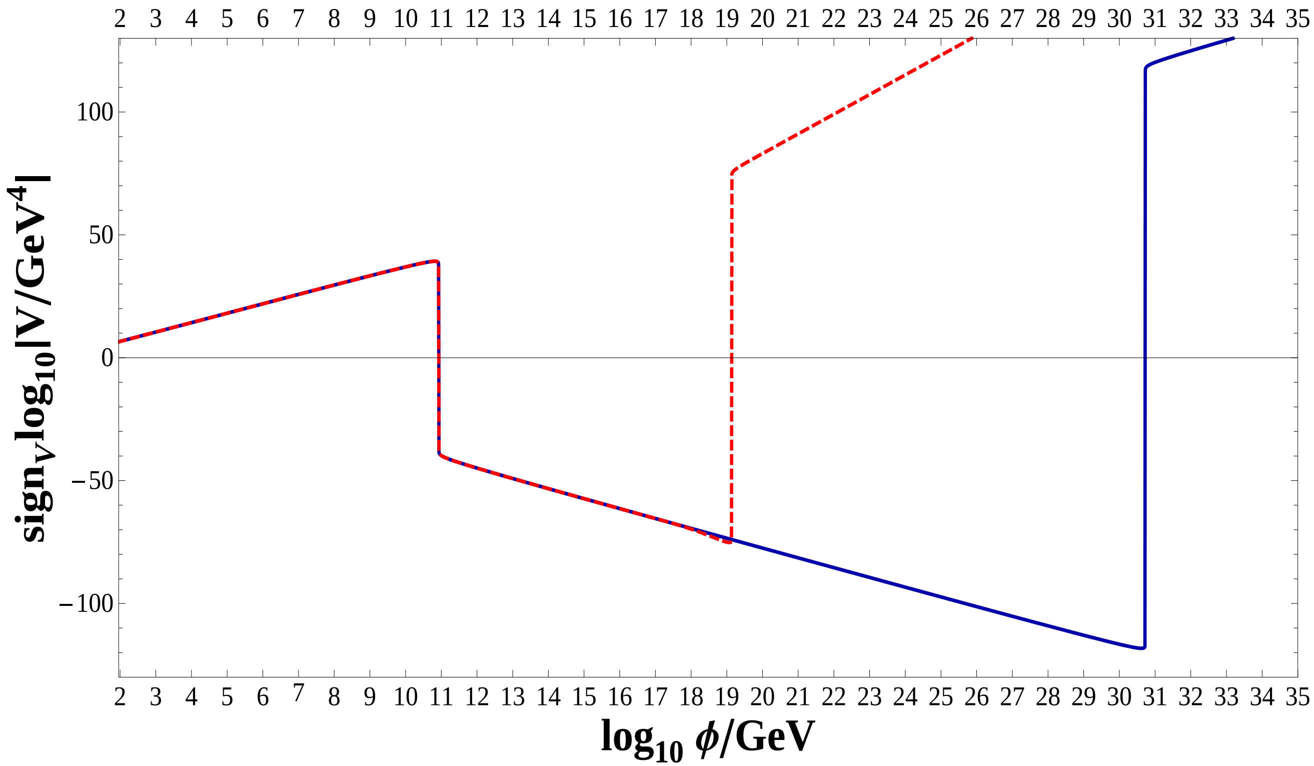
Let us consider New Physics at M_P

Add ϕ^6 and ϕ^8 in such a way to implement the stabilization of the SM Higgs potential at M_P :

$$V(\phi) \;=\; \frac{\lambda}{4}\phi^4 + \frac{\lambda_6}{6}\frac{\phi^6}{M_P^2} + \frac{\lambda_8}{8}\frac{\phi^8}{M_P^4}$$

$$V_{eff}^{new}(\phi) = V_{eff}(\phi) + \frac{\lambda_6(\phi)}{6M_P^2}\xi(\phi)^6\phi^6 + \frac{\lambda_8(\phi)}{8M_P^4}\xi(\phi)^8\phi^8$$

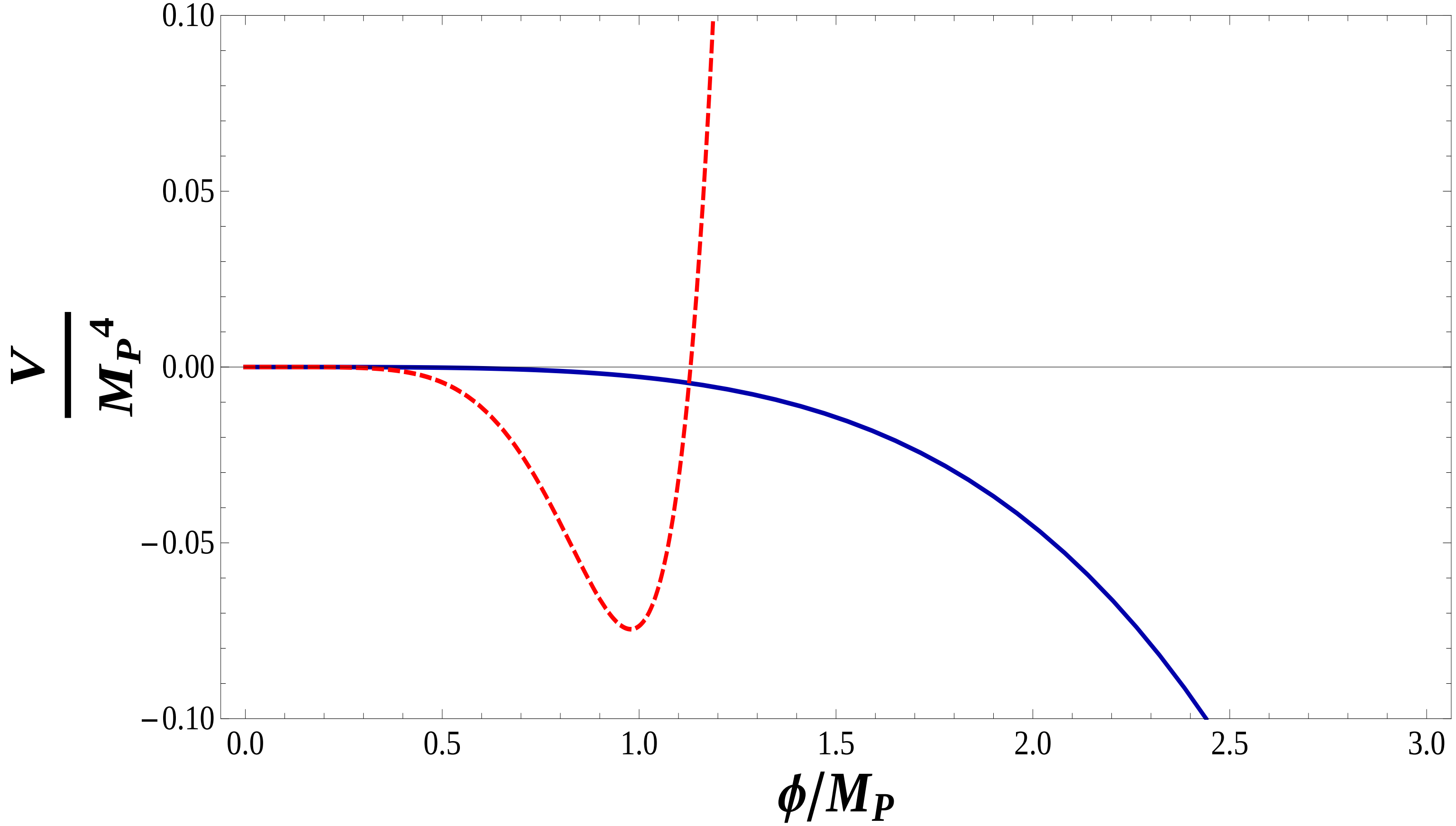
Effective Potential $M_H \sim 126$ $M_t \sim 173$ Log-Log Plot



Blue line : $V_{eff}(\phi)$ **no higher order terms**

Red line : $V_{eff}^{new}(\phi)$ **with** $\lambda_6(M_P) = -2$ $\lambda_8(M_P) = 2.1$

Zoom around the Planck scale



Blue line : $V_{eff}(\phi)$ **no new physics terms (SM alone)**

Red line : $V_{eff}^{new}(\phi)$ **with** $\lambda_6(M_P) = -2$ $\lambda_8(M_P) = 2.1$

We have a New Potential ⇒ we have to find the new bounce configurations and consider them for the computation of the tunnelling time

$$V(\phi) = \frac{\lambda}{4}\phi^4 + \frac{\lambda_6}{6}\frac{\phi^6}{M_P^2} + \frac{\lambda_8}{8}\frac{\phi^8}{M_P^4}$$

It turns out that in the computation of the EW vacuum lifetime :

Competition between

Old Bounce $\phi_b^{(Old)}(r)$ **and the New Bounce** $\phi_b^{(New)}(r)$

New Physics not included : Only $\phi_b^{(old)}$ **(Literature case)**

$$\Gamma = \frac{1}{\tau} = \frac{1}{T_U}\left[\frac{S[\phi_b^{(old)}]^2}{4\pi^2}\,\frac{T_U^4}{R_M^4}\,e^{-S[\phi_b^{(old)}]}\right]\times\left[e^{-\Delta S_1}\right]$$

New Physics included : $\phi_b^{(new)}$ and $\phi_b^{(old)}$ **(Our case)**

$$\Gamma = \Gamma_1 + \Gamma_2 = \frac{1}{\tau_1} + \frac{1}{\tau_2} = \frac{1}{T_U}\left[\frac{S[\phi_b^{(old)}]^2}{4\pi^2}\,\frac{T_U^4}{R_M^4}\,e^{-S[\phi_b^{(old)}]}\right]\times\left[e^{-\Delta S_1}\right] + \frac{1}{T_U}\left[\frac{S[\phi_b^{(new)}]^2}{4\pi^2}\,\frac{T_U^4}{\overline{R}^4}\,e^{-S[\phi_b^{(new)}]}\right]\times\left[e^{-\Delta S_2}\right]$$

Neglecting for a moment the ΔS (quantum) contributions

Literature : $S[\phi_b^{(old)}] \sim 1800 \Rightarrow \quad \tau \sim 10^{600}\, T_U$

Our case : $S[\phi_b^{(new)}] \sim 82 \quad \Rightarrow \quad \tau \sim 10^{-208}\, T_U$

Contribution from $\phi_b^{(old)}$ exponentially suppressed !

New Physics Interactions at High Scales (Planck) do matter !

Quantum fluctuations do not change significantly these “classical” results

	Literature : Loop contributions to τ
$e^{\Delta S_H}$	2.87185
$e^{\Delta S_t}$	1.20708×10^{-18}
$e^{\Delta S_{gg}}$	1.26746×10^{50}

	Our case : Loop contributions to τ
$e^{\Delta S_H}$	2.82295×10^{10}
$e^{\Delta S_t}$	8.62404×10^{-5}
$e^{\Delta S_{gg}}$	4.97869×10^{9}

How comes that new physics can have such an impact on τ ?

Why the arguments on the suppression of new physics do not apply ?

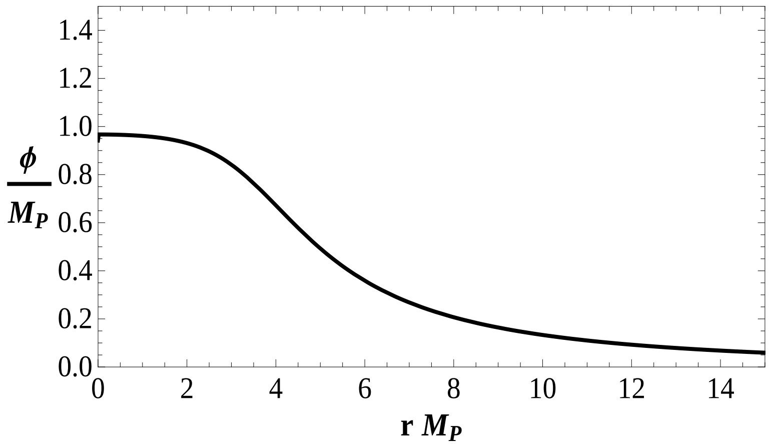
1. New physics appears in terms of higher dimension operators, and people expected their contribution to be suppressed as $(\frac{\Lambda_{inst}}{M_P})^n$

But: Tunnelling is a non-perturbative phenomenon. We first select the saddle point, i.e. compute the bounce (tree level), and then compute the quantum fluctuations (loop corrections) on the top of it.

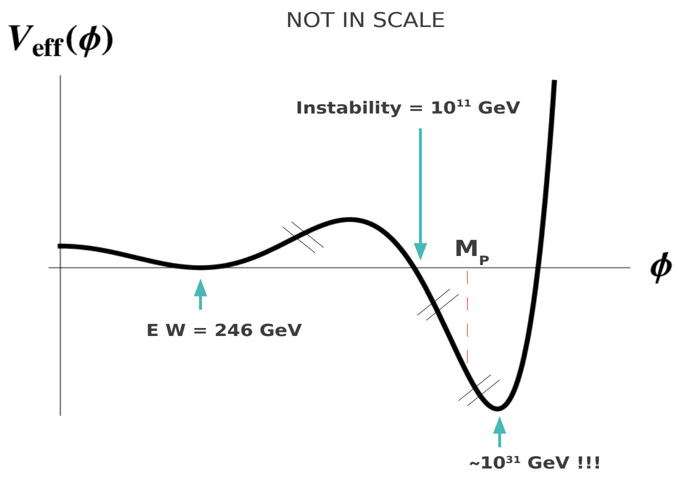
Suppression in terms of inverse powers of M_P (power counting theorem) concerns the loop corrections, not the selection of the saddle point (tree level).

Remember : $$\tau \sim e^{S[\phi_b]}$$

New bounce $\phi_b^{(2)}(r)$, New action $S[\phi_b^{(2)}]$, New τ



2. Height of the barrier and turning points...



This is QFT with "very many" dof, not 1 dof QM $\Rightarrow$ the potential is not $V(\phi)$ in figure with 1 dof, but...

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) = \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}(\vec{\nabla}\phi)^2 - V(\phi) = \frac{1}{2}\dot{\phi}(\vec{x},t)^2 - U(\phi(\vec{x},t))$$

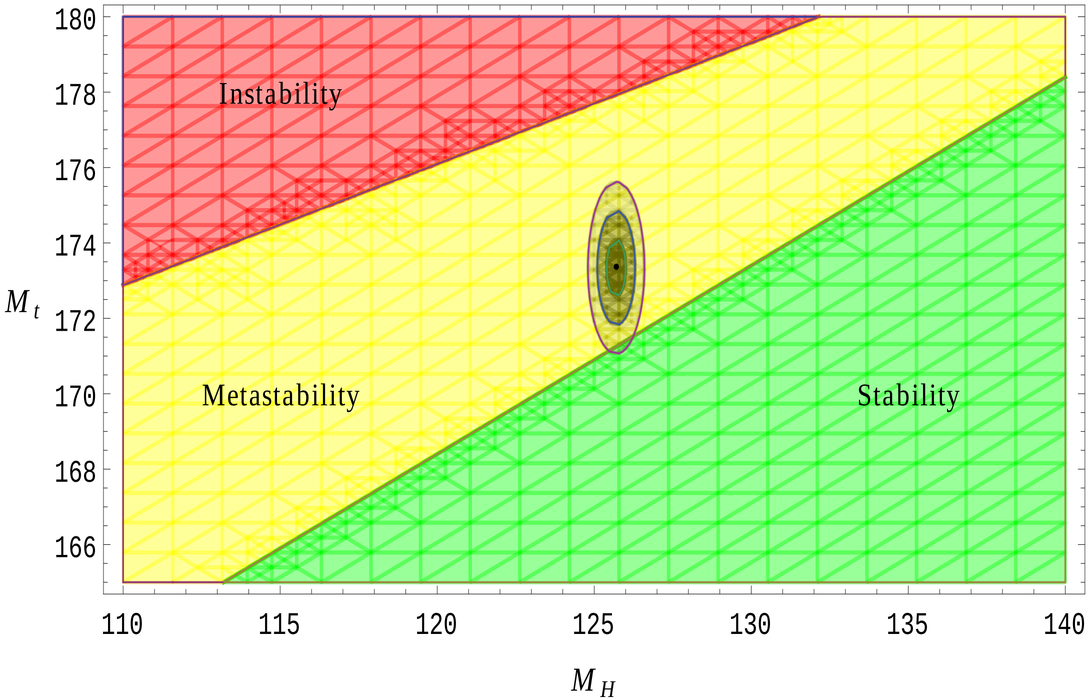
where $U(\phi(\vec{x},t))$ is : $\quad U(\phi(\vec{x},t)) = V(\phi(\vec{x},t)) + \frac{1}{2}(\vec{\nabla}\phi(\vec{x},t))^2$

Very many dof, not 1 dof... The Potential is : $\sum_{\vec{x}} U(\phi(\vec{x},t))$

The bounce is not a constant configuration ... Gradients do matter a lot!

Let us move now to Phase Diagrams...

Phase diagram with $\lambda_6 = 0$ and $\lambda_8 = 0$ - Literature case



This is the well known Stability Diagram ... According to it :

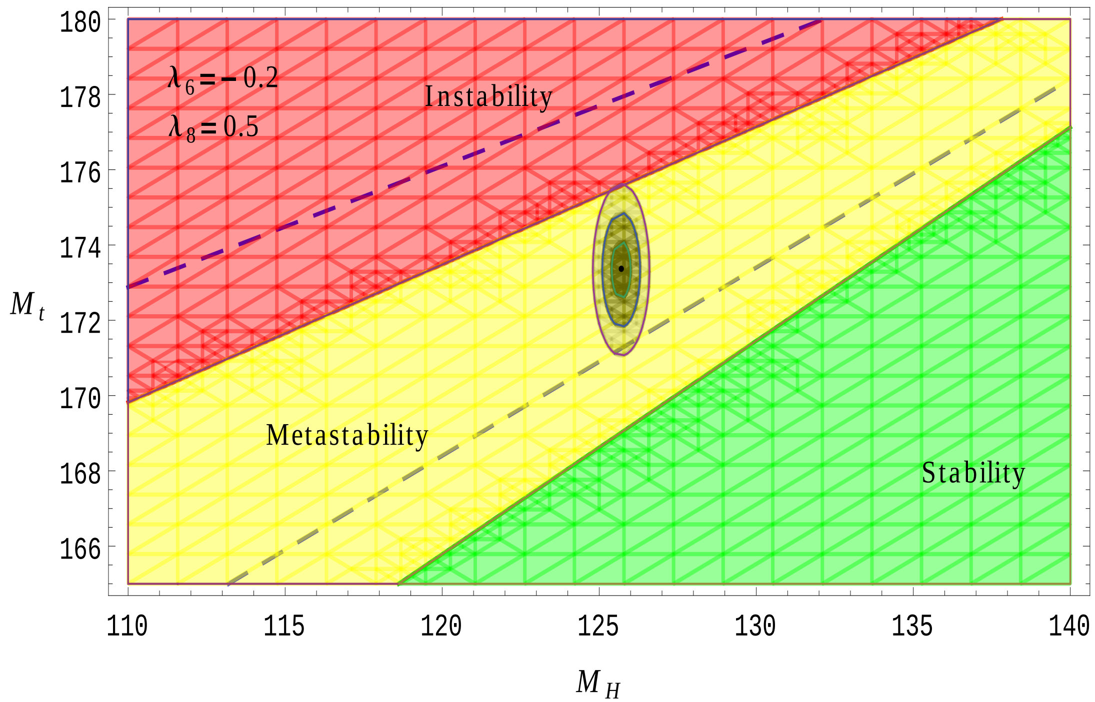
(1) For $M_H \sim 125 - 126$ GeV and $M_t \sim 173$ we live in a metastable state ;

(2) $3\,\sigma$ close to the stability line (Criticality) ;

(3) Precision measurements of the top mass should allow to discriminate between stable, metastable, or critical EW vacuum ...

Phase diagram with $\lambda_6 = -0.2$ and $\lambda_8 = 0.5$

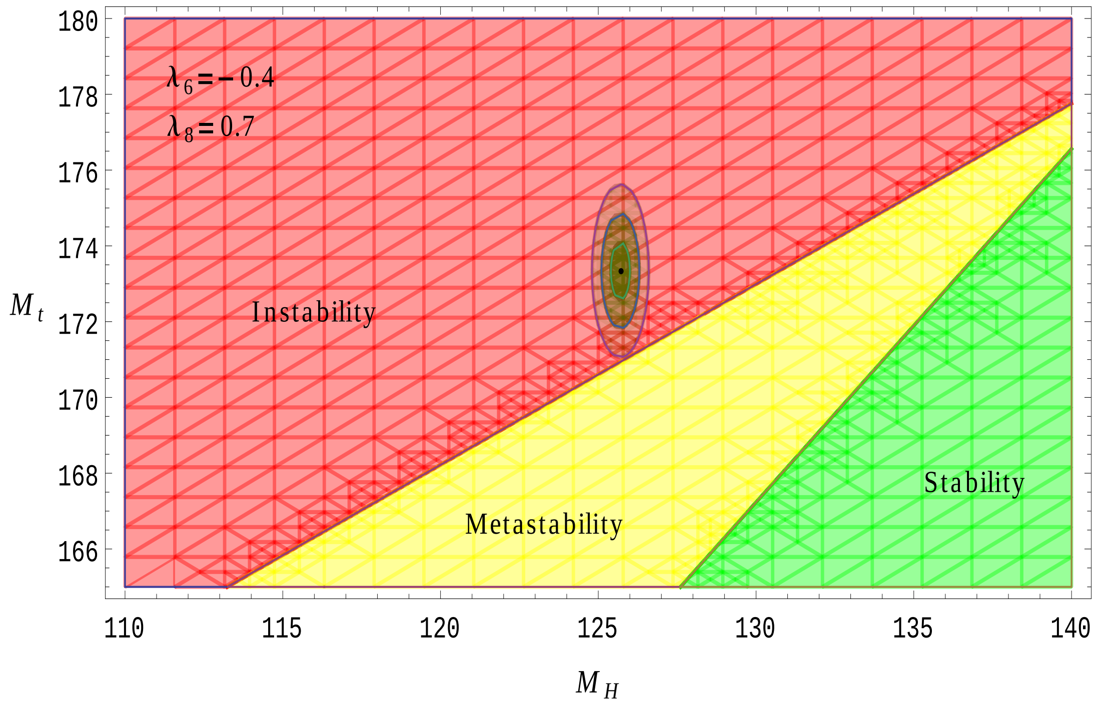
(Please note : Natural values for the coupling constants)



The strips move downwards ... The Experimental Point no longer at $3\,\sigma$ from the stability line !!! ... Stability Diagram depends on new physics !

Phase diagram with $\lambda_6 = -0.4$ and $\lambda_8 = 0.7$

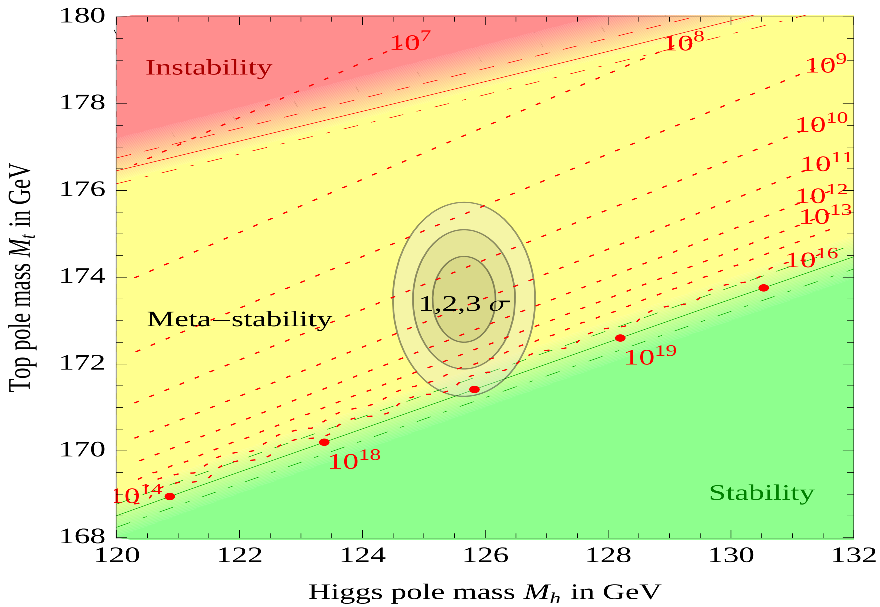
(Please note : Natural values for the coupling constants)



Even worse !

Lessons

The Phase Diagram

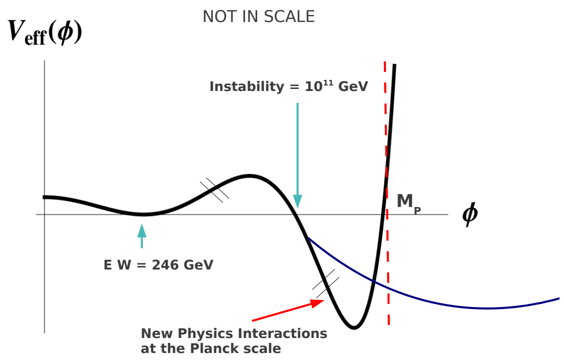


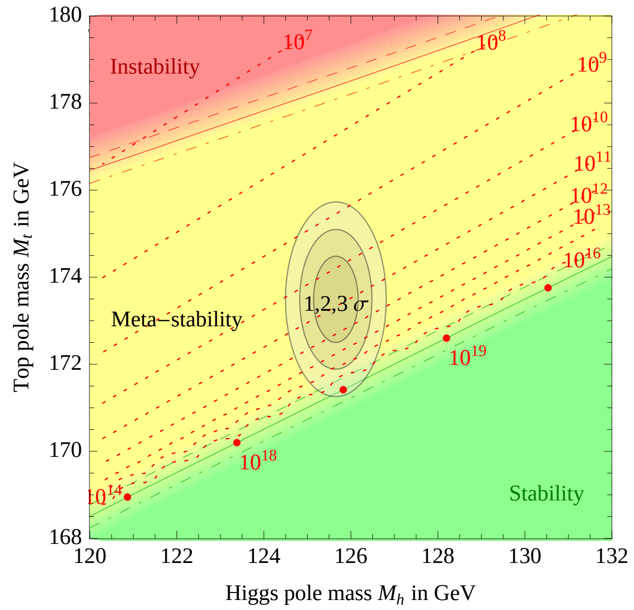
in not Universal !

... one out of different possibilities ....

The two statements :

(1) - There should be new physics at the Planck scale that stabilizes the potential

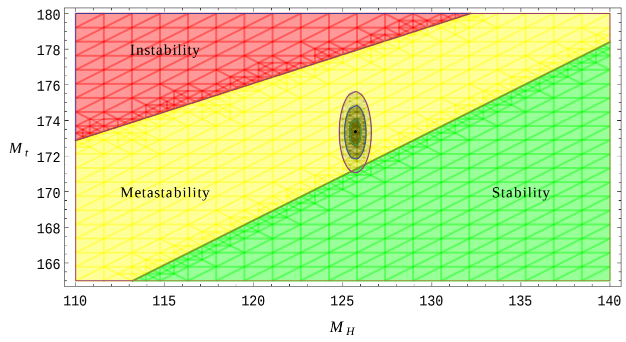


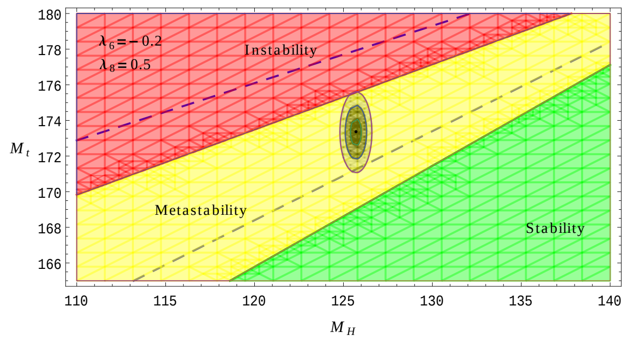


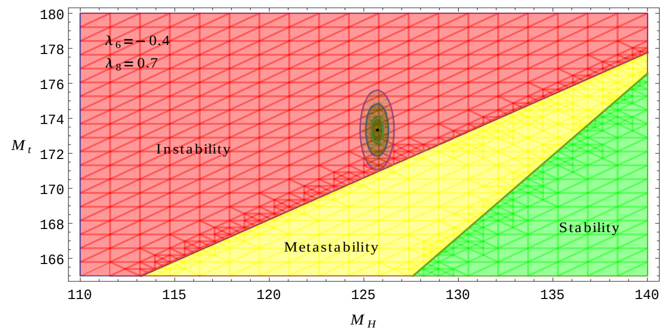
(2) - The stability phase diagram in independent on this new physics

Cannot be true at the same time

"Precision Measurements of M_t"

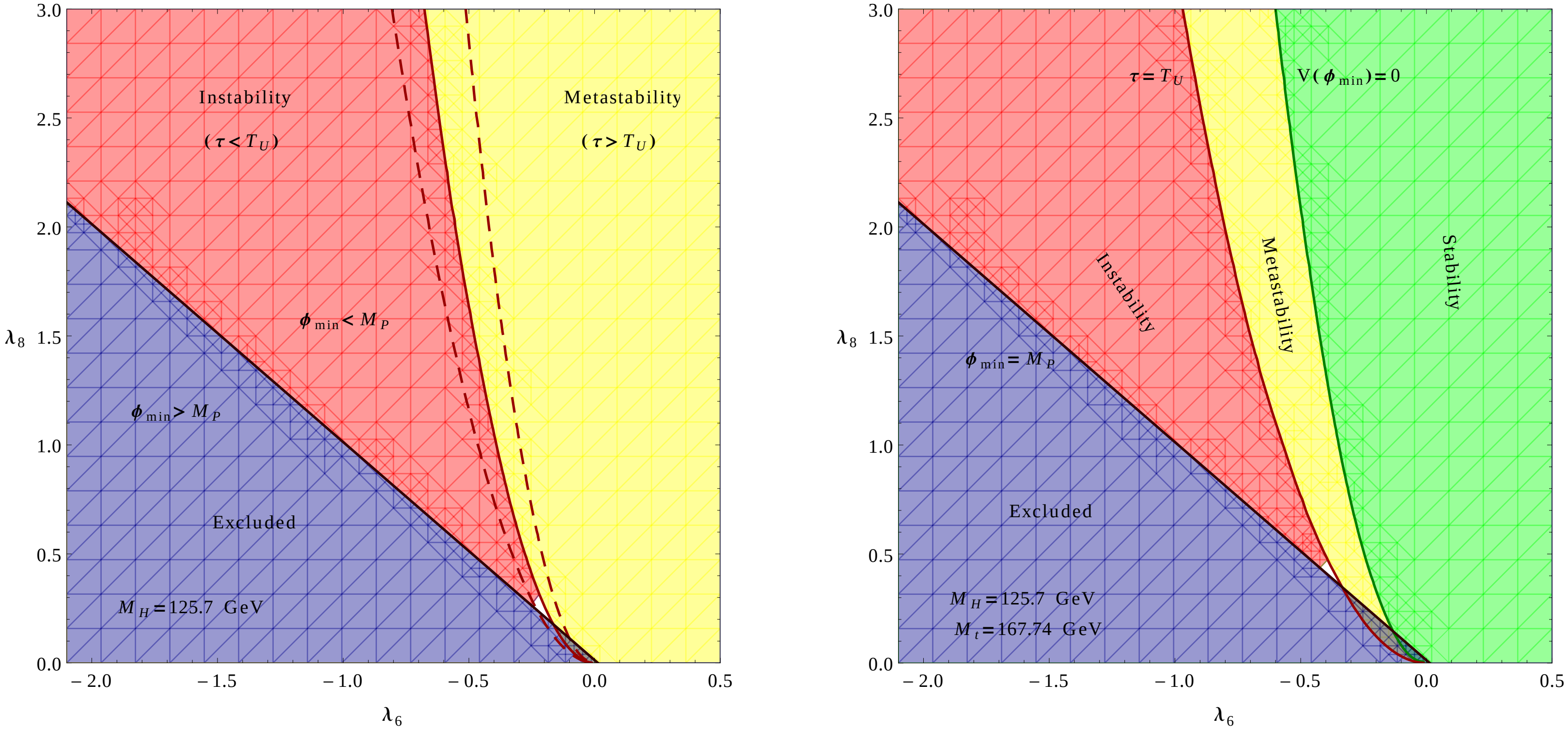






Precision measurements of M_t (and M_H) cannot discriminate between stability, metastability or criticality ... The knowledge of M_t and M_H alone is not sufficient to decide of the EW vacuum stability condition. We need informations on NEW PHYSICS in order to asses this question ...

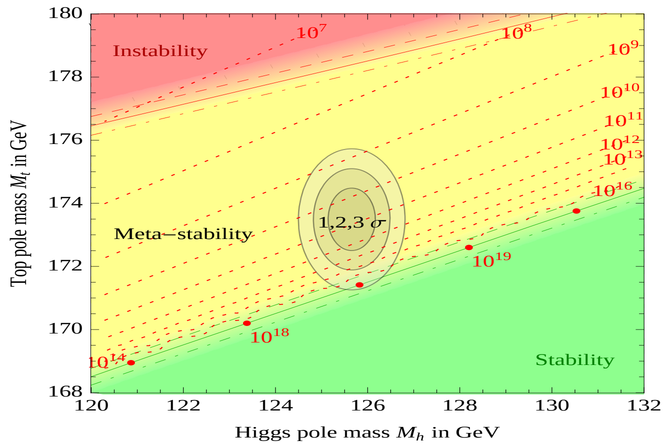
“Precision Measurements of M_t”



Constraints in the parameter space of New Physics Theories

BSM “Stability Test”

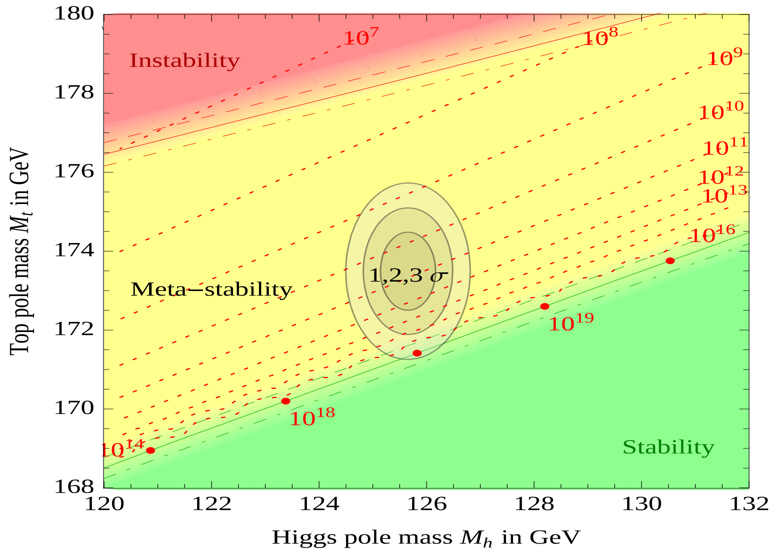
“Near-Criticality”



Somebody considers this near-criticality of the SM vacuum as the most important message so far from experimental data on the Higgs boson

But : This “near-criticality” picture (technically $\lambda(M_P) \sim 0$ and $\beta(\lambda(M_P)) \sim 0$) can be easily screwed up by even small seeds of new physics ... Strong sensitivity to new physics, No Universality.

Higgs Inflation



The Higgs inflation scenario (Shaposhnikov - Bezrukov) strongly relies on the realization of criticality ($\lambda(M_P) \sim 0$ and $\beta(\lambda(M_P)) \sim 0$). But ... even a little seed of new physics can screw up this picture

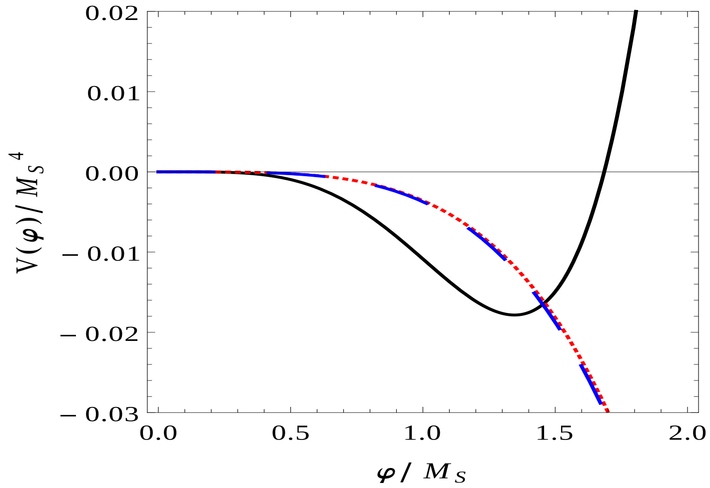
A Renormalizable (Toy) Model for New Physics

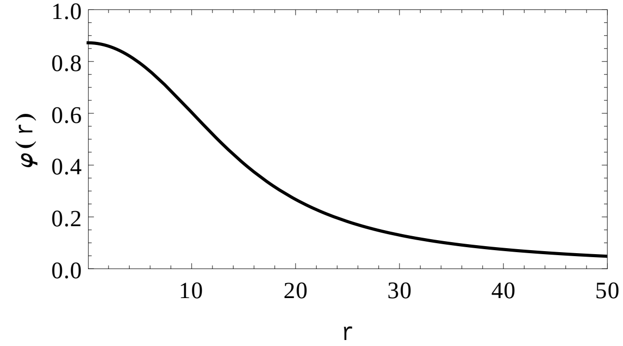
Consider the following UV completion for the SM :

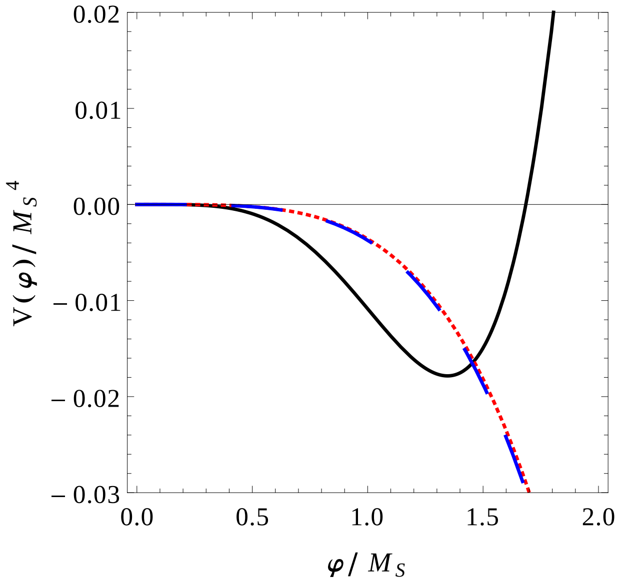
$$\Delta V(\phi, S, \psi) = \frac{M_S^2}{2} S^2 + \frac{\lambda_S}{4} S^4 + \frac{g_S}{4} \phi^2 S^2 + M_f \bar{\psi}\psi + \frac{g_f}{\sqrt{2}} \phi \bar{\psi}\psi$$

with $M_f \sim 10^{17}$ **GeV and** $M_S \sim 10^{18}$ **GeV.**

After imposing "treshold conditions" at M_f**, so that the potential for** $\phi \leq M_f$ **has the SM form, we get the Modified Higgs Potential :**







With this **New Potential** **we compute again the "bounce solution" and then the** **tunnelling time** **...**

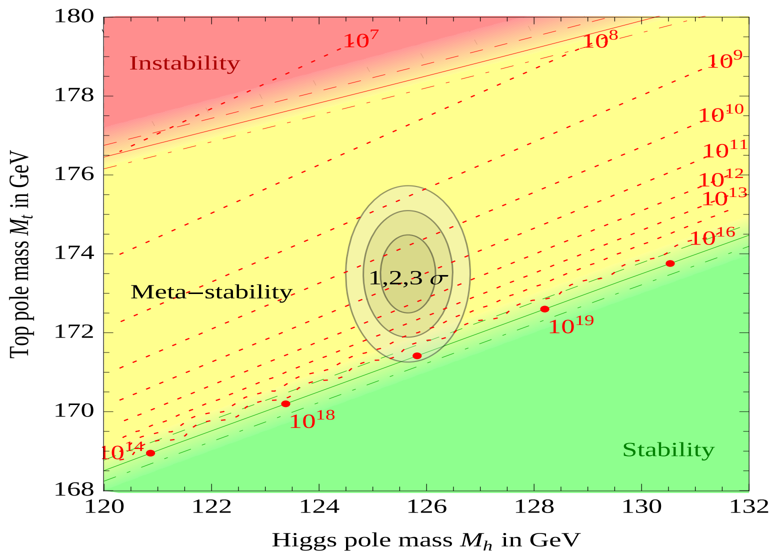
$$\tau \sim 10^{-30} T_U \quad ; \quad \tau \sim 10^{15} T_U \quad ; \quad ...$$

To be compared with : $\tau \sim 10^{600} T_U$ **(without new physics)**

... Remember ...

$$\tau \sim 10^{600} T_U$$

is the value associated with the experimental point in this stability diagram, where it is assumed that the UV completion of the SM at energies much higher than the instability scale has no impact on the stability diagram



Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, Strumia, JHEP 1312 (2013) 089

Summary and Conclusions

- **The Stability Phase Diagram of the EW vacuum strongly depends on New Physics ...**

- **Precision Measurements of the Top Mass will not allow to discriminate between stability, metastability or criticality of the EW vacuum. Phase Diagram too sensitive to New Physics ...**

- **Higgs Inflation ?? ... Any small seed of new physics screws up the conditions**

$$\lambda(M_P) \sim 0 \quad \text{and} \quad \beta(\lambda(M_P)) = \left(\mu \frac{d\,\lambda(\mu)}{d\,\mu}\right)_{\mu=M_P} \sim 0$$

- **Our results provide a "BSM stability test". A BSM is acceptable if it provides either a stable EW vacuum or a metastable one, with lifetime larger than the age of the universe (No $\tau << T_U$!!).**

- **This analysis can be repeated even if the new physics scale lies below the Planck scale (for instance, GUT scale), or above ... transplanckian physics ...**