



DARK MATTER

EVIDENCE | CANDIDATES | SEARCHES

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OUTLINE

Part I. Evidence for dark matter

- Brief historical perspective
- Clusters of galaxies
- Galaxies (including our own)
- Cosmology
- Dark matter fact sheet

Part II. Dark matter candidates

- Tour of the zoo
- Thermal decoupling
- WIMP paradigm

Part III. Dark matter searches

- Overview of detection strategies
- Direct searches: idea, techniques, status
- Indirect searches: idea, techniques, status

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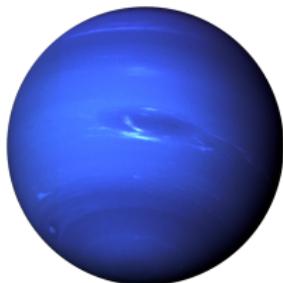
I. BRIEF HISTORICAL PERSPECTIVE

Ever since Newton (and Kepler) found the laws of gravitation in the 17th century, any anomaly in the trajectory of objects in the sky has been interpreted in one of two ways: “there is an unseen object nearby”, or “the theory is wrong”.

Solution A: “unseen object” (aka “missing mass”)

The trajectory observed is perturbed by a nearby object yet to be discovered.

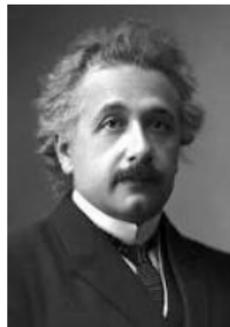
Example: The erratic behaviour of Uranus lead Bouvard, Adams and Le Verrier to hypothesise the existence of Neptune, later discovered by Galle in 1846.



Solution B: “wrong theory” (aka “new physics”)

Newton's laws of gravitations breakdown, leading to a different trajectory.

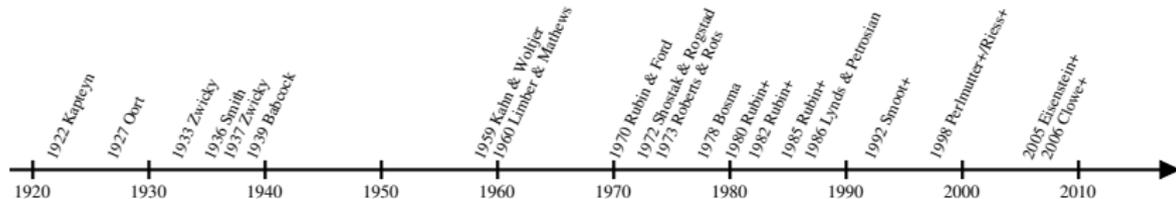
Example: The anomalous precession of Mercury's perihelion lead to the conjecture of a new planet Vulcan, but was explained with general relativity by Einstein in 1916.



In a sense, the modern concept of dark matter arises in the same context as the examples above.

I. BRIEF HISTORICAL PERSPECTIVE

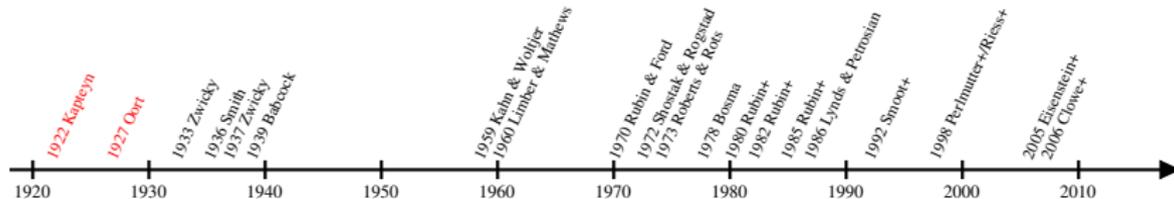
The evidence for dark matter gradually mounted throughout the 20th century, and by now we are convinced that our universe is filled with dark matter at various scales. Note that all evidence for dark matter is of gravitational origin; non-gravitational evidence is yet to be discovered.



We can identify four key revolutions in the history of dark matter:

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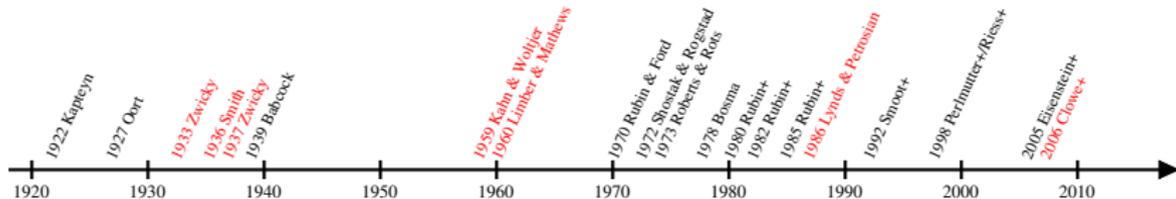
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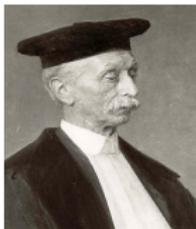
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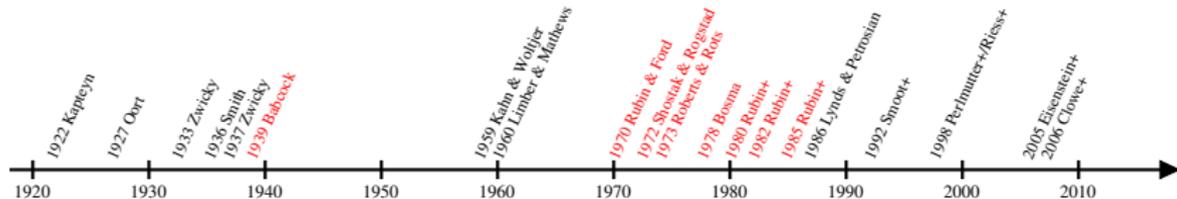
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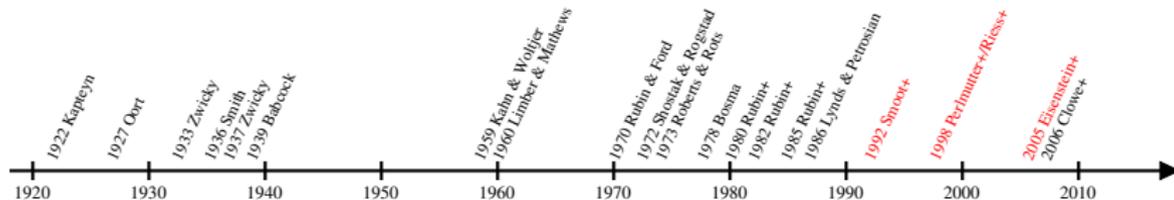
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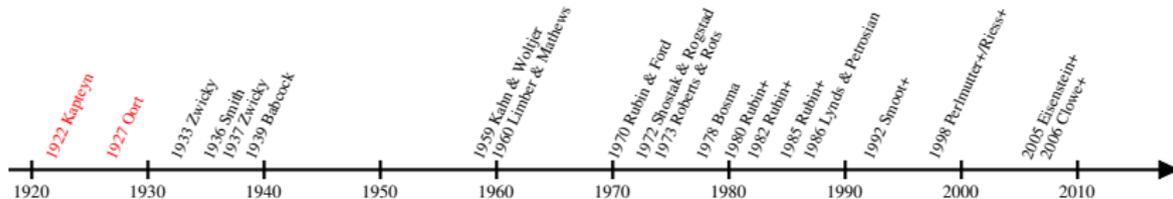
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I. PRE-DISCOVERY PHASE

Dark matter is first mentioned by Jacobus Kapteyn in 1922 when making a census of the stars in our own Galaxy.

FIRST ATTEMPT AT A THEORY OF THE ARRANGEMENT AND MOTION OF THE SIDEREAL SYSTEM¹

BY J. C. KAPTEYN²

ABSTRACT

First attempt at a general theory of the distribution of masses, forces, and velocities in the stellar system.—(1) Distribution of stars. Observations are fairly well represented, at



Remark. Dark matter. It is important to note that what has here been determined is the total mass within a definite volume, divided by the number of luminous stars. I will call this mass the average effective mass of the stars. It has been possible to include the luminous stars completely owing to the assumption that at present we know the luminosity-curve over so large a part of its course that further extrapolation seems allowable.

Now suppose that in a volume of space containing l luminous stars there be dark matter with an aggregate mass equal to Kl average luminous stars; then, evidently the effective mass equals $(l+K) \times$ average mass of a luminous star.

We therefore have the means of estimating the mass of dark matter in the universe. As matters stand at present it appears at once that this mass cannot be excessive. If it were otherwise, the average mass as derived from binary stars would have been very much lower than what has been found for the effective mass.

I. PRE-DISCOVERY PHASE

A few years later, in 1927, Jan Oort estimates the so-called Oort's constants and in the process also mentions dark matter...

BULLETIN OF THE ASTRONOMICAL INSTITUTES
OF THE NETHERLANDS.

1927 April 14

Volume III.

No. 120.

COMMUNICATION FROM THE OBSERVATORY AT LEIDEN.

Observational evidence confirming Lindblad's hypothesis of a rotation of the galactic system, by *J. H. Oort*.



In order to explain the rotation there must be near the centre an attracting mass of at least 8×10^{10} times the mass of the sun. There remains the difficulty why we do not observe this large mass. Near 6000 parsecs KAPTEYN and VAN RHIJN find an almost negligible density, whereas it *should* be very much greater than in our neighbourhood. Part of the dis-

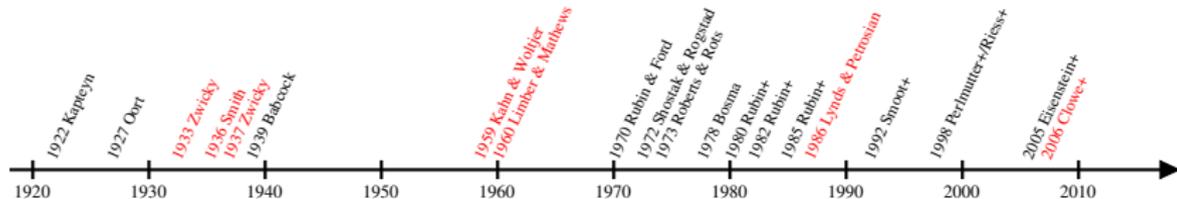
crepancy may have resulted from the approximative character of their solution, in which all galactic longitudes were combined. Discussing various galactic regions separately KREIKEN finds indications of a centre near 314° longitude, at a distance of 2270 parsecs *) which is in the right direction, but certainly at too small a distance and too little defined. **)

The most probable explanation is that the decrease of density in the galactic plane indicated for larger distances is mainly due to obscuration by dark matter.

Such a hypothesis receives considerable support from the marked avoidance of the galactic plane by the globular clusters, a phenomenon for which up to the present time no other well defensible explanation has been put forward. ***)

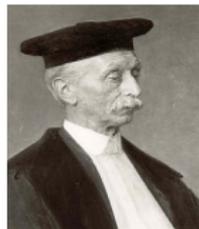
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I. CLUSTERS OF GALAXIES

Clusters of galaxies are sets of 100s to 1000s of galaxies and constitute the largest gravitationally bound objects in the universe. There many tens of thousands of known clusters up to redshift $z \sim 2$.

Coma



$N > 1000$
 $d \simeq 100 \text{ Mpc}$ ($z \simeq 0.023$)

Virgo



$N > 1300$
 $d \simeq 18 \text{ Mpc}$ ($z \simeq 0.0038$)

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Coma



The actual discovery of dark matter is usually attributed to Fritz Zwicky, who discovered in 1933 an anomalously large mass-to-light ratio in the Coma cluster.

Die Rotverschiebung von extragalaktischen Nebeln

von F. Zwicky.

(16. II. 33.)

Inhaltsangabe. Diese Arbeit gibt eine Darstellung der wesentlichsten Merkmale extragalaktischer Nebel, sowie der Methoden, welche zur Erforschung derselben gedient haben. Insbesondere wird die sog. Rotverschiebung extragalaktischer Nebel eingehend diskutiert. Verschiedene Theorien, welche zur Erklärung dieses wichtigen Phänomens aufgestellt worden sind, werden kurz besprochen. Schliesslich wird angedeutet, inwiefern die Rotverschiebung für das Studium der durchdringenden Strahlung von Wichtigkeit zu werden verspricht.



Zwicky applied the virial theorem to the observed motion of the galaxies in Coma to estimate the total mass of the cluster. Remember the virial theorem?

I. CLUSTERS OF GALAXIES

The virial theorem (in galactic dynamics) states that in an isolated self-gravitating system in equilibrium the time-averaged total kinetic energy T and the time-averaged total potential energy U are related as

$$\boxed{T = -\frac{1}{2}U} \quad T = \frac{1}{2} \sum_i m_i v_i^2 = \frac{1}{2} M \langle v^2 \rangle \quad , \quad U = -\frac{1}{2} \sum_{i,j \neq i} \frac{G m_i m_j}{|\mathbf{x}_i - \mathbf{x}_j|} = -GM^2 \langle \frac{1}{r} \rangle$$

Problem #1: Convince yourself that a typical cluster of galaxies is in equilibrium.

A typical cluster has size $R \sim 1$ Mpc and the galaxies therein move with random velocities $v \sim 10^3$ km/s, so the crossing time is

$$t_{\text{crossing}} = R/v \sim 10^9 \text{ yr} < t_{\text{cluster}} \sim 10^{10} \text{ yr}.$$

Therefore, a galaxy in a cluster has had time to cross the system several times so that a equilibrium configuration should have been established.

Problem #2: (a) Compute U for a homogeneous sphere of mass M and radius R .

In a system with spherical symmetry the potential felt by a particle of mass m_i at radius r_i is simply $U_i = -GM(< r_i)m_i/r_i$. So the total gravitational potential reads

$$\begin{aligned} U &= - \sum_i \frac{GM(< r_i)m_i}{r_i} \rightarrow - \int d^3r \rho(r) \frac{GM(< r)}{r} = - \int_0^R dr 4\pi r^2 \rho \frac{G}{r} \left(\frac{4\pi}{3} r^3 \rho \right) \\ &= -3G \left(\frac{4\pi}{3} \rho \right)^2 \int_0^R dr r^4 \Leftrightarrow U = -\frac{3}{5} \frac{GM^2}{R} \end{aligned}$$

(b) Use the virial theorem to express $\langle v^2 \rangle$ in terms of M and R .

$$\boxed{\langle v^2 \rangle = \frac{3}{5} \frac{GM}{R}}$$

I. CLUSTERS OF GALAXIES

Let us return to 1933... With 800 known galaxies in Coma and a typical luminous mass of $10^9 M_{\odot}$ for each galaxy, Zwicky computed the mass of the system and the expected velocity dispersion through the virial theorem.

förmig über den Raum verteilt ist. Der Haufen besitzt einen Radius R von ca. einer Million Lichtjahren (gleich 10^{24} cm) und enthält 800 individuelle Nebel von je einer Masse entsprechend 10^9 Sonnenmassen. Die Gesamtmasse M des Systems ist deshalb

$$M \sim 800 \times 10^9 \times 2 \times 10^{23} = 1.6 \times 10^{45} \text{ gr.} \quad (5)$$

Daraus folgt für die totale potentielle Energie Ω :

$$\Omega = -\frac{3}{5} \Gamma \frac{M^2}{R} \quad (6)$$

Γ = Gravitationskonstante

oder
$$\bar{\epsilon}_p = \Omega/M \sim -64 \times 10^{12} \text{ cm}^2 \text{ sek}^{-2} \quad (7)$$

und weiter

$$\bar{\epsilon}_k = \bar{v}^2/2 = -\epsilon_p/2 = 32 \times 10^{12} \text{ cm}^2 \text{ sek}^{-2} \quad (8)$$

$$(\bar{v}^2)^{1/2} = 80 \text{ km/sek.}$$

Scheinbare Geschwindigkeiten im Comahaufen.

$v = 8500$ km/sek	6900 km/sek
7900	6700
7600	6600
7000	5100 (?)

So he concluded:

Um, wie beobachtet, einen mittleren Dopplereffekt von 1000 km/sek oder mehr zu erhalten, müsste also die mittlere Dichte im Comasystem mindestens 400 mal grösser sein als die auf Grund von Beobachtungen an leuchtender Materie abgeleitete¹⁾. Falls sich dies bewahrheiten sollte, würde sich also das überraschende Resultat ergeben, dass dunkle Materie in sehr viel grösserer Dichte vorhanden ist als leuchtende Materie.

I. CLUSTERS OF GALAXIES

In 1937 Zwicky turns the argument around.

$$\mathcal{M} > \frac{3R\bar{v}_0^2}{5\Gamma^2}.$$

From the observations of the Coma cluster so far available we have, approximately,⁵

$$\bar{v}_0^2 = 5 \times 10^{15} \text{cm}^2 \text{sec}^{-2}.$$

Combining (33) and (34), we find

$$\mathcal{M} > 9 \times 10^{46} \text{gr}.$$

The Coma cluster contains about one thousand nebulae. The average mass of one of these nebulae is therefore

$$\bar{M} > 9 \times 10^{43} \text{gr} = 4.5 \times 10^{10} M_{\odot}.$$

Inasmuch as we have introduced at every step of our argument inequalities which tend to depress the final value of the mass \mathcal{M} , the foregoing value (36) should be considered as the lowest estimate for the average mass of nebulae in the Coma cluster. This result is somewhat unexpected, in view of the fact that the luminosity of an average nebula is equal to that of about 8.5×10^7 suns. According to (36), the conversion factor γ from luminosity to mass for nebulae in the Coma cluster would be of the order

$$\gamma = 500,$$

as compared with about $\gamma' = 3$ for the local Kapteyn stellar system.



I. CLUSTERS OF GALAXIES

In the meantime, in 1936, Sinclair Smith finds a similar result for the Virgo cluster.

THE MASS OF THE VIRGO CLUSTER*

SINCLAIR SMITH

ABSTRACT

The lists of radial velocities now include results for thirty-two members of the Virgo Cluster, thus giving for the first time sufficient data to determine some of the physical characteristics of a cluster of nebulae.



DR. SINCLAIR SMITH 1899-38

move in circular orbits with a speed of 1500 km/sec. Hence we can write either

$$m = \frac{v^2 r}{2G} \quad \text{or} \quad \frac{v^2 r}{G}$$

the difference being of small importance.

Taking the circular orbit form, and assuming for radius of the cluster 2×10^5 parsecs (i.e., 0.1 times its distance), we find for the mass

$$2 \times 10^{47} \text{ grams} \quad \text{or} \quad 10^{14} \odot .$$

Assuming 500 nebulae in the cluster and no internebular material, we find for the mean mass of a single nebula

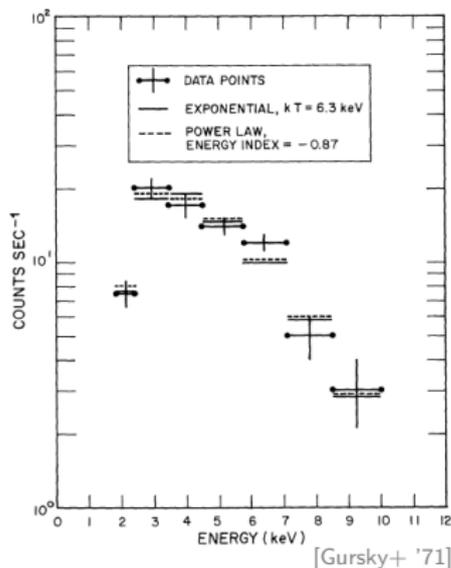
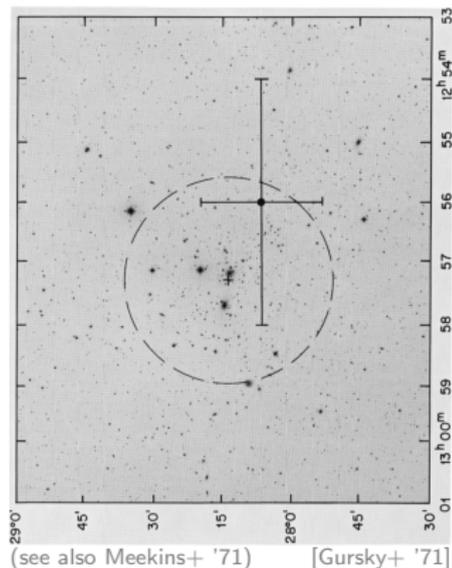
$$4 \times 10^{44} \text{ grams} \quad \text{or} \quad 2 \times 10^{11} \odot .$$

This value is some two hundred times Hubble's³ estimate of $10^9 \odot$ for the mass of an average nebula. The cause of the discrepancy is not clear. In the determination of the mass of the cluster, the only

At the time it was not known, however, that the intergalactic space in clusters is filled with heaps of gas...

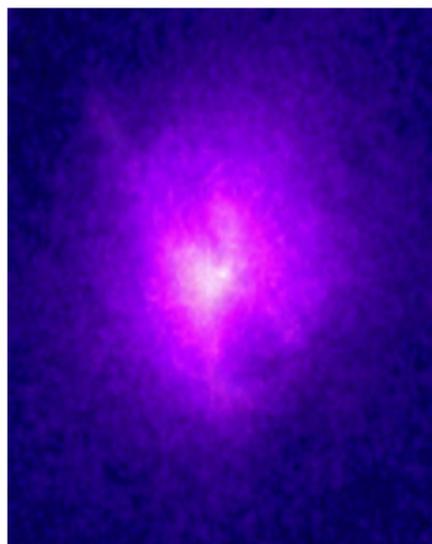
I. CLUSTERS OF GALAXIES

Soon after the launch of the Uhuru satellite in 1970, it was realised that many clusters shine in X rays.

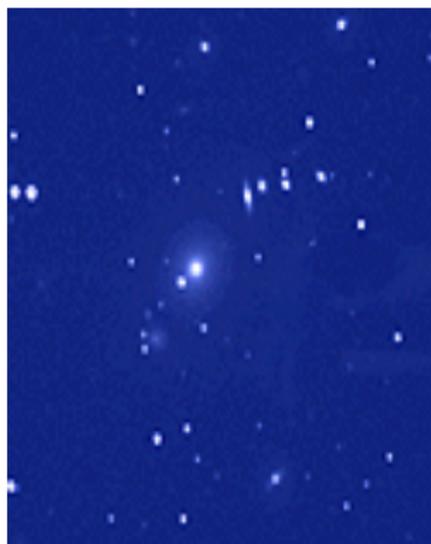


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[NASA/CXC/SAO '99, X-ray]



[La Palma/B. McNamara, optical]

This emission was readily interpreted as bremsstrahlung from hot intracluster gas. By measuring the bremsstrahlung emission,

$$\kappa_\nu = \rho_g^2 \exp(-\nu/T_g) \quad ,$$

across the cluster, it is possible to infer both the gas density $\rho_g(r)$ and its temperature $T_g(r)$. From $\rho_g(r)$ we can also compute the gas enclosed mass $M_g(< r)$.

I. CLUSTERS OF GALAXIES

Problem #3: Write down the hydrostatic equilibrium condition for a perfect gas in a spherical cluster. Solve for the total mass enclosed.

If the gas is in hydrostatic equilibrium, its pressure gradient exactly equals the gravitational force exerted on it. For a spherical cluster, we have

$$\frac{d\rho_g}{dr} = -\rho_g \frac{GM_{\text{tot}}(< r)}{r^2}.$$

For a perfect gas, $p_g = \rho_g k_B T_g / \mu m_H$ and so

$$\frac{k_B}{\mu m_H} \left(\frac{d\rho_g}{dr} T_g + \rho_g \frac{dT_g}{dr} \right) = -\frac{GM_{\text{tot}}(< r)\rho_g}{r^2}$$

$$M_{\text{tot}}(< r) = -\frac{k_B T_g r}{G\mu m_H} \left(\frac{d \ln \rho_g}{d \ln r} + \frac{d \ln T_g}{d \ln r} \right).$$

With observations typically indicating $k_B T_g(r) \simeq 10 \text{ keV} = \text{const}$ and $\rho_g \propto r^{-2}$,

$$M_{\text{tot}}(< r) \sim 10^{15} M_{\odot} \left(\frac{T_g}{10 \text{ keV}} \right) \left(\frac{r}{1 \text{ Mpc}} \right) \left(\frac{0.6}{\mu} \right),$$

in agreement with the virial estimates.

I. CLUSTERS OF GALAXIES

There is yet another method to trace the total mass of clusters: look for the deflection of light from background sources behind the clusters. This was already proposed by Zwicky in his 1937 paper.

ABSTRACT

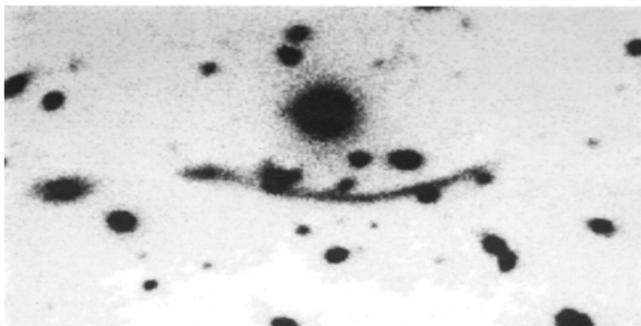
Present estimates of the masses of nebulae are based on observations of the *luminosities* and *internal rotations* of nebulae. It is shown that both these methods are unreliable; that from the observed luminosities of extragalactic systems only lower limits for the values of their masses can be obtained (sec. i), and that from internal rotations alone no determination of the masses of nebulae is possible (sec. ii). The observed internal motions of nebulae can be understood on the basis of a simple mechanical model, some properties of which are discussed. The essential feature is a central core whose internal *viscosity* due to the gravitational interactions of its component masses is so high as to cause it to rotate like a solid body.

In sections iii, iv, and v three new methods for the determination of nebular masses are discussed, each of which makes use of a different fundamental principle of physics.

Method iii is based on the *virial theorem* of classical mechanics. The application of this theorem to the Coma cluster leads to a minimum value $\bar{M} = 4.5 \times 10^{15} M_{\odot}$ for the average mass of its member nebulae.

Method iv calls for the observation among nebulae of certain *gravitational lens* effects.

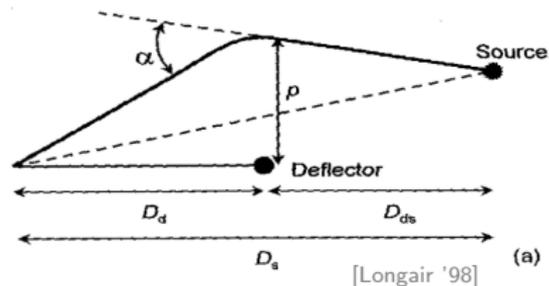
The effect was not observed until 1986/7.



(see also Lynds & Petrosian '86) [Soucail+ '87]

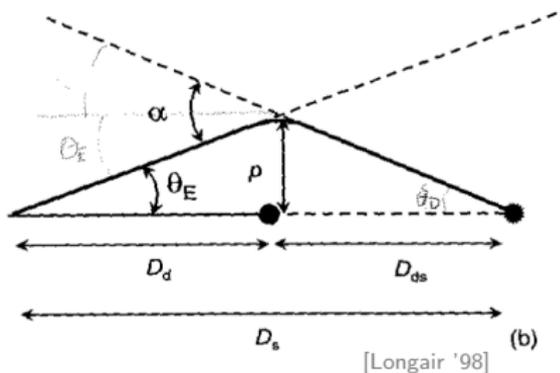
I. CLUSTERS OF GALAXIES

Let us now derive the basic lensing formulae.



In general relativity, the light from a background source is bent due to a point mass M at impact parameter ρ by an angle

$$\alpha = \frac{4GM}{pc^2}.$$



Problem #4: In a colinear lensing configuration, the observed image is the so-called Einstein ring, whose angular radius is defined as the Einstein angle, θ_E . Compute θ_E .

$$\theta_E \simeq \rho/D_d \quad \theta_0 \simeq \rho/D_{ds} \simeq \theta_E D_d/D_{ds}$$

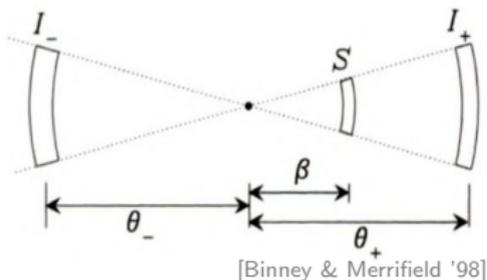
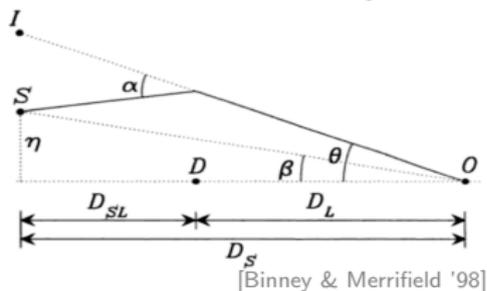
$$\alpha = \theta_E + \theta_0 \Leftrightarrow \theta_E = \frac{\alpha}{1 + D_d/D_{ds}} = \alpha \frac{D_{ds}}{D_s}$$

$$\theta_E^2 = \frac{4GM}{c^2} \frac{1 - D_d/D_s}{D_d}$$

For a cluster with $M = 10^{15} M_\odot$ at $D_d = 100$ Mpc, cosmological sources are lensed with $\theta_E \simeq 300''$.

I. CLUSTERS OF GALAXIES

We now return to a generic lensing configuration.



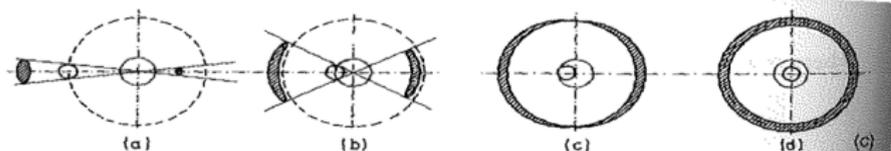
Because the angles involved are small,

$$\eta \simeq D_S \beta \simeq D_S \theta - D_{S1} \alpha \quad \alpha = \frac{4GM}{pc^2} \quad p \simeq (D_1 \theta)$$

$$\beta \simeq \theta - \frac{\theta_E^2}{\theta}$$

This is the so-called lensing equation, whose solutions are

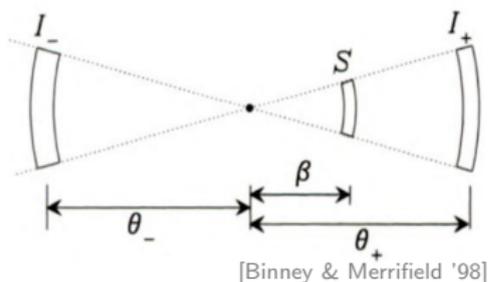
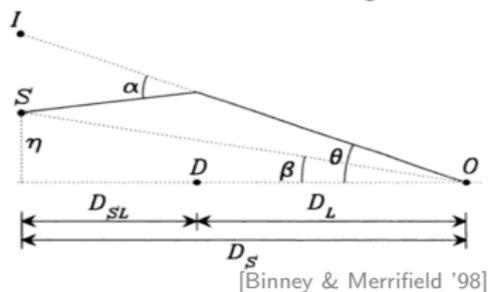
$$\theta_{\pm} = \frac{1}{2} \left(\beta \pm \sqrt{\beta^2 + 4\theta_E^2} \right)$$



[Longair '98]

I. CLUSTERS OF GALAXIES

We now return to a generic lensing configuration.



$$\alpha = \frac{4GM(< p)}{pc^2}$$

$$\theta_{\pm} \rightarrow p_{\pm}, (\beta, \theta_E) \rightarrow \alpha \rightarrow M(< p)$$

The total mass of clusters measured by lensing agrees to within 10% with the virial estimates.

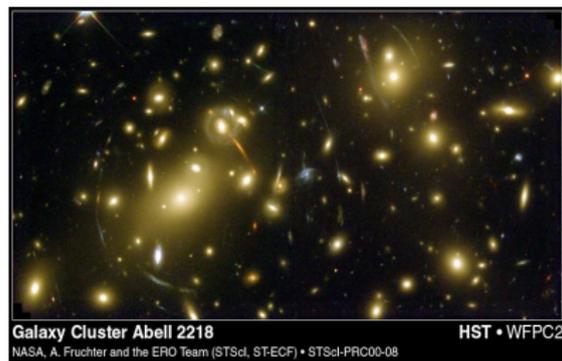
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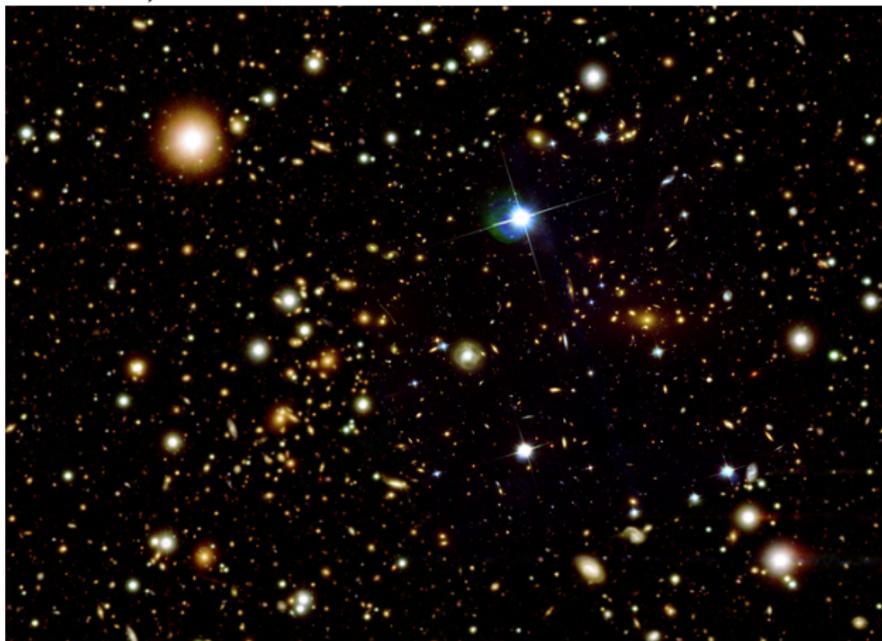
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$$\theta_{\pm} = \frac{1}{2} \left(\beta \pm \sqrt{\beta^2 + 4\theta_E^2} \right)$$



I. CLUSTERS OF GALAXIES

A beautiful example of all the phenomena discussed above is the so-called bullet cluster (1E 0657-558).



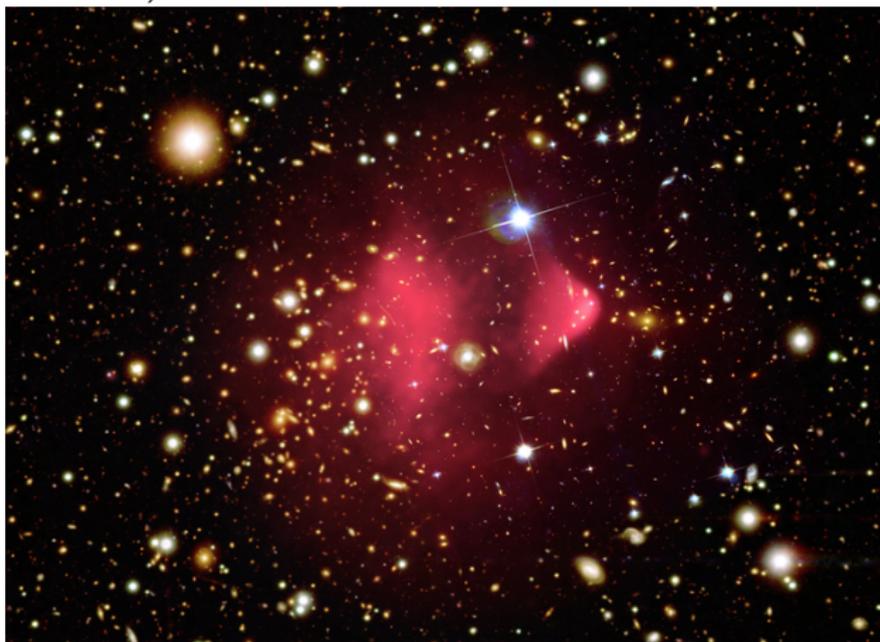
[Optical: NASA/STScI; Magellan/U.Arizona/D.Clowe et al.]

We learn that:

1. the total gravitational potential does not follow the baryons (galaxies + gas); and
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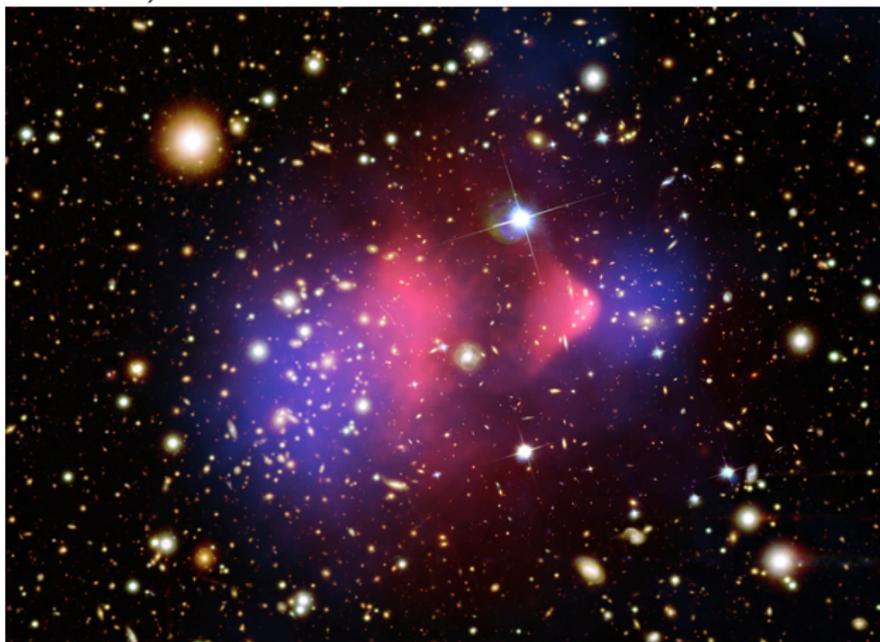
[X-ray: NASA/CXC/CfA/M.Markevitch et al.]

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[Lensing: NASA/STScI; ESO WFI; Magellan/U.Arizona/D.Clowe et al.]

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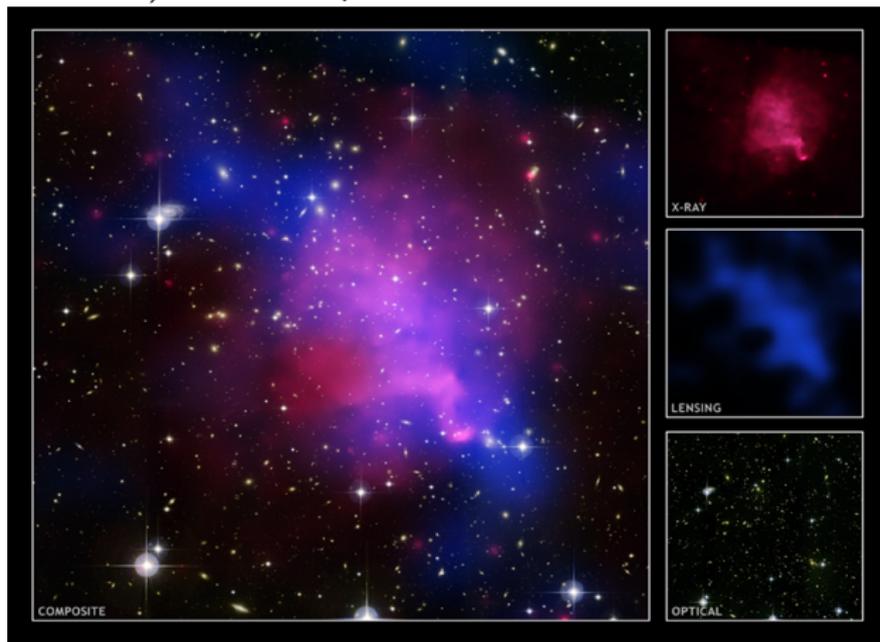


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[Optical/Lensing: CFHT/UVic./A.Mahdavi et al.]
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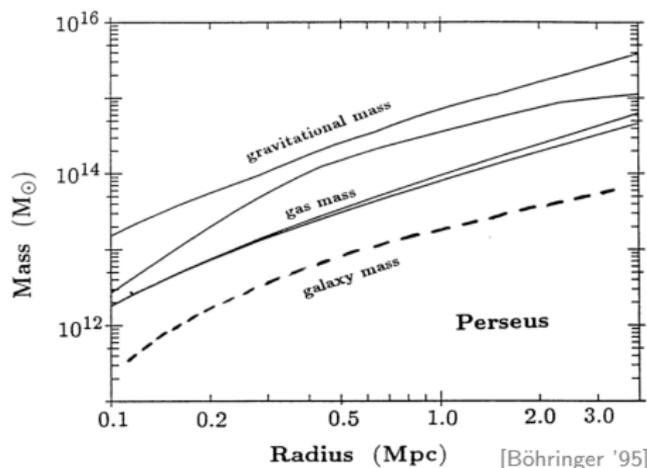
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I. CLUSTERS OF GALAXIES

Let us summarise then. The typical mass budget in clusters of galaxies reads:

$$M_{\text{tot}} = 5 \times 10^{14} - 5 \times 10^{15} M_{\odot}$$

5% galaxies
10-30% hot gas
65-85% dark matter



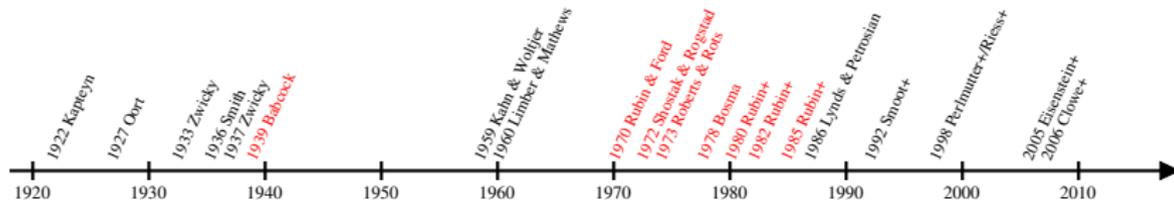
Back in the 1930s, Zwicky (and Smith) did not know about the presence of a lot of gas in clusters, but even this is not enough to explain the total gravitational potential of these objects. Dark matter is still needed in ubiquitous amounts.

Other evidence for dark matter from systems of galaxies:

- local group $M/L \sim 100 M_{\odot}/L_{\odot}$ (Kahn & Woltjer '59)
- binary galaxies $M/L \sim 300 M_{\odot}/L_{\odot}$ (Holmberg '37)
- groups of galaxies $M/L \sim 100 - 600 M_{\odot}/L_{\odot}$ (Limber & Matthews '60)

I. BRIEF HISTORICAL PERSPECTIVE

The evidence for dark matter gradually mounted throughout the 20th century, and by now we are convinced that our universe is filled with dark matter at various scales. Note that all evidence for dark matter is of gravitational origin; non-gravitational evidence is yet to be discovered.



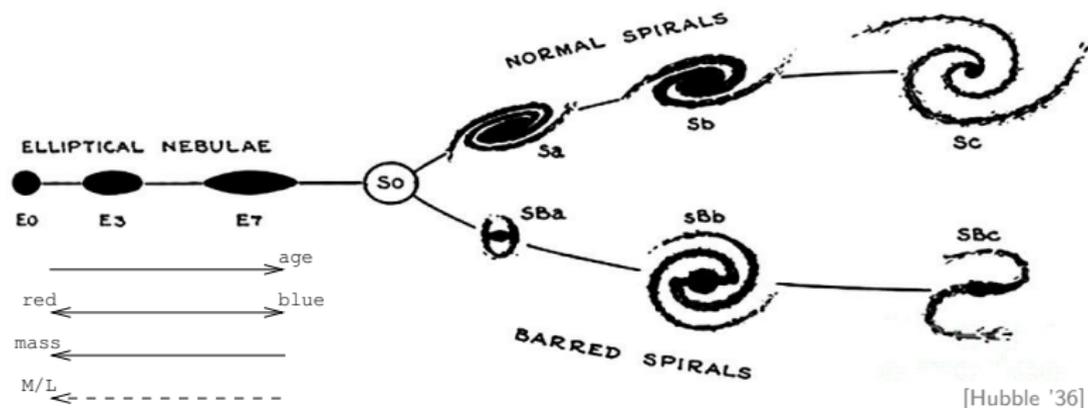
We can identify four key revolutions in the history of dark matter:

1. dark matter is first mentioned by Kapteyn;
2. dark matter is found in the Coma cluster by Zwicky;
3. dark matter is found in spiral galaxies by Rubin & Ford; and
4. dark matter is found at cosmological scales by many.



I. GALAXIES

It was Hubble that set in 1936 the main scheme for the classification of galaxies in the universe.

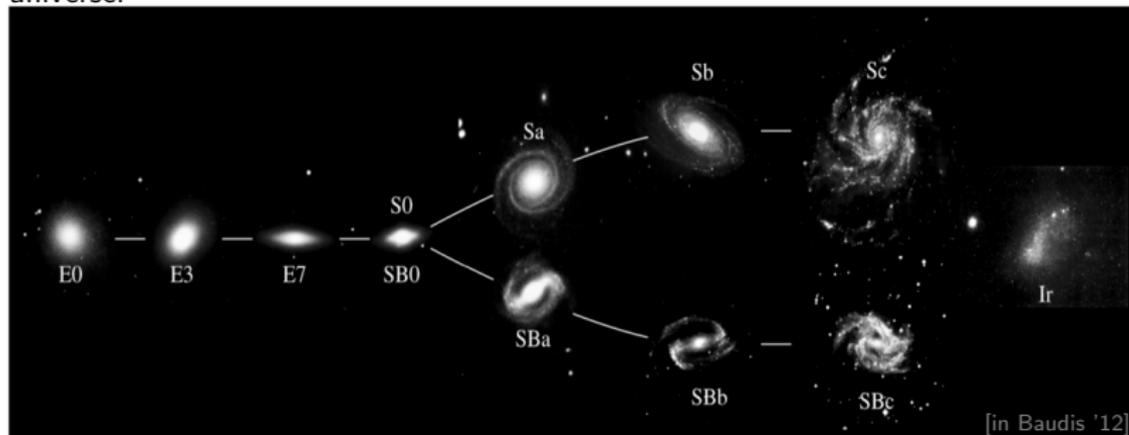


The (revised) Hubble sequence contains:

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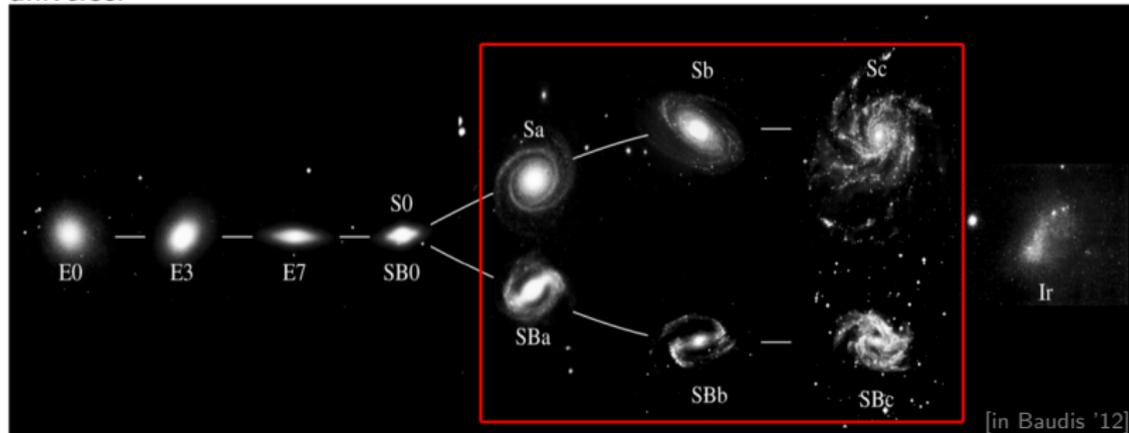
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I. GALAXIES

Let us start the journey in our sister galaxy Andromeda (or M31), the closest spiral.



I. GALAXIES

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I. GALAXIES

In a seminal article from 1939 (no typo there!), Horace Babcock used the Doppler shift of emission and absorption lines to study the rotation of Andromeda.

LICK OBSERVATORY BULLETIN

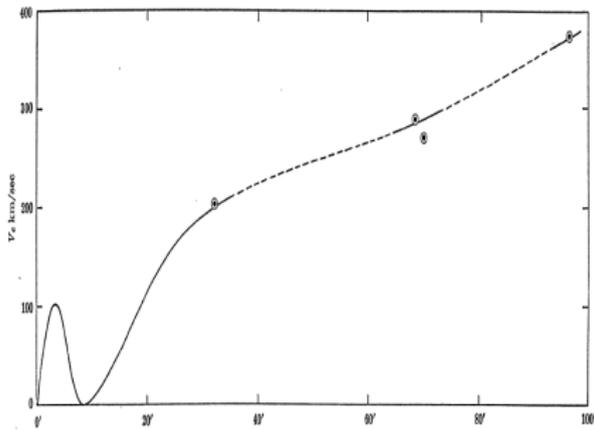
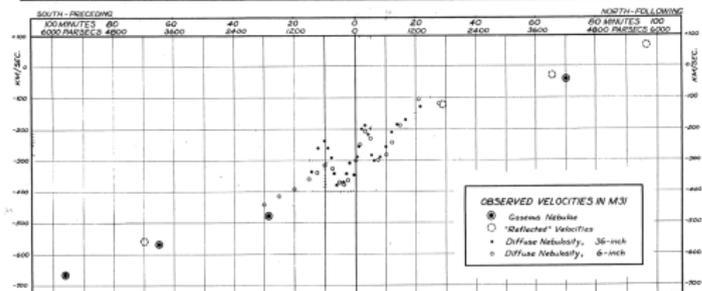
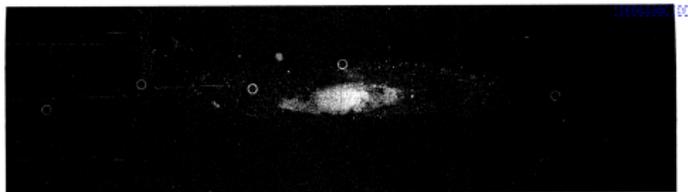
NUMBER 498

THE ROTATION OF THE ANDROMEDA NEBULA*

BY
HORACE W. BABCOCK



He found surprisingly high velocities across the galactic plane...



I. GALAXIES

... and correspondingly large mass-to-light ratios.

Imagine a column of cross-section one square parsec

The coefficients for the mass-luminosity relation given

TABLE 5
MASS-LUMINOSITY RELATIONS IN M31

x (distance from nucleus)	0'	0.5'	1.5'	5.0'	8.0'
Mass of column (\odot)		11000	7900	4100	2200
Volume (cu. psc.)		5400	5300	4500	2400
Mass density (\odot /cu. psc.)		2.1	1.5	0.9	0.9
Log I (Redman and Shirley)	(2.00)	1.29	0.10	9.44	9.00
I	(100)	19.5	1.26	0.276	0.100
Luminosity density (\odot /cu. psc.)		1.25	0.0827	0.021	0.0144
M/L	0.001	1.6	18	43	62

(Hubble)

in the table evidently indicate little more than orders of magnitude. The mass densities are especially uncertain in the central core, where they are probably too small, so that the corresponding mass-luminosity coefficients near the nucleus may be considered minimum values. Nevertheless, the great range in the calculated ratio of mass to luminosity in proceeding outward from the nucleus suggests that absorption plays a very important rôle in the outer portions of the spiral, or, perhaps, that new dynamical considerations are required, which will permit of a smaller relative mass in the outer parts.

strate that, in a wide region around the sun, circular velocities of the stars decrease with distance from the center.

The Andromeda Nebula and the Galaxy have many well-known features in common, but one outstanding discrepancy between the two systems has hitherto been in their diameters. The spiral arms of M31 can hardly be traced to a radius of more than 1.6 or 6 kiloparsecs. Beyond this radius, no stars, comparable to the brighter stars in the vicinity of the sun, have been reported, although Hubble has discovered some outlying globular

However, a confirmation of this hint had to wait for over 3 decades.

I. GALAXIES

In 1970, Vera Rubin and Kent Ford measured $H\alpha$ shifts across Andromeda.

THE ASTROPHYSICAL JOURNAL, Vol. 159, February 1970
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ROTATION OF THE ANDROMEDA NEBULA FROM A SPECTROSCOPIC SURVEY OF EMISSION REGIONS*

VERA C. RUBIN† AND W. KENT FORD, JR.†
Department of Terrestrial Magnetism, Carnegie Institution of Washington and
Lowell Observatory, and Kitt Peak National Observatory‡
Received 1969 July 7; revised 1969 August 21



They found a flat rotation curve up to 24 kpc, way beyond the luminous matter:

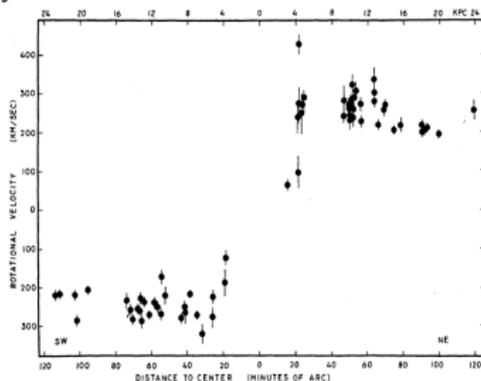
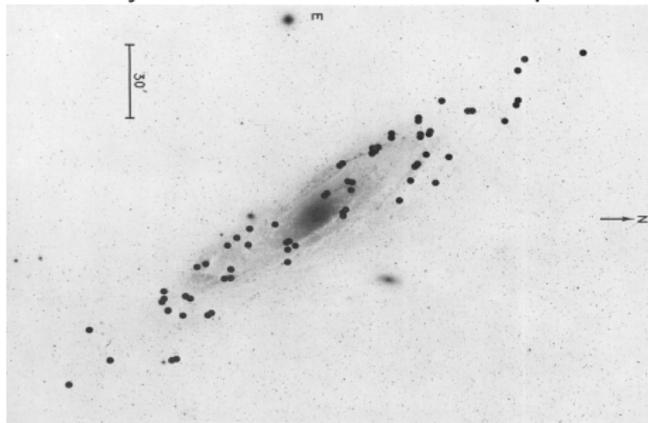


FIG. 3.—Rotational velocities for sixty-seven emission regions in M31, as a function of distance from the center. Error bars indicate average error of rotational velocities.

This paper was the turning point for the dark matter paradigm. Soon afterwards, the same result was confirmed by 21 cm line observations (Rogstad & Shostak '72).

I. GALAXIES

The 1970s and 1980s witnessed a steady growth in the number of spirals with flat rotation curves. Several authors contributed to this effort...

THE ASTROPHYSICAL JOURNAL, 238:471-487, 1980 June 1

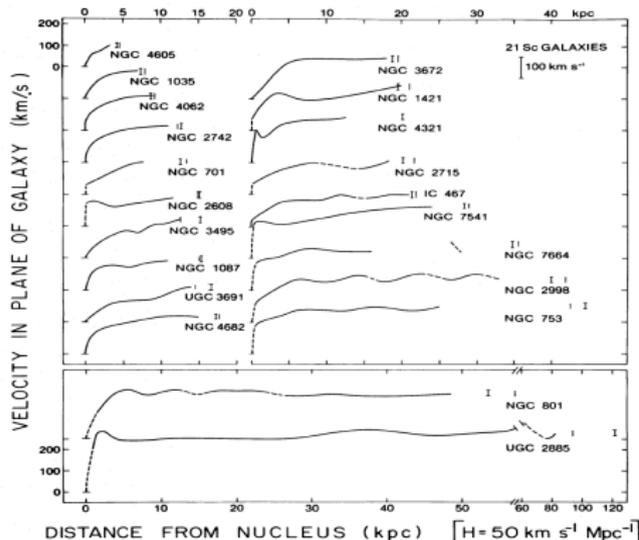
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ROTATIONAL PROPERTIES OF 21 Sc GALAXIES WITH A LARGE RANGE OF LUMINOSITIES AND RADII, FROM NGC 4605 ($R = 4$ kpc) TO UGC 2885 ($R = 122$ kpc)

VERA C. RUBIN,^{1,2} W. KENT FORD, JR.,¹ AND NORBERT THONNARD

Department of Terrestrial Magnetism, Carnegie Institution of Washington

Received 1979 October 11; accepted 1979 November 29



VIII. DISCUSSION AND CONCLUSIONS

We have obtained spectra and determined rotation curves to the faint outer limits of 21 Sc galaxies of high inclination. The galaxies span a range in luminosity from 3×10^9 to $2 \times 10^{11} L_{\odot}$, a range in mass from 10^{10} to $2 \times 10^{12} M_{\odot}$, and a range in radius from 4 to 122 kpc. In general, velocities are obtained over 83% of the optical image (defined by $25 \text{ mag arcsec}^{-2}$), a greater distance than previously observed. The major conclusions are intended to apply only to Sc galaxies.

1. Most galaxies exhibit rising rotational velocities at the last measured velocity; only for the very largest galaxies are the rotation curves flat. Thus the smallest Sc's (i.e., lowest luminosity) exhibit the same lack of a Keplerian velocity decrease at large R as do the high-luminosity spirals. This form for the rotation curves implies that the mass is not centrally condensed, but that significant mass is located at large R . The integral mass is increasing at least as fast as R . The mass is not converging to a limiting mass at the edge of the optical image. **The conclusion is inescapable that non-luminous matter exists beyond the optical galaxy.**

Inescapable? Why is that so?

I. GALAXIES

Let us go one step back to Newton's laws. Consider the motion of a test particle under the influence of a point mass (e.g., the Sun-Earth system). The gravitational potential per unit mass reads

$$\phi = -\frac{GM}{r}.$$

The force per unit mass acted upon the particle in a circular orbit is

$$|F| = \frac{v_c^2}{r} = \frac{d\phi}{dr} = \frac{GM}{r^2} \Rightarrow \boxed{v_c^2 = \frac{GM}{r}} \text{ circular speed}$$

But a galaxy cannot really be approximated by a point-like distribution of matter.

Problem #5: Consider an extended spherical mass distribution with density $\rho(r)$.

(a) Write down the gravitational potential ϕ at radius r .

There are two terms contributing to ϕ , one from matter interior to r and another from matter exterior to r .

$$\phi_{r < r'} = -GM/r' \quad \phi_{r > r'} = -GM/r'$$
$$\phi(r) = -\int_0^r d^3r' \frac{G\rho(r')}{r} - \int_r^\infty d^3r' \frac{G\rho(r')}{r'} = -4\pi G \left(\frac{1}{r} \int_0^r dr' \rho(r') r'^2 + \int_r^\infty dr' \rho(r') r' \right)$$

(b) Calculate the circular speed.

$$v_c^2 = r \frac{d\phi}{dr} = -4\pi G r \left(-\frac{1}{r^2} \int_0^r dr' \rho(r') r'^2 + \frac{1}{r} \rho(r) r^2 - \rho(r) r \right) \Rightarrow \boxed{v_c^2 = \frac{GM(< r)}{r}}$$

Problem #6*: Repeat #5 but for an axisymmetric mass distribution.

Go through Binney & Tremaine, Sec. 2.3.

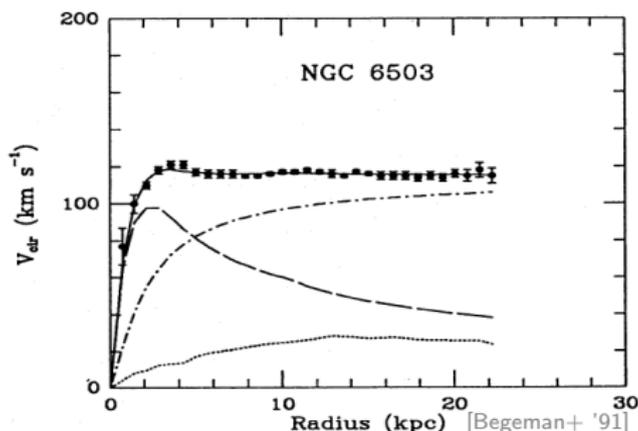
I. GALAXIES

point-like:
$$v_c^2 = \frac{GM}{r}$$

spherical:
$$v_c^2 = \frac{GM(< r)}{r}$$

We learn that:

1. for a point mass or for $M(< r) \simeq \text{const}$, $v_c \propto r^{-1/2}$ (Keplerian fall-off); and
2. $v_c \simeq \text{const}$ implies $M(< r) \propto r$.

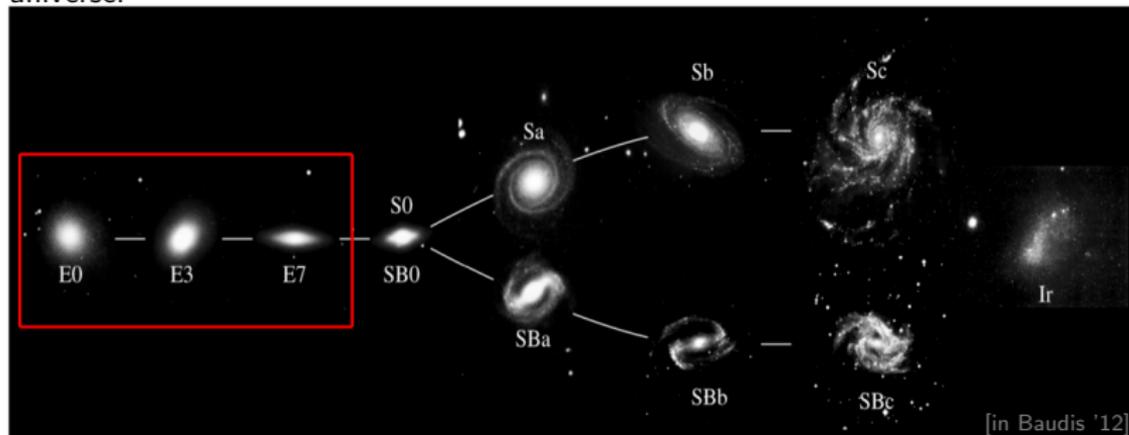


Therefore, the flat rotation curves observed in spiral galaxies indicate $M(< r) \propto r$ in a region where the luminous mass enclosed is barely increasing. This is striking evidence for dark matter.

Note: virtually in no spiral has a Keplerian fall-off been observed, which means we are not able to actually infer the total mass of these systems...

I. GALAXIES

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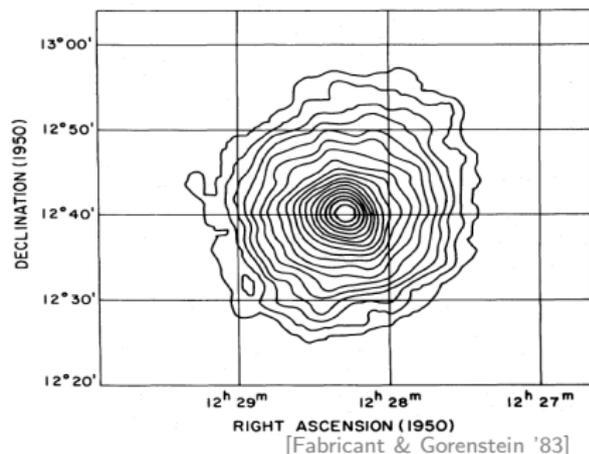
Our Galaxy is an SBb.



I. GALAXIES

Many ellipticals were found to shine in X rays, pretty much like clusters of galaxies. This is evidence for hot gas emitting bremsstrahlung, and the measured density and temperature profiles can be used to track the total mass.

$$M_{\text{tot}}(< r) = -\frac{k_B T_g r}{G\mu m_H} \left(\frac{d \ln \rho_g}{d \ln r} + \frac{d \ln T_g}{d \ln r} \right)$$



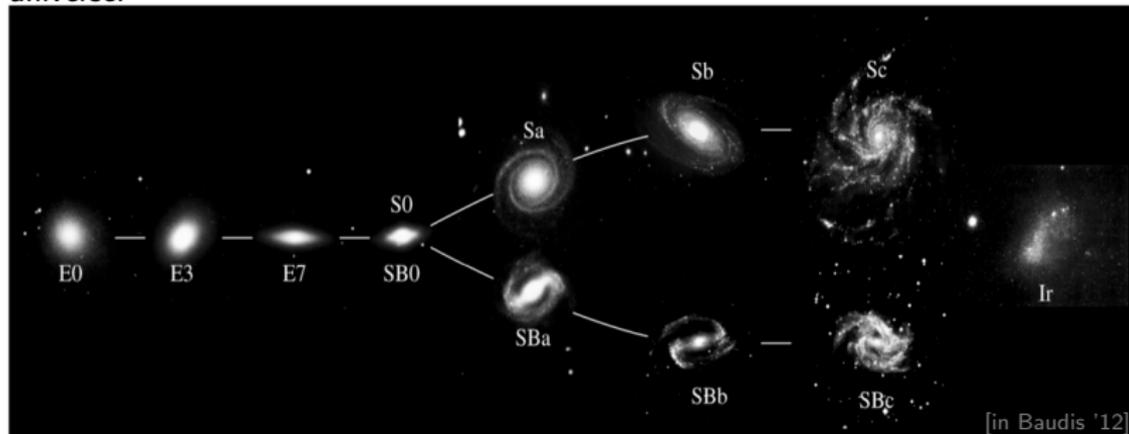
Perhaps the most notable example is M87, where $M_{\text{tot}}(< 300 \text{ kpc}) = 3 \times 10^{13} M_{\odot}$, or $M/L = 750 M_{\odot}/L_{\odot}$. Dark matter contributes $> 99\%$ of the mass budget of this giant elliptical.

Other evidence for dark matter from galaxies: (see e.g. Bertone+ '04)

- kinematics of Magellanic stream
- lensing by ellipticals
- lensing of distant galaxies
- kinematics of dwarf galaxies

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I. OUR OWN GALAXY

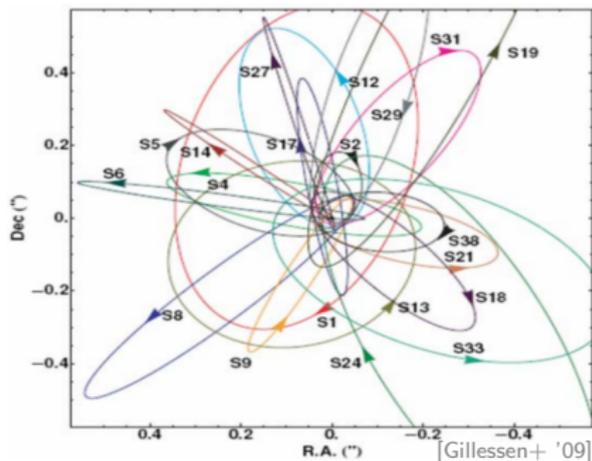
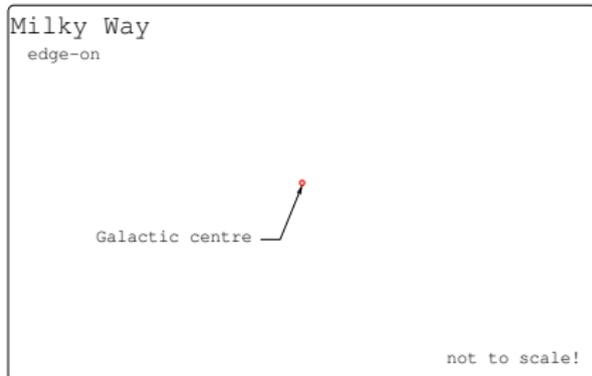
The Milky Way is a complex bound system of stars, gas, dust and dark matter.

I. OUR OWN GALAXY

The Milky Way is a complex bound system of stars, gas, dust and dark matter.

We can identify the following main components:

- supermassive black hole, with mass $4 \times 10^6 M_{\odot}$;

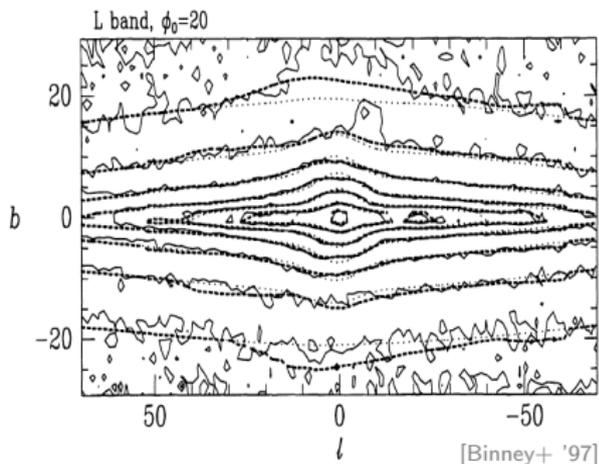
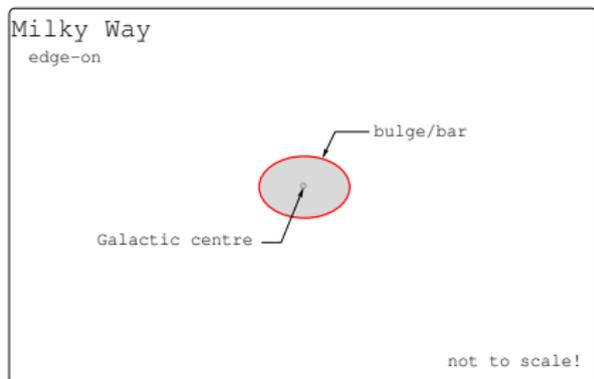


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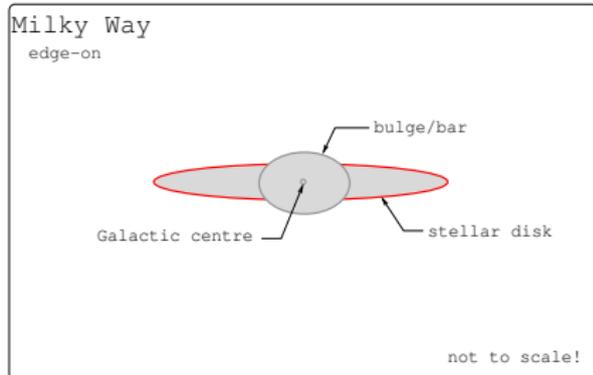
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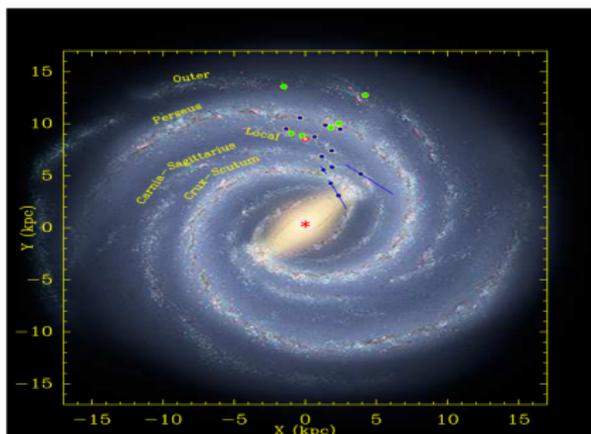
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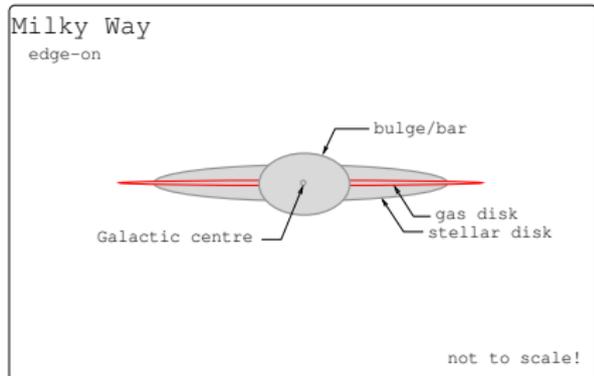
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- stellar disc, decomposed into thin and thick components of scale length 10 kpc and total mass $10^{10} M_{\odot}$ with a marked spiral structure;



[Reid+ '09]

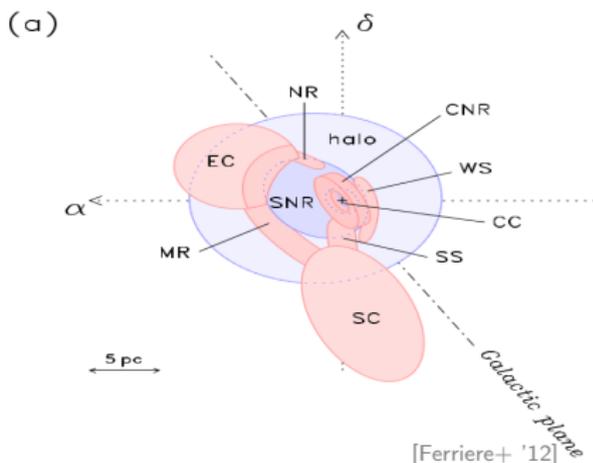
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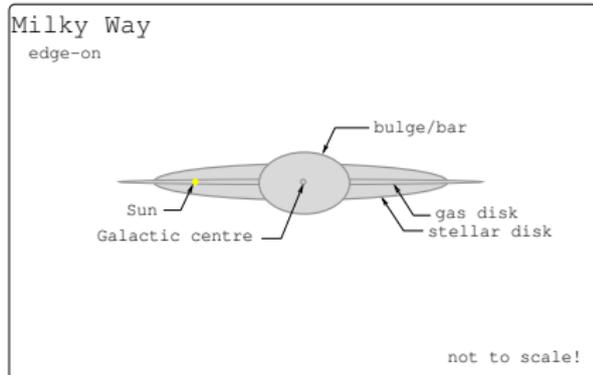
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- gas, in molecular, atomic and ionised phases (mainly H) with a patchy distribution towards the centre and a disk-like structure otherwise; and



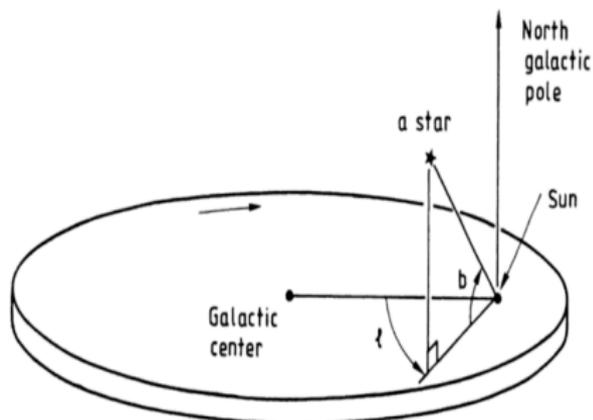
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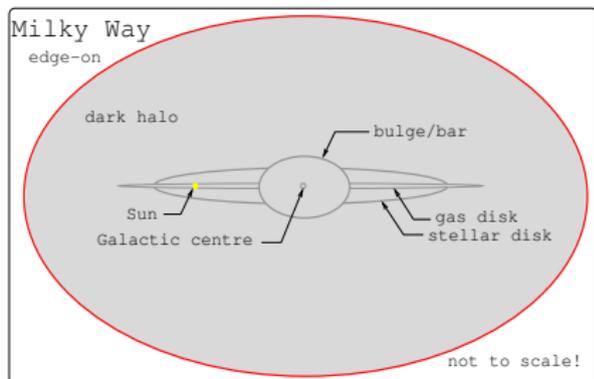


[Binney & Tremaine+ '87]

The Sun is located slightly above the Galactic plane at $R_0 \simeq 8$ kpc from the Galactic centre, in between two major spiral arms, and travels together with the local standard of rest, and about 220 km/s in a roughly circular orbit.

I. OUR OWN GALAXY

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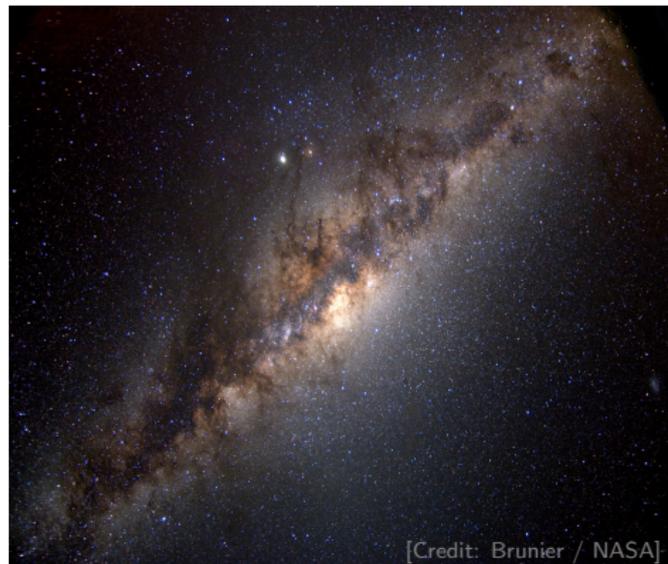
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- gas, in molecular, atomic and ionised phases (mainly H) with a patchy distribution towards the centre and a disk-like structure otherwise; and
- dark matter halo, extending hundreds of kpc.



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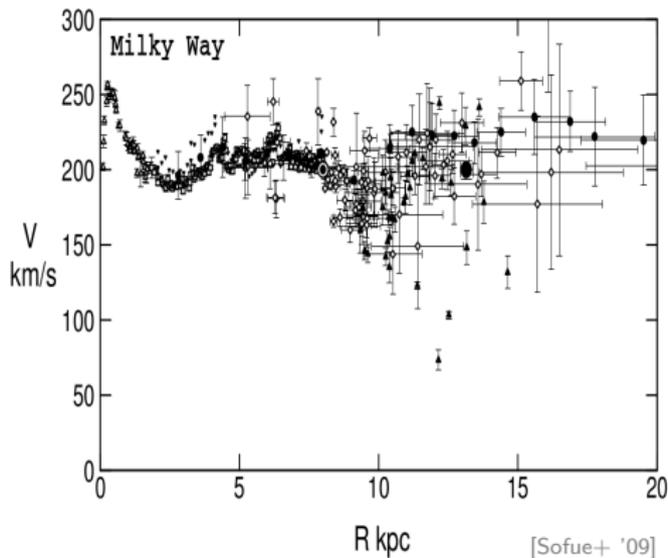
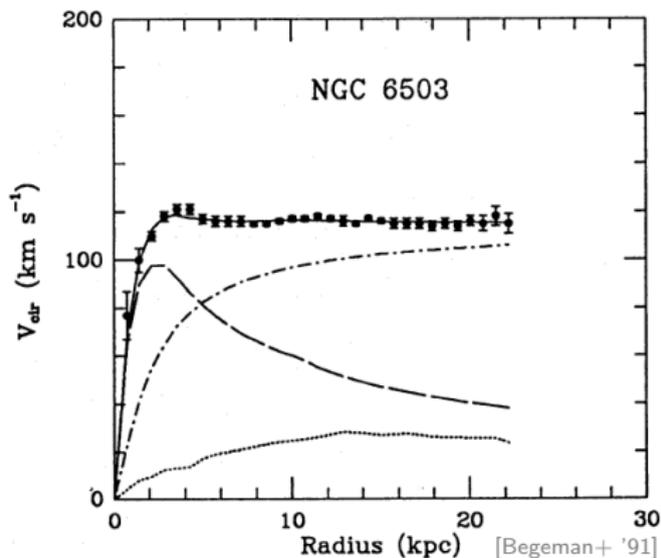
I. OUR OWN GALAXY

As for external galaxies, the prime observable to infer dark matter in our Galaxy is the rotation curve. However, this is challenging to measure given our peculiar position inside the Galaxy.



I. OUR OWN GALAXY

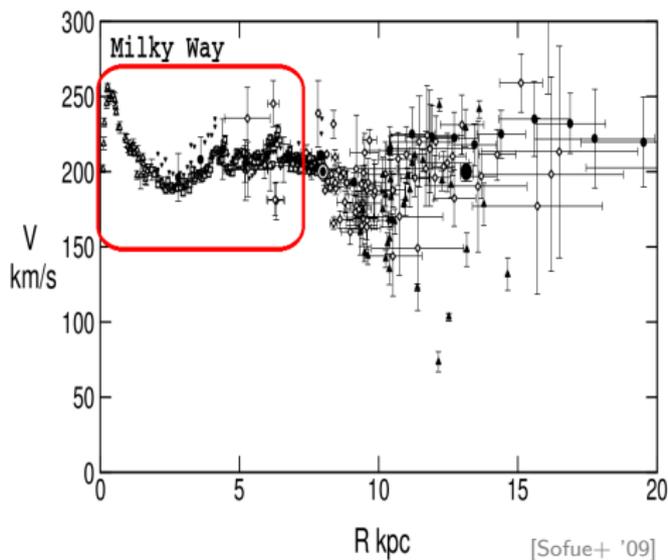
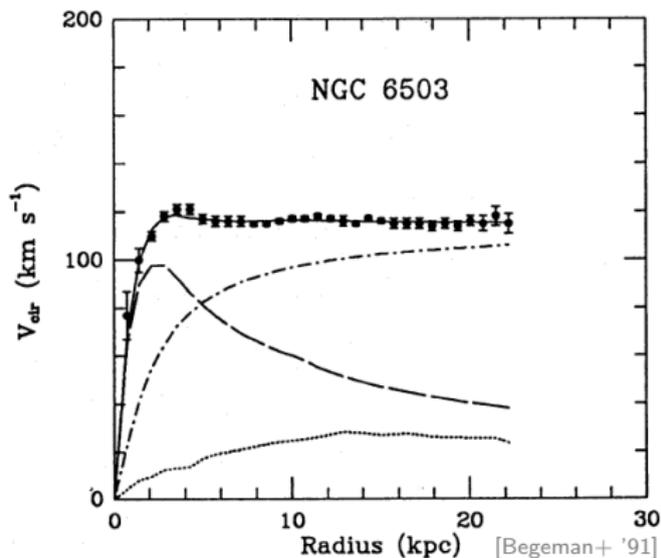
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There are three important kinematic tracers used to constrain the rotation curve across our Galaxy:

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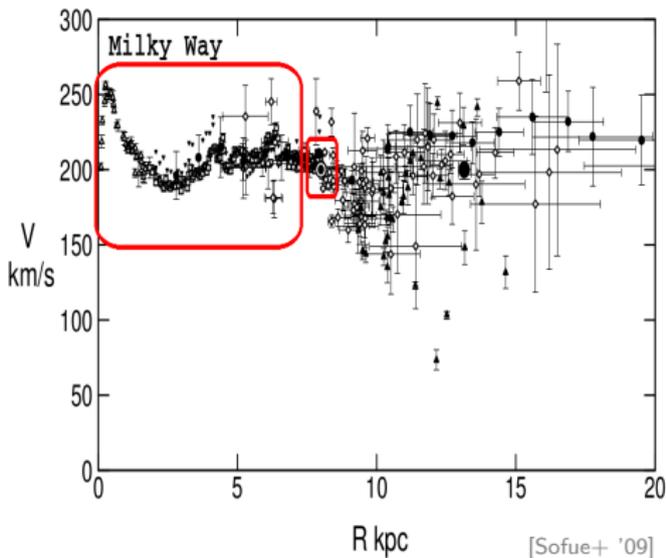
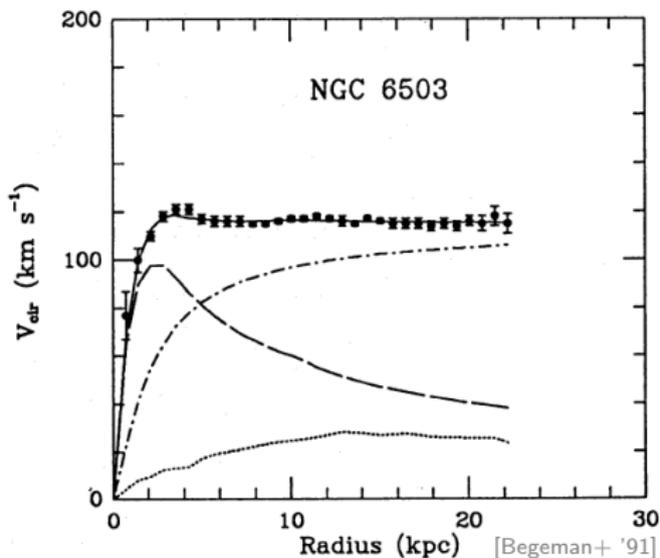


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1. **gas, for the inner Galaxy;**

I. OUR OWN GALAXY

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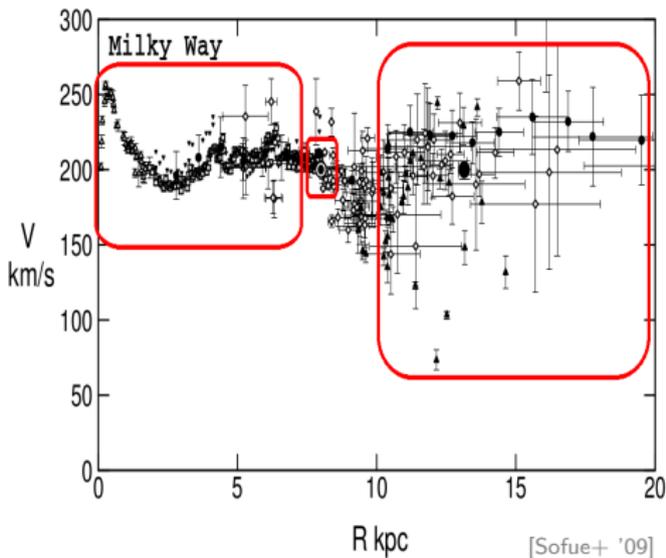
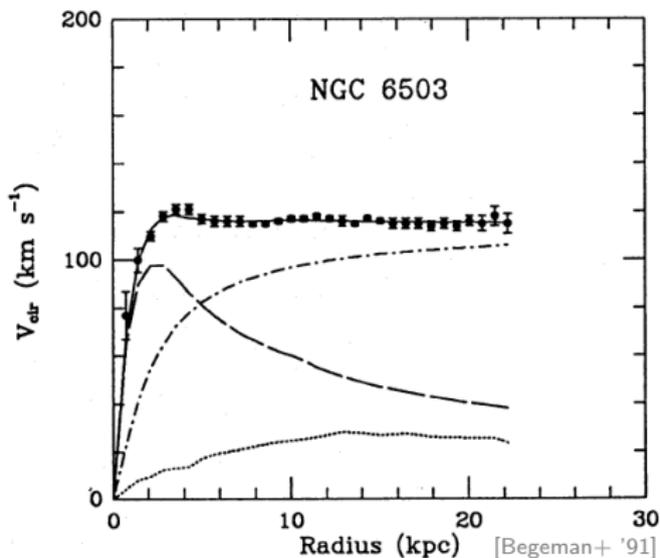


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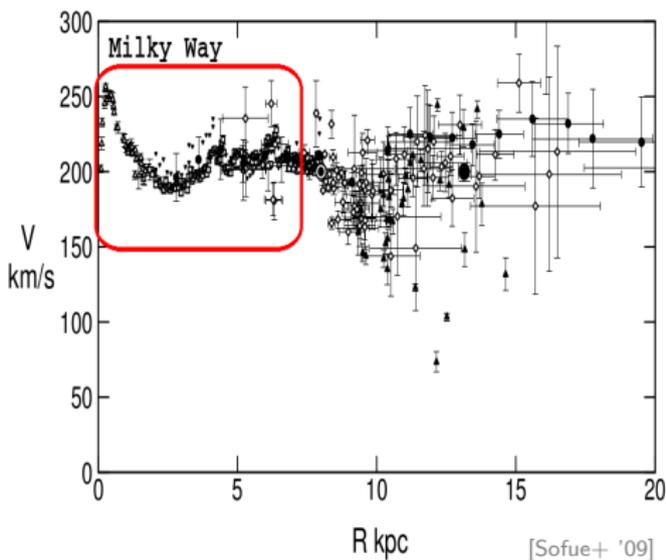
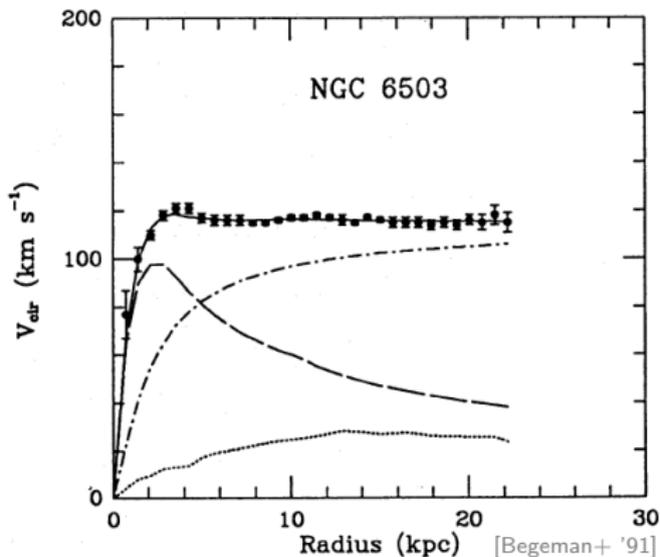


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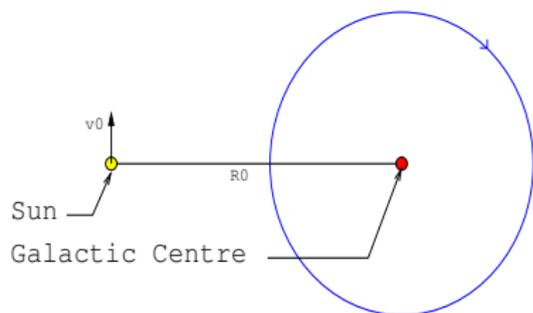


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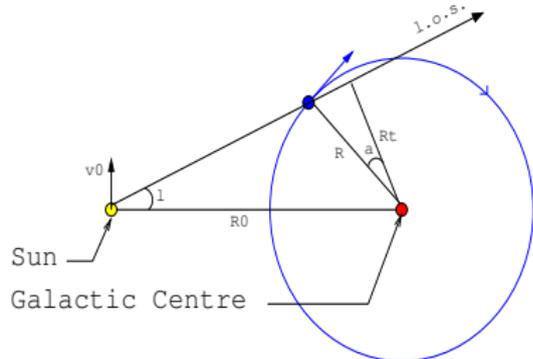
I. OUR OWN GALAXY

Let us start by considering a circular ring of gas of radius R rotating at velocity $v(R)$ about the centre of the Galaxy.



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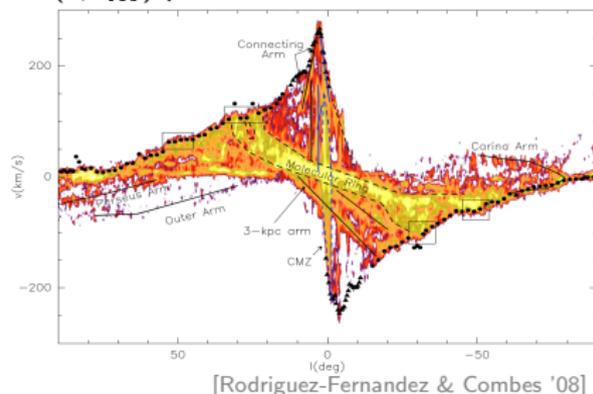
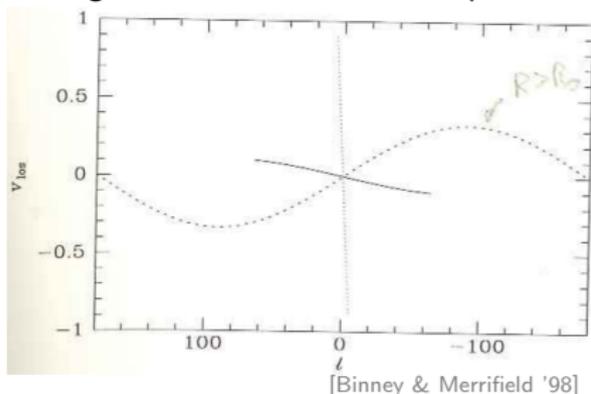


Problem #7: Find the line-of-sight velocity of the gas as observed from the local standard of rest.

$$v_{\text{los}} = v(R) \cos a - v_0 \sin l = v(R) \frac{R_t}{R} - v_0 \sin l$$

$$v_{\text{los}} = \left(v(R) \frac{R_0}{R} - v_0 \right) \sin l$$

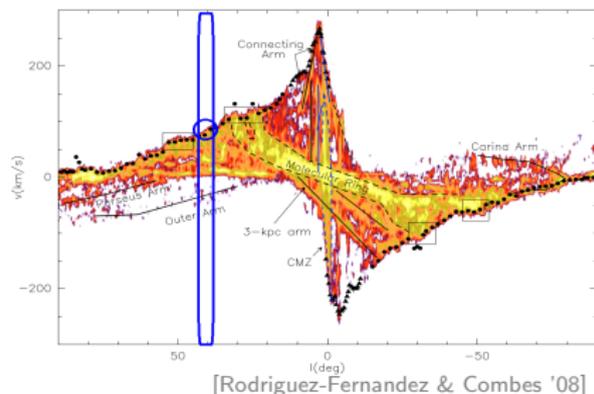
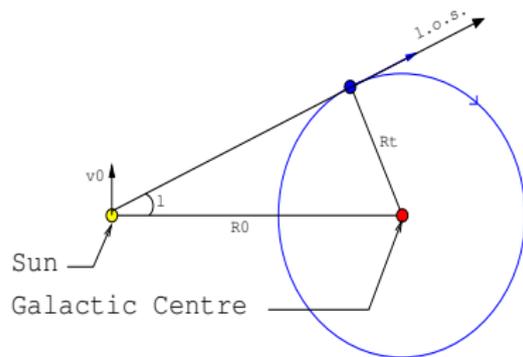
Rings of different radii leave a specific trace in a (l, v_{los}) plot.



These patterns are observed in the HI and CO line maps of the gas in our Galaxy.

I. OUR OWN GALAXY

Now say we measure the Doppler shift along a given line of sight. Then, each ring contributes with a different v_{los} .



However, the highest v_{los} detected, the so-called terminal velocity v_t , corresponds to the innermost ring, which is tangent to the line of sight.

Problem #8: Express the circular velocity at the tangent point as a function of v_t .

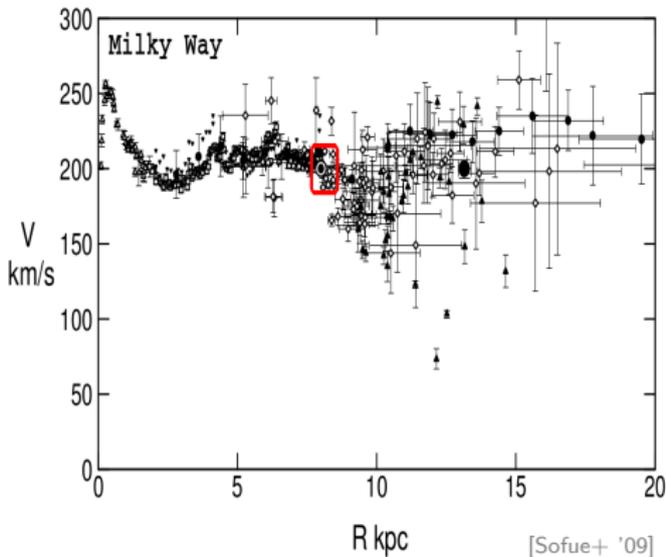
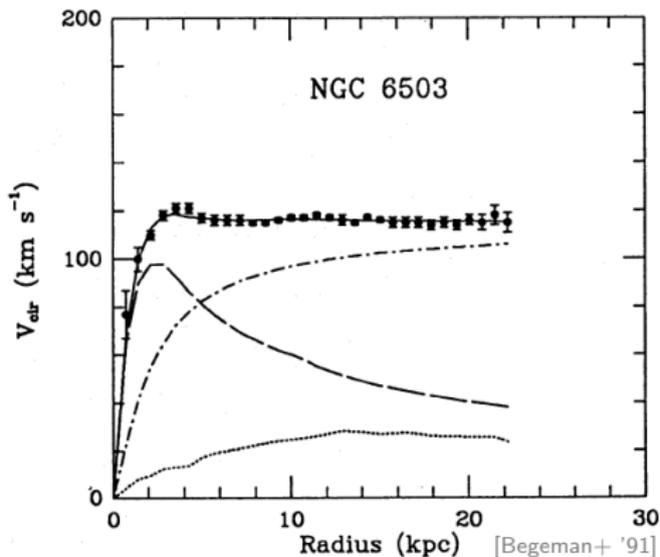
Using the result of #7 and $R \equiv R_t = R_0 \sin \ell$, we have for $\ell > 0$

$$v(R) = v_t(\ell) + v_0 \sin \ell$$

Therefore, measuring the terminal velocity at different longitudes we can construct the rotation curve of our Galaxy at $R < R_0$.

I. OUR OWN GALAXY

As for external galaxies, the prime observable to infer dark matter in our Galaxy is the rotation curve. However, this is challenging to measure given our peculiar position inside the Galaxy.



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I. OUR OWN GALAXY

The terminal velocity method is very powerful, but it is only applicable in the inner Galaxy at $R < R_0$. In the solar neighbourhood alternative tracers are needed.

We could for instance measure the line-of-sight velocities of local stars. We have then the same expression as before,

$$v_{\text{los}} = \left(v(R) \frac{R_0}{R} - v_0 \right) \sin \ell,$$

but now we are interested in the regime $R \sim R_0$.

Problem #9: Taylor-expand the expression above in powers of the distance to the star.

First, let us expand $\Omega \equiv v/R$: $\Omega(R) \simeq \Omega_0 + (R - R_0)\Omega'_0$, and therefore

$$v_{\text{los}} = \left(\Omega_0 R_0 + (R - R_0)\Omega'_0 R_0 - v_0 \right) \sin \ell = (R - R_0)R_0\Omega'_0 \sin \ell \equiv -2A(R - R_0) \sin \ell,$$

having defined the Oort's constants $A = -\frac{1}{2}R_0\Omega'_0 = \frac{1}{2} \left(\frac{v_0}{R_0} - v'_0 \right)$, $B = -\frac{1}{2} \left(\frac{v_0}{R_0} + v'_0 \right)$.

From simple geometry, $\vec{R} = \vec{R}_0 + \vec{d} \Rightarrow R^2 = R_0^2 + d^2 - 2dR_0 \cos \ell \simeq R_0^2 - 2dR_0 \cos \ell$, so

$$R - R_0 = \frac{R^2 - R_0^2}{R + R_0} \simeq \frac{R^2 - R_0^2}{2R_0} \simeq \frac{-2dR_0 \cos \ell}{2R_0} = -d \cos \ell.$$

Replacing in the expression of v_{los} , we obtain finally $v_{\text{los}} = -Ad \sin 2\ell$.

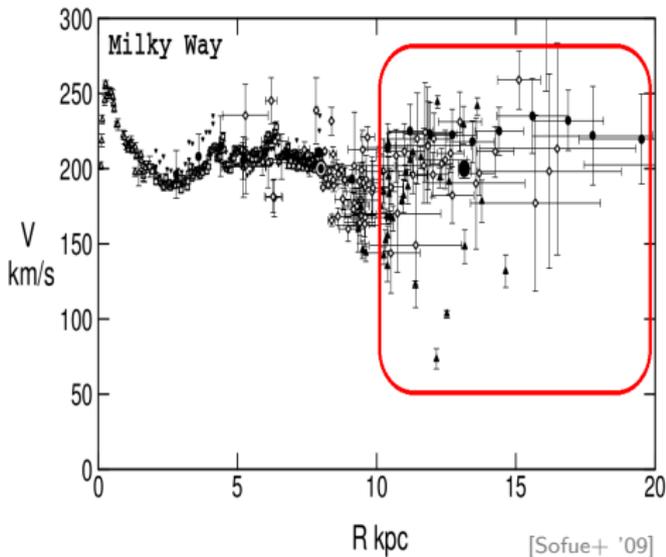
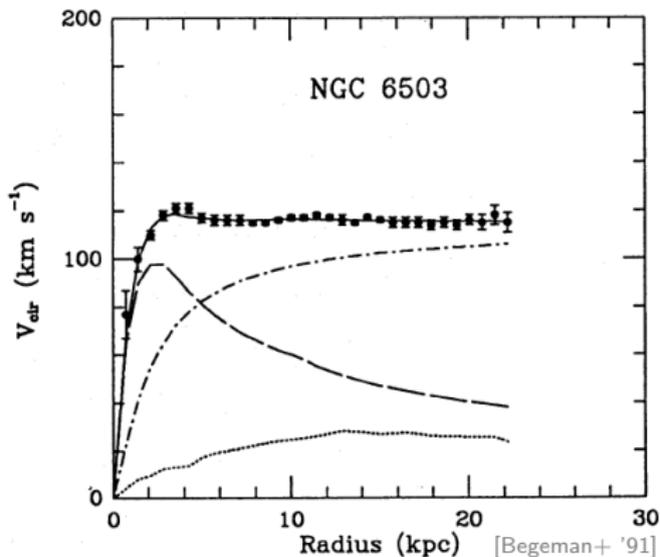
Problem #10*: Do the same as in #9 but for the transverse velocity.

Follow Binney & Merrifield, Sec. 10.3.3 to obtain $\mu_\ell = B + A \cos 2\ell$.

The bottomline is that by measuring the motion of local stars we can constrain A, B and thus the local circular velocity: $v_0 = R_0(A - B) \simeq 220 \text{ km/s}$.

I. OUR OWN GALAXY

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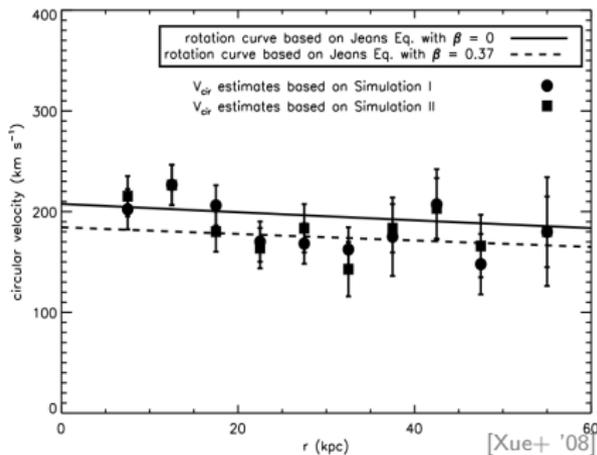
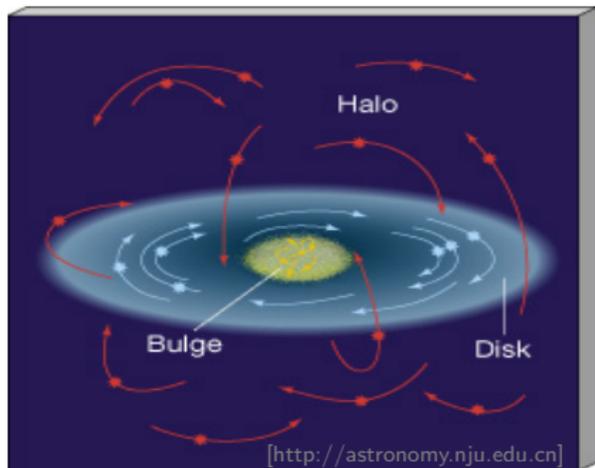
I. OUR OWN GALAXY

The outer Galaxy is perhaps the most challenging region to constrain the rotation curve due to the lack of suitable tracers. One possibility is to use old halo stars. Since these do not follow circular orbits the conversion of their motion to the circular velocity is not trivial.

We need the so-called Jeans equations (derived from the collisionless Boltzmann equation), e.g. in the spherical case:

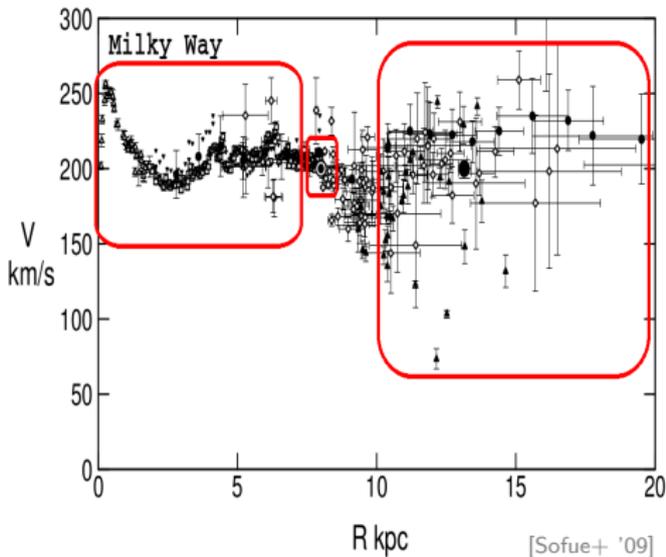
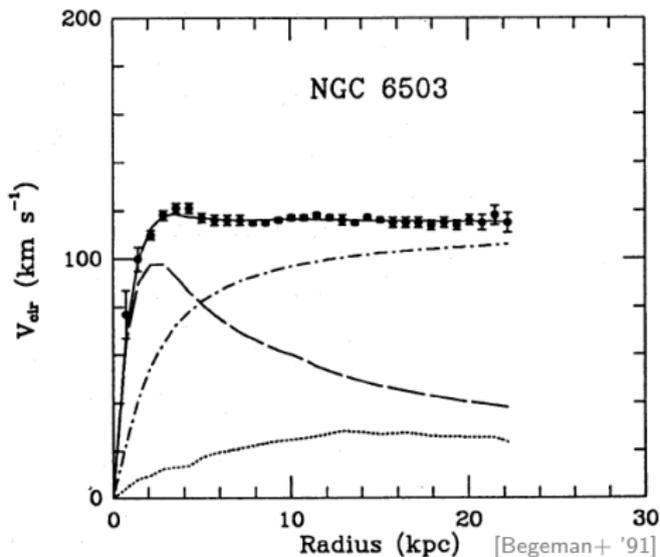
$$\frac{d(\nu\sigma_r^2)}{dr} + \frac{2\beta\nu\sigma_r^2}{r} = -\nu\frac{d\phi}{dr} \propto \frac{v_c^2}{r},$$

where ν is the density of stars, σ_r^2 the radial velocity dispersion and $\beta = 1 - \sigma_\theta^2/\sigma_r^2$ is the anisotropy parameter. In this way with measured ν, σ_r^2 of a given population of stars we can constrain the total gravitational potential in the outer Galaxy.



I. OUR OWN GALAXY

As for external galaxies, the prime observable to infer dark matter in our Galaxy is the rotation curve. However, this is challenging to measure given our peculiar position inside the Galaxy.



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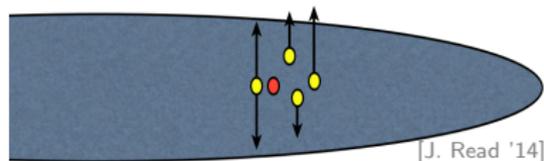
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I. OUR OWN GALAXY

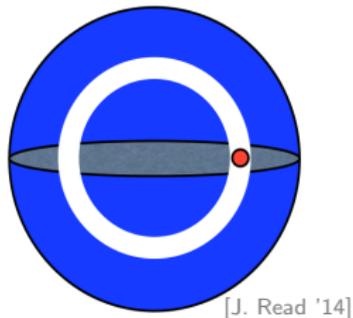
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There are two types of methods to derive ρ_0 :

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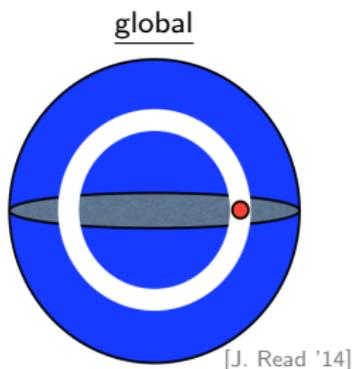
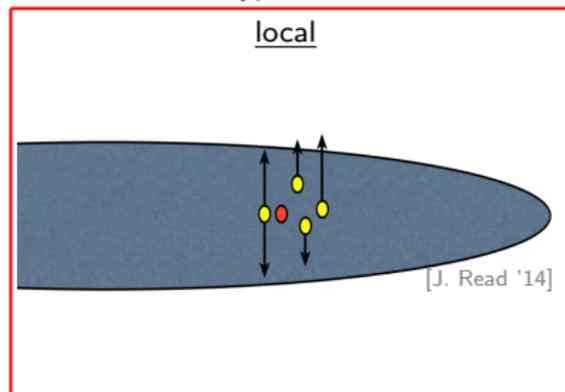
global



I. OUR OWN GALAXY

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We start by writing down Poisson's and Jeans equation in cylindrical coordinates:

$$\nabla^2 \phi = \frac{1}{R} \frac{\partial}{\partial R} \left(R \frac{\partial \phi}{\partial R} \right) + \frac{\partial^2 \phi}{\partial z^2} = 4\pi G \rho \quad , \quad \frac{\partial(\nu \overline{v_z^2})}{\partial z} = -\nu \frac{\partial \phi}{\partial z} ,$$

where ρ is the total mass density. Eliminating the potential,

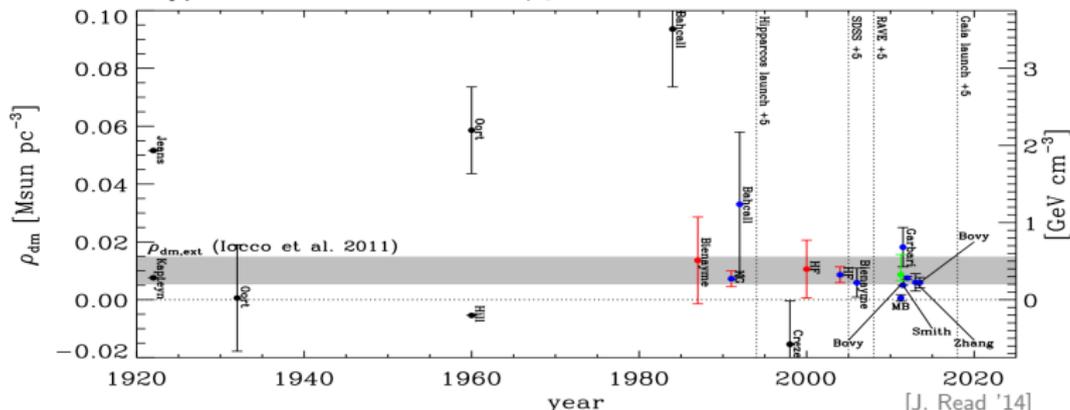
$$\frac{\partial}{\partial z} \left(\frac{1}{\nu} \frac{\partial(\nu \overline{v_z^2})}{\partial z} \right) = -4\pi G \rho .$$

This is a lot to ask from the data! Then, you still have to subtract off the baryonic contribute to have the local dark matter density ρ_0 .

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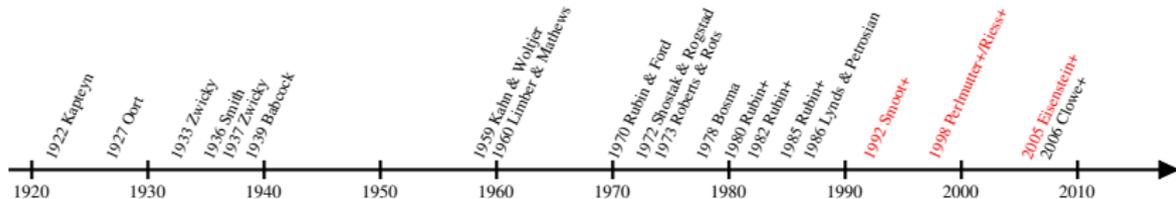
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I. BRIEF HISTORICAL PERSPECTIVE

The evidence for dark matter gradually mounted throughout the 20th century, and by now we are convinced that our universe is filled with dark matter at various scales. Note that all evidence for dark matter is of gravitational origin; non-gravitational evidence is yet to be discovered.



We can identify four key revolutions in the history of dark matter:

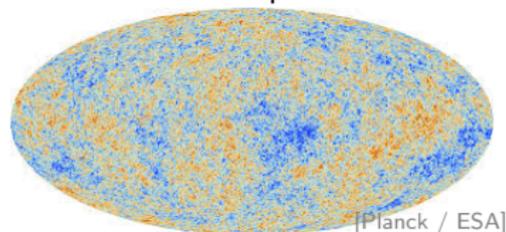
1. dark matter is first mentioned by Kapteyn;
2. dark matter is found in the Coma cluster by Zwicky;
3. dark matter is found in spiral galaxies by Rubin & Ford; and
4. dark matter is found at cosmological scales by many.



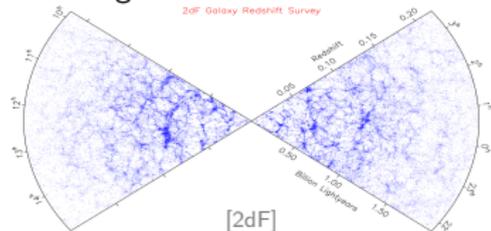
I. COSMOLOGY

The evidence for dark matter at large small scales is very robust. It comes mainly from three probes:

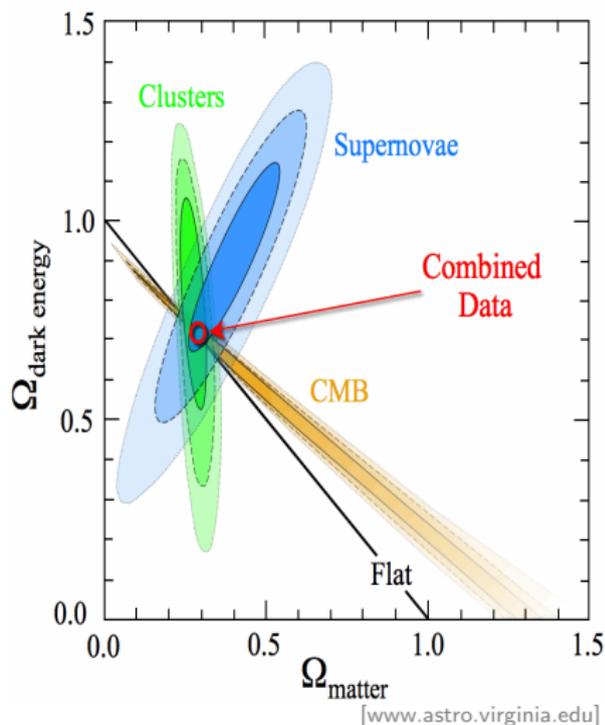
1. CMB anisotropies



2. large scale structure



3. Supernovae Type Ia

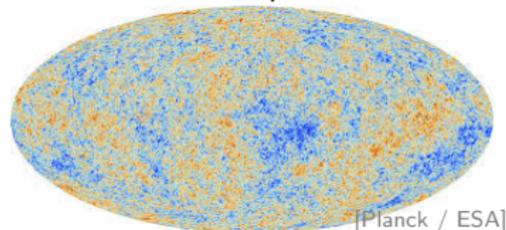


Λ CDM model
 $\Omega_m = 0.315, \Omega_b h^2 = 0.022, \Omega_\Lambda = 0.685$

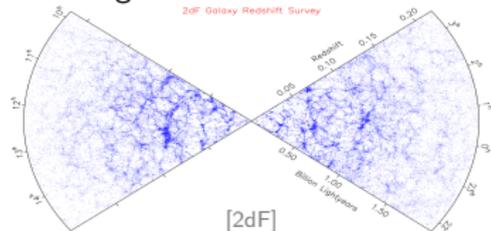
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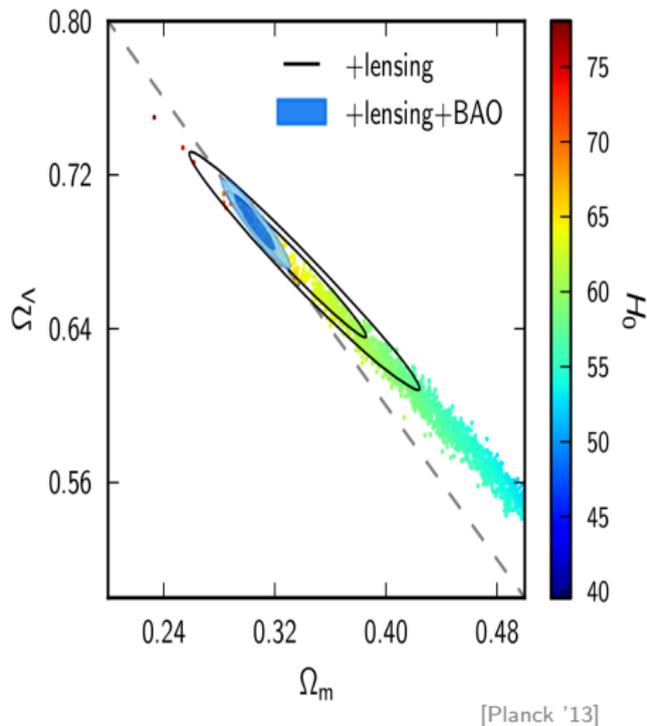
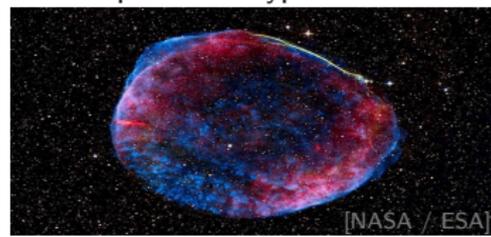
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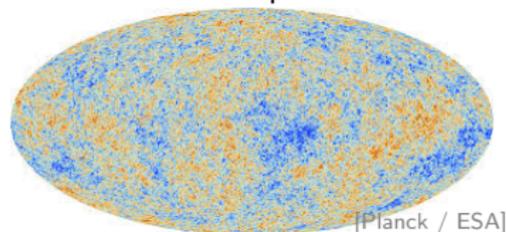


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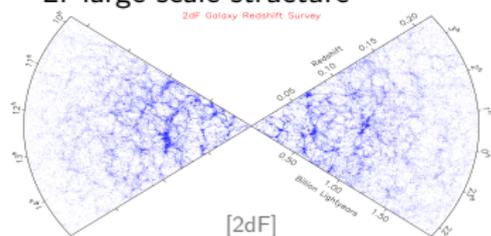
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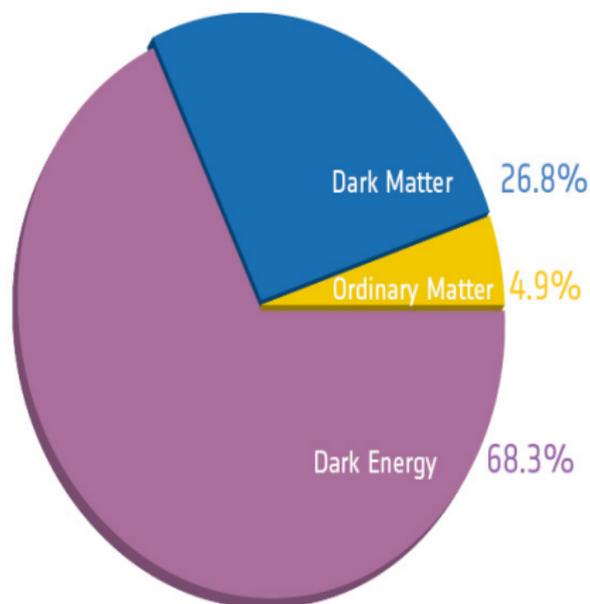
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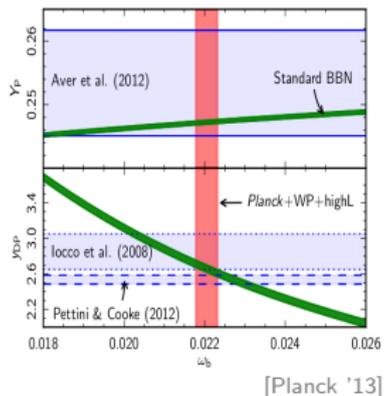
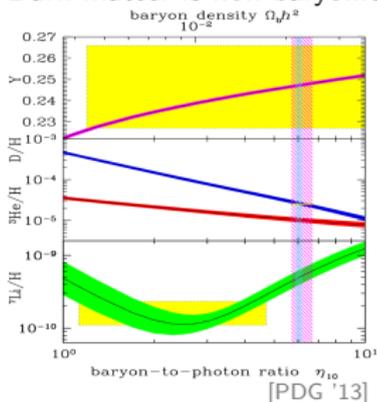
[Planck / ESA]

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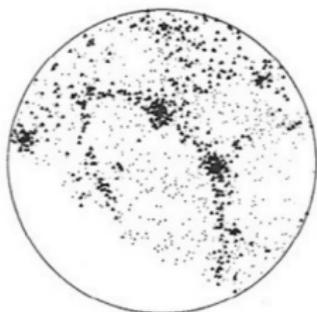
I. COSMOLOGY

There are two other important lessons from cosmology:

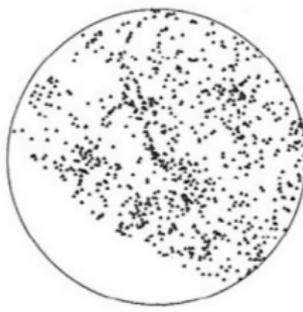
1. Dark matter is non-baryonic.



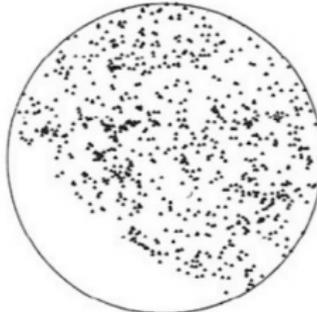
2. Dark matter is "cold" (i.e., non-relativistic at the onset of structure formation).



HDM

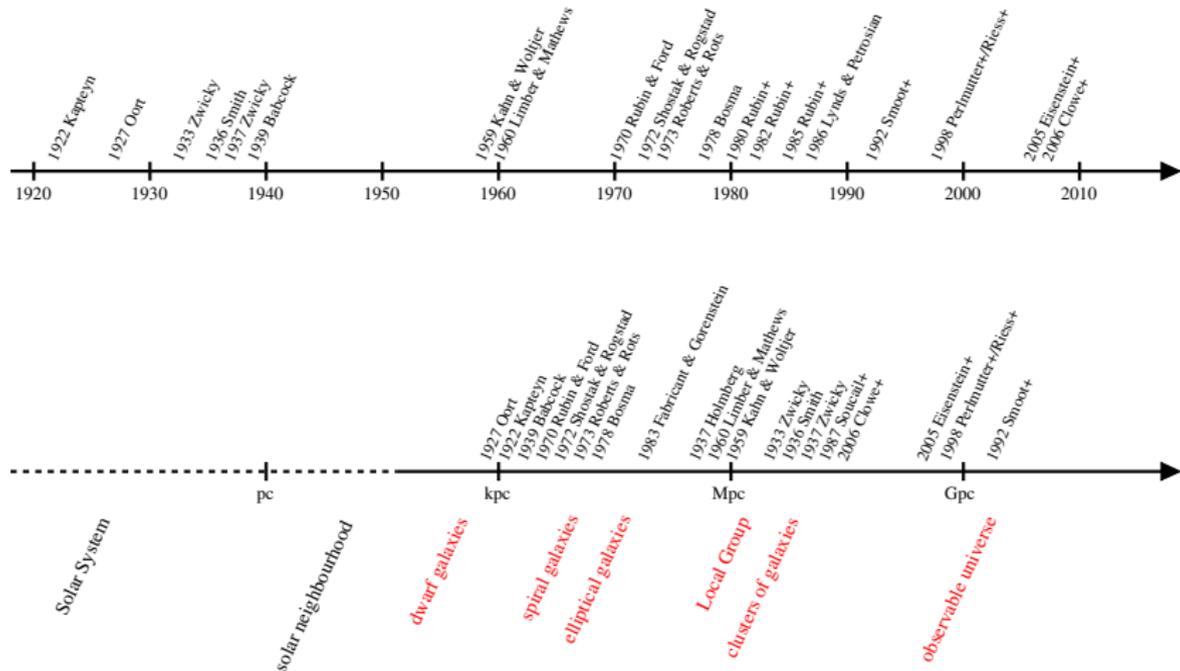


Observed Galaxy Distribution



CDM

I. BRIEF HISTORICAL PERSPECTIVE



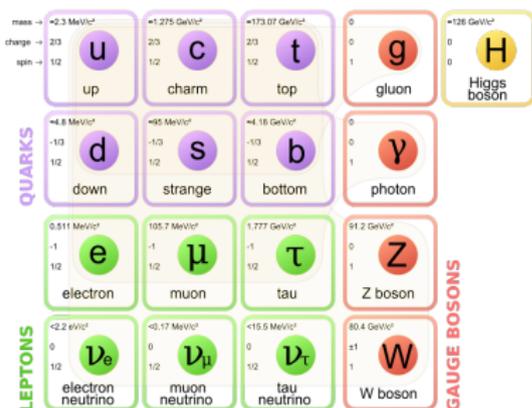
The evidence for dark matter is very compelling and spans all scales from dwarf galaxies up to the edge of the observable universe...

I. DARK MATTER FACT SHEET

What do we know for sure about dark matter?

1. It exists across the universe, past and present.
2. It contributes 26% of the energy density budget (or 85% of the matter budget).
3. It is dark, i.e. no electric charge or color.
4. It is (essentially) collisionless.
5. It is cold.
6. It is non-baryonic.
7. It has a lifetime longer than the age of universe.

cold non-baryonic dark matter



There is no suitable candidate
in the Standard Model



evidence for physics
beyond the Standard Model

OUTLINE

Part I. Evidence for dark matter

- Brief historical perspective
- Clusters of galaxies
- Galaxies (including our own)
- Cosmology
- Dark matter fact sheet

Part II. Dark matter candidates

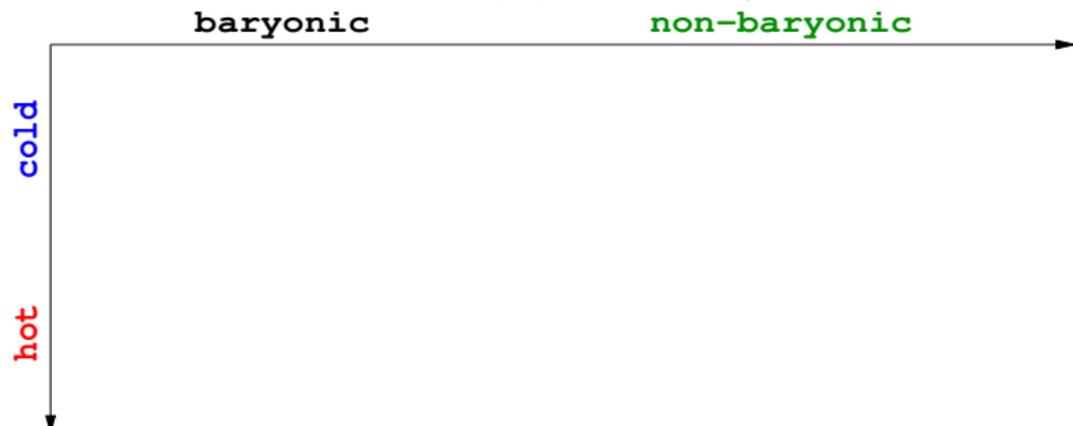
- Tour of the zoo
- Thermal decoupling
- WIMP paradigm

Part III. Dark matter searches

- Overview of detection strategies
- Direct searches: idea, techniques, status
- Indirect searches: idea, techniques, status

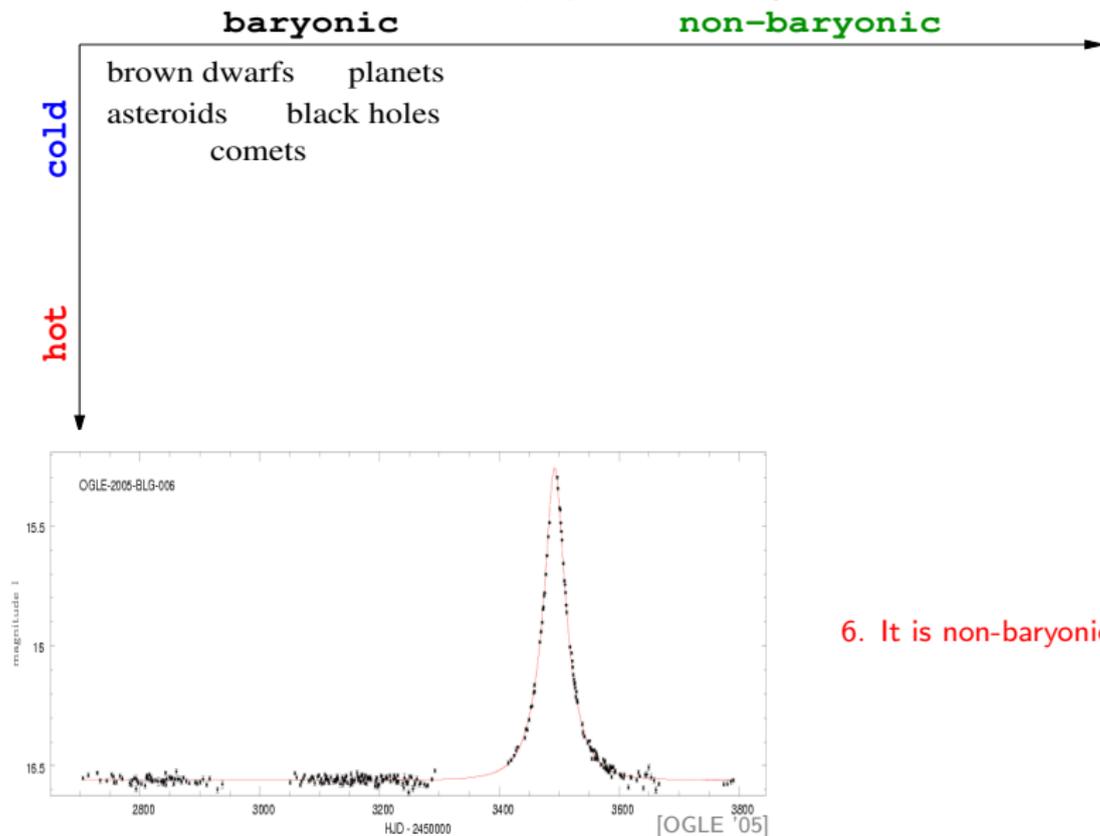
II. DARK MATTER CANDIDATES

Before focussing on cold non-baryonic dark matter, let us take a historical tour of the dark matter candidates that have been proposed over the years.



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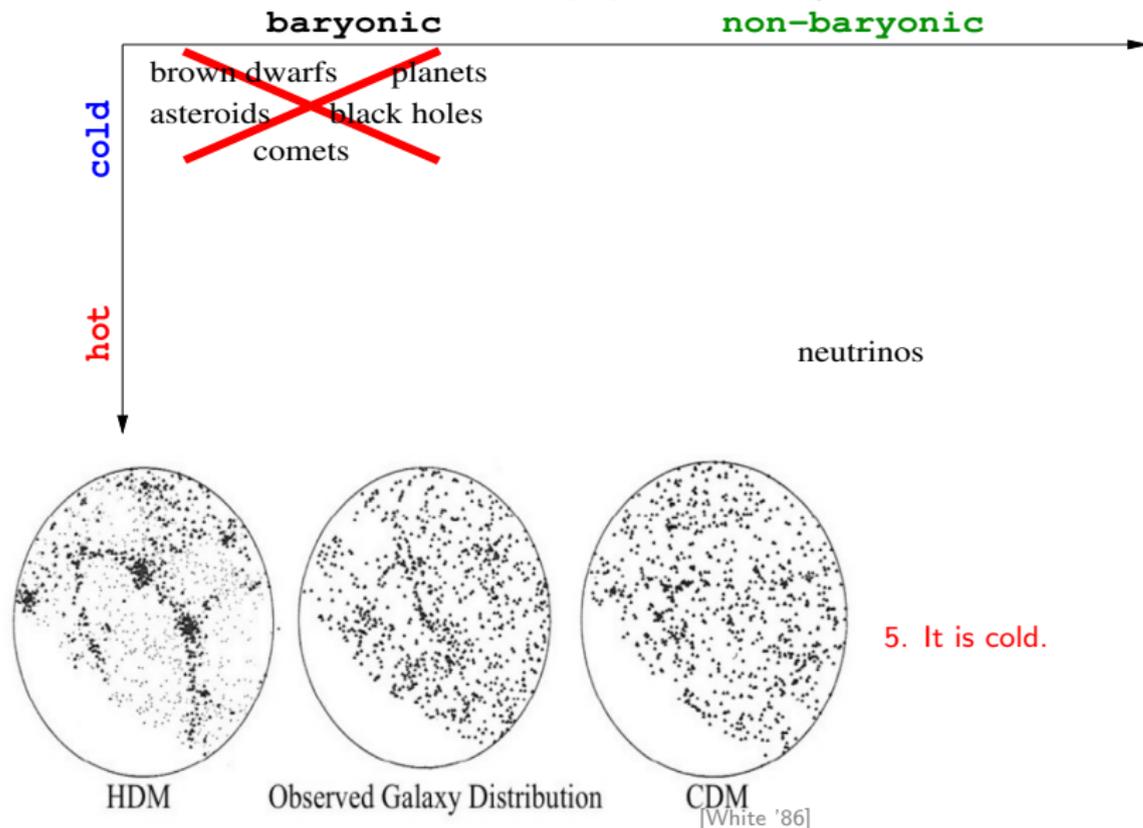
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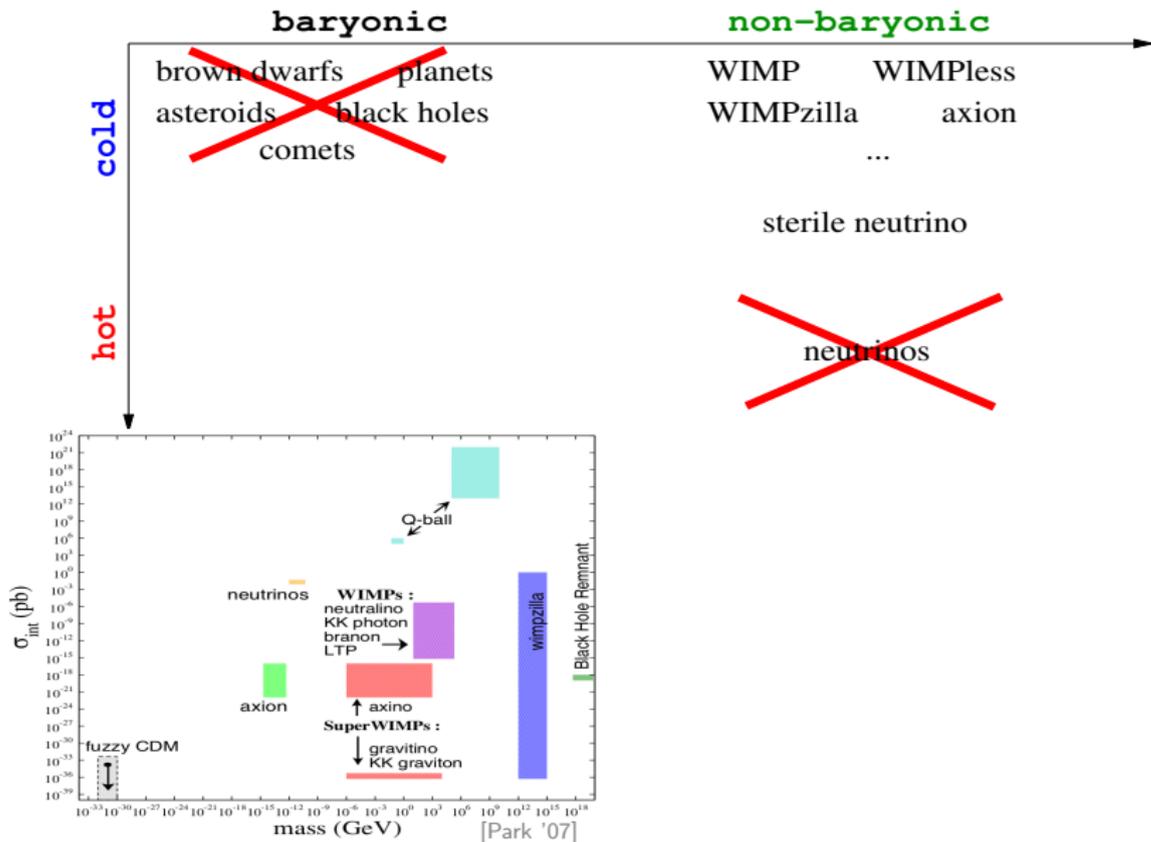
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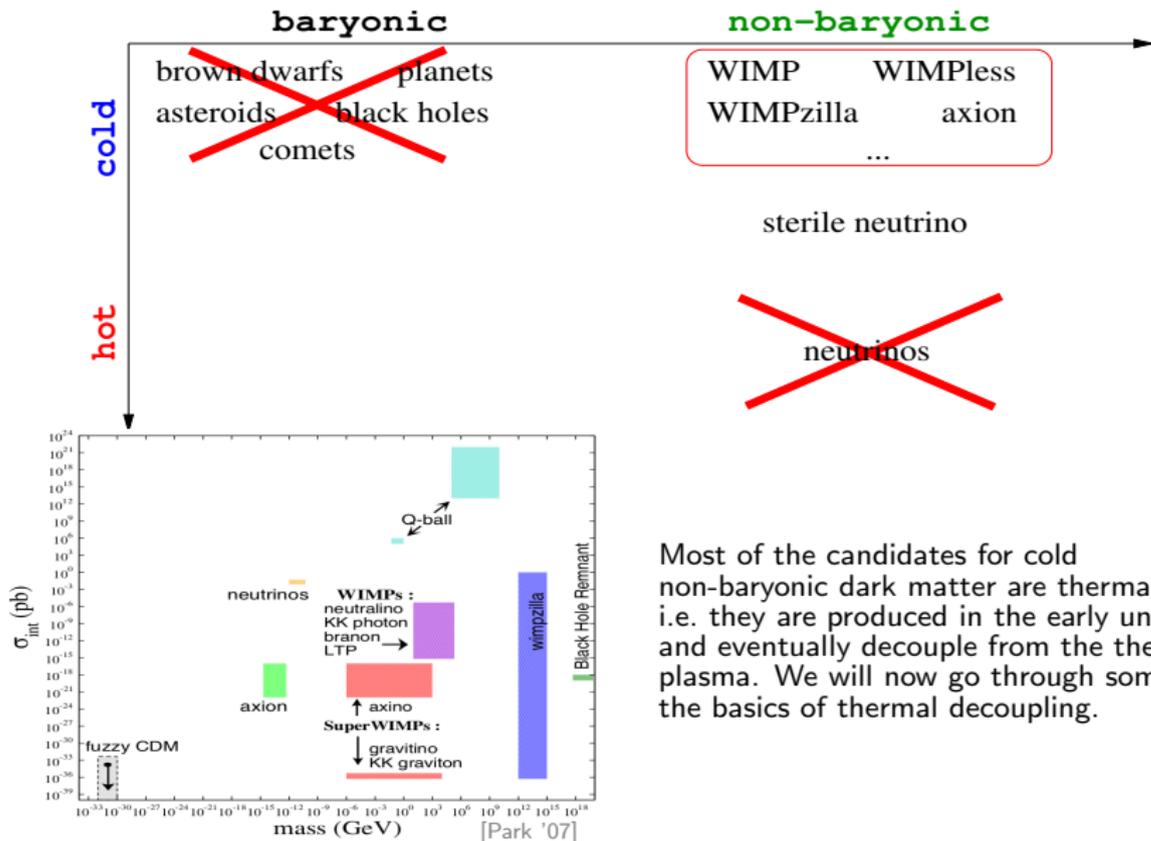
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Most of the candidates for cold non-baryonic dark matter are thermal, i.e. they are produced in the early universe and eventually decouple from the thermal plasma. We will now go through some of the basics of thermal decoupling.

II. THERMAL DECOUPLING

First, refresh your memory from the cosmology notes:

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho \quad , \quad s = \frac{2\pi^2}{45}g_s T^3 \quad , \quad \rho = \frac{\pi^2}{30}g_\rho T^4$$
$$\Omega_\chi^0 = \rho_\chi^0 / \rho_{\text{crit}}^0 \quad , \quad \rho_{\text{crit}} = \frac{3H^2}{8\pi G} \quad .$$

Now consider a Majorana particle χ with interactions such that it has reached chemical equilibrium with the photons and other particles early on in the universe. The evolution of the density of χ is given by the Boltzmann equation:

$$\frac{dn_\chi}{dt} + 3Hn_\chi = \langle\sigma v\rangle (n_{\chi,\text{eq}}^2 - n_\chi^2) \quad .$$

Problem #11*: Go back to your cosmology notes and deduce this.

See e.g. Kolb & Turner, Secs. 5.1 and 5.2.

The meaning of the Boltzmann equation is very simple:

$$\frac{dn_\chi}{dt} + 3Hn_\chi = \frac{1}{a^3} \frac{d}{dt} (n_\chi a^3) \quad \text{evolution of comoving density}$$
$$\langle\sigma v\rangle n_{\chi,\text{eq}}^2 \quad \text{production } ij \rightarrow \chi\chi$$
$$-\langle\sigma v\rangle n_\chi^2 \quad \text{annihilation } \chi\chi \rightarrow ij$$

Furthermore, entropy conservation implies

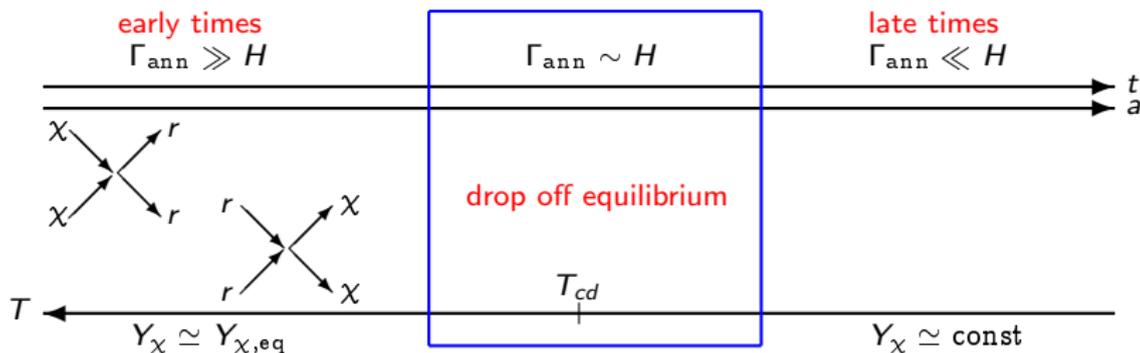
$$\frac{d}{dt} (sa^3) = 0 \Leftrightarrow \dot{s}a^3 + 3sa^2\dot{a} = 0 \Leftrightarrow \boxed{\dot{s} = -3sH} \Leftrightarrow \dots \Leftrightarrow \boxed{\dot{T} = -3H \left(\frac{d \ln g_s}{dT} + \frac{3}{T} \right)^{-1}} \quad .$$

II. THERMAL DECOUPLING

$$\frac{dn_X}{dt} + 3Hn_X = \langle \sigma v \rangle (n_{X,\text{eq}}^2 - n_X^2) \quad \dot{s} = -3sH \quad \dot{T} = -3H \left(\frac{d \ln g_s}{dT} + \frac{3}{T} \right)^{-1}$$

Problem #12: Make the substitutions $x = m_X/T$, $Y_X = n_X/s$, $\Gamma_{\text{ann}} \equiv \langle \sigma v \rangle n_{X,\text{eq}}$ and use the expressions above to obtain dY_X/dx .

$$\frac{x}{Y_{X,\text{eq}}} \frac{dY_X}{dx} = -\frac{\Gamma_{\text{ann}}}{H} \left[1 + \frac{1}{3} \frac{d \ln g_s}{d \ln T} \right] \left[\left(\frac{Y_X}{Y_{X,\text{eq}}} \right)^2 - 1 \right]$$



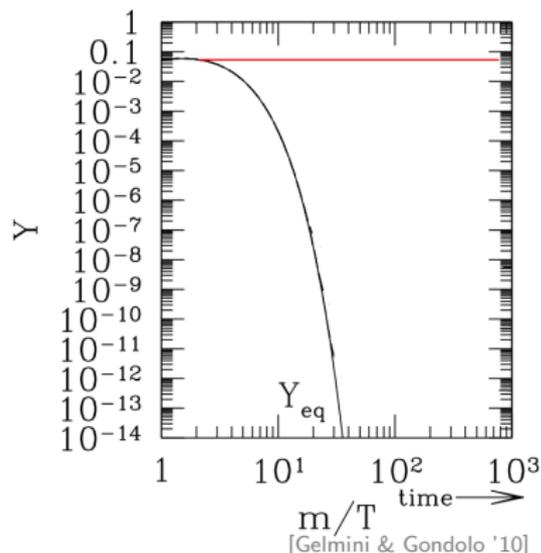
relativistic species	$x \ll 3$	$Y_{X,\text{eq}}(x) = \frac{45\zeta(3)}{2\pi^4} \frac{g_{\text{eff}}}{g_s(x)}$
non-relativistic species	$x \gg 3$	$Y_{X,\text{eq}}(x) = \frac{45}{4\sqrt{2}\pi^{7/2}} \frac{g}{g_s(x)} x^{3/2} e^{-x}$

II. THERMAL DECOUPLING

Let us now treat the two cases: hot relics and cold relics.

$$\frac{x}{Y_{\chi,\text{eq}}} \frac{dY_{\chi}}{dx} = -\frac{\Gamma_{\text{ann}}}{H} \left[1 + \frac{1}{3} \frac{d \ln g_s}{d \ln T} \right] \left[\left(\frac{Y_{\chi}}{Y_{\chi,\text{eq}}} \right)^2 - 1 \right]$$

Hot relics $Y_{\chi,\text{eq}} \simeq \text{const}$



At early times $Y_{\chi} \simeq Y_{\chi,\text{eq}} \simeq \text{const}$, while at late times $Y_{\chi} \simeq \text{const}$, so today we have

$$Y_{\chi,0} \simeq Y_{\chi,\text{eq}}(x_f) = \frac{45\zeta(3)}{2\pi^4} \frac{g_{\text{eff}}}{g_s(x_f)},$$

which implies a relic abundance

$$\Omega_{\chi}^0 h^2 = \frac{m_{\chi} s_0 Y_{\chi,0}}{\rho_{\text{crit}}^0 / h^2} \sim \left(\frac{m_{\chi}}{90 \text{ eV}} \right)$$

for $g_{\text{eff}} = 2 \times (3/4)$ and $g_s(x_f) = 10.75$ as for neutrinos.

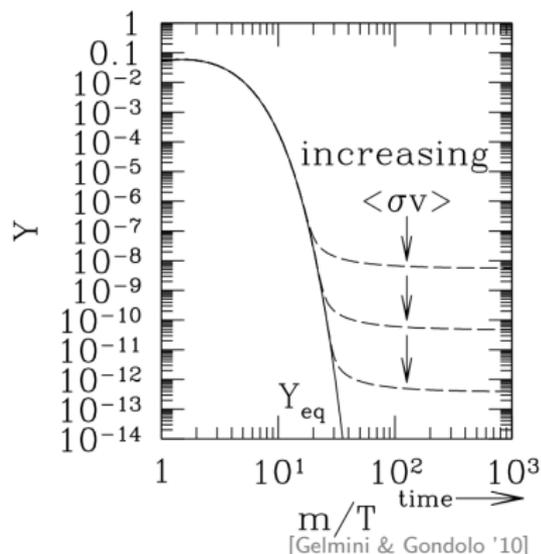
Hot relics would need a mass $m_{\chi} \sim 9 \text{ eV}$ in order to explain the observed dark matter abundance $\Omega_m h^2 \simeq 0.1$. Neutrinos cannot therefore be all of the dark matter since current limits indicate $m_{\nu} \lesssim 1 \text{ eV}$.

II. THERMAL DECOUPLING

Let us now treat the two cases: hot relics and cold relics.

$$\frac{x}{Y_{X,\text{eq}}} \frac{dY_X}{dx} = -\frac{\Gamma_{\text{ann}}}{H} \left[1 + \frac{1}{3} \frac{d \ln g_s}{d \ln T} \right] \left[\left(\frac{Y_X}{Y_{X,\text{eq}}} \right)^2 - 1 \right]$$

Cold relics $Y_{X,\text{eq}} \propto x^{3/2} e^{-x}$



Since in this case $Y_{X,\text{eq}}$ varies quickly with x , a very simple solution as for hot relics cannot be found.

But let us start at early times where the deviation from the equilibrium abundance, Δ , is small. Then $Y_X = (1 + \Delta)Y_{X,\text{eq}}$ and

$$\frac{d \ln(1 + \Delta)Y_{X,\text{eq}}}{d \ln x} = -\frac{\Gamma_{\text{ann}}}{H} \left[(1 + \Delta)^2 - 1 \right] \Leftrightarrow$$

$$\frac{\Delta(2 + \Delta)}{1 + \Delta} \simeq \frac{H}{\Gamma_{\text{ann}}} \left(x - \frac{3}{2} \right) \propto \left(x - \frac{3}{2} \right) x^{-1/2} e^x.$$

Eventually, Δ starts to grow. A possible definition of freeze-out is $\frac{\Delta_f(2+\Delta_f)}{1+\Delta_f} = 1 \Leftrightarrow \Delta_f = 0.618$.

Problem #13: Find an iterative formula for x_f .

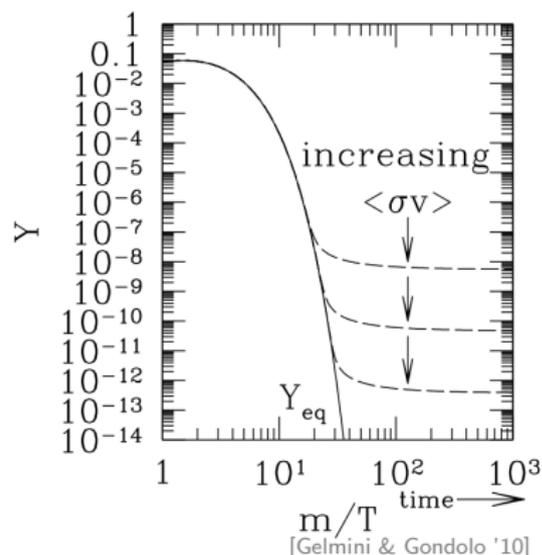
$$x_f + \ln \left(x_f - \frac{3}{2} \right) - \frac{1}{2} \ln x_f = 19.8 + \ln g + \ln \left(\frac{\langle \sigma v \rangle}{10^{-26} \text{ cm}^3/\text{s}} \right) + \ln \left(\frac{m_X}{\text{GeV}} \right) - \frac{1}{2} \ln g_\rho$$

II. THERMAL DECOUPLING

Let us now treat the two cases: hot relics and cold relics.

$$\frac{x}{Y_{X,\text{eq}}} \frac{dY_X}{dx} = -\frac{\Gamma_{\text{ann}}}{H} \left[1 + \frac{1}{3} \frac{d \ln g_s}{d \ln T} \right] \left[\left(\frac{Y_X}{Y_{X,\text{eq}}} \right)^2 - 1 \right]$$

Cold relics $Y_{X,\text{eq}} \propto x^{3/2} e^{-x}$



Having x_f we finally get the abundance at freeze-out: $Y_{X,f} = (1 + \Delta_f) Y_{X,\text{eq}}(x_f)$.

After freeze-out, Δ grows quickly and the Boltzmann equation is roughly

$$\frac{dY_X}{dx} = -\frac{s\langle\sigma v\rangle}{Hx} Y_X^2,$$

which can be integrated to give

$$Y_{X,0} = \frac{Y_{X,f}}{1 + AY_{X,f}}, \quad A = \int_{x_f}^{\infty} dx \frac{s\langle\sigma v\rangle}{Hx} \propto \frac{m_X \langle\sigma v\rangle}{x_f}.$$

$$\Omega_X^0 h^2 = \frac{m_X s_0 Y_{X,0}}{\rho_{\text{crit}}^0 / h^2} \sim 10^{-27} x_f g_{\rho,f}^{-1/2} \left(\frac{\text{cm}^3/\text{s}}{\langle\sigma v\rangle} \right)$$

II. THERMAL DECOUPLING

From Problem #13 we find typical values $x_f \simeq 20 - 30$. And from the latest Planck measurements $\Omega_x^0 h^2 \simeq 0.11$. Rewriting the expression in the last slide,

$$\Omega_x^0 h^2 = \frac{m_x s_0 Y_{x,0}}{\rho_{\text{crit}}^0 / h^2} \sim 0.1 \left(\frac{x_f}{25} \right) \left(\frac{3 \times 10^{-26} \text{ cm}^3/\text{s}}{\langle \sigma v \rangle} \right).$$

The “magic” annihilation rate $3 \times 10^{-26} \text{ cm}^3/\text{s}$ gives you the observed relic abundance. Let us see the implications of this.

II. THERMAL DECOUPLING

From Problem #13 we find typical values $x_f \simeq 20 - 30$. And from the latest Planck measurements $\Omega_\chi^0 h^2 \simeq 0.11$. Rewriting the expression in the last slide,

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The “magic” annihilation rate $3 \times 10^{-26} \text{ cm}^3/\text{s}$ gives you the observed relic abundance. Let us see the implications of this. First, find the typical velocity of the relic at freeze-out:

$$x_f \sim 25 \Leftrightarrow T_f \sim m_\chi/25 \quad , \quad \langle v \rangle \sim \sqrt{3T_f/m_\chi} \sim \sqrt{3/25} \sim 0.35c \simeq 10^{10} \text{ cm/s}.$$

Problem #14: Compute the cross section σ needed to have $\langle \sigma v \rangle = 3 \times 10^{-26} \text{ cm}^3/\text{s}$ at freeze-out.

$$\sigma \sim \frac{\langle \sigma v \rangle}{\langle v \rangle} \sim 3 \times 10^{-36} \text{ cm}^2 \sim \mathcal{O}(1) \text{ pb}$$

This is the electroweak scale!

Now, with $\sigma \sim g^4/m_\chi^2$ and assuming a weak scale coupling $g \sim g_{\text{weak}} \sim 0.1$, we get $m_\chi \sim \mathcal{O}(10 - 1000) \text{ GeV}$.

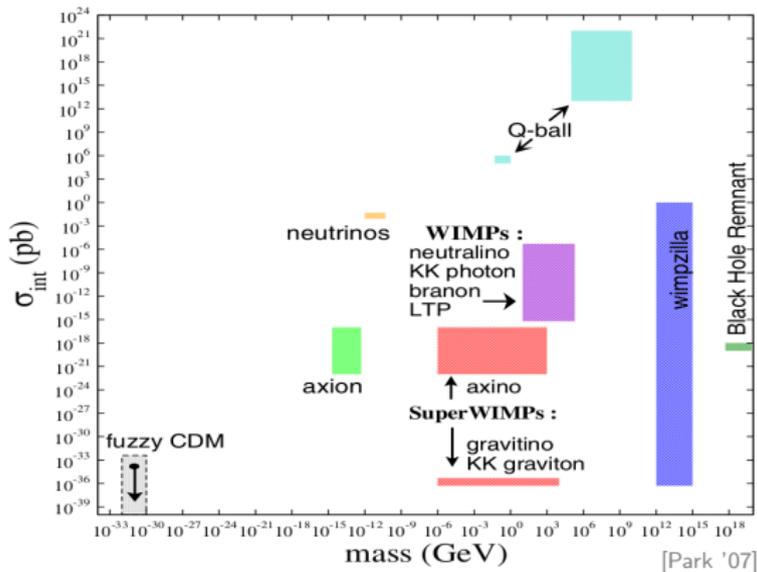
We learn that weakly interacting massive particles (WIMPs) give naturally the observed relic abundance. Incidentally, such particles are predicted in beyond the Standard Model theories, motivated by completely different reasons. This is what people dubbed the “WIMP miracle”.

Note that to obtain the observed relic abundance you do not need a WIMP necessarily – other particles with appropriate combination m_χ, g, σ can do the job.

II. WIMP WISH LIST

What do we wish in a prototypical WIMP?

1. $\langle \sigma v \rangle \sim 3 \times 10^{-26} \text{ cm}^3/\text{s}$ (weakly interacting)
2. $m_\chi \sim \mathcal{O}(10 - 1000) \text{ GeV}$ (massive)
3. $T_f \sim m_\chi/25$ (non-relativistic, cold)
4. $\Omega_\chi^0 h^2 \sim 0.11$ (*the* dark matter)



Disclaimer: There are many other dark matter candidates (thermal and non-thermal), but from now on I shall focus on WIMPs and their rich phenomenology.

OUTLINE

Part I. Evidence for dark matter

- Brief historical perspective
- Clusters of galaxies
- Galaxies (including our own)
- Cosmology
- Dark matter fact sheet

Part II. Dark matter candidates

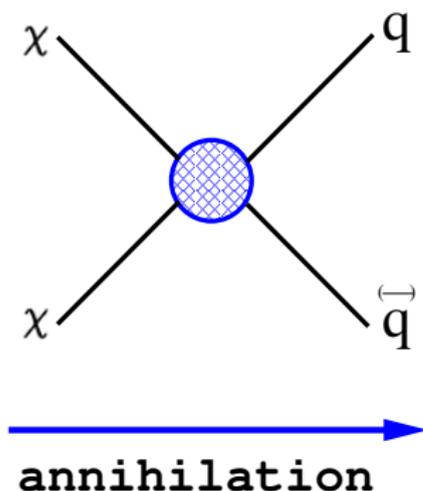
- Tour of the zoo
- Thermal decoupling
- WIMP paradigm

Part III. Dark matter searches

- Overview of detection strategies
- Direct searches: idea, techniques, status
- Indirect searches: idea, techniques, status

III. DARK MATTER SEARCHES

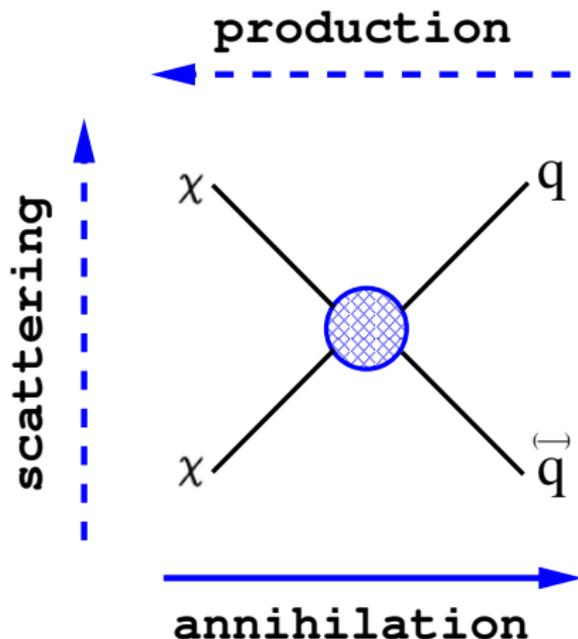
The process at the core of the WIMP paradigm is the annihilation into Standard Model particles, presumably occurring in the early universe.



There is no reason why this process does not happen at present in our Galaxy and beyond. So we can set out to search for the products of this annihilation today!

III. DARK MATTER SEARCHES

The process at the core of the WIMP paradigm is the annihilation into Standard Model particles, presumably occurring in the early universe.



There is no reason why this process does not happen at present in our Galaxy and beyond. So we can set out to search for the products of this annihilation today!

Plus, given a specific WIMP model, we can rotate the diagram and look for scattering and production.

III. DARK MATTER SEARCHES

This sets the stage for a three-prong strategy for WIMP searches.

direct searches

WIMP scattering off nuclei underground

indirect searches

yields of WIMP annihilation or decay

WIMP production in the lab

collider searches

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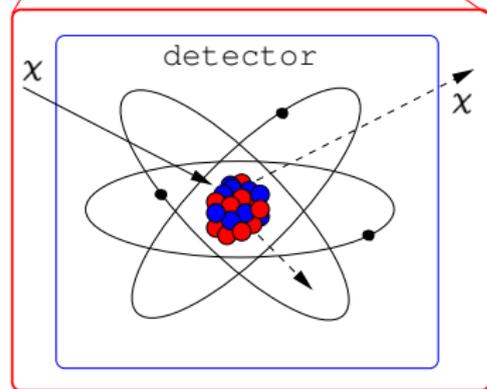
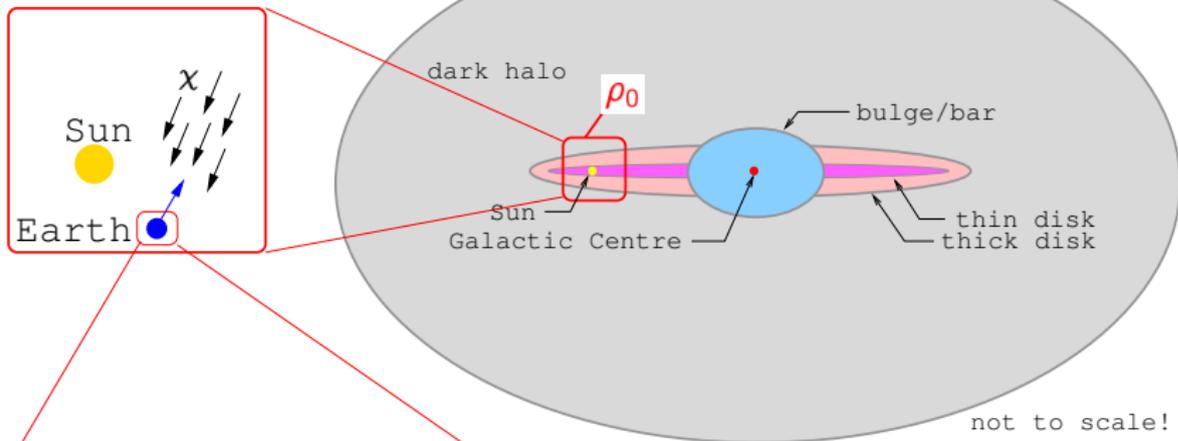
WIMP production in the lab

collider searches

2. DIRECT DARK MATTER SEARCHES

Milky Way

edge-on



III. DIRECT SEARCHES

Let us start with a warm-up problem.

Problem #15: (a) Compute how many WIMPs cross your laptop per second.

In Part I we have seen that the local dark matter mass density is $\rho_0 \sim 0.3 \text{ GeV}/\text{cm}^3$ and that the velocity of the Solar System in the Galactic frame is $v_0 \sim 220 \text{ km/s}$. With a typical WIMP mass $m_\chi \sim 100 \text{ GeV}$ as motivated in Part II, the WIMP “wind” is crossing the Earth with a flux

$$\phi_\chi \simeq \frac{\rho_0}{m_\chi} v_0 \sim 6.6 \times 10^4 \text{ cm}^{-2} \text{ s}^{-1}.$$

Taking a 15" laptop of $36 \text{ cm} \times 24 \text{ cm}$, we have $A_{\text{laptop}} \sim 864 \text{ cm}^2$ and finally

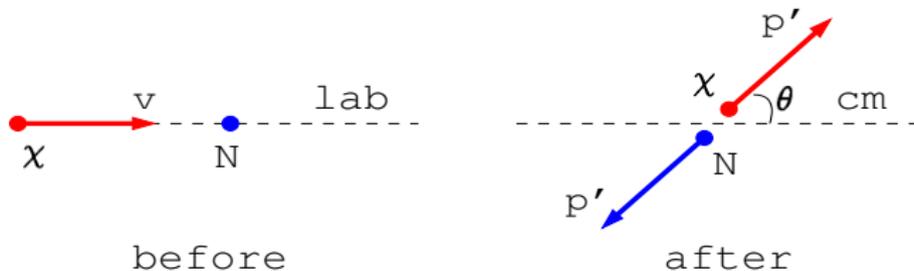
$$\frac{dN_\chi}{dt} = \phi_\chi A_{\text{laptop}} \sim 5.7 \times 10^7 \text{ s}^{-1}.$$

(b) And what is the typical WIMP momentum?

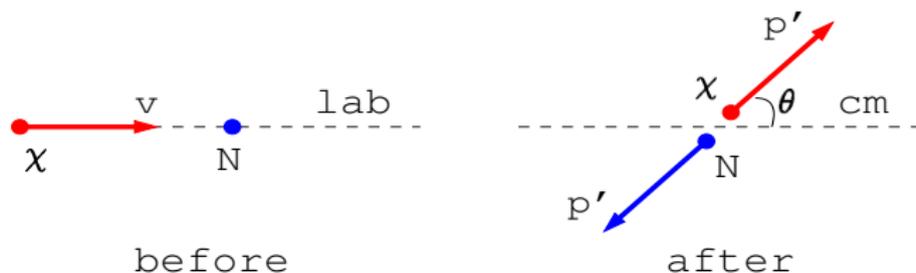
$$p_\chi \simeq m_\chi v_0 \sim 73 \text{ MeV}$$

That is a lot of speedy WIMPs! Of course, they interact very feebly so you do not have to worry, but this gives us the hint and hope to eventually detect WIMPs.

Now we go a step further. Say we have a box of atoms at rest. Given enough time, a WIMP will scatter off an atom in the box.



III. DIRECT SEARCHES



Conservation of energy gives

$$s_i^{\text{lab}} = s_f^{\text{cm}} \Leftrightarrow (E_\chi + m_N, \vec{p}_\chi)^2 = (E'_\chi + E'_N, \vec{0})^2 \Leftrightarrow p'^2 = \frac{m_N^2 p_\chi^2}{m_N^2 + m_\chi^2 + 2E_\chi m_N}.$$

Boosting p' back to the lab frame, we get the target recoil energy:

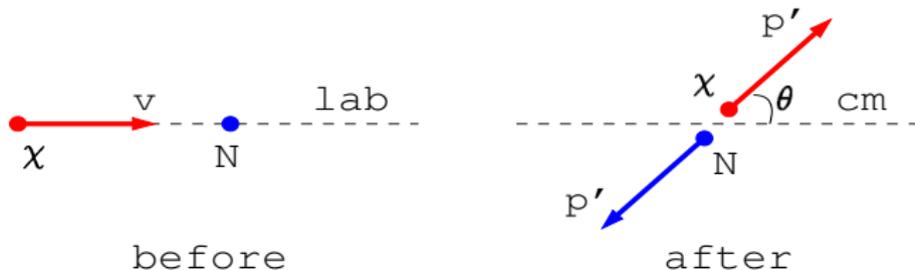
$$E_R = \frac{p_N^2}{2m_N} = \frac{\mu_N^2 v^2 (1 - \cos \theta)}{m_N} \simeq \frac{p_\chi^2}{m_N} (1 - \cos \theta) \sim \mathcal{O}(10) \text{ keV} (1 - \cos \theta).$$

Problem #16: What is the minimal WIMP velocity to produce a recoil at energy E_R ?

$$\cos \theta = -1 \Leftrightarrow v_{\min} = \sqrt{m_N E_R / (2\mu_N^2)}$$

Note that for $m_\chi \gg m_N$ v_{\min} is independent of m_χ .

III. DIRECT SEARCHES



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Let us get a bit more serious. The WIMP scattering rate off nuclei is

$$\frac{dR}{dE_R} = \frac{1}{m_N} \int_{v_{\min}}^{\infty} dv \frac{\rho_0 v}{m_\chi} f(\vec{v} + \vec{v}_e) \frac{d\sigma_{\chi-N}(v, E_R)}{dE_R}.$$

astrophysics

$$\rho_0 \sim 0.3 \text{ GeV/cm}^3$$

$$f(w) \propto \exp(-w^2/2\sigma^2)$$

nuclear physics

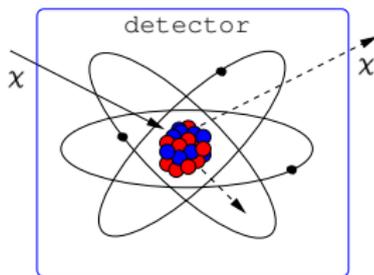
$$\sigma_{\text{SI}} \propto A^2$$

$$\sigma_{\text{SD}} \propto J/(J+1)$$

III. DIRECT SEARCHES

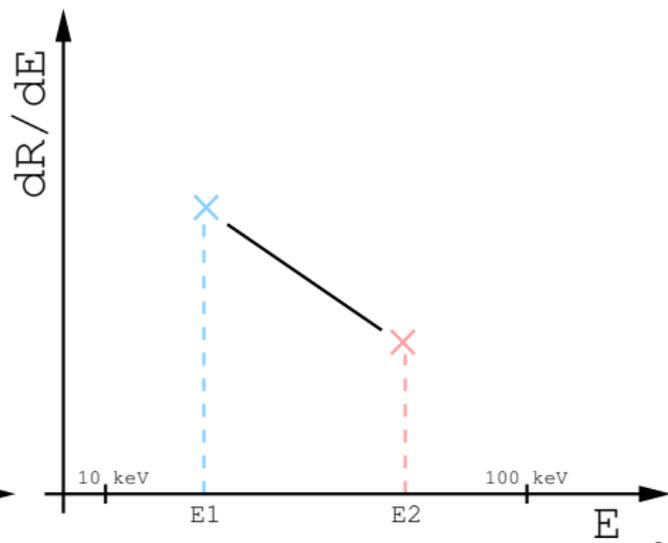
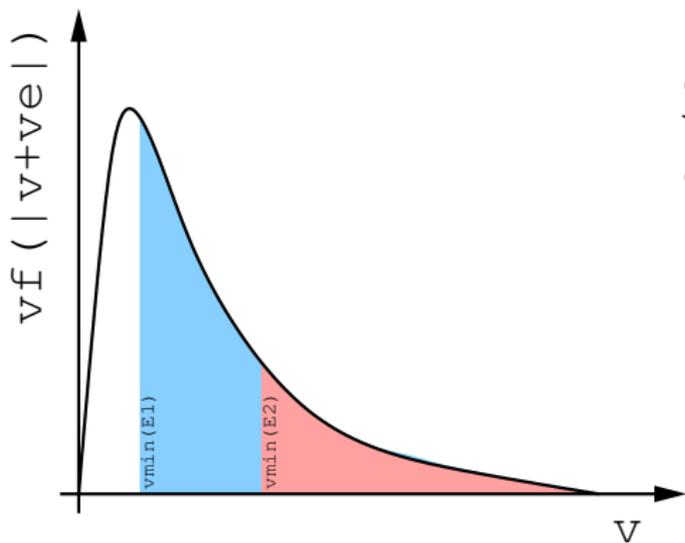
There are three main WIMP signatures:

1. exponential recoil spectrum



$$v_{\min} = \sqrt{\frac{m_N E_R}{2\mu_N^2}}$$

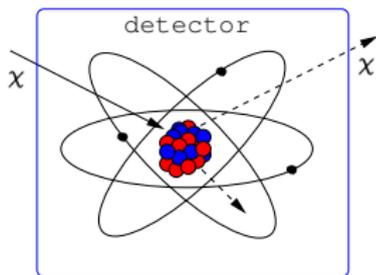
$$f(w) \propto \exp(-w^2/2\sigma^2)$$



III. DIRECT SEARCHES

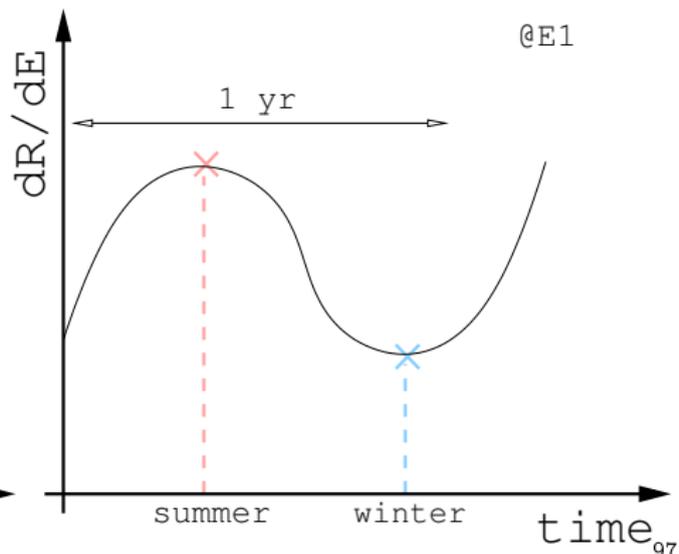
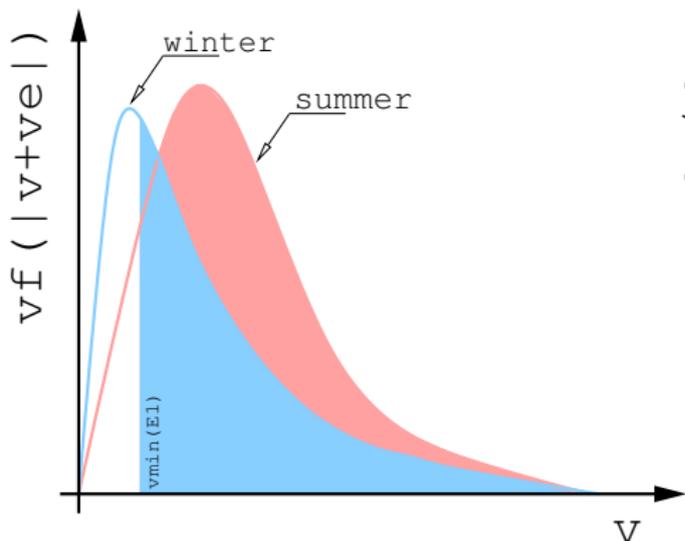
There are three main WIMP signatures:

1. exponential recoil spectrum
2. **annual modulation**



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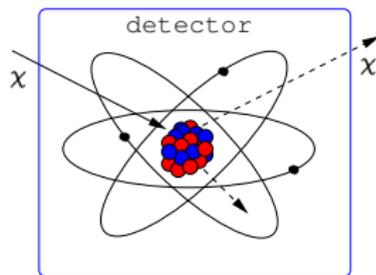
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III. DIRECT SEARCHES

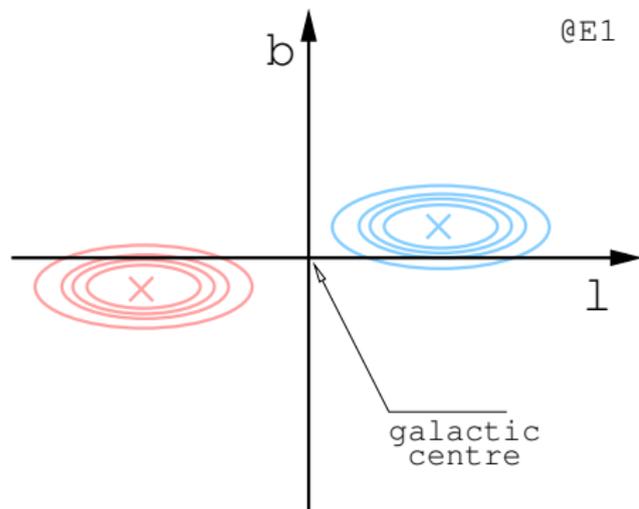
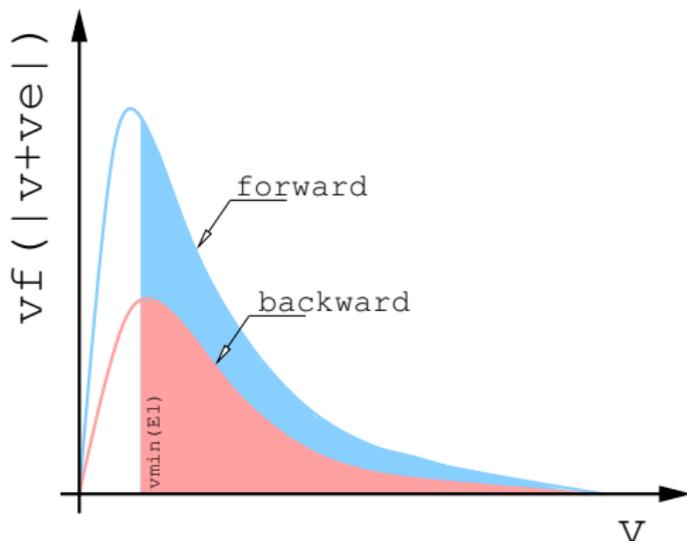
There are three main WIMP signatures:

1. exponential recoil spectrum
2. annual modulation
3. **directional pattern**



$$v_{\min} = \sqrt{\frac{m_N E_R}{2\mu_N^2}}$$

$$f(w) \propto \exp(-w^2/2\sigma^2)$$



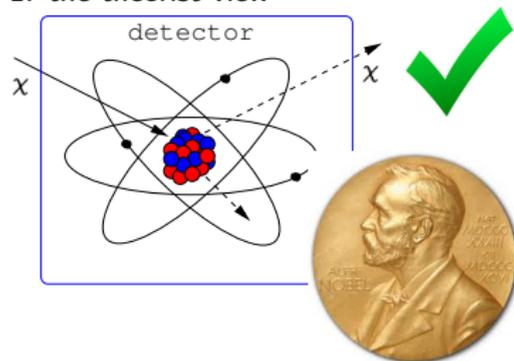
III. DIRECT SEARCHES

How can we detect these signatures?

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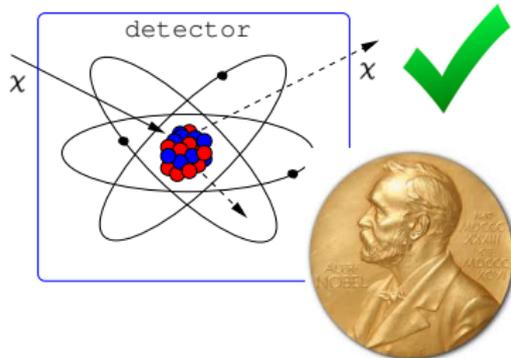
1. the theorist view



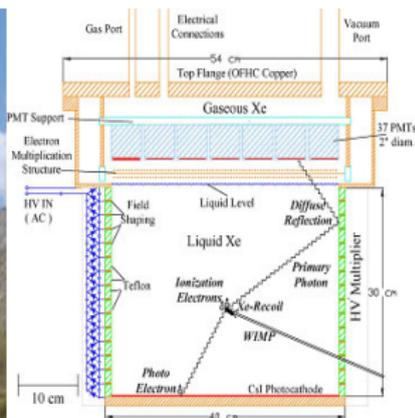
III. DIRECT SEARCHES

How can we detect these signatures?

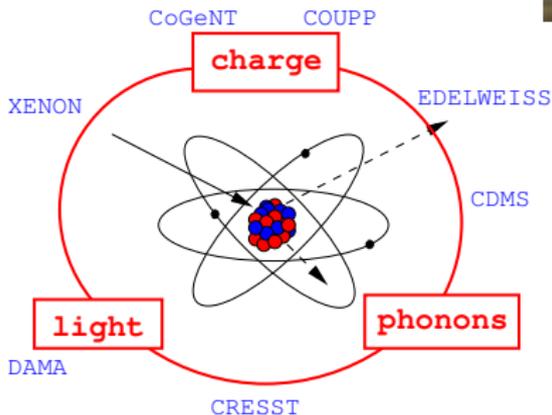
1. the theorist view



2. the experimentalist view (aka real world)



The recoils can be detected by scintillation (i.e. photons), ionisation (i.e. charge) and/or heat (i.e. phonons) triggered in the target material. Using two of these signals is crucial to discriminate background.



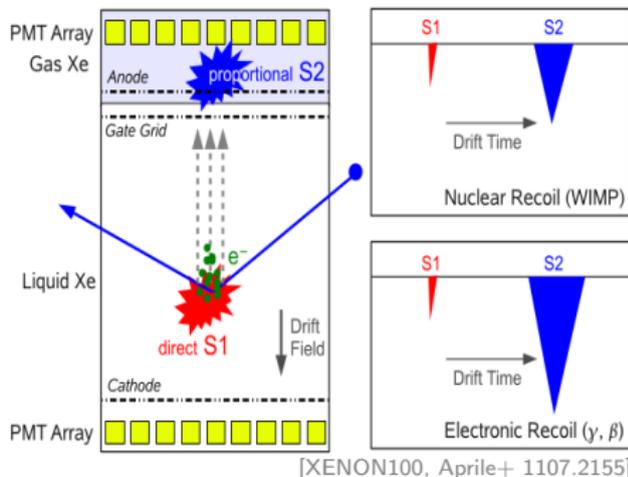
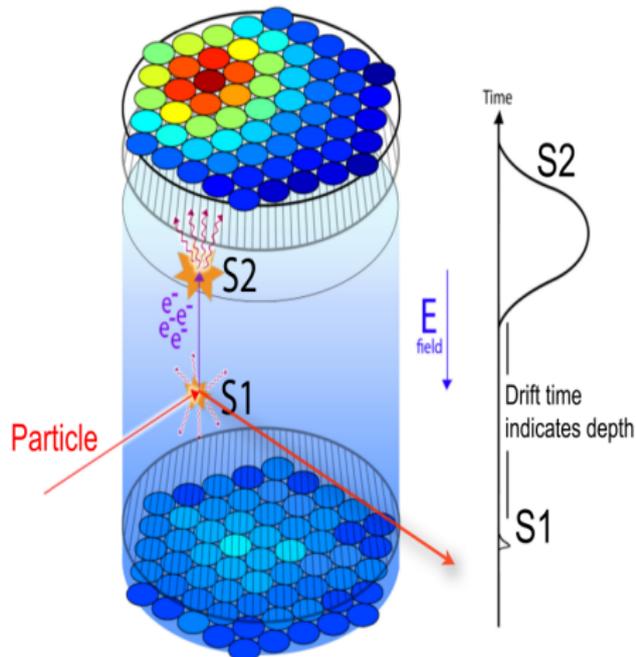
III. DIRECT SEARCHES

The list of direct detection experiments is very long...

Experiment	Location	Readout (γ, ϕ, q)	T (K)	M (kg)	Target	Search Dates
NAIAD	Boulby	γ	300	50	NaI	2001–2005
DAMA/NaI	Gran Sasso	γ	300	87	NaI	1995–2002
DAMA/LIBRA	Gran Sasso	γ	300	233	NaI	2003–
ANAIS	Canfranc	γ	300	11	NaI	2000–2005
ANAIS	Canfranc	γ	300	100	NaI	2011–
KIMS	Yangyang	γ	300	35	CsI	2006–2007
KIMS	Yangyang	γ	300	104	CsI	2008–
CDMS II	Soudan	ϕ, q	< 1	1	Si	2001–2008
				3	Ge	2001–2008
SuperCDMS	Soudan	ϕ, q	< 1	12	Ge	2010–2012
SuperCDMS	SNOLAB	ϕ, q	< 1	120	Ge	2013–2016
GEODM	DUSEL	ϕ, q	< 1	1200	Ge	2017–
EDELWEISS I	Modane	ϕ, q	< 1	1	Ge	2000–2004
EDELWEISS II	Modane	ϕ, q	< 1	4	Ge	2005–
CRESST II	Gran Sasso	ϕ, γ	< 1	1	CaWO ₄	2000–
EURECA	Modane	ϕ, q	< 1	50	Ge	2012–2017
		ϕ, γ	< 1	50	CaWO ₄	2012–2017
SIMPLE	Rustrel	Threshold	300	0.2	Freon	1999–
PICASSO	Sudbury	Threshold	300	2	Freon	2001–
COUPP	Fermilab	Threshold	300	2	Freon	2004–2009
COUPP	Fermilab	Threshold	300	60	Freon	2010–
TEXONO	Kuo-Sheng	$q, \beta\beta$	77	0.02	Ge	2006–
CoGeNT	Chicago	$q, \beta\beta$	77	0.3	Ge	2005–
	Soudan	$q, \beta\beta$	77	0.3	Ge	2008–
MAJORANA	Sanford	$q, \beta\beta$	77	60	Ge	2011–
ZEPLIN III	Boulby	γ, q	150	7	LXe	2004–
LUX	Sanford	γ, q	150	100	LXe	2010–
XMASS	Kamioko	γ, q	150	3	LXe	2002–2004
XMASS	Kamioko	γ, q	150	100	LXe	2010–
XENON10	Gran Sasso	γ, q	150	5	LXe	2005–2007
XENON100	Gran Sasso	γ, q	150	50	LXe	2009–
WArP	Gran Sasso	γ, q	86	3	LAr	2005–2007
WArP	Gran Sasso	γ, q	86	140	LAr	2010–
ArDM	CERN	γ, q	86	850	LAr	2009–
DEAP-1	SNOLAB	γ	86	7	LAr	2008–
MiniCLEAN	SNOLAB	γ	86	150	LAr	2012–
DEAP-3600	SNOLAB	γ	86	1000	LAr	2013–
DRIFT-1	Boulby	Direction	300	0.17	CS ₂	2002–2005
DRIFT-2	Boulby	Direction	300	0.34	CS ₂	2005–
NEWAGE	Kamioko	Direction	300	0.01	CF ₄	2008–
MIMAC	Saclay	Direction	300	0.01	many	2006–
DMTPC	MIT	Direction	300	0.01	CF ₄	2007–

III. DIRECT SEARCHES

Liquid xenon experiments are designed to detect a primary scintillation signal, S1, and a secondary ionisation signal, S2. The ratio $S2/S1$ gives a good discriminant of electronic recoils, the main background in the search for WIMPs.



→ ionization electrons

→ UV scintillation photons (~175 nm)

[LUX, Phelps APS 2013]

III. DIRECT SEARCHES

What have we detected (or not) so far?

First off, direct searches have not found dark matter yet (at least beyond doubt).
There are however several hints of signal...

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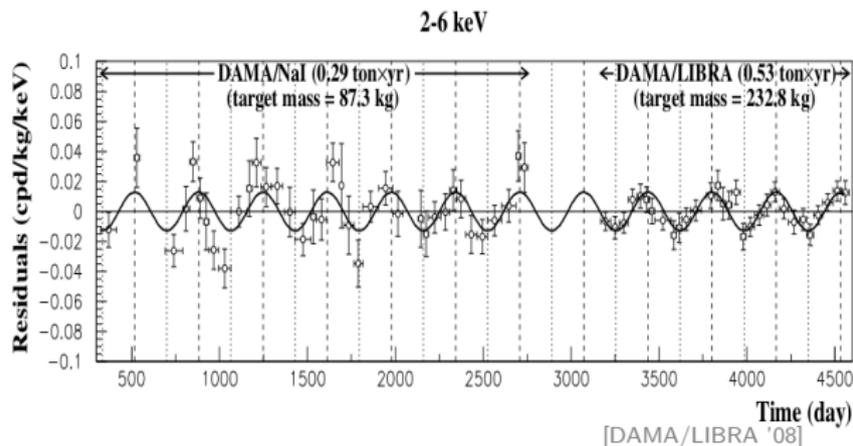
DAMA/LIBRA

250 kg NaI

0.82 ton.yr

$E_{\text{thr}} \simeq 2 \text{ keV}$

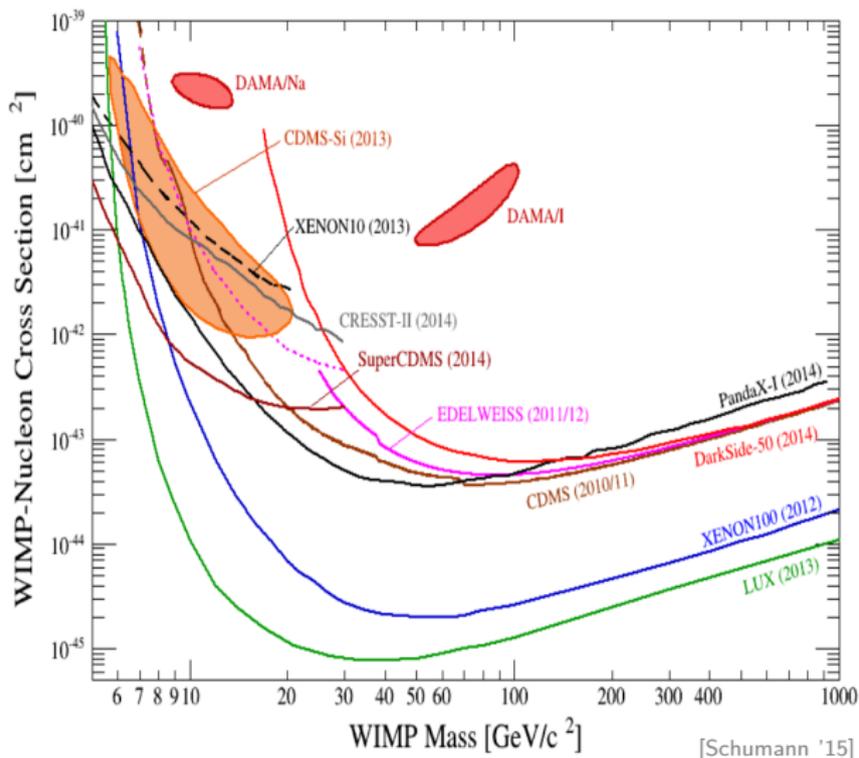
scintillation



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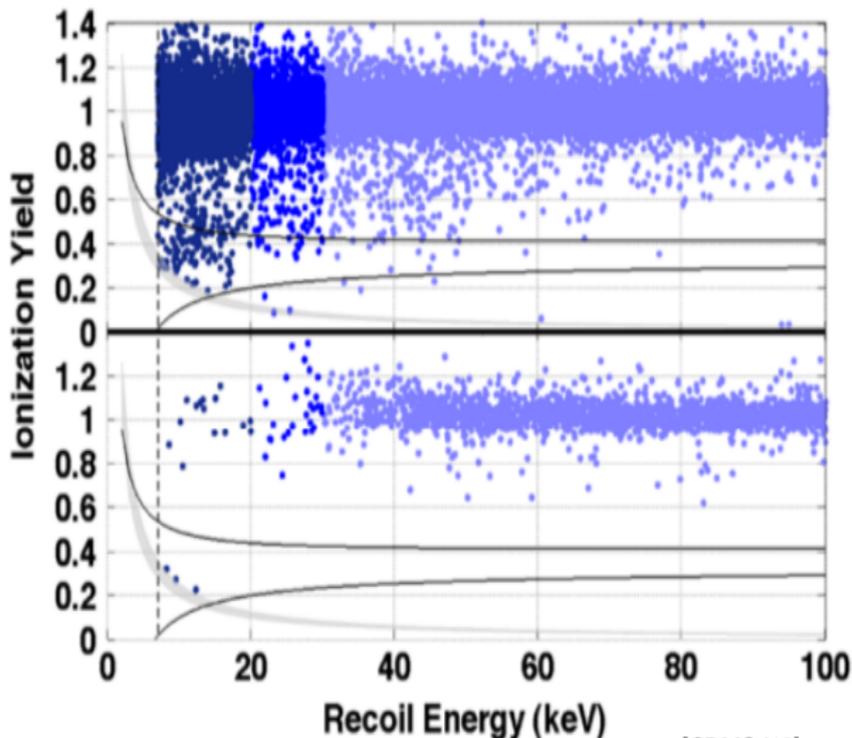
CDMS-Si

4.6 kg Ge, 1.2 kg Si

140.2 kd.day (Si)

$E_{\text{thr}} \simeq 7 \text{ keV}$

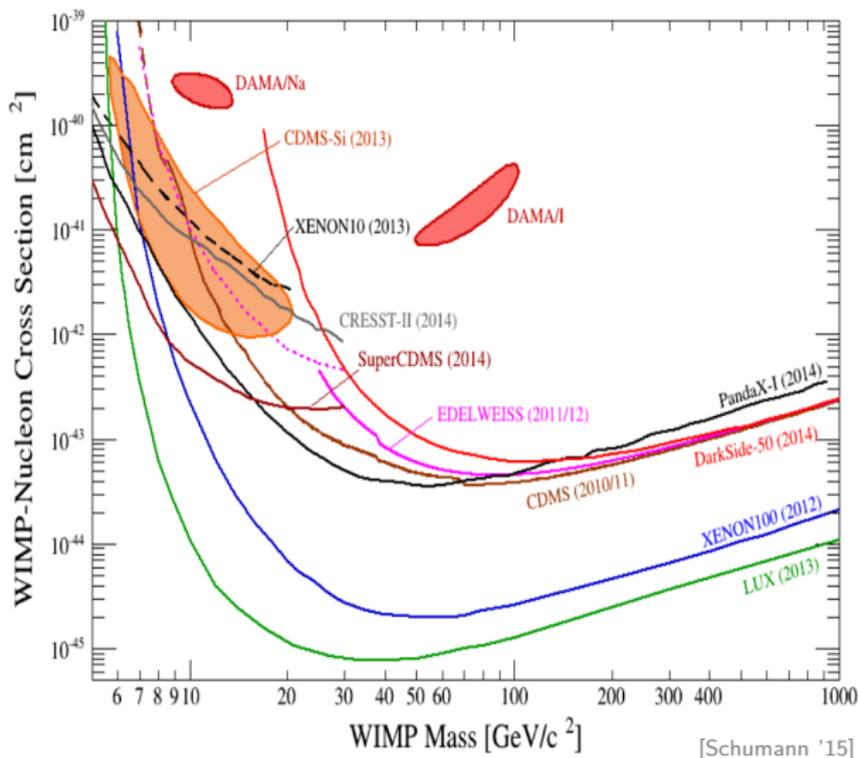
ionisation, heat



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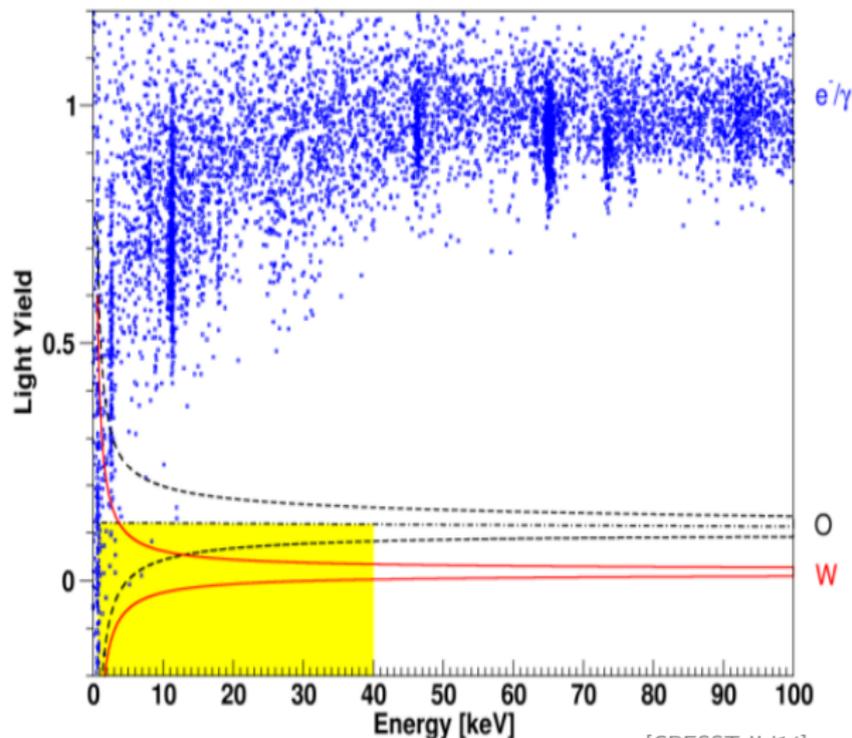
CRESST-II

CaWO_4

29.35 kg.day

$E_{\text{thr}} \simeq 1 \text{ keV}$

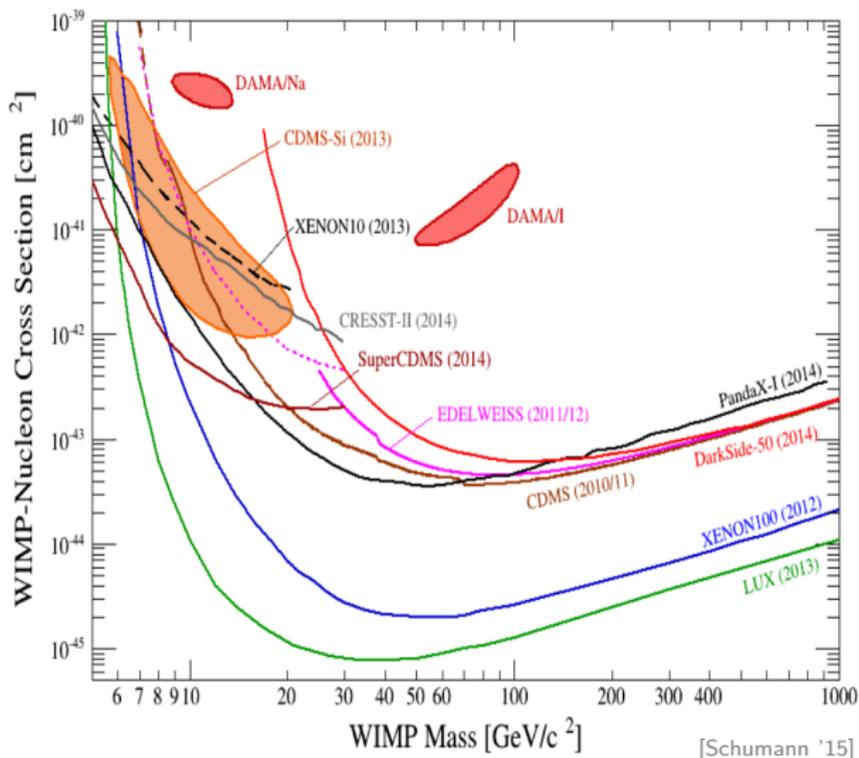
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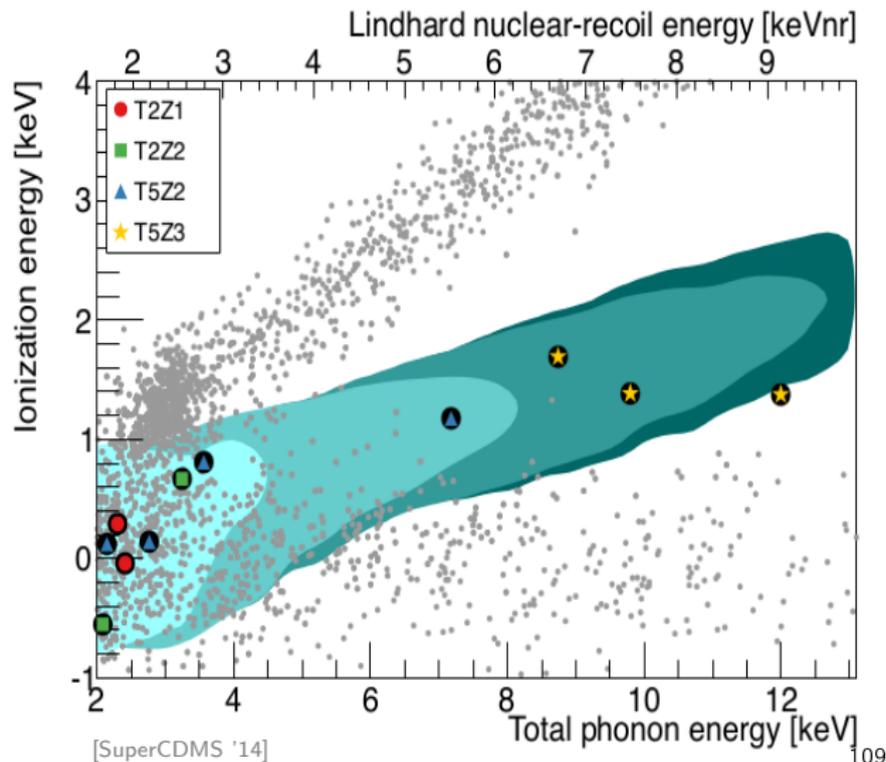
SuperCDMS

9 kg Ge

577 kg.day

$E_{\text{thr}} \simeq 1.6$ keV

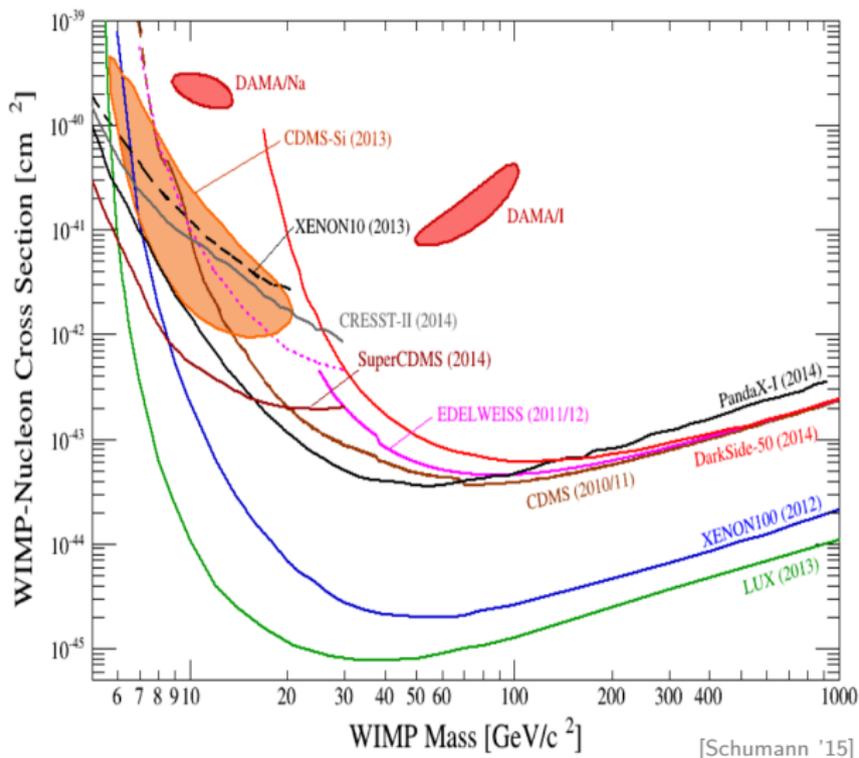
ionisation, heat



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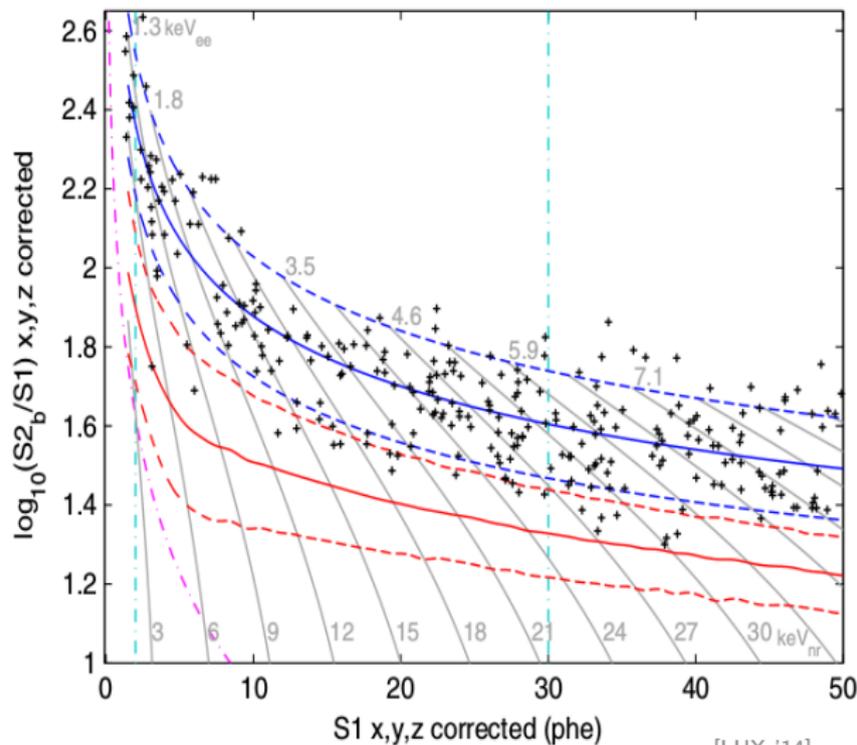
LUX

250 kg LXe

10.1 ton.day

$E_{\text{thr}} \simeq 3 \text{ keV}$

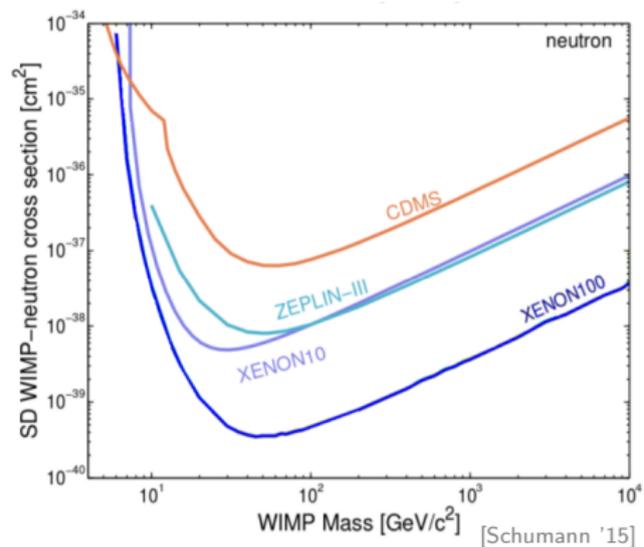
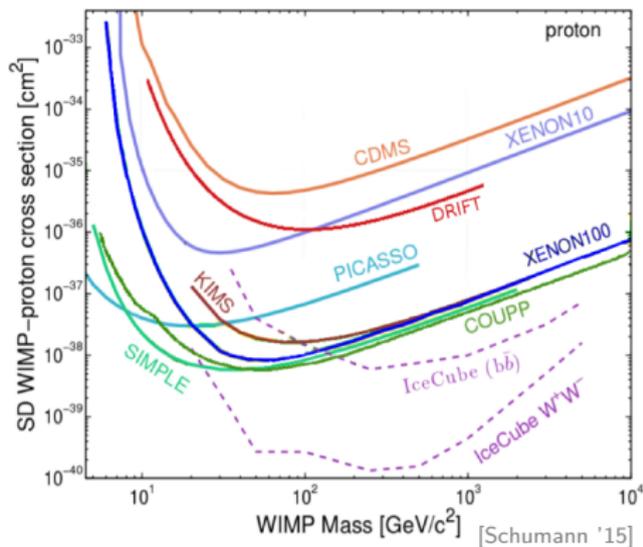
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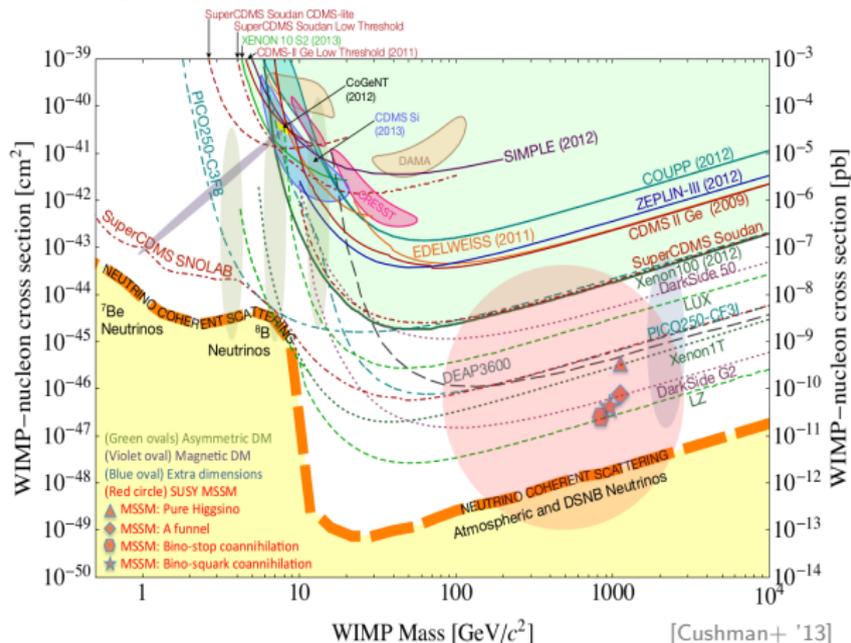


III. DIRECT SEARCHES

Summing up:

1. the hints are very difficult to interpret as WIMP scattering; and
2. the upper limits start to probe many viable WIMP candidates.

Is the WIMP paradigm dead? Not yet.



When can we declare WIMPs dead? Hard to say. But if the next generation of experiments finds nothing, then WIMPs will decline as a dark matter candidate.

III. DARK MATTER SEARCHES

This sets the stage for a three-prong strategy for WIMP searches.

direct searches

WIMP scattering off nuclei underground

indirect searches

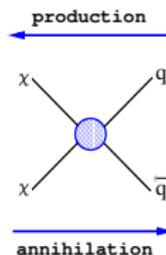
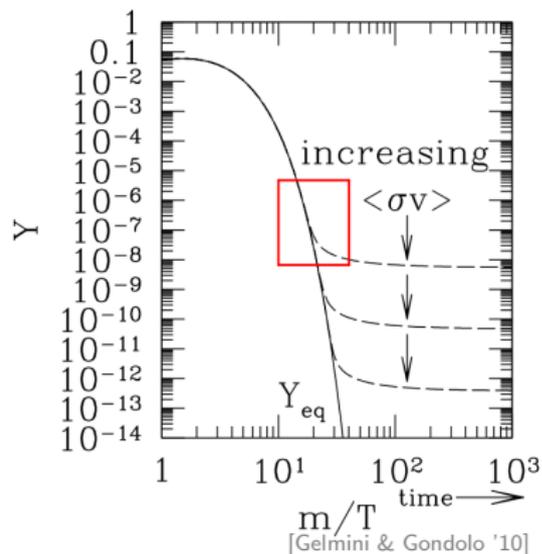
yields of WIMP annihilation or decay

WIMP production in the lab

collider searches

III. INDIRECT SEARCHES

The idea behind indirect searches is intimately related to the freeze-out process.

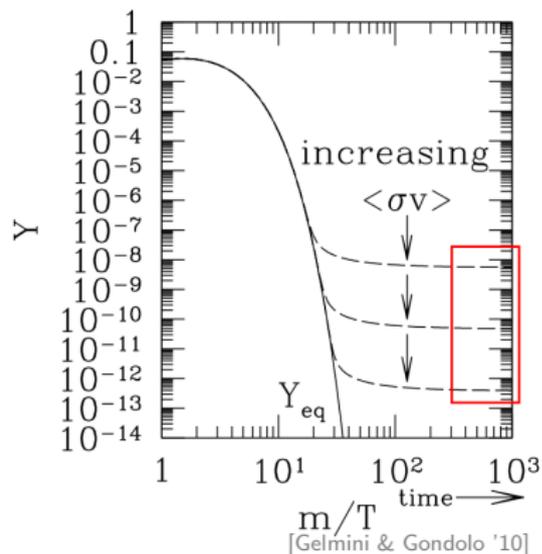


early universe

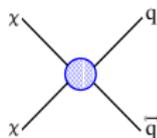
$$\frac{dn_X}{dt} + 3Hn_X = \langle \sigma v \rangle (n_{X,eq}^2 - n_X^2)$$

III. INDIRECT SEARCHES

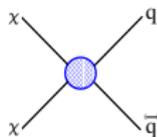
The idea behind indirect searches is intimately related to the freeze-out process.



production



annihilation



annihilation

early universe

$$\frac{dn_X}{dt} + 3Hn_X = \langle \sigma v \rangle (n_{X,eq}^2 - n_X^2)$$

today

$$\frac{dn_X}{dt} = -\langle \sigma v \rangle_0 n_X^2$$

It is thus legitimate to ask if we can detect the annihilation in the present universe.

Problem #17: Compute the spectrum of X particles produced at the annihilation site.

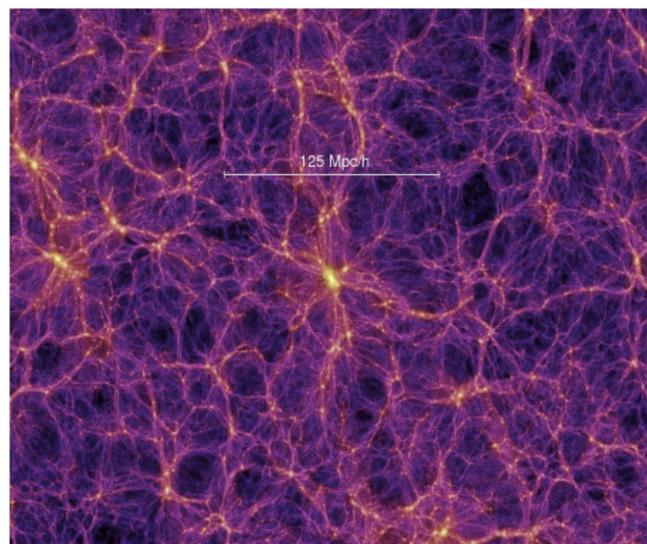
$$\frac{d^2 n_X}{dE_X dt} = \langle \sigma v \rangle_0 \frac{\rho_X^2}{2m_X^2} \times \frac{dN_X}{dE_X}$$

Problem #18: Repeat #17 for a dark matter decay.

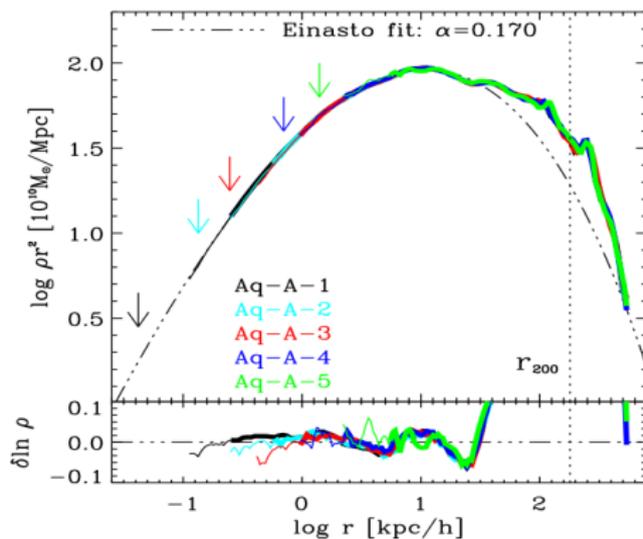
III. INDIRECT SEARCHES

$$\frac{d^2 n_X}{dE_X dt} = \frac{\langle \sigma v \rangle_0}{2m_X^2} \frac{dN_X}{dE_X} \times \rho_X^2 \quad \text{astrophysics}$$

As we have seen in Part I, it is hard to infer the dark matter density distribution to a high degree of precision. A precious help comes from N-body simulations that can follow the evolution of structures of all scales in the universe.



[Millennium simulation]



[Navarro+ '10]

Disclaimer: these are dark-matter-only simulations. Simulations including baryons do exist, but the effect of baryons is still not fully understood.

III. INDIRECT SEARCHES

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Navarro-Frenk-White

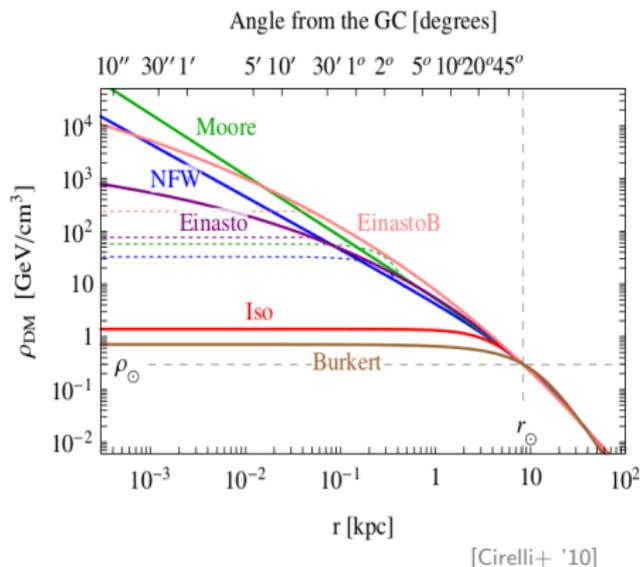
$$\rho_X \propto \frac{1}{(r/r_s)(1+r/r_s)^2}$$

Einasto

$$\rho_X \propto \exp[-2((r/r_s)^\alpha - 1)/\alpha]$$

cored isothermal

$$\rho_X \propto \frac{1}{1+(r/r_s)^2}$$



III. INDIRECT SEARCHES

$$\frac{d^2 n_X}{dE_X dt} = \frac{\langle \sigma v \rangle_0}{2m_X^2} \frac{dN_X}{dE_X} \times \rho_X^2 \quad \text{particle physics}$$

There are three main annihilation or decay products to search for:

photons
(all wavelengths)

EGRET
Fermi-LAT
H.E.S.S.
MAGIC
EUVE
SAX

cosmic rays
(mainly antimatter)

PAMELA
ATIC
BESS
AMS-02

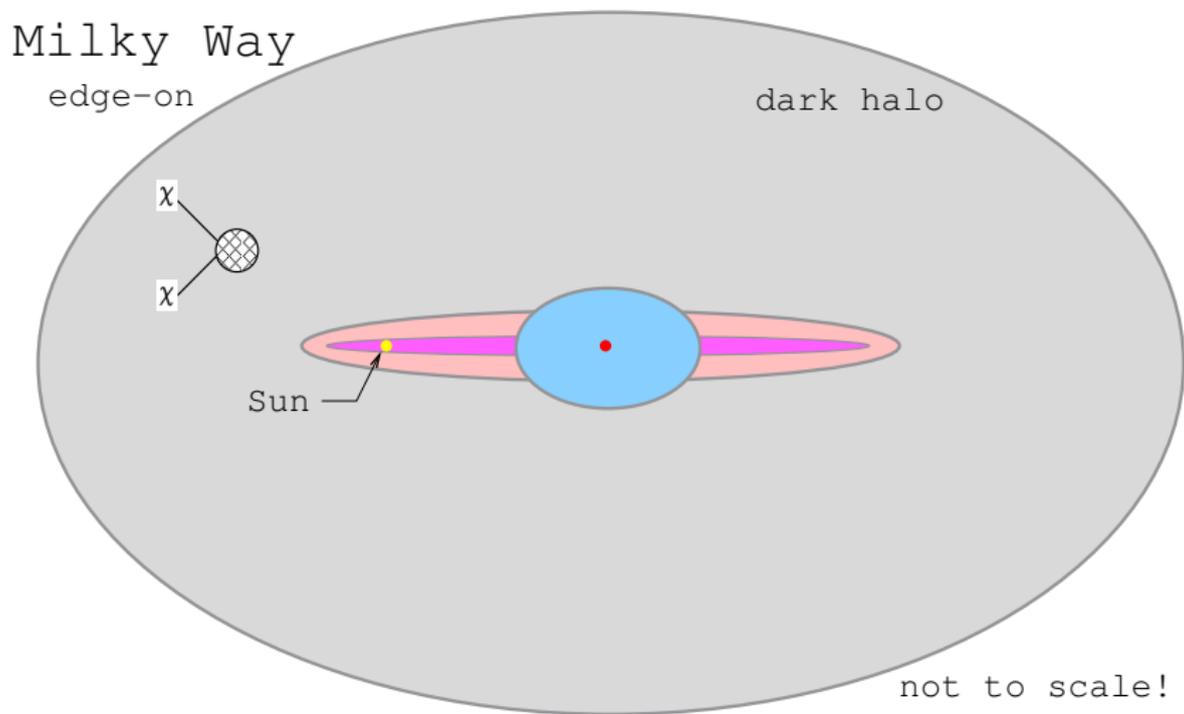
neutrinos
(and antineutrinos)

SuperK
IceCube
Antares

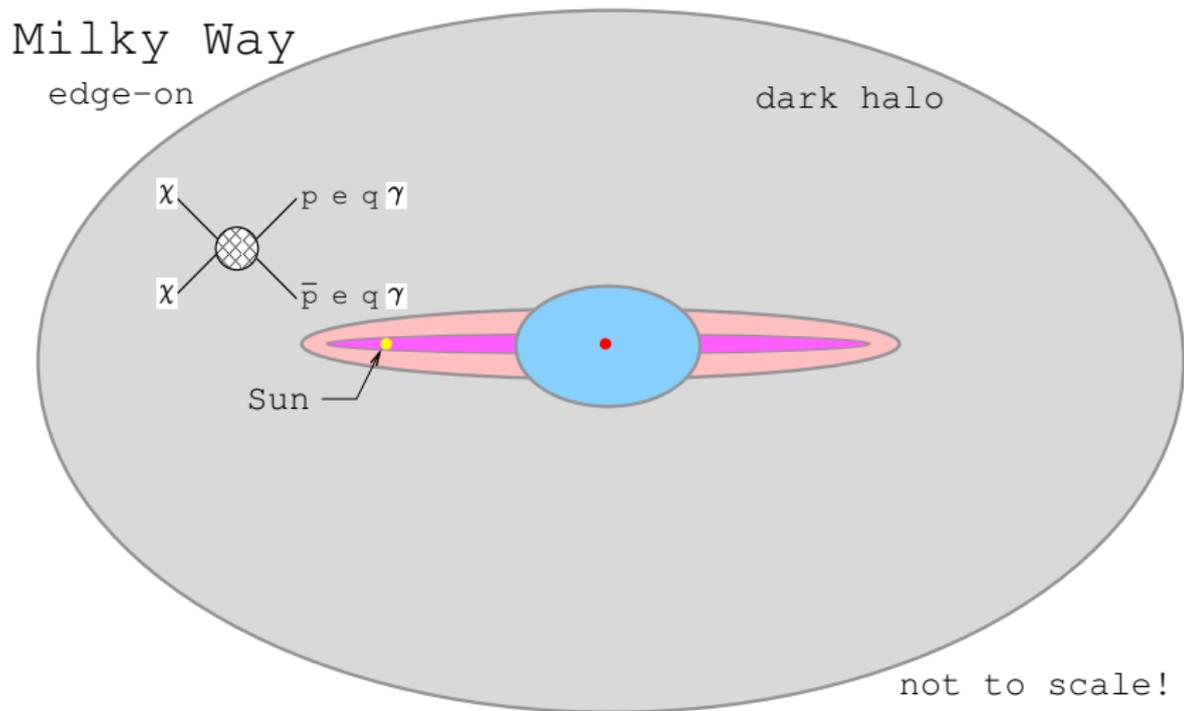
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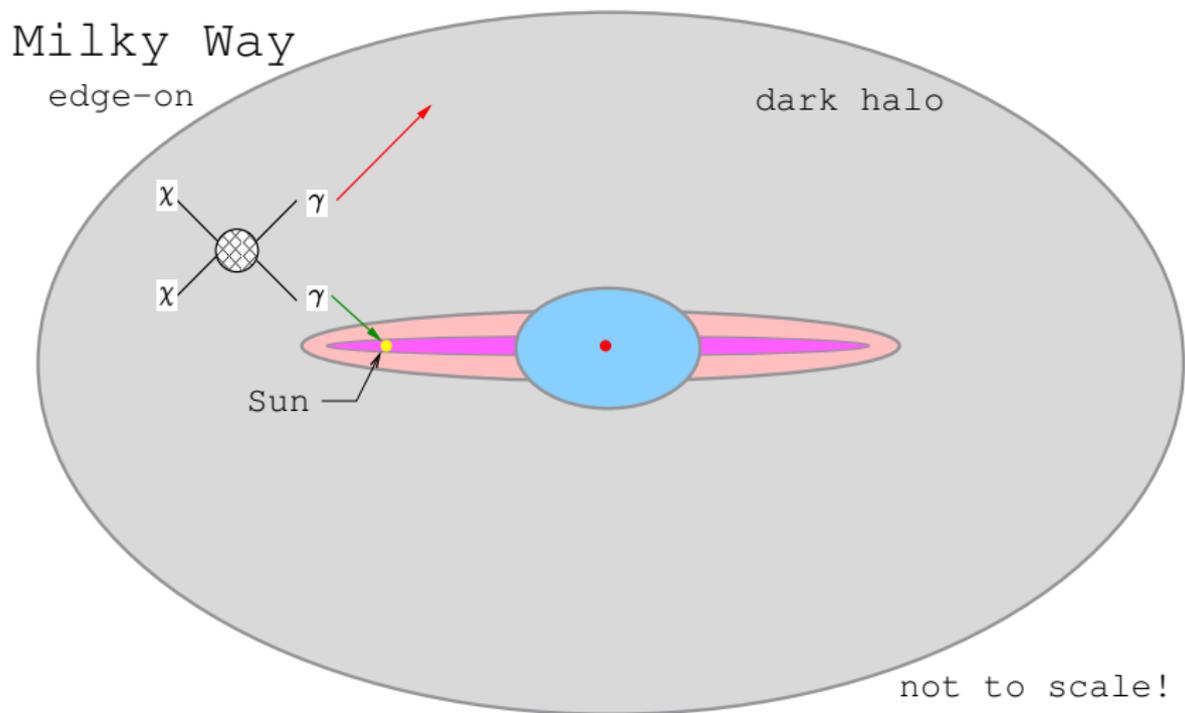
III. INDIRECT SEARCHES



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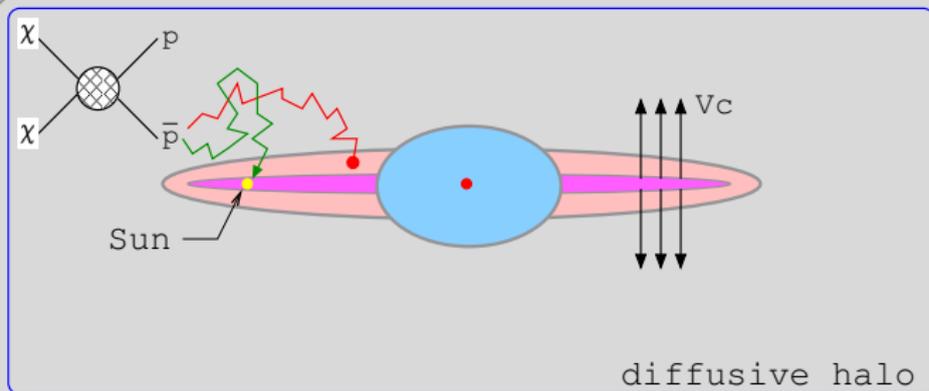


III. INDIRECT SEARCHES

Milky Way

edge-on

dark halo



diffusive halo

not to scale!

III. INDIRECT SEARCHES

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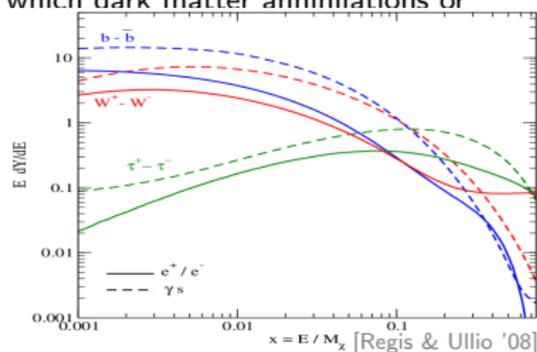
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There are three possible mechanisms through which dark matter annihilations or decays yield photons:

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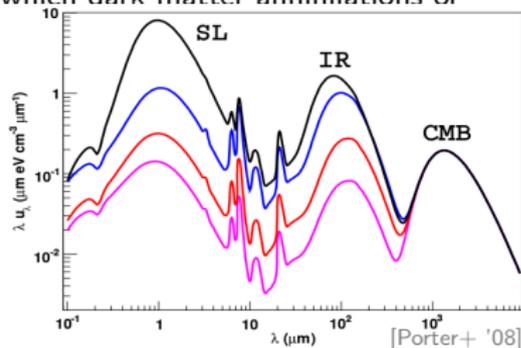
1. prompt emission
 $\chi(\chi) \rightarrow q, \ell \rightarrow \gamma$



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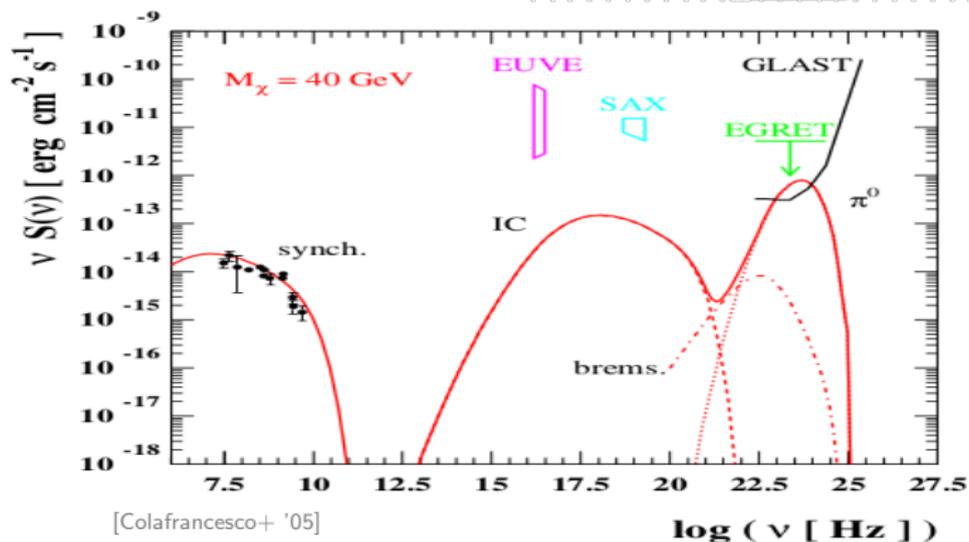
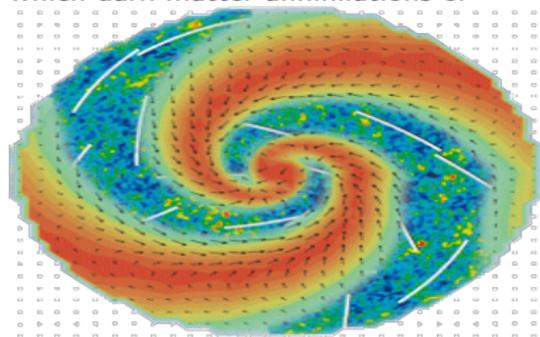
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 $\chi(\chi) \rightarrow e^{\pm}; e^{\pm}\gamma_{\text{bkg}} \rightarrow e^{\pm}\gamma_{\text{HE}}$



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2. inverse Compton scattering
 $\chi(\chi) \rightarrow e^{\pm}; e^{\pm} \gamma_{\text{bkg}} \rightarrow e^{\pm} \gamma_{\text{HE}}$
3. synchrotron emission
 $\chi(\chi) \rightarrow e^{\pm}; e^{\pm} \vec{B} \rightarrow e^{\pm} \gamma$



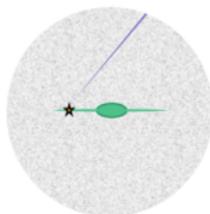
III. INDIRECT SEARCHES

The prompt emission is a particularly interesting channel since it tracks the distribution of dark matter directly. Plus, for the energy of interest (1 GeV–10 TeV) photons are not absorbed and travel in straight lines across the universe.

Problem #19: (a) Consider a far-away galaxy at distance d (but redshift $z \simeq 0$) with a given distribution of dark matter ρ_χ . Compute the photon flux at Earth induced by dark matter annihilation.

$$\frac{d\phi_\gamma}{dE_\gamma} = \frac{1}{4\pi d^2} \int_{\text{gal}} dV \frac{d^2 n_\gamma}{dE_\gamma dt} = \frac{\langle \sigma v \rangle_0}{8\pi m_\chi^2} \frac{dN_\gamma}{dE_\gamma} \frac{1}{d^2} \int_{\text{gal}} dV \rho_\chi^2$$

(b) Now do the same for our Galaxy along a given line of sight.



$$\frac{d\phi_\gamma}{dE_\gamma} = \frac{1}{4\pi l^2} dl dS \frac{d^2 n_\gamma}{dE_\gamma dt}, \quad dS = l^2 d\Omega$$

$$\frac{d^2 \phi_\gamma}{dE_\gamma d\Omega} = \frac{\langle \sigma v \rangle_0}{8\pi m_\chi^2} \frac{dN_\gamma}{dE_\gamma} \underbrace{\int_{\text{los}} dl \rho_\chi^2}_{\text{J-factor}}$$

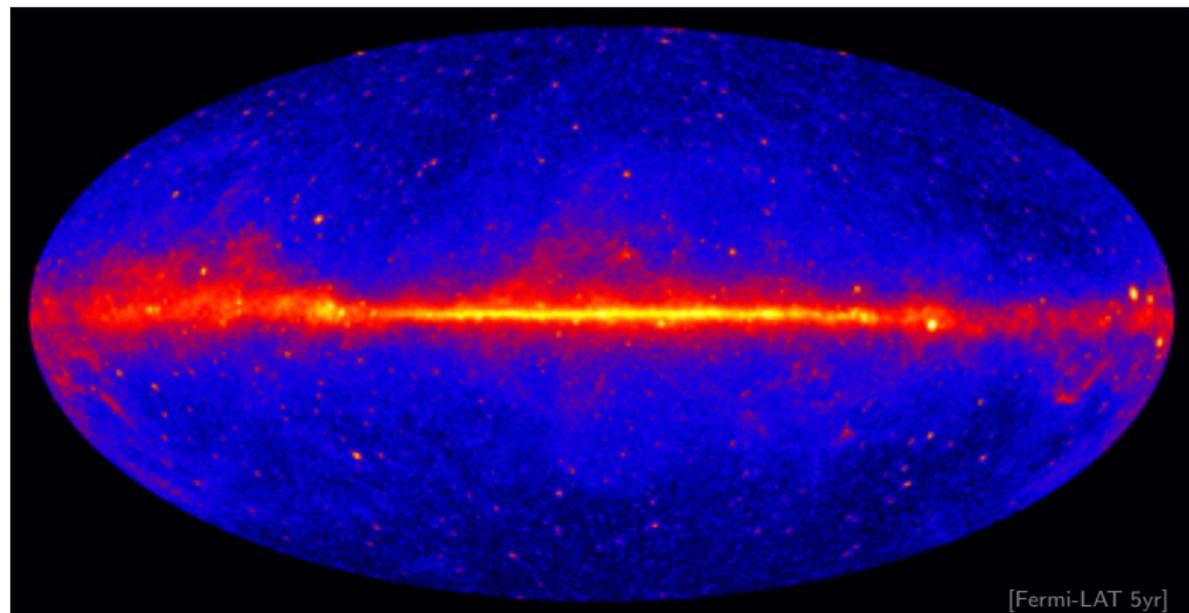
Problem #20: Repeat #19 for decaying dark matter.

$$\frac{d^2 \phi_\gamma}{dE_\gamma d\Omega} = \frac{\Gamma}{4\pi m_\chi} \frac{dN_\gamma}{dE_\gamma} \int_{\text{los}} dl \rho_\chi$$

Ok, this was all very neat, but where should we look at to find these gamma rays?

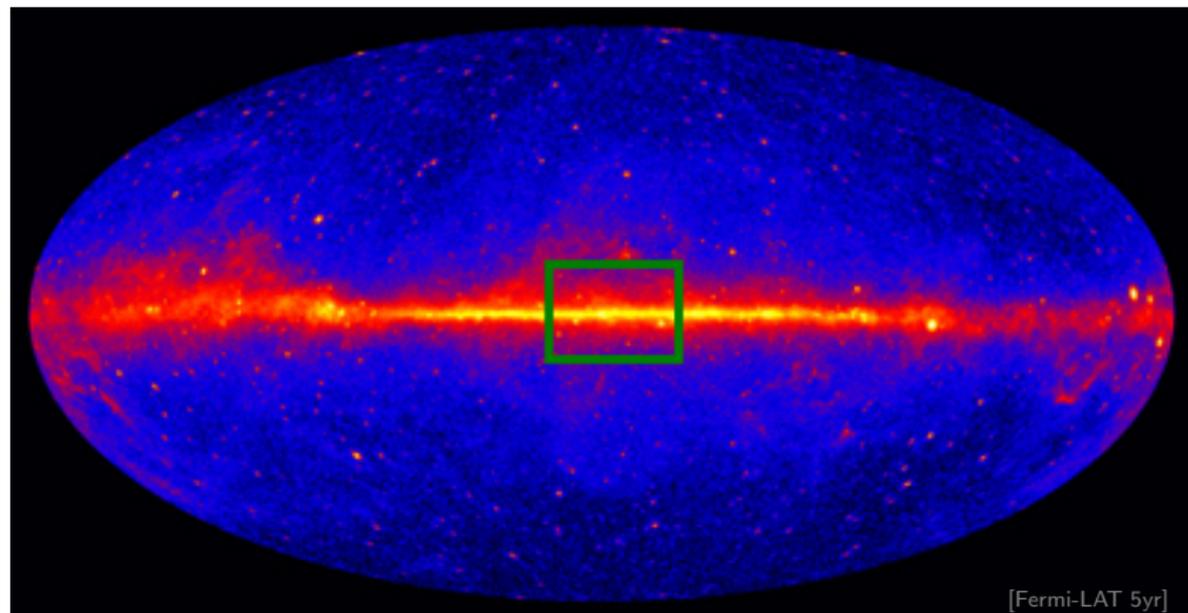
III. INDIRECT SEARCHES

The best target should meet two conditions: high J-factor *and* low background.



III. INDIRECT SEARCHES

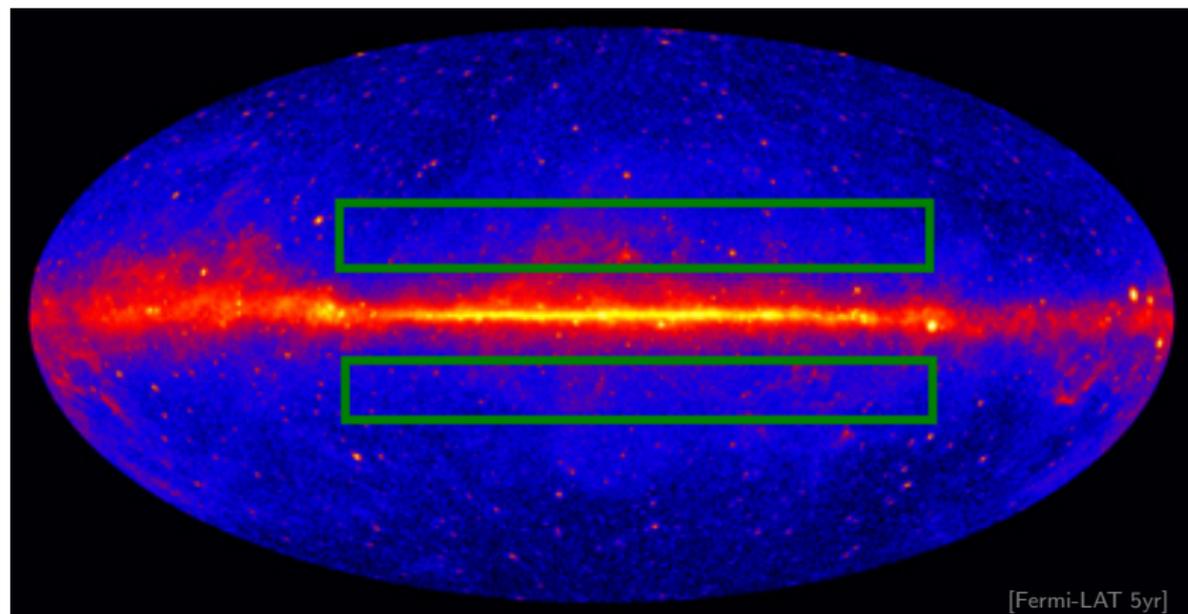
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galactic centre
very high J
high bkg

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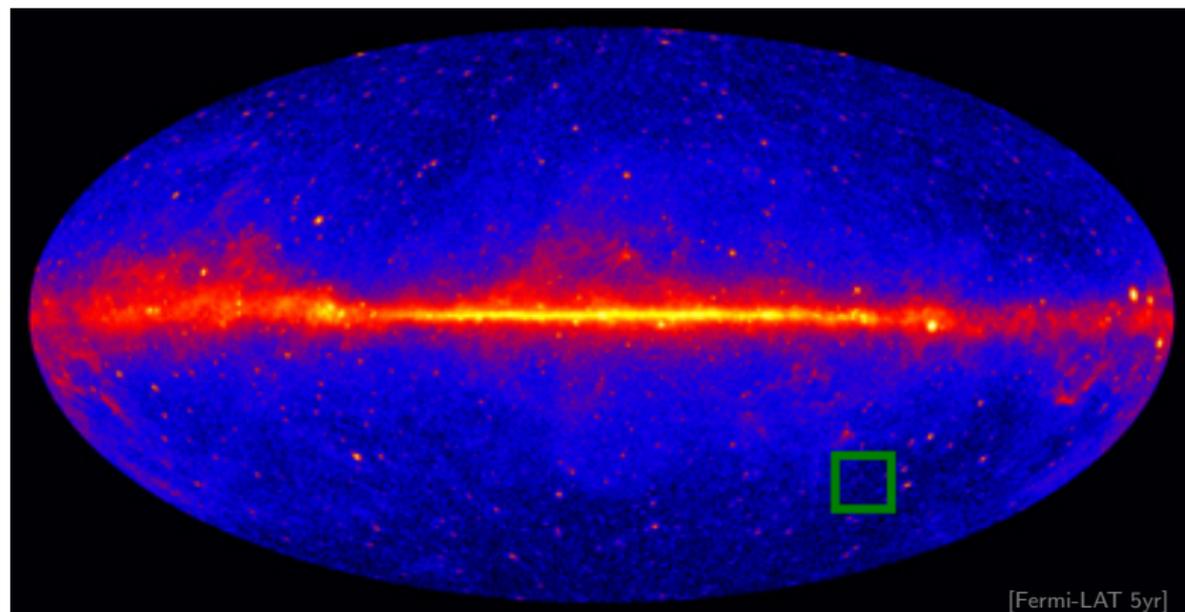


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galactic halo
medium J
low bkg

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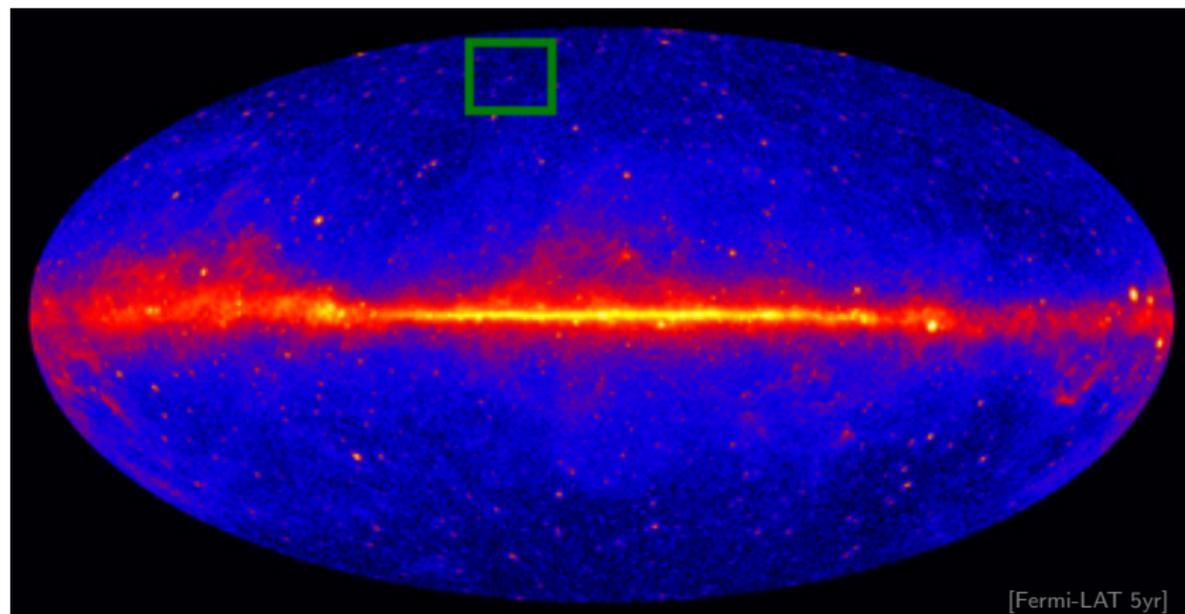
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high bkg

galactic halo
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satellites
high J
very low bkg

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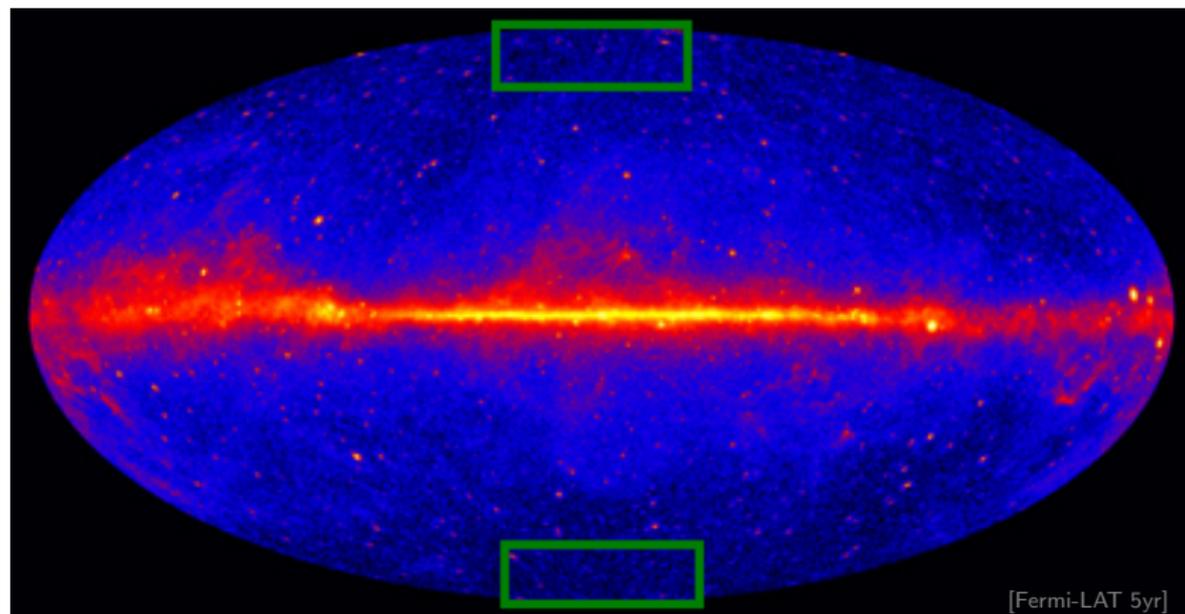
galactic halo
medium J
low bkg

satellites
high J
very low bkg

galaxy clusters
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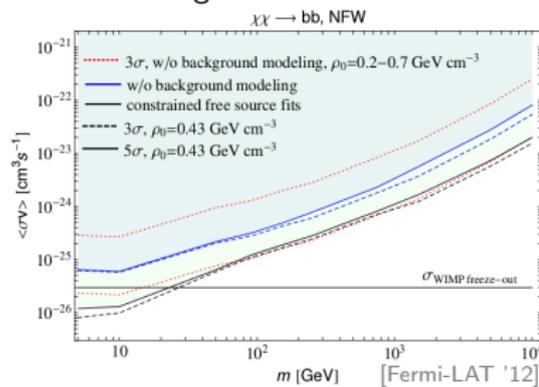
galaxy clusters
high J
high bkg

extragalactic
medium J
medium bkg

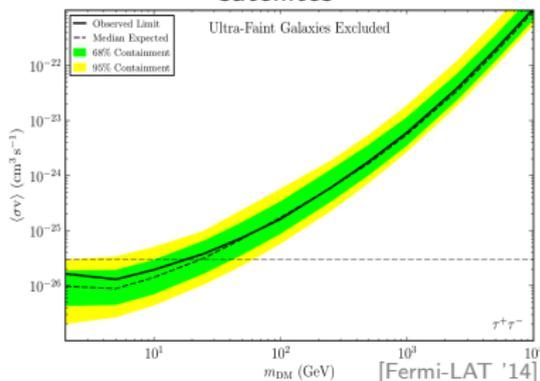
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Up to now no dark matter signal has been firmly detected in indirect searches. The upper limits imposed by the non-observation of a signal are very powerful, though.

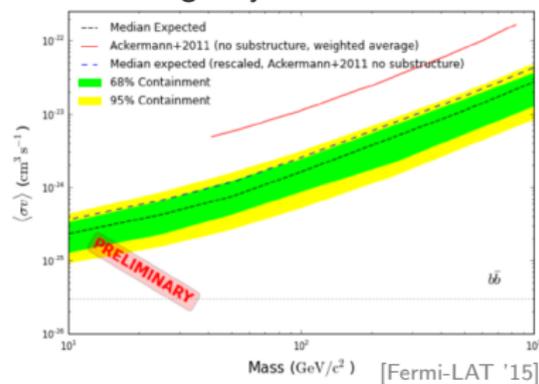
galactic halo



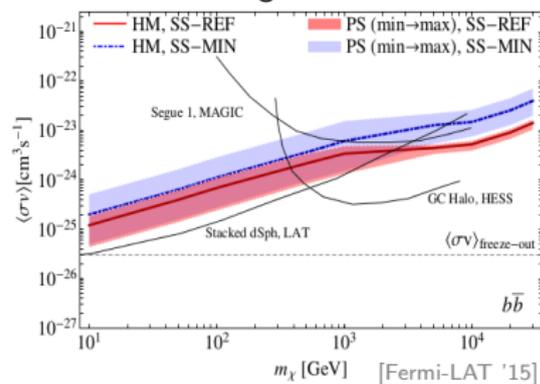
satellites



galaxy clusters

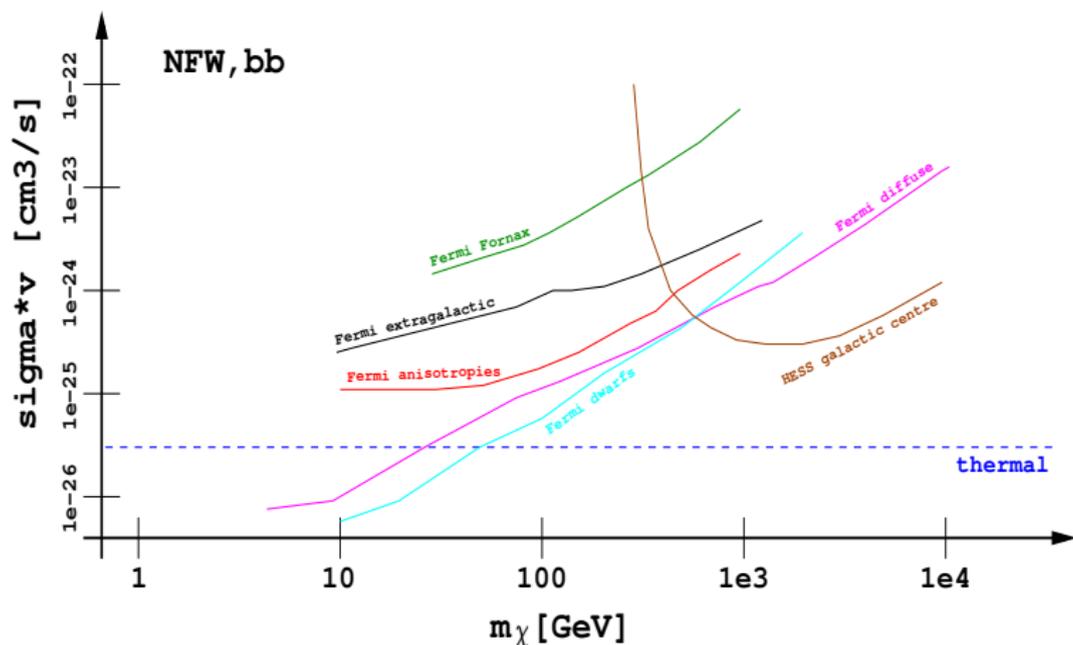


extragalactic



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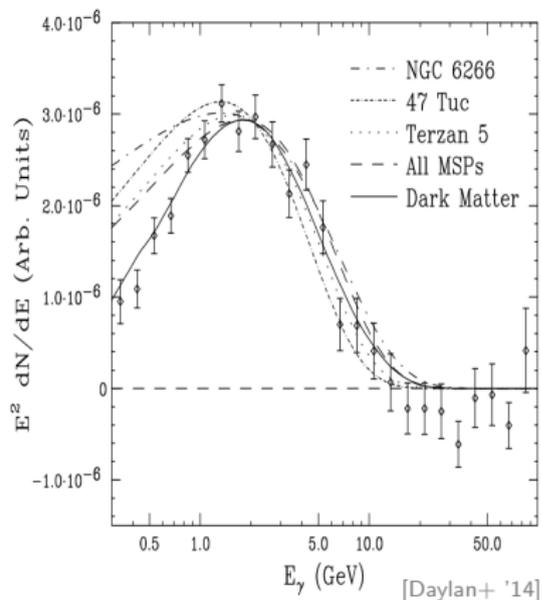
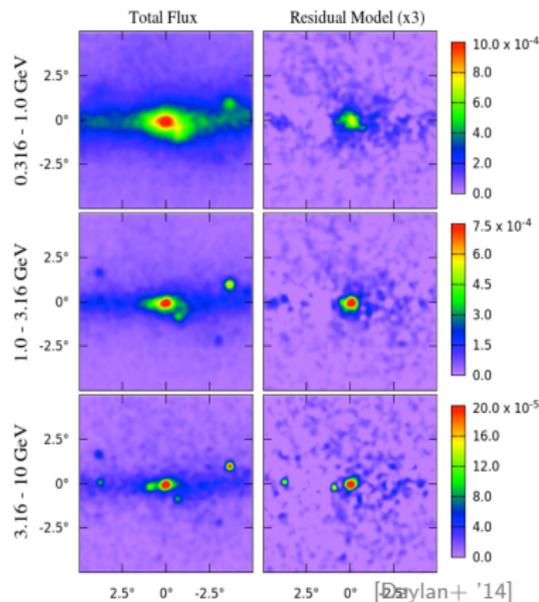


For certain final states and certain profiles, the limits are touching the “WIMP miracle” and putting lower limits on the mass, of order 100 GeV. After decades of theoretical speculation, this means we are living exciting times!

As in direct searches, there are also hints of a signal...

III. INDIRECT SEARCHES

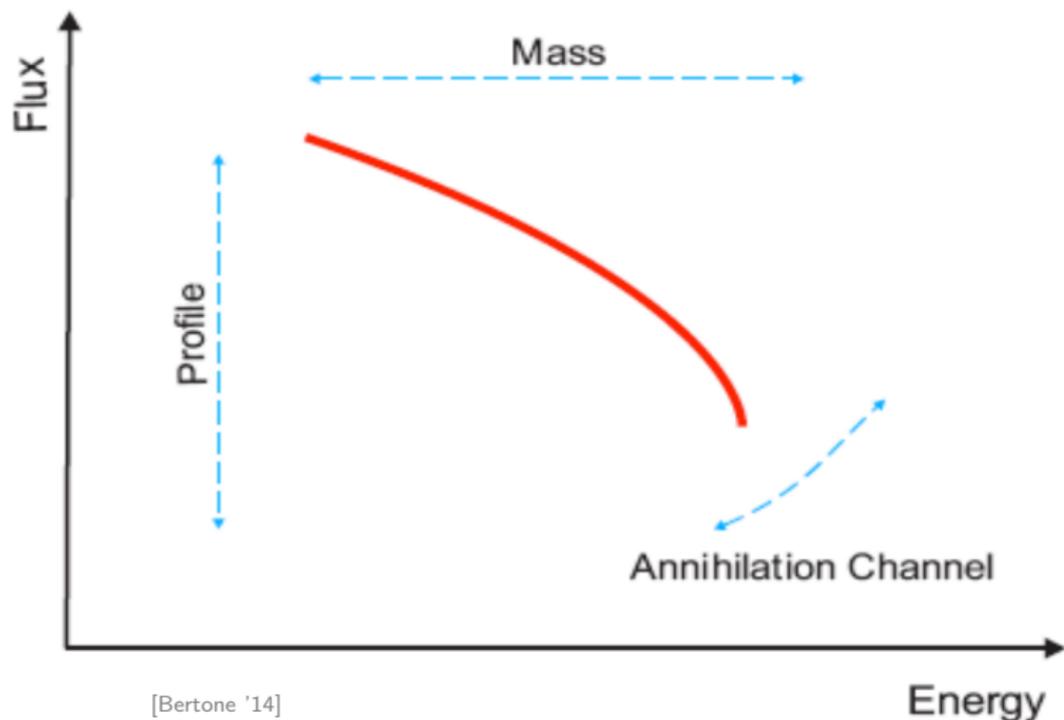
... in the galactic centre.



This is a topic under intense discussion right now. Only time (and data) will tell if this is a false alarm or not!

III. INDIRECT SEARCHES

But be advised: more than predictive, prompt emission from dark matter annihilation or decay is flexible enough to fit (almost) any signal!



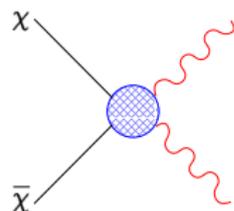
[Bertone '14]

We definitely need smoking-gun signatures before we can even dream to claim a dark matter discovery in indirect searches.

III. INDIRECT SEARCHES

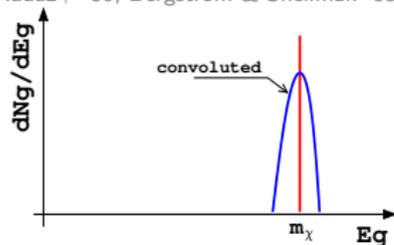
Fortunately, there are such smoking-gun signatures: gamma-ray spectral features.

1 lines

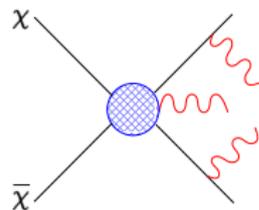


$$\frac{dN_\gamma}{dE_\gamma} \propto \delta(E_\gamma - m_\chi)$$

[Srednicki+ '86, Rudaz+ '86, Bergström & Snellman '88]



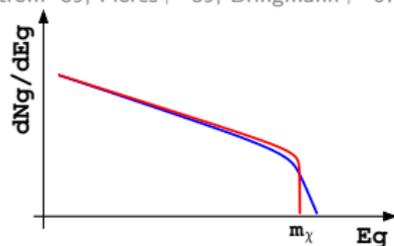
2 internal bremsstrahlung



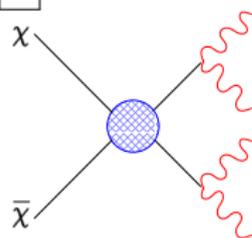
$$\frac{dN_\gamma}{dE_\gamma} \propto E_\gamma^{-1}$$

"hard" in astroph

[Bergström '89, Flores+ '89, Bringmann+ '07]

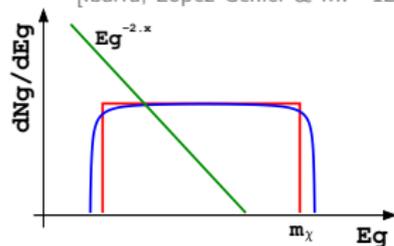


3 boxes



$$\frac{dN_\gamma}{dE_\gamma} \propto \text{const}$$

[Ibarra, López Gehler & MP '12]



III. INDIRECT SEARCHES

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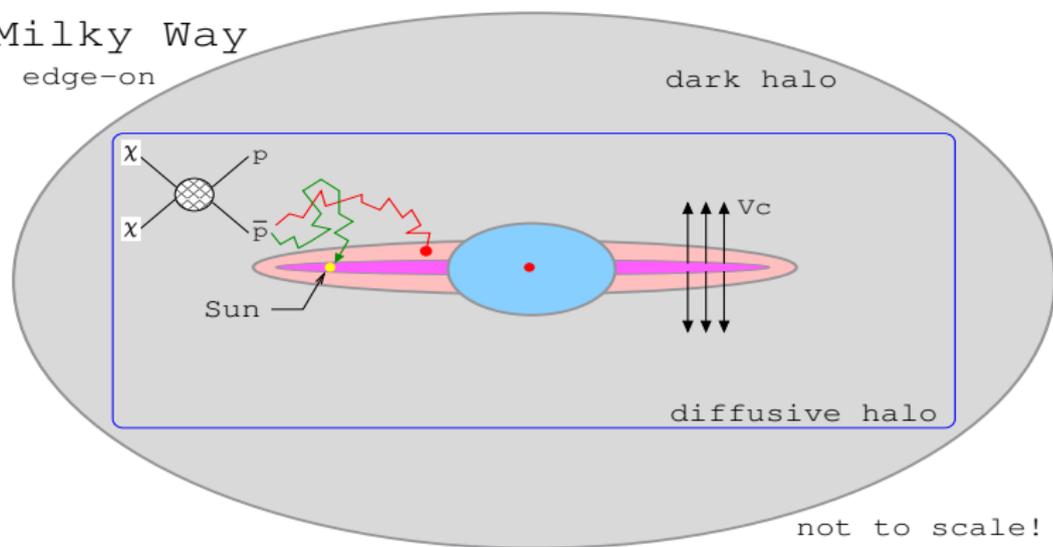
III. INDIRECT SEARCHES

$$\frac{d^2 n_X}{dE_X dt} = \frac{\langle \sigma v \rangle_0}{2m_X^2} \frac{dN_X}{dE_X} \times \rho_X^2$$

Remember X can be e^\pm , \bar{p} , \bar{D} instead of a photon. The first task is to propagate these particles until the Earth.

Milky Way

edge-on

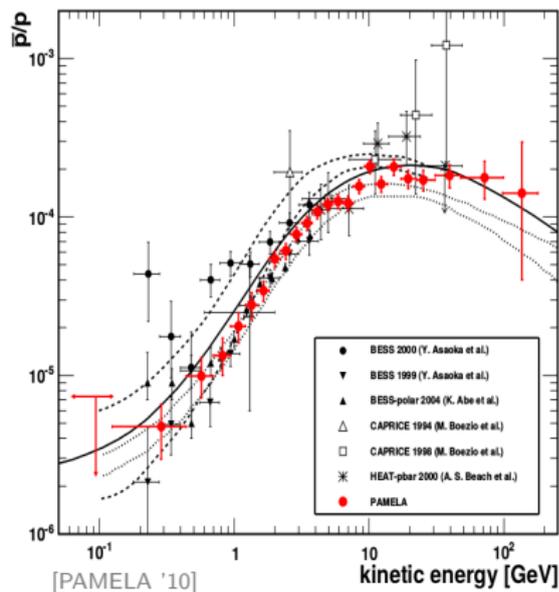
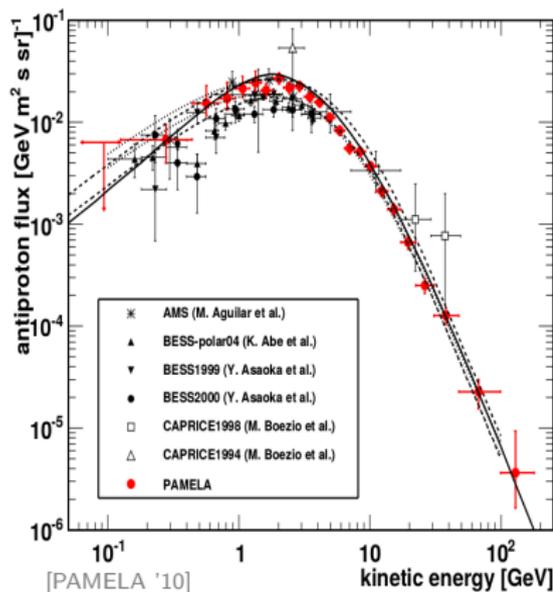


$$0 = \frac{\partial f_{\bar{p}}}{\partial t} = \nabla \cdot (K(T, \vec{r}) \nabla f_{\bar{p}}) - \nabla \cdot (\vec{V}_c(\vec{r}) f_{\bar{p}}) - 2h\delta(z) \Gamma_{\text{ann}} f_{\bar{p}} + Q(T, \vec{r})$$

III. INDIRECT SEARCHES

Note that cosmic rays and produced and accelerated in supernova remnants (and other objects) - these constitute background for dark matter searches. The best channel to look for dark matter is antimatter since the fluxes from cosmic-ray propagation are lowest (but non negligible).

1. antiprotons

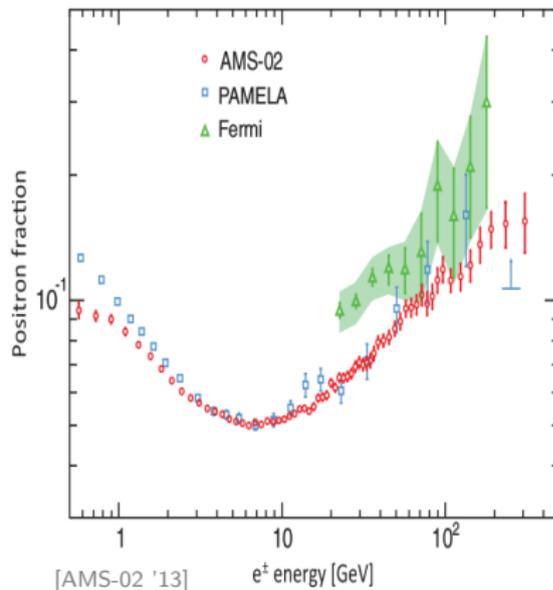
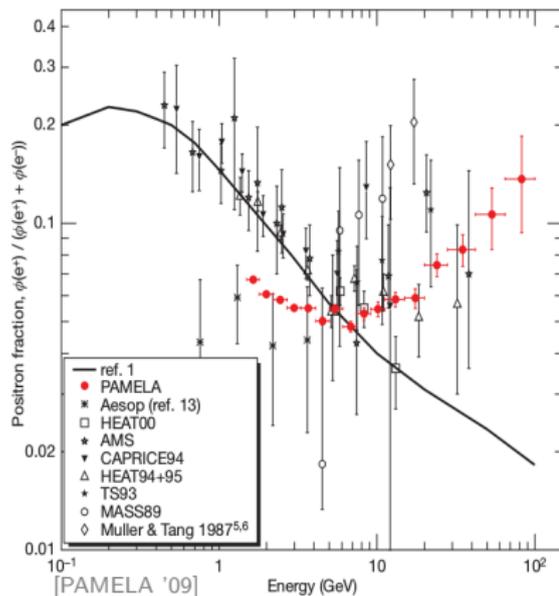


No room for dark matter \rightarrow strong constraints for “hadronic” WIMPs

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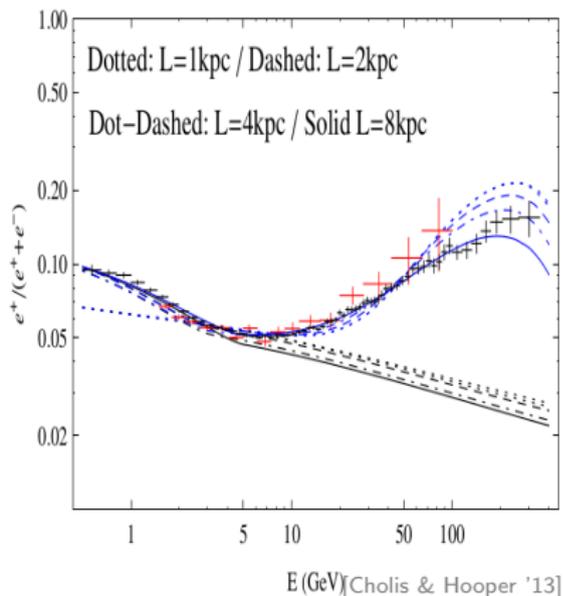
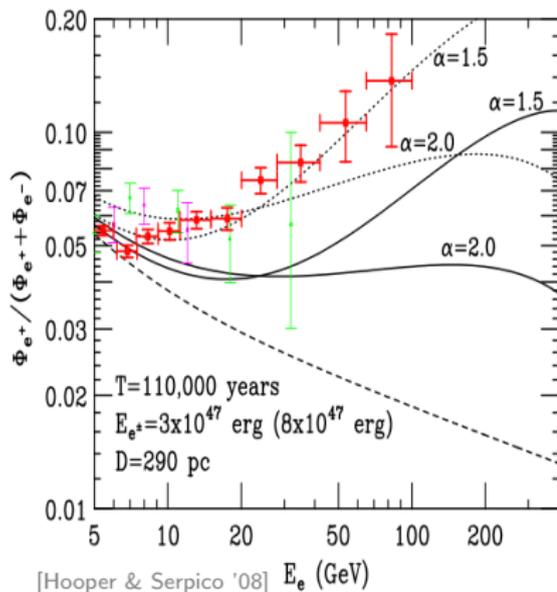


So have we found dark matter?

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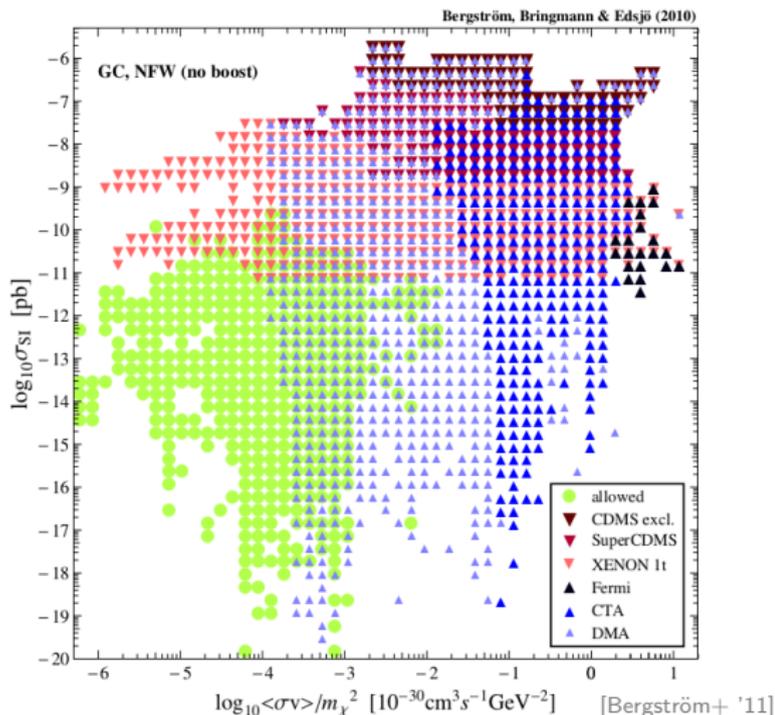
1. positrons



So have we found dark matter? No... This is a hot topic at the moment, but it will be hard to claim a discovery just on the basis of electron-positron data.

III. OVERVIEW OF DIRECT AND INDIRECT SEARCHES

We are probing swiftly through the parameter space of WIMP models. In this context, complementarity plays a crucial role.



But keep in mind that a sizeable corner of the parameter space will remain unconstrained for a very very long time. This means a WIMP can still be out there, but the WIMP paradigm will no longer be “natural”.

CONCLUSION

Scheinbare Geschwindigkeiten im Comahaufen.

$v = 8500$ km/sek	6900 km/sek
7900	6700
7600	6600
7000	5100 (?)

[Zwicky '33]

We have come a long way since Zwicky inferred the need for dark matter from a couple of Doppler velocities in the Coma cluster of galaxies (and a lot of intuition).

We now know dark matter is present from the scale of the smallest galaxies to cosmological scales.

But we have no clue what it is. This should be enough motivation for us (read *you*) to keep on looking for it.

DISCLAIMER

The field of dark matter research is simply huge. Some topics I just touched upon, other I did not even mention. There are excellent books and reviews covering the different topics at all levels of detail. Here is a good list to start with.

- A: astro-oriented P: particle-oriented C: cosmology-oriented *: advanced
- Books
- A Binney & Merrifield, Galactic Astronomy (1981).
 - A* Binney & Tremaine, Galactic Dynamics (1987).
 - A Longair, High-energy astrophysics, Vol I (1992).
 - A Longair, High-energy astrophysics, Vol II (1994).
 - AC Longair, Galaxy Formation (1998).
 - C Kolb & Turner, The Early Universe (1990).
 - C* Peebles, Principles of Physical Cosmology (1993).
 - C Padmanabhan, Structure Formation in the Universe (1993).
 - C* Dodelson, Modern Cosmology (2003).
 - APC Bergström & Goobar, Cosmology and Particle Astrophysics (2004).
 - APC Bertone (ed.), Particle Dark Matter: Observations, Models and Searches (2010).
- Reviews
- P* Jungman, Kamionkowski & Griest, arXiv:hep-ph/9506380.
 - A Lewin & Smith, APh, Vol 6, 87 (1996).
 - PC Bergström, arXiv:hep-ph/0002126.
 - P Muñoz, arXiv:hep-ph/0309346.
 - APC Bertone, Hooper & Silk, arXiv:hep-ph/0404175.
 - PC Bergström, arXiv:1205.4882.

My advice: if you are curious about a topic, do not stop until you understand it!

“I have no special talent. I am only passionately curious.”

Problem #21: Figure out who said so!